Certification in Concentrated Markets

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Abstract

Who does, and who should initiate certification by a third party under asymmetric quality information, the buyer or the seller? Our univocal answer — the seller — follows from an elementary but non-trivial insight: Buyer-induced certification acts as an inspection device, whence seller-induced certification acts as a signalling device. Based on this difference alone, we show that equilibrium involves, and social welfare is larger under, seller-induced certification. Thus, the party better informed about the object should, all other things equal, initiate certification. We motivate our results with many diverse examples; amongst them the examination of patents, automotive parts, and financial products.

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1 Introduction

In many, if not most markets, the seller of a product knows better its quality than the buyer – yet often cannot convey it credibly. An independent expert is called for to certify such a product's quality. Important examples are intermediate products, such as patents, or complex parts into complex final products such as automobiles or airplanes. A further, very topical example is modern financial products that are certified by rating agencies.

There is demand for certification from both sides of such markets. The seller asks for certification in order to obtain an appropriately high price for a high quality product. The buyer asks for certification because she does not want to overspend on a low quality product.

In this situation and all other things equal, does it make a difference whether the buyer or the seller initiates certification? No, strikes us as the spontaneous answer. We show, however, that it does. The reason is that the economic role of certification differs drastically and systematically, depending on whether the informed or the uninformed party initiates it. Whereas buyer-induced certification acts as an inspection device, seller-induced certification acts as a signalling device.

Our question has both a positive and a normative component: Who derives more value from the removal of informational asymmetries, the buyer or the seller? Putting it differently: Is it more profitable for the certifier to sell its service to the seller, or the buyer? More fundamentally, which choice is appropriate from a welfare point of view?

In order to obtain clear cut answers, we start our analysis with a stylized model. In particular, we show that already in a straightforward Akerlof setup, it is unclear *a priori* to whom certification is more valuable, and therefore, from whom the certifier can extract more rents. Moreover, it is unclear whether this rent extraction perspective leads to a socially desirable outcome. Our answers are, nevertheless, unequivocal. Seller–induced certification is *both* more profitable to the certifier – *and* preferred from a welfare point of view.

From the model developed below, we derive our reasoning as follows: With buyer-induced certification, certification is used to dissuade a low quality seller to claim a high price. As a result, the low quality product is certified with positive probability, yet not sold in the market because its price is too high. By contrast, under seller-induced certification, certification ascertains value because only the high quality product is certified, and both high and low quality products are sold at the appropriate price. With this, both the wasteful certification of the low quality product is avoided, and all surplus from the trade is exploited.

With these answers, we contribute to a number of topical debates. First of all, our results shed light on the current debate about who should initiate the certification of financial products. In the aftermath of the 2008 financial crisis, the claim is that, due to concerns of capture, the buyers, rather than the sellers of financial products should initiate their certification. Our contribution is to provide a benchmark which points out that the current norm of seller-induced certification is optimal from a welfare point of view, when one abstracts away the problem of capture. Hence, any change from this norm will involve welfare costs and should therefore be well motivated.[1]

A second important application of our results involves the debate about certifying inventions in the form of patents. The U.S. American and the currently discussed European patent examination systems – in particular the German one – differ roughly by the fact that in the U.S., patent applications are registered, whence in Germany, they are tightly examined. In the former case, the examination is ultimately relegated to the courts, invoked by the user challenging the patent. This case therefore corresponds to buyer-induced certification. In the latter case, the patent office examines the patent upon application by the inventor who can then either license off the patent or use it to create an innovative, protected product himself. This corresponds to seller-induced certification. On the basis of our analysis, we claim that the latter procedure is preferable from a welfare point of view, without

[1] It is, for instance, not clear that a switch from seller-induced to buyer-induced certification solves the problem of capture, because also buyers have an incentive to capture certifiers.

even considering the fact that U.S. style court proceedings involved in buyer-induced certification are, on average, much more costly than certification by the patent granting institution, as induced by the inventor.

The main economic insight of our analysis is that certification to the seller vs. the buyer serves fundamentally different functions. If the buyer wants to check the seller's claim about the quality of his product implicit in his price quotation, certification acts as an *inspection device*. By contrast, if the seller wants to prove high product quality to the buyer, certification acts as a *signalling device*. Our contribution is to identify and, to the best of our knowledge for the first time, to directly compare these two drastically different roles of certification within a unified framework, and to show that, and how they lead to two fundamentally different economic games.

Indeed, when the *buyer* initiates certification, the buyer and the seller play an inspection game. The certifier then picks a price for its service that maximizes its profit in the mixed strategy equilibrium in that game. By contrast, when the *seller* initiates certification, the buyer and the seller play a signalling game. The certifier then also picks a price that maximizes its profit, but that price must ensure that certification is an effective signalling device, by separating high quality from low quality sellers. Hence, the certifier must not only ensure that the price of certification is low enough so that the high quality producer wants to signal high quality via certification, but also high enough so that the low quality seller does not find it worthwhile to buy certification and mimic the high quality seller.

We further demonstrate that the equilibrium outcome in the signalling game is more efficient than the equilibrium outcome in the associated inspection game. Again, the reason is that the mixed strategy equilibria of the inspection game yield two inefficiencies not shared by the outcome of the signalling game: first, certification is sometimes unnecessarily demanded for the low quality good; and second, the low quality good is not always traded. Hence, in the inspection game, certification is sometimes wasteful, and gains from trade are not always exhausted. We therefore conclude that seller–induced certification is socially more desirable. It is interesting to see that it is also more profitable for the certifier, and thus, in equilibrium, preferred to buyer-induced certification. In all, our baseline analysis, extended later without losing this central insights, brings us to the general conclusion that *truthful certification should be induced by the party that is better informed about the quality of the good, no matter whether adverse selection, or moral hazard are involved in its provision.*

In the ensuing Section 2, we first relate our results to the pertinent literature. In Section 3, we describe the model. In Section 4, we derive the results for buyer–induced certification. Section 5 contains the results for seller–induced certification, as well as the comparison between the two from the point of view of the certifier. In Section 6 we evaluate that outcome from a welfare point of view. In Section 7 we discuss many extensions of our baseline model and show the results to be robust. In Section 8 we discuss empirical examples involving certification. We summarize and conclude with Section 9. All proofs are relegated to the Appendix.

2 Literature

Dranove and Jin (2010) provide a comprehensive survey of how different economic institutions, such as certification, warranties, buy-back guarantees, or seller reputation, mitigate inefficiencies due to informational asymmetries between buyers and sellers. While explicitly listed in their set of central questions, the question we focus on, whether certification is, or should be initiated alternatively by the buyer or the seller, is not discussed in their survey. We, therefore, concentrate here on the literature pertinent to our subject.

Viscusi (1978) was the first to point out formally that, in Akerlof's (1970) lemons market, there exist gains of trade for an external certifier, who reduces asymmetric information by providing quality certification.[2] Biglaiser (1993) extends this result to a dynamic adverse selection setting, and derives conditions under which an expert improves upon welfare by taking possession

[2]See also De and Nabar (1991), who point out that inaccurate certification technologies may yield quantitatively different results than the perfect certification framework as in Viscusi (1978).

of the good(s) and trading with the typical buyer. Because the expert acts as a middleman rather than as a certifier, the model differs from ours.

Faulhaber and Yao (1989) focus on how, in a dynamic framework, the possibility of certification impacts reputational concerns. For the sake of our focus, we keep the model simple and static and so do not address reputation. Albano and Lizzeri (2001) consider a moral hazard problem and show how certification can provide the correct incentives for the production of high but costly quality. Yet unlike in our model, the certifier sells by assumption only to the seller and not, alternatively, to the buyer. This is also assumed by Bolton, Freixas and Shapiro (2012). Unlike our focus, they analyze the consequences of a number of market imperfections related to the credit rating industry.

More in line with our research questions, Fasten and Hofmann (2010) discuss the provision of certification to a buyer or a seller, but concentrate on asymmetries in information disclosure: The seller wants public information, the buyer private one. These issues do not arise in our context. Bouvard and Levy (2012) show that in spite of reputation effects involved in certification, the certifier does not necessarily fully disclose information, an aspect, again for simplicity, not discussed in our comparison.

We follow the aforementioned literature in the assumption that certifiers reveal honestly all their information. Yet there is also a literature on the strategic disclosure of a certifier's information and straight–out fraudulent experts. Lizzeri (1999) focuses on the strategic manipulation of information by a monopolistic certifier and shows that, in its quest of maximizing returns, the certifier minimizes the amount of information provided. Guerra (2001) demonstrates in a slightly modified version that more than a minimal information serves that objective. Peyrache and Quesada (2004) extend Lizzeri's analysis of the strategic disclosure of information by certifiers, to include reputation and differentiation effects between sellers.

Wolinsky (1993) shows how buyers' search for multiple opinions disciplines fraudulent certifiers. Emons (1997) discusses whether in markets for experts, the market mechanism induces non–fraudulent behavior. Strausz

(2004) discusses how reputation in a repeated game can induce non–fraudulent behavior, even if a seller can induce dishonest certification by bribing the certifier. He also shows that honest certification exhibits economies of scale and constitutes a natural monopoly.[3] While we can use the latter result in our model to justify our assumption of a certifier monopoly, this strand of literature is very different in spirit and intention to ours.

We became aware only lately of an unpublished paper by Durbin (1999) in which a similar question to one of ours is addressed, namely "who pays for certification". The comparison is between selling private information to buyers e.g. via guidebooks, and selling public information paid by the seller. We follow a similar line in one of our extensions. Yet in our model, the price for the good is set by the seller *before*, rather than *after* certification can be bought. Our mode, arguably much more common when it comes to the certification of final products rather than inputs, induces the inspection game analyzed in the sequel. Durbin also assumes identical cost of producing high and low quality which excludes adverse selection, an aspect we feel important in the informational asymmetry whose consequences are analyzed here. We contrast our approach to Durbin's in more detail when discussing possible extensions of our model in Section 7.

3 Model

Consider a seller offering one unit of a good at price p whose quality, before certification, is revealed only to him and is unobservable to a single buyer.[4] From the buyer's point of view, the seller's quality is high, q_h, with probability λ and low, $q_l > 0$, with probability $1 - \lambda$, where $\Delta q \equiv q_h - q_l > 0$. The good's quality is identified with the buyer's willingness to pay, which is public information.The risk neutral buyer is therefore willing to pay up to a price that equals expected quality $\bar{q} \equiv \lambda q_h + (1 - \lambda)q_l$. If not buying at all, his reservation payoff is zero. The high quality seller has a production cost

[3] See Hvide (2004) for a model with several certifiers, who compete in prices but are ranked by the difficulty at which their test is passed. Broadly speaking the author shows that the matching of sellers and certifiers is assortative.

[4] In Section 6, we argue robustness of our results to the case of many buyers.

$c_h > 0$, and the low quality seller has a production cost $c_l = 0$. If not producing the seller's reservation payoff is zero.

We assume that producing the high quality good delivers higher economic rents: $q_h - c_h > q_l - c_l = q_l > 0$, yet its production cost exceeds the average quality, $c_h > \bar{q}$.[5] This creates a lemon's problem and leads to adverse selection: without certification, a high quality seller would not want to offer his good in the market, and thus the market outcome with informational asymmetry would be inefficient. Without the informational asymmetry, however, the high quality seller could sell his good for the price $q_h > c_h$. Consequently, the high quality seller has demand for certification that reveals the good's true quality to the buyer. Clearly, the high quality seller is willing to pay the certifier at most $q_h - c_h$.

Yet also the buyer has a demand for certification. Whenever the seller quotes a price higher than that appropriate for the low quality good, the buyer may demand certification ascertaining that the good has indeed high quality, so that the high price is justified.

Summarizing, both the buyer and the seller have a demand for certification. For a monopolistic certifier this brings the question as to whom it should offer its services.[6] By assumption, the certifier has the technology to perfectly detect the seller's quality at a cost $c_c \in [0, q_h - c_h)$ and to announce it publicly. Note that the specification considered here includes any bimodal certification, and in particular the frequently observed certification of minimum quality.[7]

The certifier's problem is as follows. In an initial stage, it has to decide whether to offer its services to the buyer or the seller. After this decision, it sets a price p_c at which the buyer or the seller, respectively, can obtain certification. If not offering certification at all, his reservation payoff is zero. We focus on honest certification where the certifier cannot be bribed.

[5] This implies that λ is strictly smaller than 1.

[6] For obvious reasons, the certifier cannot sell to both parties at the same time. In Section 7, we consider many empirical cases exactly reflecting this structure.

[7] In the Extension Section 6, we discuss the implications of an imperfect certification technology.

Our research question is twofold, namely whether — all other things equal — the monopolistic certifier is better off servicing the uninformed buyer or the informed seller, and whether its decision conforms to that taken by a welfare seeker who compares his decision by looking at the sum of consumer and producer surplus. In order to answer these questions, we separately study "buyer–induced", and "seller–induced" certification, and contrast their outcomes from both the certifier's and a social welfare point of view. Again, the analysis of the two cases can be easily linked, by adding the certifier's decision whom to address in its decision tree. The separate analysis should clarify the substantive difference in the way certification works through these two demand channels.

4 Buyer–Induced Certification

Here we consider the certification problem when induced by the buyer. Before analyzing the formal model, it is helpful to provide an intuition on the role of certification and the certifier's motive in this setup.

Buyer–induced certification enables the buyer to check the seller's quality claim. In particular, certification offers the buyer protection against a low quality seller who pretends to have high quality. From the buyer's perspective, therefore, certification is an inspection device to detect low quality sellers.

The game underlying buyer–induced certification, therefore, is an *inspection game*. A mixed strategy equilibrium is typical for this type of game. Indeed, a pure strategy equilibrium in which the buyer never buys certification cannot exist, because it would give the low quality seller an incentive to always claim high quality – yet against this claim the buyer would have a strong incentive to buy certification. Likewise, an equilibrium in which the buyer always buys certification cannot exist either, because it keeps the low quality seller from claiming high quality – yet against such behavior certification is only wasteful for the buyer. Consequently, we typically have a mixed strategy equilibrium, where the low quality seller cheats with some probability and claims to offer high quality, and consequently the buyer initiates certification with some probability.

Hence, buyer–induced certification plays the role of reducing cheating. The buyer's demand for certification will therefore be high when the problem of cheating is large. This reasoning suggests that a monopolistic certifier, who targets his services towards the buyer, will choose a certification price that maximizes the buyer's cheating problem.

A closer look reveals that the buyer's cheating problem depends on two factors: the buyer's uncertainty and the seller's price quotation. First, the buyer's cheating problem is the bigger the less certain she is about the true quality offered by the seller. Second, checking true quality through certification is especially worthwhile for intermediate prices of the good. Indeed, for a low price the buyer would not lose much from simply buying the good uncertified. By contrast, when the price is high, the buyer would not lose much from not buying the good at all. Hence, the buyer's willingness to pay for certification is largest for intermediate prices that are neither too low nor too high. With the ensuing formal analysis we show that this intuitive reasoning is correct, yet not trivial.

With buyer–induced certification, the parties play the following game:

- t=1 The certifier sets a price p_c for his service.
- t=2 Nature selects the quality $q_i, i \in \{l, h\}$, of the good offered, and conveys it to the seller.
- t=3 The seller offering the good of quality q_i at cost c_i decides about the price p at which he offers the good.
- t=4 The buyer decides whether or not to demand certification of the good.
- t=5 The buyer decides whether or not to buy the good.

Note that we assume that if the seller q_i sets a price in stage 3, he incurs the production cost c_i for sure, even though the buyer may decide not to buy the good in stage 5. This assumption is natural under several forms of certification.

First, certification may mean that the certifier inspects the actual good the buyer is interested in. In this case, the good must already be produced in order for the certifier to inspect it, and the seller must therefore have incurred the production cost even if the buyer decides not to acquire it. A second possibility is that the certifier determines the seller's product quality by inspecting his production facility, and certifying his production technology. In this case, the production cost c_h may be interpreted as a fixed cost of installing production technology that differs between high and low quality sellers.[8] Under both interpretations, the seller incurs the cost even if the buyer, in the end, does not buy the product.

By letting the seller determine the price of the good before certification takes place, we allude to empirical cases from the consumer industry. We also exclude more complex forms of price setting; for instance conditioning the price on revealed quality - yet comment on this in the Extension Section 6.

We focus on the Perfect Bayesian Equilibria (PBE) of the game described above. Note that after the certifier has set its price p_c, a proper subgame, $\Gamma(p_c)$, starts with nature's decision about the quality of the seller's product. The subgame $\Gamma(p_c)$ is a signalling game where the seller's price p may or may not reveal his private information about the quality of the good.

In the subsequent analysis, we first consider the PBE of the subgame $\Gamma(p_c)$. A PBE specifies three components: First, the seller's pricing strategy as a function his private knowledge about the good's type q_i; second, the buyer's belief $\mu(p)$ after observing the price p; third, the buyer's behavior; in particular whether or not to buy certification and/or the actual good.

We allow the seller to randomize over prices. In order to circumvent measure–theoretical complications, we assume that the seller can randomize over any infinite but countable set of prices. Consequently, we can express the strategy of quality q_i's seller by the function $\sigma_i : R_+ \to [0, 1]$, with the interpretation that $\sigma_i(p_j)$ denotes the probability that the seller with quality

[8] In the two most extreme examples discussed in Section 7, the first form applies to financial products, and both forms apply to automotive parts.

q_i chooses the price p_j. Thus, for both $i \in \{h, l\}$,

$$\sum_j \sigma_i(p_j) = 1.$$

The buyer's decisions are based on his belief specified as a function $\mu : \mathbb{R}_+ \to [0, 1]$ with the interpretation that, after observing price p, the buyer believes that the seller is of the high quality type q_h with probability $\mu(p)$.

We can express the buyer's behavior after observing the price p and possessing belief μ by the following six actions:

1. Action s_{nn}: The buyer does not buy certification nor buy the good. This action yields the payoff
$$U(s_{nn}|p, \mu) = 0.$$

2. Action s_{nb}: The buyer does not buy certification, but buys the product. This action yields the expected payoff
$$U(s_{nb}|p, \mu) = \mu q_h + (1 - \mu)q_l - p.$$

3. Action s_{ch}: The buyer buys certification and buys the product only when the certifier reveals high quality. This action yields the expected payoff
$$U(s_{ch}|p, \mu) = \mu(q_h - p) - p_c.$$

4. Action s_{cb}: The buyer buys certification and buys the product irrespective of the outcome of certification. This action yields the expected payoff
$$U(s_{cb}|p, \mu) = \mu(q_h - p) + (1 - \mu)(q_l - p) - p_c.$$
Clearly, $U(s_{cb}|p, \mu) < U(s_{nb}|p, \mu)$ for any $p_c > 0$ so that the action s_{nb} dominates the action s_{cb}.

5. Action s_{cl}: The buyer buys certification and buys the product only when the certifier reveals low quality. This action yields the expected payoff
$$U(s_{cl}|p, \mu) = (1 - \mu)(q_l - p) - p_c.$$
Clearly, $U(s_{cl}|p, \mu) \leq U(s_{nb}|p, \mu)$ for $p \leq q_h$ and $U(s_{cl}|p, \mu) \leq U(s_{nn}|p, \mu)$ for $p > q_h$. Hence, also the action s_{cl} is weakly dominated.

6. Action s_{cn}: The buyer demands certification and does not buy the product. This action yields the expected payoff

$$U(s_{cn}|p,\mu) = -p_c.$$

Clearly, $U(s_{cn}|p,\mu) < U(s_{nn}|p,\mu)$ for any $p_c > 0$ so that the action s_{cn} is dominated.

To summarize, only the first three actions s_{nn}, s_{nb}, s_{ch} are not (weakly) dominated for some combination (p,μ). The intuition is straightforward: the role of certification is to enable the buyer to discriminate between high and low quality. It is therefore only worthwhile to buy certification when the buyer uses it to screen out bad quality.[9]

In the following, we delete the weakly dominated actions from the buyer's action space. Consequently, we take the buyer's action space as $S \equiv \{s_{nn}, s_{nb}, s_{ch}\}$. To express a buyer's mixed strategy, we let $\sigma(s|p,\mu) \in [0,1]$ represent the probability that the buyer takes action $s \in S = \{s_{nn}, s_{nb}, s_{ch}\}$ given price p and belief μ. Thus

$$\sum_{s \in S} \sigma(s|p,\mu) = 1.$$

A PBE in our subgame $\Gamma(p_c)$ can now be described more specifically: it is a tuple of functions $\{\sigma_l, \sigma_h, \mu, \sigma\}$ satisfying the following three equilibrium conditions. First, seller type i's pricing strategy σ_i must be optimal with respect to the buyer's strategy σ. Second, the buyer's belief μ must be consistent with the sellers' pricing strategy, whenever possible. Third, the buyer's strategy σ must be a best response given the observed price p and her beliefs μ.

We start our analysis of the Perfect Bayesian Equilibria of $\Gamma(p_c)$ by studying first the last requirement: the optimality of the buyer's strategy given a price p and beliefs μ.

[9]Observe that the strategy s_{ch} is not renegotiation proof, because even after certification has revealed low quality, gains could be realized by trading the low quality product. In Section 6, we will consider the simple extension to include renegotiation.

Fix a price $\bar{p}$ and a belief $\bar{\mu}$. Then the pure strategy s_{nn} is a best response whenever $U(s_{nn}|\bar{p},\bar{\mu}) \geq U(s_{nb}|\bar{p},\bar{\mu})$ and $U(s_{nn}|\bar{p},\bar{\mu}) \geq U(s_{ch}|\bar{p},\bar{\mu})$. It follows that the strategy s_{nn} is a best response whenever

$$(\bar{p},\bar{\mu}) \in S(s_{nn}|p_c) \equiv \{(p,\mu)|p \geq \mu q_h + (1-\mu)q_l \ \wedge \ p_c \geq \mu(q_h - p)\}\,.$$

Likewise, the pure strategy s_{nb} is (weakly) preferred whenever $U(s_{nb}|\bar{p},\bar{\mu},p_c) \geq U(s_{nn}|\bar{p},\bar{\mu},p_c)$ and $U(s_{nb}|\bar{p},\bar{\mu},p_c) \geq U(s_{ch}|\bar{p},\bar{\mu},p_c)$. It follows that the strategy s_{nb} is a best response whenever

$$(\bar{p},\bar{\mu}) \in S(s_{nb}|p_c) \equiv \{(p,\mu)|p \leq \mu q_h + (1-\mu)q_l \ \wedge \ p_c \geq (1-\mu)(p-q_l)\}\,.$$

Finally, the pure strategy s_{ch} is (weakly) preferred whenever $U(s_{ch}|\bar{p},\bar{\mu},p_c) \geq U(s_{nn}|\bar{p},\bar{\mu},p_c)$ and $U(s_{ch}|\bar{p},\bar{\mu},p_c) \geq U(s_{nb}|\bar{p},\bar{\mu},p_c)$. It follows that the strategy s_{ch} is a best response whenever

$$(\bar{p},\bar{\mu}) \in S(s_{ch}|p_c) \equiv \{(p,\mu)|p_c \leq \mu(q_h - p) \wedge p_c \leq (1-\mu)(p-q_l)\}\,.$$

Since a mixed strategy is optimal only if it randomizes among those pure strategies that are a best response, we arrive at the following result:

Lemma 1 *In any Perfect Bayesian Equilibrium* $(\sigma_l^*,\sigma_h^*,\mu^*,\sigma^*)$ *of the subgame* $\Gamma(p_c)$ *we have for any* $s \in S = \{s_{nn}, s_{nb}, s_{ch}\}$*,*

$$\sigma^*(s|p,\mu) > 0 \Rightarrow (p,\mu^*(p)) \in S(s|p_c). \tag{1}$$

Figure 1 illustrates the buyer's behavior for a given certification price p_c. For low product prices p, the buyer buys the good uncertified, $(p,\mu) \in S(s_{nb})$, whereas for high prices p the buyer refrains from buying, $(p,\mu) \in S(s_{nn})$. It turns out that as long as $p_c < \Delta q/4$, there is an intermediate range of prices p and beliefs μ such that the buyer demands certification, i.e. $(p,\mu) \in S(s_{ch})$. In this case, the buyer only buys the product when certification reveals that it has high quality. Intuitively, the buyer demands certification to ensure that the highly priced product is indeed of high quality. Note that apart from points on the thick, dividing lines, the buyer's optimal buying behavior

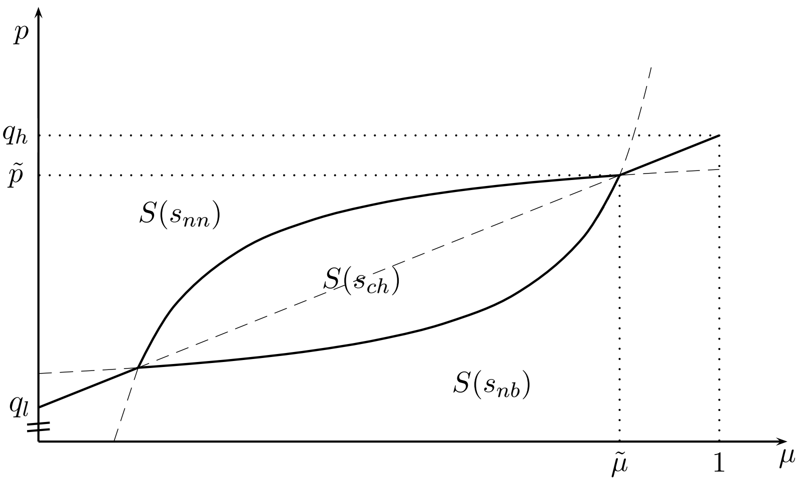

Figure 1: Buyer's buying behavior for given $p_c < \Delta q/4$.

of both certification services and the product is uniquely determined, and mixing does not take place.

For future reference we define

$$\tilde{p} \equiv \left(q_h + q_l + \sqrt{\Delta q(\Delta q - 4p_c)}\right)/2$$

and

$$\tilde{\mu} \equiv \left(1 + \sqrt{1 - 4p_c/\Delta q}\right)/2.$$

If the seller prices at $\tilde{p}$ and the buyer has beliefs $\tilde{\mu}$, the buyer is indifferent between all three decisions, namely not to buy the good, s_{nn}, to buy the good uncertified, s_{nb}, or to buy the good only after it has been certified as high quality, s_{ch}.

We previously argued that the monopolistic certifier benefits from high buyer uncertainty and an intermediate price of the good. We now can give precision to this statement. The buyer's willingness to pay for certification is the difference between her payoff from certification and the next best alternative, namely either to buy the good uncertified, or to not buy the good at all. More precisely, given her beliefs are μ, the difference in the buyer's expected payoffs between buying the high quality good when certified and buying any good uncertified is

$$\Delta U^1 \equiv \mu(q_h - p) - (\bar{q} - p).$$

Similarly, the difference in the buyer's expected payoffs between buying the good only when certified and buying the good not at all is

$$\Delta U^2 = \mu(q_h - p).$$

Hence, the buyer's willingness to pay for certification is maximized for a price $\hat{p}$ and a belief $\hat{\mu}$ that solves

$$\max_{p,\mu} \ \min\{\Delta U^1, \Delta U^2\}.$$

The solution is $\hat{\mu} = 1/2$ and $\hat{p} = (q_h + q_l)/2$. We later demonstrate that, with buyer–induced certification, the certifier chooses a price p_c for certification to induce this outcome as closely as possible.

Next, we address the optimality of type i seller's strategy $\sigma_i(p)$. For a given strategy σ of the buyer and a fixed belief μ, a seller with quality q_h expects the following payoff from setting a price p:

$$\Pi_h(p, \mu|\sigma) = [\sigma(s_{nb}|p, \mu) + \sigma(s_{ch}|p, \mu)]p - c_h.$$

A specific strategy σ_h yields seller q_h, therefore, an expected profit of

$$\bar{\Pi}_h(\sigma_h) = \sum_i \sigma_h(p_i)\Pi_h(p_i, \mu(p_i)|\sigma).$$

Likewise, a seller with quality q_l obtains the payoff

$$\Pi_l(p, \mu|\sigma) = \sigma(s_{nb}|p, \mu)p$$

and any strategy σ_l yields

$$\bar{\Pi}_l(\sigma_l) = \sum_i \sigma_l(p_i)\Pi_l(p_i, \mu(p_i)|\sigma).$$

It follows that in a PBE $(\sigma_h^*, \sigma_l^*, \mu^*, \sigma^*)$ the high quality seller q_h's and the low quality seller q_l's payoffs, respectively, are

$$\Pi_h^* = \sum_i \sigma_h^*(p_i)\Pi_h(p_i, \mu^*(p_i)|\sigma^*) \ \text{ and } \Pi_l^* = \sum_i \sigma_l^*(p_i)\Pi_l(p_i, \mu^*(p_i)|\sigma^*).$$

The next lemma makes precise the intuitive result that the seller's expected profits increase when the buyer is more optimistic about the good's quality.

Lemma 2 *In any PBE* $(\sigma_l^*, \sigma_h^*, \mu^*, \sigma^*)$ *of the subgame* $\Gamma(p_c)$ *with* $p_c > 0$ *the payoffs* $\Pi_h(p, \mu|\sigma^*)$ *and* $\Pi_l(p, \mu|\sigma^*)$ *are non–decreasing in* μ.

Seller type i's pricing strategy σ_i is an optimal response to the buyer's behavior (σ^*, μ^*) exactly if, for any p', we have

$$\sigma_i^*(p) > 0 \;\Rightarrow\; \Pi_i(p, \mu^*(p)|\sigma^*) \geq \Pi_i(p', \mu^*(p')|\sigma^*). \tag{2}$$

Because the buyer's beliefs depend on the observed price p, it affects the buyer's behavior and, therefore, the belief function μ^* plays a role in condition (2).

Finally, a PBE demands that the buyer's beliefs μ^* have to be consistent with equilibrium play. In particular, they must follow Bayes' rule:

$$\sigma_i^*(p) > 0 \;\Rightarrow\; \mu^*(p) = \frac{\lambda \sigma_h^*(p)}{\lambda \sigma_h^*(p) + (1-\lambda)\sigma_l^*(p)}. \tag{3}$$

The next lemma shows the implications on PBEs that are due to Bayes' rule. In particular, it shows that the seller, no matter his type, never sets a price below q_l, and the low quality seller never sets a price above q_h. The lemma also shows that, in equilibrium, the low quality seller never loses from the presence of asymmetric information, since he can always guarantee himself the payoff q_l that he obtains with observable quality. By contrast, the high quality seller loses from the presence of asymmetric information; his payoff is strictly smaller than $q_h - c_h$. We should emphasize the im0plication of this: *If, under buyer certification, the seller would face the choice between producing the high or the low quality good, he would never choose to produce high quality*!

Lemma 3 *In any PBE* $(\sigma_l^*, \sigma_h^*, \mu^*, \sigma^*)$ *of the subgame* $\Gamma(p_c)$ *we have i)* $\sigma_l^*(p) = 0$ *for all* $p \notin [q_l, q_h]$ *and* $\sigma_h^*(p) = 0$ *for all* $p < q_l$*; ii)* $\Pi_l^* \geq q_l$*; iii)* $\Pi_h^* < q_h - c_h$.

As is well known, the concept of Perfect Bayesian Equilibrium places only weak restrictions on admissible beliefs. In particular, it does not place any restrictions on the buyer's beliefs for prices that are not played in equilibrium; any out–of–equilibrium belief is allowed. Hence, as is typical for signalling games, without any restrictions on out–of–equilibrium beliefs we cannot pin down behavior in the subgame $\Gamma(p_c)$ to a specific equilibrium. Especially by the use of pessimistic out–of–equilibrium beliefs, one can sustain many equilibrium pricing strategies.

In order to reduce the arbitrariness of equilibrium play, it is necessary to strengthen the solution concept of PBE by introducing more plausible restrictions on out–of–equilibrium beliefs. A standard belief restriction is the intuitive criterion of Cho–Kreps (1987), which in its standard formulation only has bite in an equilibrium where the signalling player reveals himself fully so that $\mu \in \{0, 1\}$ results. Bester and Ritzberger (2001) propose the following extension of the intuitive criterion to intermediate beliefs $\mu \notin \{0, 1\}$.

Belief Restriction (B.R.): A Perfect Bayesian Equilibrium $(\sigma_h^*, \sigma_l^*, \mu^*, \sigma^*)$ satisfies the Belief Restriction if, for any $\mu \in [0, 1]$ and any out–of–equilibrium price p, we have

$$\Pi_l(p, \mu) < \Pi_l^* \wedge \Pi_h(p, \mu) > \Pi_h^* \;\Rightarrow\; \mu^*(p) \geq \mu.$$

The belief restriction states intuitively that if a pessimistic belief μ gives only the q_h seller an incentive to deviate, then the restriction requires that the buyer's actual belief should not be even more pessimistic than μ. It extends the intuitive criterion of Cho–Kreps, because the criterion obtains for the special case for $\mu = 1$. Indeed, the restriction extends the logic of the Cho–Kreps criterion to a situation where a deviation to a price p is profitable only for the q_h seller when the buyer believes that the deviation originates from the q_h seller with probability μ. As we may have $\mu < 1$, the restriction considers more pessimistic beliefs than the Cho–Kreps criterion.

The next lemma characterizes equilibrium outcomes that satisfy the belief restriction (B.R.). It, in particular, shows that the refinement implies that the high quality seller can sell his product for a price of at least $\tilde{p}$.

Lemma 4 *Any Perfect Bayesian Equilibrium* $(\sigma_l^*, \sigma_h^*, \mu^*, \sigma^*)$ *of the subgame* $\Gamma(p_c)$ *that satisfies B.R. exhibits i)* $\sigma_h^*(p) = 0$ *for all* $p < \tilde{p}$ *and ii)* $\Pi_h^* \geq \tilde{p} - c_h$.

By combining the previous two lemmata we are now able to characterize the equilibrium outcome.

Proposition 1 *Consider a PBE* $(\sigma_l^*, \sigma_h^*, \mu^*, \sigma^*)$ *of the subgame* $\Gamma(p_c)$ *that satisfies B.R. Then*

i) for $\lambda < \tilde{\mu}$ *and* $c_h < \tilde{p}$ *it exhibits unique pricing behavior by the seller and unique buying behavior by the buyer. In particular, the high quality seller sets the price* $\tilde{p}$ *with certainty, and the low quality seller randomizes between the price* $\tilde{p}$ *and* q_l*. Observing the price* $\tilde{p}$ *the buyer buys certification with positive probability. The certifier's equilibrium profit equals*

$$\Pi_c(p_c) = \frac{\lambda(\tilde{p} - q_l)}{\tilde{\mu}\tilde{p}}(p_c - c_c). \tag{4}$$

ii) For $\lambda > \tilde{\mu}$ *or* $c_h > \tilde{p}$ *we have* $\Pi_c(p_c) = 0$ *in any equilibrium.*

iii) For $\lambda \leq \tilde{\mu}$ *and* $c_h \leq \tilde{p}$ *there exists an equilibrium outcome, in which the certifier's profits equal expression (4).*

The Proposition shows that the buyer and the low quality seller play the mixed strategies that reflect the typical outcome of an inspection game. Indeed, by choosing the low price q_l, a low quality seller honestly signals his low quality. In contrast, we may interpret a low quality seller, who sets a high price $\tilde{p}$, as trying to cheat. Hence, whenever the buyer observes the price $\tilde{p}$, she is uncertain whether the good is supplied by the high quality or the low quality seller. She therefore wants the good inspected by buying certification with positive probability. Through inspection, the buyer tries to dissuade the low quality seller to set the "cheating" price $\tilde{p}$.

Yet, as typical in an inspection game, the buyer has only an incentive to buy certification and inspect when the low quality seller cheats "often enough". This gives rise to the use of mixed strategies: the buyer's certification probability is such that the low quality seller is indifferent between cheating, i.e., setting the high price $\tilde{p}$, and honestly signaling his low quality by setting the price q_l. On the other hand, the probability with which the low quality seller chooses the high price $\tilde{p}$ is such that the buyer is indifferent between buying the good uncertified and asking for certification.

Proposition 1 also completely describes the certifier's profits in the subgame $\Gamma(p_c)$. The certifier anticipates this outcome when choosing its price p_c for certifying the good's quality. When the certifier maximizes its profits Π_c with respect to the certification price p_c, it must take into account that $\tilde{\mu}$ depends on p_c itself and the certifier therefore anticipates that the very case distinction $\lambda \lesseqgtr \tilde{\mu}$ and $c_h \gtreqless \tilde{p}$ depends on its choice of p_c. The following proposition shows that the certifier's equilibrium profit (4) is increasing in p_c. Hence, the certifier picks the largest price such that $\lambda \leq \tilde{\mu}$ and $c_h \leq \tilde{p}$.

Proposition 2 *Consider the full game with buyer–induced certification.*
i.) Suppose that $\lambda \leq 1/2$ and $c_h \leq (q_h + q_l)/2$. Then the certifier sets a price $p_c^b = \Delta q/4$ and obtains a profit of

$$\Pi_c^b = \frac{\lambda \Delta q}{2(q_h + q_l)}(\Delta q - 4c_c).$$

ii.) Suppose that $\lambda > 1/2$ or $c_h > (q_h + q_l)/2$. Then the certifier sets the price $p_c^b = (q_h - c_h)(c_h - q_l)/\Delta q$ and obtains a profit of

$$\Pi_c^b = \frac{\lambda[(q_h - c_h)(c_h - q_l) - \Delta q c_c]}{c_h}.$$

We argued that the monopolistic certifier benefits from a relatively high uncertainty for the buyer and an intermediate price of the good; we also showed that the buyer's willingness to pay for certification is maximized for $\hat{\mu} = 1/2$ and $\hat{p} = (q_h + q_l)/2$. A comparison demonstrates that, for the parameter constellation $\lambda \leq 1/2$ and $c_h \leq (q_h + q_l)/2$, the equilibrium induces exactly this outcome. Indeed, the certifier's optimal price $p_c = \Delta q/4$ leads to a price $p = (q_h + q_l)/2$ and a belief $\mu = 1/2$ and maximizes the expression

$$\min\{\Delta U^1, \Delta U^2\}.$$

For $c_h > (q_h + q_l)/2$, the price $p = (q_h + q_l)/2$ would imply a loss to the high quality seller and, intuitively, the certifier cannot induce this maximum degree of uncertainty. For $\lambda > 1/2$, the ex ante belief of the buyer about the product exceeds $1/2$. Consequently, the certifier is unable to induce the belief $\mu = 1/2$. Instead, the certifier is restricted and maximizes the expression $\min\{\Delta U^1, \Delta U^2\}$ under a feasibility constraint. That is, the certifier's price maximizes the buyer's uncertainty about the seller's quality and, thereby, her willingness to pay.

5 Seller Induced Certification

In this section we consider the case where the seller instead of the buyer may buy certification. Here certification plays a different role. Rather than giving the buyer the possibility to protect herself from bad quality, it enables a high quality seller to ascertain the quality of his product to the buyer. Although the distinction seems small, it has a major impact on the equilibrium outcome, primarily because only the high quality seller is interested in certification. Because of this, we can show that seller–induced certification is simpler and easier to control by the certifier.

Under seller–induced certification the parties play the following game:

t=1 The certifier sets a price p_c.

t=2 Nature selects the quality $q_i, i \in \{l, h\}$ of the good offered by the seller.

t=3 The seller offering the good at quality q_i and cost c_i decides about the price p at which he offers the good.

t=4 The seller decides whether or not to demand certification for his good.

t=5 The buyer decides whether or not to buy the good.

Thus, in comparison to the model described in the previous section, we only change stage four by letting the seller, rather than the buyer, decide about purchasing certification. Note that the sequence of stages 3 and 4 is immaterial. Our setting where the seller first chooses his price and then decides about certification is strategically equivalent to the situation where he simultaneously takes both decisions, or reverses their order.

We again focus on Perfect Bayesian Equilibria of this game. Note again that after the certifier has set his price p_c a proper subgame, $\Gamma(p_c)$, starts with nature's decision about the quality offered by the seller. The subgame $\Gamma(p_c)$ is a pure signalling game if the seller does not buy certification in stage 4. In contrast, if the seller does decide to certify, the quality is revealed to the buyer, and there is no asymmetric information. In the subsequent subgame, the q_h seller sells his good at price $p = q_h$, whence the low quality seller sells his good at a price $p = q_l$.

In order to capture the seller's option to certify, we expand the actions open to the seller by an action c that represents the seller's option to certify and to charge the maximum price q_i. Hence, the seller's payoff associated with the action c are $\Pi_h(c) = q_h - c_h$ and $\Pi_l(c) = q_l$ for a high and low quality seller, respectively. Let $\sigma_i(c)$ denote the probability that the q_i seller buys certification. We further adopt the notation of the previous section. Then we may express a mixed strategy of the seller q_i over certification and a, possibly, infinite but countable number of prices by probabilities $\sigma_i(p_j)$ such that

$$\sigma_i(c) + \sum_j \sigma_i(p_j) = 1. \tag{5}$$

In contrast to the previous section, the buyer can no longer decide to buy certification so that her actions are now constrained to s_{nn} and s_{nb}. As before let $\mu(p)$ represent the buyer's belief upon observing a non–certified good priced at p. Consequently, s_{nb} is individually rational whenever

$$\mu(p)\Delta q + q_l \geq p$$

and s_{nn} is individually rational whenever

$$\mu(p)\Delta q + q_l \leq p.$$

Proposition 3 *For any price of certification $p_c < q_h - c_h$, the equilibrium outcome in the subgame $\Gamma(p_c)$ is unique. The high quality seller certifies with probability 1 and obtains the profit $\Pi_h^* = q_h - c_h - p_c > 0$, whereas the* *low quality seller does not certify and obtains the payoff* $\Pi_l^* = q_l$*. For any price* $p_c > q_h - c_h$*, any equilibrium outcome of the subgame* $\Gamma(p_c)$ *involves no certification. For* $p_c = q_h - c_h$*, the subgame* $\Gamma(p_c)$ *has an equilibrium in which high quality seller certifies with probability 1 and obtains the profit* $\Pi_h^* = 0$*, whereas the low quality seller does not certify and obtains the payoff* $\Pi_l^* = q_l$*.*

The proposition completely characterizes the equilibrium outcome of the subgame $\Gamma(p_c)$. From this characterization, we can derive the equilibrium of the overall game of seller–induced certification.

Proposition 4 *The full game with seller–induced certification has the unique equilibrium outcome* $p_c = q_h - c_h$ *with equilibrium payoffs* $\Pi_c^s = \lambda(q_h - c_h - c_c)$*,* $\Pi_h^* = 0$*, and* $\Pi_l^* = q_l$*.*

Comparing the outcome of seller–induced certification with the outcome under buyer–induced certification we get the following result.

Proposition 5 *The certifier obtains a higher profit under seller–induced than under buyer–induced certification:* $\Pi_c^s > \Pi_c^b$*.*

The certifier is better off when it sells certification to the seller. The intuition behind this result is that if the buyer decides whether or not to certify, the decision to certify cannot be made contingent on the actual quality. This is different from when the seller has the right to decide about certification. Clearly, a seller with low quality q_l will never demand certification. In contrast, we showed that, in any equilibrium, the seller q_h always certifies. The intuition is that if seller q_h does not certify at a price p_c quoted by the certifier, then the certifier gets zero profits from the seller. It, therefore, does strictly better by lowering the certification price to a level where it is worthwhile for the seller to demand certification.

6 Social Welfare

Certification enables the high quality seller to sell his good and this increases social welfare both under buyer- and seller-induced certification by the same degree. From an efficiency perspective, the difference between the two regimes therefore only relates to the difference in the probability at which the low quality good is sold and in the frequency of costly certification.

First, under seller–induced certification the low quality good is always sold in equilibrium. This is different under buyer–induced certification, where the good is not sold when the low quality seller picks the high price $\tilde{p}$ and the buyer certifies. This happens with probability

$$\omega = \sigma_l^*(\tilde{p})\sigma^*(s_{ch}|\tilde{p}, \mu^*(\tilde{p})).$$

Thus, under buyer–induced certification an efficiency loss of q_l occurs with probability $(1 - \lambda)\omega$.

Second, the different regimes may lead to different intensities of certification and therefore differences in expected certification costs. In particular, under buyer–induced certification, the probability of certification is

$$x^b = [\lambda + (1 - \lambda)\sigma_l^*(\tilde{p})]\sigma(s_{ch}|\tilde{p}, \mu^*(\tilde{p})).$$

Remember that the buyer demands certification only if the seller quotes a high price. The cornered bracket contains the probabilities at which this is the case, which include the probability λ at which he sells the high quality product, and the probability $(1 - \lambda)\sigma_l^*(\tilde{p})$ by which he has a low quality product but quotes the high price.

By comparison, under seller–induced certification the probability of certification is

$$x^s = \lambda.$$

Let $WF^i, i = b, s$ denote social welfare under buyer and seller–induced certification, respectively. As usual, it is defined as the sum of consumer and producer surplus. Then, social welfare under buyer–induced certification is

$$WF^b = \lambda(q_h - c_h) + (1 - \lambda)(1 - \omega)q_l - x^b c_c,$$

whereas under seller–induced certification, it is

$$WF^s = \lambda(q_h - c_h) + (1 - \lambda)q_l - x^s c_c.$$

Consequently, the difference in social welfare between the two regimes is

$$\Delta WF = WF^s - WF^b = (1-\lambda)\omega q_l + (x^b - x^s)c_c,$$

In Proposition 5 we have established that the profits of a monopolistic certifier are larger under seller certification. The certifier will therefore have a preference for seller–induced certification. We now check whether these preferences are aligned with social efficiency. Clearly, when certification costs are zero, this follows immediately. The more interesting case is therefore when the cost of certification, c_c, is strictly positive. In this case, the certifier's preferences are still in line with social efficiency, when the probability of certification is smaller when that is induced by the seller. In the next lemma we compare the probabilities of certification in both regimes.

Lemma 5 *For $\lambda > 1/2$ or $c_h > (q_h + q_l)/2$ the probability of certification under seller–induced certification, x^s, is lower than under buyer–induced certification, x^b. For $\lambda \leq 1/2$ and $c_h \leq (q_h + q_l)/2$ the probability of certification under seller–induced certification, x^s, is higher than under buyer–induced certification, x^b, if and only if $q_h < 3q_l$.*

The lemma identifies a case where the probability of certification is higher under seller–induced certification than under buyer–induced certification. This still leaves open the possibility that the decision of a monopolistic certifier to offer its services to the seller rather than the buyer is not in the interest of social efficiency. In particular, if certification costs, c_c, are large, the certifier's decision may be suboptimal. The following proposition shows that this possibility does not arise. Whenever the certifier's profit under buyer–induced certification is non–negative, social welfare is larger under seller–induced certification, despite a possibly higher probability of certification.

Proposition 6 *Social welfare is higher under seller–induced certification than under buyer–induced certification.*

Again, the reasons are that under seller-induced certification, the wasteful revelation of low quality does not arise, yet the low quality good is always sold.

7 Extensions

We derived our central result that the certifier is better off selling its services to the better informed party, and that its decision is also preferred from a social welfare point of view, in a very stylized model. In this section, we discuss the most important extensions to argue that our result is robust.

To begin, we assumed that, if certification reveals low quality, the buyer does not purchase the good in spite of gains from trade, because the seller has quoted an inappropriately high price. Implicit in this is the idea that the seller does not renegotiate because of its high cost. Our results do not depend on this absence of renegotiation. To see this, consider the other extreme where renegotiation is costless so that, after certification, the buyer and a low quality seller always renegotiate to trade the low quality good at the price $p = q_l$. In this case, the low quality seller always has an incentive to quote the higher price for the low quality good before certification, because he is ensured the low quality price even when the buyer demands certification. Hence, ex post renegotiation actually worsens the outcome of the inspection game by raising the seller's cheating incentives - yet it does not change the outcome of the signalling game.[10]

Our results are also robust to the introduction of imperfect certification technologies. Consider a certification technology that reveals the correct quality only with probability $\pi > 1/2$, whereas it identifies the wrong quality with corresponding probability $(1 - \pi) > 0$. Although the imperfect certification technology reduces the profitability of buyer–induced certification, it does not qualitatively change the equilibrium. Intuitively, a less informative certification technology shrinks the intermediate area in Figure 1, where $S(s_{ch})$ is optimal, in a continuous way.

Imperfect certification also does not change the nature of the equilibrium outcome with seller–induced certification. In particular, an equilibrium exists where the certifier charges the certification price $p_c = \pi q_h - c_h$, the high quality seller certifies and charges the price q_h, and the low quality seller sells

[10] Costless renegotiation yields a framework more conform Durbin (1999), where the price is set after the certification outcome is known. Durbin (1999) however does not account for the problem of adverse selection.

the good uncertified at a price q_l. As in the baseline model, the equilibrium is sustained by a buyer who buys the good at the price q_h only if it is certified as of high quality and, consistent with equilibrium play, only believes that the good has high quality when it is certified and the price is q_h. Hence, as shown in Strausz (2010), the equilibrium outcome remains separating also with imperfect certification. Consequently, the equilibrium outcomes under buyer– and seller–induced certification are continuous in π. As a result, our results are robust to imperfect certification technologies that are not completely uninformative.

Starting from an industrial organization perspective, we assumed that buyer, seller, and certifier can only use prices rather than sophisticated contracts to coordinate their exchange. This raises the question whether more complicated contracts, such as prices that condition on the certification outcome, can change our ranking between seller–induced and buyer–induced certification. As one can formally show with optimal mechanism design, this is not the case. The intuition is that with seller–induced certification, the certifier extracts all the rents from certification, and hence, the certifier cannot do better than in our context with seller–induced certification. Stated more formally, the equilibrium payoffs under the mechanism that maximizes the certifier's payoff coincides with the equilibrium payoffs in our certification game with seller–induced certification.

This argument also provides a justification for our equilibrium result not to change, when the buyer's preference parameter θ is private information. By a ballpark argument, this restricts the certifier's potential to extract rents from the buyer but leaves unchanged the conditions for seller certification, so that the equilibrium (or mechanism design) result by which the certifier chooses seller rather than buyer certification because of higher rents extracted this way is upheld.

In the baseline model, the seller can produce only one fixed quality. Suppose alternatively that a high quality producer actually has the choice to produce alternatively high or low quality, whence a low quality producer can produce only low quality. In this case, the high quality seller's next best alternative to producing high quality and having this certified is to sell low quality without certification. This changes the outside option of the high quality seller from zero to q_l and limits the certifier's possibility to exploit him. Nevertheless, all our qualitative results are upheld. In particular, the certifier obtains the higher profits from seller–induced certification, because, as explained, it enables it to extract all rents from certification – even though the rents from certification are now smaller. Similarly, welfare is higher under seller–induced certification.

The bilateral seller–buyer framework, within which we have developed our argument, is also not essential for our result to be upheld. As a particular example, consider a setting that applies particularly well to the financial market, where one seller can sell n units of the good to n identical buyers. Essentially, there are two possible information structures. A first one in which buyers cannot share the certification result but each individually must buy certification.[11] Under buyer certification, our formal results carry through and, hence, the certifier's profits are simply multiplied by n. Under seller certification, Proposition 3 is changed so that the profits from selling the product are also multiplied by n, and $p_c = n[q_h - c_h]$. Because the certifier's profits from selling to buyers and sellers are both multiplied by n, both the ranking of seller–induced vs. buyer–induced certification by the certifier and from a welfare point of view are as in our baseline model.

The second information structure is one in which buyers collude to collectively initiate certification. Under buyer certification, the market structure remains as in the baseline model, yet with n times the buyer's benefit that can be exploited by the certifier. Under seller certification, the same change of Proposition 3 takes place as above. Again, the results remain unchanged.

Finally, in our baseline model, the certifier, as the residual claimant, captures all rents. Suppose alternatively and quite naturally, that the rents are shared between the seller and the certifier based on a bargaining subgame. Such a situation could arise, for instance, when the certifiers compete imperfectly. Then, our equilibrium result according to which certification is always initiated by the seller will only survive if the certifier's bargaining power is strong, as – because of strong economies of scale and scope involved

[11] This is similar to Durbin's (1999) "guidebook" example.

in certification – is not infrequently the case. Yet, as it is independent of the distribution of rents, our welfare result is sustained.

8 Empirical Examples

When looking at empirical examples, we should recollect the situation characterized in our model: If the price quoted by the seller is (appropriately) high, the buyer needs to know the quality of the good before taking her purchase decision - yet the seller cannot credibly commit to that. Only the certifier can truthfully evaluate and communicate its quality. Here, we document that this characterization includes a class of particularly relevant economic transactions.[12]

First of all, our baseline model and results apply one–to–one to situations in which certification is both product and customer specific. An example in point is parts procurement in the automotive industry.[13] The development and production of a complex part for a premium automobile is typically done by only one supplier — our seller, whom the automotive producer — our buyer — selects explicitly. Because the part is customer specific, the buyer–seller relationship is a bilateral monopoly.

Before the so called null–series production, information between the buyer and the seller about the quality of the part is asymmetric. Independent certifiers mediate this informational asymmetry.[14] Due to significant economies

[12] We should also keep in mind, however, that our model does *not* relate to cases in which consumers can share their experience about the quality, as reflected, for instance, in consumer reports.

[13] The evidence is taken from Mueller et al. (2008), and from a large scale study conducted in 2007/08 by Stahl et al. for the German Association of Automotive Manufacturers (VDA) on Upstream Relationships in the Automotive Industry. Survey participants were car producers and their upstream suppliers. All German car producers and 13 first tier counterparts were questioned as to their procurement relationships. A description of the data base is found in Koenen et al. (2012)

[14] An example is EDAG, an engineering company centering on the development and prototype–construction of cars, as well as on independent certification of car modules and systems. In this function it serves all major car producers world wide. See http://www.edag.de/produkte/prueftechnik/automotive/index_html

of scope involving the analytical instruments, the certification industry is highly concentrated. One of the key test criteria is the fulfilment of safety norms. It turns out that the testing of car modules and systems is almost always performed on the request of the upstream supplier rather than the buyer. Moreover, the buyer conditions her actual purchase on the outcome of the certification process. Our model, therefore, captures the procurement relationships in the automotive industry, and our results are consistent with the observations in this industry.

Whereas our model applies particularly well to cases in which certification is both product and customer specific, the results also help us understanding purely product specific certification. Examples range from the certification of foodstuff for production without herbicides or pesticides; to the certification of toys for production without aggressive chemicals, to the certification of building materials, of ecologically correct inputs in the production of particular products, or of the fire–resistance of safes. An example close to our academic activity is the certification process induced by the editors of academic journals, on request of the producers of academic articles.

A particularly timely and controversially discussed example that fits our last extension is the certification of financial products. Certification is produced in a heavily concentrated rating industry. The fact that many actual buyers now admit that they poorly understood the products' complexities underscores the pervasive informational asymmetry in this market, and the rating agencies' principle role in reducing it. Before the crisis, and consistent with our result, certification was initiated by the issuers of financial products – our sellers, who paid the rating agencies for their services. A controversial claim is that seller–induced certification led to certifiers' capture and inflated ratings, which precipitated the financial crisis. Proponents of this claim, therefore, argue for a regulatory response to transfer the rating decision from sellers to buyers.

Due to the superior welfare properties of seller-induced certification, our results caution against regulatory pressure in favor of buyer-induced certification. Since capture is an undeniable issue, regulatory initiative should concentrate on directly preventing this, by designing a certification system in which capture is minimized or excluded.[15] A particularly successful example is the German "Stiftung Warentest", originally founded and subsidized by the German Federal Government to initiate the unbiased certification of consumer products, and to prevent capture. Yet the design of an efficient, capture–proof regulatory mechanism addressing certification in financial markets lies beyond the scope of this paper.

9 Conclusion

Under asymmetric quality information, demand for certification tends to arise from both buyers and sellers. Buyers do not want to be cheated if offered a good of unknown quality at a high price. In turn, sellers want to offer the good at a high price – especially if it is of high quality. So to whom does, and, from a welfare point of view, to whom should a credible certifier sell his services, to the buyer or to the seller? Within a parsimonious model, we give straightforward answers to these questions, namely that certification should be induced by the party that is better informed about the product, typically the seller. While this answer appears deceptively simple, its justification needs an elaborate argument.

Ultimately, our analysis provides new, elementary insights in the economic role of certification. Within a unified framework it shows that certification to the buyer and certification to the seller differ fundamentally. They lead to different games and, therefore, outcomes. Buyer-induced certification leads to an inspection game with the typical mixed strategy equilibrium. Seller-induced certification leads to a signalling game with a fully separating equilibrium in the case of seller-induced certification.

Our result is consistent with real life situations, in which buyers cannot observe a product's quality before it is traded, and sellers cannot credibly announce it either, so an independent certifier has to remove the informational asymmetry between sellers and buyers. Particularly relevant examples dis-

[15] Because also buyers have an incentive to capture the certifier, a naive switch from seller- to buyer–induced certification may also result in a switch of the capture problem from sellers to buyers.

cussed are sales of parts into the production of technically complex products such as automobiles, and sales of modern financial products.

Our analysis leads to a clear policy implication, that is relevant especially in the ongoing discussion about certification in financial markets. Against a current argument about transferring the certification initiative to buyers, we argue in favor of seller–initiated certification - as long as that is not subject to capture. In view of this, policy makers should think of means to prevent certifier capture by sellers, rather than simply reverting from seller– to buyer–induced certification.

We also demonstrated the robustness of our results by considering many extensions. Clearly, further extensions and refinements of the approach are possible. In order to focus on our central point, we have purposively excluded seller reactions to certification, such as adapting quality, as this is discussed in other papers. For the same reason, we also have excluded certifier capture by the seller. Finally, we excluded competition between many sellers, or many certifiers. Arguably, the latter is less important, in view of the technical economies of scale and reputation effects associated with certification. The former, competition between sellers, enhances sellers's demand for certification, but tends not to change our insights qualitatively.

Appendix

The appendix contains all formal proofs to our Lemmata and Propositions.

Proof of Lemma 1: Follows directly from the text. Q.E.D.

Proof of Lemma 2: To show that $\Pi_h(p,\mu|\sigma^*)$ is non–decreasing in μ we first establish that, in any PBE, $\sigma^*(s_{nn}|p,\mu)$ is weakly decreasing in μ. Suppose not, then we may find $\mu_1 < \mu_2$ such that $0 \leq \sigma^*(s_{nn}|p,\mu_1) < \sigma^*(s_{nn}|p,\mu_2) \leq 1$. Lemma 1 implies that $(p,\mu_2) \in S(s_{nn}|p_c)$. That is,

$$p \geq \mu_2 q_h + (1-\mu_2)q_l \tag{6}$$

and

$$p_c \geq \mu_2(q_h - p). \tag{7}$$

Now since $\sigma^*(s_{nn}|p,\mu_1) < 1$ we have either $\sigma^*(s_{nb}|p,\mu_1) > 0$ or $\sigma^*(s_{ch}|p,\mu_1) > 0$. Suppose first $\sigma^*(s_{nb}|p,\mu_1) > 0$, then by Lemma 1 we have $p \leq \mu_1 q_h + (1-\mu_1)q_l$. But from $\mu_2 > \mu_1$ and $q_h > q_l$ it then follows that $\mu_2 q_h + (1-\mu_2)q_l > p$, which contradicts (6). Suppose therefore that $\sigma^*(s_{ch}|p,\mu_1) > 0$, then by Lemma 1 we have $\mu_1(q_h - p) \geq p_c > 0$. This requires $q_h > p$. But then, due to $\mu_2 > \mu_1$, we get $\mu_2(q_h - p) > p_c$, which contradicts (7).

Hence, we establish that $\sigma^*(s_{nn}|p,\mu)$ is weakly decreasing in μ and therefore $\sigma^*(s_{nb}|p,\mu) + \sigma^*(s_{ch}|p,\mu)$ must be weakly increasing in μ. Consequently, $\Pi_h(p,\mu|\sigma^*)$ is weakly increasing in μ.

Next we show that in any PBE $\sigma^*(s_{nb}|p,\mu)$ is weakly increasing in μ. Suppose not, then we may find $\mu_1 < \mu_2$ such that $1 \geq \sigma^*(s_{nb}|p,\mu_1) > \sigma^*(s_{nb}|p,\mu_2) \geq 0$. Since $\sigma^*(s_{nb}|p,\mu_1) > 0$, Lemma 1 implies that $(p,\mu_1) \in S(s_{nb}|p_c)$. That is,

$$p \leq \mu_1 q_h + (1-\mu_1)q_l \tag{8}$$

and

$$p_c \geq (1-\mu_1)(p - q_l). \tag{9}$$

Now since $\sigma^*(s_{nb}|p,\mu_2) < 1$ we have either $\sigma^*(s_{nn}|p,\mu_2) > 0$ or $\sigma^*(s_{ch}|p,\mu_2) > 0$. Suppose first $\sigma^*(s_{nn}|p,\mu_2) > 0$, then by Lemma 1 this implies $p \geq$

$\mu_2 q_h+(1-\mu_2)q_l$. But due to $\mu_2 > \mu_1$ and $q_h > q_l$ we get $p > \mu_1 q_h+(1-\mu_1)q_l$. This contradicts (8). Suppose therefore that $\sigma^*(s_{ch}|p,\mu_2) > 0$, then by Lemma 1 we have $(1-\mu_2)(p-q_l) \geq p_c > 0$. This requires $p > q_l$. But then, due to $\mu_2 > \mu_1$, we get $(1-\mu_1)(p-q_l) > p_c$. This contradicts (9). Hence, $\sigma^*(s_{nb}|p,\mu)$ must be weakly increasing in μ. Consequently, $\Pi_l(p,\mu|\sigma^*)$ is weakly increasing in μ. Q.E.D.

Proof of Lemma 3: i) For any $\bar{p} < q_l$, $\mu \in [0,1]$ we have $(\bar{p},\mu) \notin S(s_{nn})$, $(\bar{p},\mu) \notin S(s_{ch})$ and $(\bar{p},\mu) \in S(s_{nb})$. Hence, $\sigma^*(s_{nb}|\bar{p},\mu^*(\bar{p})) = 1$. Now suppose for some $\bar{p} < q_l$ we have $\sigma_i^*(\bar{p}) > 0$. This would violate (2), because instead of charging $\bar{p}$ seller q_i could have raised profits by $\varepsilon\sigma_i(\bar{p})$ by charging the higher price $\bar{p}+\varepsilon < q_l$ with $\varepsilon \in (0,(q_l-\bar{p}))$. At $\bar{p}+\varepsilon < q_l$ the buyer always buys, because, as established, $\sigma^*(s_{nb}|\bar{p}+\varepsilon,\mu) = 1$ for all μ and in particular for $\mu = \mu^*(\bar{p}+\varepsilon)$.

For any $\bar{p} > q_h$, $\mu \in [0,1]$ we have $(\bar{p},\mu) \in S(s_{nn})$, $(\bar{p},\mu) \notin S(s_{ch})$ and $(\bar{p},\mu) \notin S(s_{nb})$. Hence, $\sigma^*(s_{nn}|\bar{p},\mu^*(\bar{p})) = 1$. Now suppose we have $\sigma_l(\bar{p}) > 0$. This would violate (2), because instead of charging $\bar{p}$ seller q_l could have raised profits by $(q_l-\varepsilon)\sigma_l(\bar{p})$ by charging the price $q_l-\varepsilon$.

ii) Suppose $q_l - \Pi_l^* = \delta > 0$. Now consider a price $p' = q_l - \varepsilon$ with $\varepsilon \in (0,\delta)$ then for any $\mu' \in [0,1]$ we have $(p',\mu') \in S(s_{nb})$ and $(p',\mu') \notin S(s_{nn}) \cup S(s_{ch})$ so that, by (1), we have $\sigma^*(s_{nb}|p'^*,\mu^*(p')) = 1$ and, therefore, $\Pi_l(p'^*,\mu^*(p')|\sigma^*) = p' > \Pi_l^*$. This contradicts (2).

iii) For any p such that $\sigma_h^*(p) > 0$, we have $\Pi_h^* = \Pi_h(p,\mu^*(p)|\sigma^*) = [\sigma^*(s_{nb}|p,\mu^*(p))+\sigma^*(s_{ch}|p,\mu^*(p))]p-c_h$. As argued in i), we have $\sigma^*(s_{nn}|p,\mu) = 1$ for all $p > q_h$ and $\mu \in [0,1]$. Hence, $\Pi_h(p,\mu|\sigma^*) = 0$ whenever $p > q_h$. But for any price $p \leq q_h$ we have $\Pi_h(p,\mu|\sigma^*) \leq q_h - c_h$. Hence, it follows that $\Pi_h^* \leq q_h - c_h$. Now suppose $\Pi_h^* = q_h - c_h$. Then we must have $\sigma_h^*(q_h) = 1$ and $\sigma^*(s_{nb}|q_h,\mu^*(q_h)) + \sigma^*(s_{ch}|q_h,\mu^*(q_h)) = 1$. But, due to $\mu^*(q_h)(q_h-q_h) = 0 < p_c$, we have $(q_h,\mu^*(q_h)) \notin S(s_{ch}|q_h)$ so that $\sigma^*(s_{ch}|q_h,\mu^*(q_h)) = 0$. Hence, we must have $\sigma^*(s_{nb}|q_h,\mu^*(q_h)) = 1$. This requires $(q_h,\mu^*(q_h)) \in S(s_{nb}|p_c)$ so that we must have $\mu^*(q_h) = 1$. By (3), this requires $\sigma_l^*(q_h) = 0$. But since $\Pi_l(q_h,1|\sigma^*) = \sigma^*(s_{nb}|q_h,\mu^*(q_h))q_h = q_h$ we must, by (2), have $\Pi_l^* \geq q_h$. Together with $\sigma_l^*(q_h) = 0$, it would require $\sigma_l^*(p) > 0$ for some $p > q_h$ and leads to a contradiction with i). Q.E.D.

Proof of Lemma 4: We first prove ii): Suppose to the contrary that

$\delta \equiv \tilde{p}-c_h-\Pi_h^* > 0$. Then, due to the countable number of equilibrium prices, we can find an out–of–equilibrium price $p' = \tilde{p} - \varepsilon$ for some $\varepsilon \in (0, \delta)$. Then for any belief $\mu' \in (p_c/(q_h-p'), 1-p_c/(p'-q_l)) \neq \emptyset$[16] we have $(p', \mu') \in S(\sigma_{ch})$ and $(p', \mu') \notin S(\sigma_{nn}) \cup S(\sigma_{nb})$. Consequently, $\sigma^*(s_{ch}|p', \mu') = 1$. Hence, $\Pi_h(p', \mu') = p'-c_h = \tilde{p}-c_h-\varepsilon > \tilde{p}-c_h-\delta = \Pi_h^*$ and $\Pi_l(p', \mu') = 0 < q_l \leq \Pi_l^*$. Therefore, by B.R. the buyer's equilibrium belief must satisfy $\mu^*(p') \geq \mu'$. By Lemma 2 it follows $\Pi_h(p', \mu^*(p')) \geq \Pi_h(p', \mu') = \tilde{p} - c_h - \varepsilon > \Pi_h^*$. This contradicts (2). Consequently, we must have $\Pi_h^* \geq \tilde{p} - c_h$. To show i) note that for all $p < \tilde{p}$ and $\mu \in [0, 1]$ we have $\Pi_h(p, \mu|\sigma) \leq p - c_h < \tilde{p} - c_h \leq \Pi_h^*$ so that $\sigma_h(p) > 0$ would violate (2). Q.E.D.

Proof of Proposition 1: i): First we show that for $\lambda < \tilde{\mu}$ and $c_h < \tilde{p}$ there exists no pooling, i.e., there exists no price $\bar{p}$ such that $\sigma_h^*(\bar{p}) = \sigma_l^*(\bar{p}) > 0$. For suppose there does. Then, by Lemma 4.i, we have $\bar{p} \geq \tilde{p}$ and, by Lemma 3.i, we have $\bar{p} \leq q_h$. Yet, due to (3) we have $\mu^*(\bar{p}) = \lambda < \tilde{\mu}$ so that $q_l + \mu^*(\bar{p})\Delta q - \bar{p} < q_l + \tilde{\mu}\Delta q - \tilde{p} = 0$. Moreover, $\mu^*(\bar{p})(q_h - \bar{p}) < \tilde{\mu}(q_h - \tilde{p}) = p_c$. Therefore, $\sigma^*(s_{nn}|\bar{p}, \mu^*(\bar{p})) = 1$ and $\Pi_h(\bar{p}, \mu^*(\bar{p})) = 0$. As a result, $\sigma_h^*(\bar{p}) > 0$ contradicts (2), because, by Lemma 4.ii, $\Pi_h^* \geq \tilde{p} - c_h > 0 = \Pi_h(\bar{p}, \mu^*(\bar{p}))$.

Second, suppose that for some $\bar{p} > \tilde{p}$ we have $\sigma_h^*(\bar{p}) > 0$ then, by definition of $\tilde{p}$, we have $(\bar{p}, \mu) \notin S(s_{ch})$ for any $\mu \in [0, 1]$. Hence, $\sigma^*(s_{ch}|\bar{p}, \mu^*(\bar{p})) = 0$ so that $\Pi_l(\bar{p}, \mu^*(\bar{p})) = \Pi_h(\bar{p}, \mu^*(\bar{p})) + c_h$. From Lemma 4.ii it then follows $\Pi_l(\bar{p}, \mu^*(\bar{p})) \geq \tilde{p}$ and, therefore, $\sum_{p \geq \tilde{p}} \sigma_l^*(p) = 1$. From $\bar{p} > \tilde{p}$ and $\tilde{\mu} > \lambda$ it follows $\lambda\Delta q + q_l - \bar{p} < \tilde{\mu}\Delta q + q_l - \tilde{p} = 0$ so that $\lambda\Delta q + q_l < \bar{p}$. Now take a $\bar{p} > \tilde{p}$ with $\sigma_l(\bar{p}) > 0$ then, by Lemma 3.ii and (2), $0 < q_l \leq \Pi_l^* = \Pi_l(\bar{p}, \mu^*(\bar{p})|\sigma^*) = \sigma(s_{nb}|\bar{p}, \mu^*(\bar{p}))\bar{p}$. This requires $\sigma(s_{nb}|\bar{p}, \mu^*(\bar{p})) > 0$ and therefore $(\bar{p}, \mu^*(\bar{p})) \in S(s_{nb}|p_c)$ and, hence, $\mu^*(\bar{p})\Delta q + q_l \geq \bar{p}$. Combining the latter inequality with our observation that $\lambda\Delta q + q_l < \bar{p}$ and using (3), it follows

$$\lambda\Delta q + q_l < \frac{\lambda\sigma_h^*(\bar{p})}{\lambda\sigma_h^*(\bar{p}) + (1 - \lambda)\sigma_l^*(\bar{p})}\Delta q + q_l,$$

which is equivalent to $\sigma_h^*(\bar{p}) > \sigma_l^*(\bar{p})$. Summing over all $p \geq \tilde{p}$ and using $\sum_{p \geq \tilde{p}} \sigma_l^*(p) = 1$ yields the contradiction $\sum_{p \geq \tilde{p}} \sigma_h^*(p) > 1$. Hence, we must

[16]Let $l(p) \equiv p_c/(q_h - p)$ and $h(p) \equiv 1 - p_c/(p - q_l)$. Then by the definition of $\tilde{p}$ we have $l(\tilde{p}) = h(\tilde{p})$. Moreover, for $q_l < p < q_h$ we have $l'(p) = p_c/(q_h - p)^2 > h'(p) = p_c/(p - q_l)^2 > 0$. Hence, $l(\tilde{p} - \varepsilon) < h(\tilde{p} - \varepsilon)$ for $\varepsilon > 0$ so that $\tilde{p} - \varepsilon > q_l$ and, therefore, $l(p') < h(p')$.

have $\sigma_l^*(\bar{p}) = 0$ for any $\bar{p} > \tilde{p}$. But this contradicts $\sum_{p \geq \tilde{p}} \sigma_l^*(p) = 1$ and, therefore, we must have $\sigma_h^*(\bar{p}) = 0$ for all $\bar{p} > \tilde{p}$. Hence, if an equilibrium for $\lambda < \tilde{\mu}$ and $\tilde{p} > c_h$ exists then, by Lemma 4, it exhibits $\sigma_h^*(\tilde{p}) = 1$, $\Pi_h^* = \tilde{p} - c_h$ and $\sigma^*(s_{ch}|\tilde{p}, \tilde{\mu}) + \sigma^*(s_{nb}|\tilde{p}, \tilde{\mu}) = 1$.

We now show existence of such an equilibrium and demonstrate that any such equilibrium has a unique equilibrium outcome. If $\sigma_h^*(\tilde{p}) = 1$ then (3) implies that $\mu^*(\tilde{p}) = \tilde{\mu}$ whenever

$$\sigma_l^*(\tilde{p}) = \frac{\lambda(1-\tilde{\mu})}{\tilde{\mu}(1-\lambda)},$$

which is smaller than one exactly when $\lambda < \tilde{\mu}$. By definition, $(\tilde{p}, \tilde{\mu}) \in S(s_{ch}) \cap S(s_{nb})$ so that any buying behavior with $\sigma^*(s_{ch}|\tilde{p}, \tilde{\mu}) + \sigma^*(s_{nb}|\tilde{p}, \tilde{\mu}) = 1$ is consistent in equilibrium. In particular, $\sigma^*(s_{nb}|\tilde{p}, \tilde{\mu}) = q_l/\tilde{p} < 1$ is consistent in equilibrium. Only for this buying behavior we have $\Pi_l(q_l, 0) = q_l = \Pi_l(\tilde{p}, \tilde{\mu})$ so that seller q_l is indifferent between price $\tilde{p}$ and q_l. The equilibrium therefore prescribes $\sigma_l^*(q_l) = 1 - \sigma_l^*(\tilde{p})$. Finally, let $\mu^*(q_l) = 0$ and $\sigma^*(s_{nb}|q_l, \mu^*(q_l)) = 1$ and $\mu^*(p) = 0$ for any price p larger than q_l and unequal to $\tilde{p}$. This out–of–equilibrium beliefs satisfies B.R.. Hence, the expected profit to the certifier is

$$\Pi_c(p_c) = (\lambda + (1-\lambda)\sigma_l^*(\tilde{p}))\, \sigma^*(s_{ch}|\tilde{p}, \tilde{\mu})(p_c - c_c) = \frac{\lambda(\tilde{p} - q_l)}{\tilde{\mu}\tilde{p}}(p_c - c_c).$$

ii) In order to show that, in any equilibrium of $\Gamma(p_c)$, we have $\Pi_c(p_c) = 0$ whenever $\lambda > \tilde{\mu}$, we prove that for any $\bar{p}$ such that $\sigma^*(s_{ch}|\bar{p}, \mu^*(\bar{p})) > 0$, it must hold $\sigma_h^*(\bar{p}) = \sigma_l^*(\bar{p}) = 0$. Suppose we have $\sigma^*(s_{ch}|\bar{p}, \mu^*(\bar{p})) > 0$, then $(\bar{p}, \mu^*(\bar{p})) \in S(s_{ch})$ and, necessarily, $\bar{p} \leq \tilde{p}$. But by Lemma 4.i, $\sigma_h^*(\bar{p}) > 0$ also implies $\bar{p} \geq \tilde{p}$. Therefore, we must have $\bar{p} = \tilde{p}$. But $(\tilde{p}, \mu) \in S(s_{ch})$ only if $\mu = \tilde{\mu}$. Hence, we must have $\mu^*(\tilde{p}) = \tilde{\mu}$. By (3) it therefore must hold

$$\tilde{\mu} = \mu^*(\tilde{p}) = \frac{\lambda\sigma_h^*(\tilde{p})}{\lambda\sigma_h^*(\tilde{p}) + (1-\lambda)\sigma_l^*(\tilde{p})}.$$

For $\lambda > \tilde{\mu}$ this requires $\sigma_h^*(\tilde{p}) < \sigma_l^*(\tilde{p}) \leq 1$ and therefore there is some other $p' > \tilde{p}$ such that $\sigma_h^*(p') > 0$. But if also p' is an equilibrium price, then $\Pi_h(\tilde{p}, \mu^*(\tilde{p})|\sigma^*) = \Pi_h(p', \mu^*(p')|\sigma^*)$. Yet, for any $p' > \tilde{p}$ it holds $(p', \mu) \notin S(s_{ch}|p_c)$ for any $\mu \in [0,1]$ so that $\Pi_l(p', \mu|\sigma^*) = \Pi_h(p', \mu|\sigma^*) + c_h$ and, together with our assumption $\sigma^*(s_{ch}|\bar{p}, \mu^*(\bar{p})) > 0$ yields $\Pi_l(\bar{p}, \mu^*(\bar{p})|\sigma^*) <$

$\Pi_h(\bar{p}, \mu^*(\bar{p})|\sigma^*) + c_h = \Pi_h(p', \mu^*(p')|\sigma^*) + c_h = \Pi_l(p', \mu^*(p')|\sigma^*)$ so that, by (2), $\sigma_l^*(\bar{p}) = 0$. Since $\bar{p} = \tilde{p}$, this violates $\sigma_l^*(\tilde{p}) > \sigma_h^*(\tilde{p}) \geq 0$. As a result, $\sigma^*(s_{ch}|\bar{p}, \mu^*(\bar{p})) > 0$ implies $\sigma_h^*(\bar{p}) = 0$.

In order to show that we must also have $\sigma_l^*(\bar{p}) = 0$, assume again that $\sigma^*(s_{ch}|\bar{p}, \mu^*(\bar{p})) > 0$. We have shown that his implies $\sigma_h^*(\bar{p}) = 0$. Now if $\sigma_l^*(\bar{p}) > 0$ then, by (3), it follows $\mu^*(\bar{p}) = 0$. But then $q_l + \mu^*(\bar{p})\Delta q - \bar{p} - p_c = q_l - \bar{p} - p_c < q_l - \bar{p}$ so that $(\bar{p}, \mu^*(\bar{p})) \notin S(s_{ch})$, which contradicts $\sigma^*(s_{ch}|\bar{p}, \mu^*(\bar{p})) > 0$.

In order to show that $\tilde{p} < c_h$ implies $\Pi_c(p_c) = 0$ suppose, on the contrary that, $\Pi_c(p_c) > 0$. This requires that there exists some $\bar{p}$ such that $\sigma^*(s_{ch}|\bar{p}, \mu^*(\bar{p})) > 0$ and $\sigma_i^*(\bar{p}) > 0$ for some $i \in \{l, h\}$. First note that $\sigma^*(s_{ch}|\bar{p}, \mu^*(\bar{p})) > 0$ implies $\bar{p} \leq \tilde{p}$. Now suppose $\sigma_h^*(\bar{p}) > 0$ then $\Pi_h(\bar{p}, \mu^*(\bar{p})|\sigma^*) = (\sigma^*(s_{ch}|\bar{p}, \mu^*(\bar{p})) + \sigma^*(s_{nb}|\bar{p}, \mu^*(\bar{p})))\bar{p} - c_h < 0$ so that the high quality seller would make a loss and, thus, violates (2). Therefore, we have $\sigma_h^*(\bar{p}) = 0$. Now if $\sigma_l^*(\bar{p}) > 0$ then (3) implies $\mu^*(\bar{p}) = 0$ so that $\sigma^*(s_{ch}|\bar{p}, \mu^*(\bar{p})) = 0$, which contradicts $\Pi_c(p_c) > 0$. Q.E.D.

Proof of Proposition 2: In order to express the dependence of $\tilde{\mu}$ and $\tilde{p}$ on p_c explicitly, we write $\tilde{\mu}(p_c)$ and $\tilde{p}(p_c)$, respectively. We maximize expression (4) with respect to p_c over the relevant domain

$$P = \{p_c | p_c \leq \Delta q/4 \wedge \tilde{\mu}(p_c) \geq \lambda \wedge \tilde{p}(p_c) \geq c_h\}.$$

First, we show that (4) is increasing in p_c. Define

$$\alpha(p_c) \equiv \frac{\lambda(\tilde{p}(p_c) - q_l)}{\tilde{\mu}(p_c)\tilde{p}(p_c)}$$

so that $\Pi_c(p_c) = \alpha(p_c)(p_c - c_c)$. We have

$$\alpha'(p_c) = \frac{4\lambda\Delta q^2}{\sqrt{\Delta q(\Delta q - 4p_c)}\left(q_h + q_l + \sqrt{\Delta q(\Delta q - 4p_c)}\right)^2} > 0$$

so that $\alpha(p_c)$ is increasing in p_c and, hence, $\Pi_c(p_c)$ is increasing in p_c and maximized for $\max P$.

We distinguish two cases. First, for $\lambda \leq 1/2$, it follows $\tilde{\mu}(p_c) \geq 1/2 \geq \lambda$. Therefore,

$$P = \{p_c | p_c \leq \Delta q/4 \wedge \tilde{p}(p_c) \geq c_h\}.$$

Hence, max P is either $p_c = \Delta q/4$ or such that $\tilde{p}(p_c) = c_h$. Because $\tilde{p}(\Delta q/4) = (q_h + q_l)/2$, it follows that for $\lambda \leq 1/2$ and $c_h \leq (q_h + q_l)/2$, the maximum obtains for $p_c = \Delta q/4$ with

$$\Pi_c^b = \frac{\lambda \Delta q}{2(q_h + q_l)}(\Delta q - 4c_c).$$

For $\lambda \leq 1/2$ and $c_h > (q_h + q_l)/2$ the maximum obtains for p_c such that $\tilde{p}(p_c) = c_h$, which yields $p_c = (q_h - c_h)(c_h - q_l)/\Delta q$ with

$$\Pi_c^b = \frac{\lambda[(q_h - c_h)(c_h - q_l) - \Delta q c_c]}{c_h};$$

Second, for $\lambda > 1/2$ we have

$$\tilde{\mu}(p_c) \geq \lambda \Leftrightarrow p_c \leq \lambda(1-\lambda)\Delta q.$$

Since $\lambda(1-\lambda) \leq 1/4$ the requirement $p_c < \lambda(1-\lambda)\Delta q$ automatically implies $p_c \leq \Delta q/4$. Hence for $\lambda > 1/2$ we have

$$P = \{p_c | p_c \leq \lambda(1-\lambda)\Delta q \wedge \tilde{p}(p_c) \geq c_h\}.$$

Because, $\tilde{p}(\lambda(1-\lambda)\Delta q) = \lambda q_h + (1-\lambda)q_l$, which by assumption is smaller than c_h, we have max $P = (q_h - c_h)(c_h - q_l)/\Delta q$. Note that $c_h > \lambda q_h + (1-\lambda)q_l$ and $\lambda > 1/2$ implies that $c_h > (q_h + q_l)/2$. It follows $\tilde{\mu} = (c_h - q_l)/\Delta q$ and

$$\Pi_c^b = \frac{\lambda[(q_h - c_h)(c_h - q_l) - \Delta q c_c]}{c_h};$$

Q.E.D.

Proof of Proposition 3 Fix some $p_c < q_h - c_h$. By certifying, seller q_h guarantees himself the payoff $\Pi_h(c) = q_h - c_h - p_c > 0$. Hence, in any equilibrium of the subgame $\Gamma(p_c)$ seller q_h must obtain a payoff of at least $\Pi_h(c) > 0$.

Now suppose that there exists some equilibrium in which $\sigma_h(c) < 1$. Then, by (5) there exists some price $\tilde{p}$ such that $\sigma_h(\tilde{p}) > 0$. For $\tilde{p}$ to be optimal, it is required that $\Pi_h(\tilde{p}, \mu^*(\tilde{p})|\sigma^*) = \tilde{p}\sigma(s_{nb}|\tilde{p}, \mu^*(\tilde{p})) - c_h \geq \Pi_h(c) > 0$. This implies $\Pi_l(\tilde{p}, \mu^*(\tilde{p})|\sigma^*) = \tilde{p}\sigma(s_{nb}|\tilde{p}, \mu^*(\tilde{p})) > c_h$ so that the equilibrium payoff of seller q_l is $\Pi_l^* > c_h > \bar{q}$. Consequently, $\sigma_l^*(p) = 0$ for any $p < \bar{q}$ and therefore

$$\sum_{p \geq \bar{q}} \sigma_l^*(p) = 1. \tag{10}$$

But if $\sigma_l^*(p) > 0$ then we must have $p\sigma(s_{nb}|p, \mu^*(p)) > c_h$. This requires $\sigma(s_{nb}|p, \mu^*(p)) > 0$. Therefore, s_{nb} must be an optimal response given price p and belief $\mu^*(p)$. Hence, $\mu^*(p)\Delta q + q_l \geq p > c_h > \lambda \Delta q + q_l$. As a result, $\mu^*(p) > \lambda$ and, due to (3), it holds $\sigma_h^*(p) > \sigma_l^*(p)$ for any $\sigma_l^*(p) > 0$. Together with (10) we arrive at the contradiction

$$\sum_{p \geq \bar{q}} \sigma_h^*(p) > \sum_{p \geq \bar{q}} \sigma_l^*(p) = 1. \tag{11}$$

It is straightforward to verify that for $p_c \leq q_h - c_h$, the strategies $\sigma_h(c) = 1$, $\sigma_l(q_l) = 1$, $\sigma^*(s_{nn}|p, \mu) = 1$ whenever $\mu \Delta q + q_l \geq p$ and zero otherwise together with $\mu^*(p) = q_l$ constitute an equilibrium that sustains the equilibrium outcome.

For $p_c > q_h - c_h$, certification would yield seller q_h a negative payoff: $\Pi_h(c) = q_h - c_h - p_c < 0$. Certification would yield seller q_l a payoff $\Pi_l(c) = q_l - p_c < q_l$, whereas seller q_l could guarantee himself the payoff q_l by not certifying. Q.E.D.

Proof of Proposition 4: First, suppose there exists an equilibrium in which the payoff of the certifier, Π_c^*, is strictly smaller than $\lambda(q_h - c_h - c_c)$. That is, $\delta = \lambda(q_h - c_h - c_c) - \Pi_c^* > 0$. Now note that the price $p_c = q_h - c_h - \delta/2 < q_h - c_h$ yields the certifier a payoff $\lambda(q_h - c_h + \delta/2) > \Pi_c^*$, because Proposition 3 shows that its subgame $\Gamma(p_c)$ has the unique outcome that seller q_h always certifies and seller q_l does not. Second, note that the certifier cannot obtain a profit that exceeds $\lambda(q_h - c_h - c_c)$, because it would require that the price of certification exceeds $q_h - c_h$ or that the low quality seller certifies with a strictly positive probability. Hence, in any equilibrium the certifier obtains the payoff $\lambda(q_h - c_h - c_c)$. According to Proposition 3 the certifier may become this payoff only for $p_c = q_h - c_h$ with $\sigma_h(c) = 1$. Q.E.D.

Proof of Proposition 5: For $\lambda \leq 1/2$ and $c_h \leq (q_h + q_l)/2$ we have $\Pi_c^s = \lambda(q_h - c_h - c_c) \geq \lambda(q_h - c_h - c_c)\frac{q_h - q_l}{q_h + q_l} \geq \lambda(q_h - (q_h + q_l)/2 - c_c)\frac{q_h - q_l}{q_h + q_l} =$

$\lambda(q_h - q_l - 2c_c)\frac{q_h-q_l}{2(q_h+q_l)} \geq \lambda(q_h - q_l - 4c_c)\frac{q_h-q_l}{2(q_h+q_l)} = \Pi_s^b$, where the second inequality uses $c_h \leq (q_h + q_l)/2$.

For $\lambda > 1/2$ or $c_h > (q_h+q_l)/2$ it follows that $\Pi_c^b = \frac{\lambda[(q_h-c_h)(c_h-q_l)-\Delta q c_c]}{c_h} < \frac{\lambda[(q_h-c_h)(c_h-q_l)-(c_h-q_l)c_c]}{c_h} = \lambda(q_h - c_h - c_c)\frac{c_h-q_l}{c_h} \leq \lambda(q_h - c_h - c_c) = \Pi_b^s$, where the first inequality uses $q_h > c_h$. Q.E.D.

Proof of Lemma 5: For $\lambda > 1/2$ or $c_h > (q_h + q_l)/2$, it follows

$$x_c^b = (\lambda + (1-\lambda)\sigma_l^*(\tilde{p}))\sigma(s_{ch}|\tilde{p}, \mu_h^*) = \lambda\frac{\Delta q}{c_h} \leq \lambda = x_c^s,$$

where the inequality obtains from $q_h - c_h - c_c > q_l \;\Rightarrow\; \Delta q < c_h + c_c < c_h$.

For $\lambda \leq 1/2$ and $c_h \leq (q_h + q_l)/2$, it follows

$$x_c^b = (\lambda + (1-\lambda)\sigma_l^*(\tilde{p}))\sigma(s_{ch}|\tilde{p}, \mu_h^*) = \lambda\frac{2\Delta q}{q_h + q_l}.$$

Hence, $x_c^b < x_c^s$ if and only if $2\Delta q < q_h+q_l$. This yields the condition $q_h < 3q_l$. Q.E.D.

Proof of Proposition 6: Due to Lemma 5 we need only check for the case $\lambda \leq 1/2$ and $c_h \leq (q_h + q_l)/2$ and $q_h < 3q_l$. According to Proposition 2 the certifier in this case makes non–negative profits exactly when $p_c^b = \Delta q/4 \geq c_c$. The differences in social welfare for this case is

$$\begin{aligned}
\Delta WF &= \lambda\frac{\Delta q}{q_h + q_l}q_l + \lambda\left(\frac{2\Delta q}{q_h + q_l} - 1\right)c_c && (12)\\
&= \frac{\lambda}{q_h + q_l}\left(\Delta q q_l - (3q_l - q_h)c_c\right) && (13)\\
&\geq \frac{\lambda}{q_h + q_l}\left(\Delta q q_l - (3q_l - q_h)\Delta q/4\right) && (14)\\
&= \lambda\Delta q/4 > 0. && (15)
\end{aligned}$$

Q.E.D.

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