Some Have a Talent for Bargaining and Some Don't

Ella Segev* and Tomer David and Niv Lazar † ‡

March 2008

Abstract

In Bargaining Theory players differ from one another in either their valuation for the negotiated object and/or their time preferences. We suggest a model that incorporates "bargaining ability" as another source for heterogeneity across players. This bargaining ability for the buyer determines how many offers he will be able to turn down before he walks away. For the seller this ability determines how many offers she will be willing to make before she decides to make no more offers. In order to emphasize the role of the bargaining skill as what motivates the bargainers we assume that players have no time preferences. This assumption is natural in many daily bargaining situations in which bargaining last for no more than few minutes and the time between consecutive offers is normally seconds. This paper suggests a theoretical explanation for the observed behavior in these situations. Incomplete information for both players on their opponent's bargaining talent leads the seller to lower her offers in equilibrium. We then get in equilibrium a series of decreasing turned down offers. The bargaining then either ends with trade or when the buyer decides to walk away or when the seller decides to make no more offers.

JEL classification: C78

Keywords: Bargaining; Asymmetric information; No Time Preferences

*Corresponding author. Faculty of Industrial Engineering and Management, Technion, Israel. email: esegev@ie.technion.ac.il

†Faculty of Industrial Engineering and Management Technion, Israel, emails: stomdav@t2.technion.ac.il and snivlzr@t2.technion.ac.il

‡The first author would like to thank Paul Schweinzer and Gabi Gayer for insightful comments. Financial support from the Israeli Science Foundation via grant no. ISF 726/06 is gratefully acknowledged.

1 Introduction

Bargaining theory, although a relatively young discipline, is already a fundamental theory in Economics as can be seen by the many books and reviews covering it (e.g. Muthoo (1999), Osborne and Rubinstein (1990), Ausubel, Cramton and Deneckere (2001), Binmore, Osborne and Rubinstein (1992), Fundenberg, Levine and Tirole (1985) and Kennan and Wilson (1993)). John Nash (1950, 1953) initiated two related approaches to Bargaining Theory. In his 1950 paper he initiated the axiomatic approach, which describes the desired properties of the bargaining solution and identifies it. The second approach, more relevant here, was initiated in his 1953 paper in which he considers a bargaining game and concludes that it has a unique equilibrium which corresponds to the outcome identified in the earlier paper. Following the pioneering work by Nash, Rubinstein (1982) describes an extensive form game of alternating offers with complete information and derives the unique subgame perfect equilibrium. A shortcoming of the complete information assumption is that it can not provide an explanation for turned down offers or for disagreement. Therefore many studies added incomplete information and were able to predict a positive probability for disagreement and turned down offers on the equilibrium path.

Thus a large body of literature in Bargaining Theory is devoted to developing bargaining models i.e. non-cooperative games in extensive form and deriving a well defined equilibrium each game with its own characteristics. All these bargaining models (with either incomplete or complete information) assume that, as Binmore, Osborne & Rubinstein (1992) put it: "once their preferences are given other psychological issues are irrelevant". Therefore these papers' starting point is the players' preferences over the set of outcomes. Each outcome is a pair (t, x) of a time t when the agreement was reached and the agreement price p. Players always prefer early to late - i.e. they discount time by either a constant fixed cost per period or by discount factor - δ[1]. Even though in most models results are drawn for the limit case when the time between consecutive offers becomes very short (i.e. the discount factors go to 1) it is still true that the existence of discount factors is a necessary condition for the derived results. Furthermore the players' type (which might either be common knowledge or their private information) is also one dimensional. Usually a player's type is her valuation for the object or her discount factor.

[1] Two exception (that we know of) to this is Binmore, Rubinstein & Wolinsky (1986) in which players are indifferent to the passage of time but face an exogenous "breaking down" probability p. In a complete information environment they derive a unique subgame perfect equilibrium with an outcome similar to the Nash bargaining solution. A second exception is Schweinzer (2007) which examines a common value bargaining situation with no time preferences.

We wish to introduce into the existing theory players with different bargaining skills. Therefore we introduce two dimensional types. We keep the assumption that players differ in their valuation for the object but they also differ in another aspect which describes their bargaining ability. Moreover, to be able to isolate the effect of their bargaining ability on the equilibrium behavior we further assume that the players have no time preferences during the periods they are active in. We believe that many bargaining situations in daily life can be described as situations with no time preferences. Most of our daily bargaining episodes (grocery, shoes, books or even a used car) last few minutes or less and the time between consecutive offers is only few seconds. During this short time it would be reasonable to assume that the negotiated object is still of the same value to the bargainers. We therefore suggest a bargaining game in which the value of the object to both players is constant during the bargaining. One should keep in mind that we don't argue that this model can describe all bargaining situation since in many such situations time plays a major role in the bargaining process (e.g. international negotiations, labor market negotiations).

An alternative equivalent assumption is of a very discontinuous discounting:to time - a fixed cost of zero in the earlier periods when the players are active and then an infinite cost to all following periods. Note that in our model preferences depend on the player's type. The player's type determines for how many periods the cost of bargaining will be zero for her (for the buyer her type also determines her valuation for the object). In most existing models where the type of the player is her valuation for the object, preferences are homogeneous across different types.

Another important property of our model is that it allows a bargaining to end with no trade in equilibrium even though there is common knowledge that gains from trade exist. In most existing bargaining models (e.g. Admatti and Perry (1987), Cramton (1992), Perry and Reny (1993)), the only inefficiency is due to delay - parties may reach an agreement after a long delay which will serve as a signal of their type but they will never end the bargaining with no agreement if indeed it became common knowledge that gains from trade exist. In our model, on the other hand, bargaining may end in no trade on the equilibrium path. This can happen when the bargainers have a low bargaining talent and can not bear to stay in the bargaining process for many offers due to their personality characteristics. Therefore they forgo the expected gains. We believe that the assumption of common knowledge that gains from trade exist is quite natural in the environment we have. The underlying natural assumption would be that the customer always values the object more than the seller. Still, we observe many bargaining situations that end with an impasse.

What then motivates the players to reach an agreement? The seller, who makes the offers, can not know for sure, in the beginning of the game, when it will end. A seller, facing a potential buyer, does not know if and when the buyer will decide to leave the bargaining without buying the object. Under such incomplete information she then tries to balance between her will to extract as much as possible from trade (i.e. by demanding a high price) and her fear that the customer will leave the store without buying. She can make several offers and lower the price but she might "lose" the buyer if she does not lower the price enough.

The buyer, on the other hand, does not know whether the seller is serious in her current offer i.e. if she has a low bargaining talent and will not make any other offers, or if she might lower it if he insists and turns down the current offer. Therefore he considers the risk of losing the trade (we assume that once he turned down an offer it is no longer available to him) against the chance of getting a lower price.

We now turn to describe the bargaining model with incomplete information. We have a seller who wishes to sell one indivisible object and a buyer who wishes to buy it. The value of the object to the seller is common knowledge and is normalized to be zero. The value of the object to the buyer is her private information. It is common knowledge, however, that this valuation is taken from a uniform distribution on the interval $[0, 1]$ [2].

We also have one other source of incomplete information. Both the seller and the buyer have a characteristic feature described by a certain parameter which is their private information. We call this parameter, which is an integer, the "bargaining talent" of the player. These integers parameters, which we label β^S for the seller and β^B for the buyer, describe their inherent bargaining ability. This parameter determines how many offers will they be able to make or turn down before leaving the bargaining (for the buyer) or stop making offers (for the seller). Equivalently we can say that a player has a fixed cost of zero in the beginning, in all periods before or at the β offer and afterwords an infinite cost per period. A seller or a buyer with a low value for β is an individual with a limited ability to bargain - he or she can not stand to be in a bargaining situation for more than β offers. If $\beta^S = 1$ the seller is only making take it or leave it offers - she will never make a second offer after the first one was rejected. If $\beta^B = 1$ the buyer is not capable of bargaining - if the first offer, made by the seller, is already in his initial range (i.e. below his valuation for the object) he will accept it and if not he will walk away. He has no ability to refuse an offer and still stay in the bargaining and wait for a second offer. If he had a higher β^B he could have refused the first offer even if it is within his range and wait for the seller to lower her price. In other words these parameters describe how good are the players in bargaining. We treat the bargaining talent as an inherent feature, independent of the current object. An individual is characterized by the same

[2] The assumption of a uniform distribution is made to allow for tractability and comparative statics. We discuss generalizations in section 4.

bargaining talent throughout all of his bargaining interactions and it is not strategically chosen by her. Furthermore we do not explain how these parameters came about or ask whether they are beneficial or not (from a life time experience point of view). We hope to address these question in a future work.

Intuitively a buyer with a higher bargaining talent will get a higher expected payoff than a buyer with a low bargaining talent - he can stay longer in the bargaining and therefore get better offers. Similarly, a seller with a higher bargaining talent will get a higher expected payoff since she will be able to make more offers and exploit more of the available gains from trade. This result is in contrast to many bargaining models in which a shorter horizon gives the player a stronger bargaining position. From the seller's point of view, convincing the buyer that she has a low bargaining talent is a good strategy since then the buyer will accept the offer (he won't turn it down if he thinks there is a high probability that there will be no other offers). Therefore for a range of parameters we indeed get a pooling equilibrium in which the more talented seller imitates the offer of the other type of seller in the first offer but then continuous to make offers after the less talented seller stops making them.

This project started with observations we made in real markets. The data is summarized in the following table

	No. of observations	Seller offers buyer agrees	Seller offers buyer walks away	Seller offers buyer turns down seller offers lower price buyer agrees	Other*
Shoes stand in a street market	44	29	9	3	3
Optics store (glasses)	16	4	8	1	3
Electronics store (small appliances)	16	4	9	N/A	3

* Seller offers, buyer turns down, seller offers a lower price, buyer walks away or Alternating offers.

This table suggests that if indeed our model closely resembles reality then most individuals (both sellers and buyers) have a bargaining talent equal to 1. Only a very small fraction of buyers were able to turn down two offers and stay in the bargaining. We also make several other assumptions supported by the data we collected. First, since we almost never observed buyers making offers but rather reacting to the seller's offers, in our model only the seller is making offers and the buyer can either 1) accept an offer, or 2) turn it down and stay, or 3) turn it down and leave. We also assume that once an offer was rejected it is no longer available from the seller. Moreover, our players are risk neutral and wish to maximize their expected payoff. Therefore the parameters β^S and β^B do not enter the player's utility function. It affects the results indirectly through the strategies chosen in equilibrium but not the utility (as opposed to a discount factor to time).

We analyze the case of two bargaining talent types for the seller and two bargaining talent types for the buyer. The generalization for a finite number of types is discussed in section 4. In the pooling equilibrium we find the seller makes a series of weakly decreasing offers. This also matches our intuition and observations. The seller usually lowers her offer if the buyer turned it down. We also assume that the first offer is always 1 (the highest possible valuation for the buyer). We sometimes think of this first offer as the written price of the object. The bargaining starts with that price.

The description of the game is as follows. In period zero nature chooses three parameters independently. Two parameters for the buyer: $v \in [0,1]$ - her valuation and $\beta^B \in \left\{\beta_l^B, \beta_h^B\right\}$ where $1 \leq \beta_l^B < \beta_h^B$ are integers and one for the seller $\beta^S \in \left\{\beta_l^S, \beta_h^S\right\}$ where $1 \leq \beta_l^S < \beta_h^S$ are integers as well. We assume that we have four different values $\left\{\beta_l^B, \beta_h^B, \beta_l^S, \beta_h^S\right\}$ and that these values and the following probabilities distributions are common knowledge among the players. The valuation for the buyer, v is taken from a uniform distribution on the interval $[0,1]$ while $\Pr\left(\beta^B = \beta_h^B\right) = \pi^B$ and $\Pr\left(\beta^S = \beta_h^S\right) = \pi^S$. The strictly positive probabilities $\left(\pi^S, \pi^B\right)$ describe the current population state of bargaining talents. Note that although we assume that the buyer's bargaining talent is independent of his valuation for the object we get that offers decrease in equilibrium. If we were to assume that when bargaining on objects which are worth more (or less) to him the buyer has a higher bargaining ability then we might get different solutions.

The vector $\left(\beta^S, \left\{\beta^B, v\right\}\right)$ fully characterizes the following game. In each period the seller can either make an offer for a price for the object or not. If she makes an offer then the buyer can either accept it and they exchange the object for that price, or turn it down and stay or turn it down and leave. The game either ends when the buyer accepts an offer or when the buyer leaves or when the seller makes no more offers. As long as an offer was made at the previous period and the buyer stayed the game continues. Obviously the game ends no later then after $\Lambda = \min\left\{\beta^B, \beta^S\right\}$ offers and it can end either in trade or not.

We can describe an equilibrium (under several assumptions) of this game as a function of $\left(\pi^B, \pi^S\right)$. The description of the equilibrium is straightforward. For example if $\beta_l^B <$

$\beta_l^S < \beta_h^B < \beta_h^S$ and $0 \leq \pi^S < \frac{1}{2}$ we have the following pooling equilibrium: both seller's types (β_l^S, and β_h^S) will start by offering the price of 1, in the first period, second period, third and so on up to period $\beta_l^B - 1$. All buyers turn down the offer and stay. In period β_l^B both seller's types offer $p = \frac{1}{2}$. Buyers with a low bargaining talent type β_l^B and a type $v \in \left[\frac{1}{2}, 1\right]$ will then accept. A buyer of type $v \in [0, \frac{1}{2})$ and a bargaining talent type β_l^B will turn down the offer and leave the bargaining. All other buyers turn down the offer and stay. Next, sellers continue to offer $p = \frac{1}{2}$ in all periods $\beta_l^B + 1, \beta_l^B + 2, ..., \beta_l^S - 1$ and buyers turn it down and stay. At period β_l^S both seller's types make the offer $x^* = \frac{1}{2} - \frac{1}{4}\pi^S$. Buyers ($\beta_h^B$) with $v \in \left[\frac{1}{2}, 1\right]$ accept the offer and others reject it and stay. Now, a seller of a bargaining talent type β_l^S makes no more offers and a seller of a bargaining talent type β_h^S repeats the offer x^* at periods $\beta_l^S + 1, \beta_l^S + 2, ..., \beta_h^B - 1$ and buyers turn it down and stay. Finally, at the final period β_h^B the seller offers $p = \frac{1}{4}$ and buyers with $v \in [\frac{1}{4}, \frac{1}{2})$ accept it. All other buyers turn down the offer and leave. Therefore, if we had a low bargaining ability type buyer (β_l^B) the bargaining would have end after a short procedure (β_l^B long) and it would either end with trade or not depending on the buyer's valuation. This is similar to the case of a take it or leave it offer. However if we had two very talented bargainers (β_h^S and β_h^B) and a buyer with a low valuation ($v < \frac{1}{2}$) then the bargaining would have lasted longer (β_h^B periods) and the price would have gone down from 1 to $\frac{1}{2}$ to x^* and to $\frac{1}{4}$.

This equilibrium exhibits exactly the features we observed - seller makes consecutive offers which slowly go down. Some buyers stay while others leave. We therefore have a bargaining game in which, in equilibrium, we always have a series of weakly decreasing offers made by the seller. The number of offers is determined by the bargainers' character. Moreover we usually hear people describing their bargaining talent as either good or bad when they explain why they accepted an offer or rejected it. We believe that using such a parameter to describe this property is a good approximation to the true decision making mechanism we have.

Note that if we were to assume complete information regarding the value of the object to the buyer our model predicts that the seller will extract all the gains from trade no matter what bargaining talent type she is. In such a complete information environment our model is equivalent to a model in which the seller makes a take it or leave it offer. In this case bargaining talent plays no role.

Our model predicts equilibria with multiple weakly decreasing offers. The number of offers is dependent both on the type of the seller and on her belief about the buyer's type. As opposed to our result, in which bargaining can last for many periods, in most of the above mentioned papers the bargaining ends, in equilibrium, after at most three offers (in Cramton (1992)). In most of these models (especially those with only sellers offers) the probability that the bargaining ends immediately goes to one when the time between offers goes to zero.

The paper is organized as follows: in section 2 we describe the model, in section 3 we describe the equilibrium refinement we use and characterize the equilibria of the game for two of the possible orderings of the four values $\left\{\beta_l^B, \beta_h^B, \beta_l^S, \beta_h^S\right\}$ (the other possible orderings give the same equilibria with minor changes) in section 4 we discuss certain generalizations and future research and in section 5 we conclude. The proofs are in the appendix.

2 The Model

We have a seller who wishes to sell an indivisible object she owns. The value of the object to her is common knowledge among the players and is normalized to zero. We have a buyer who wishes to buy the object. The value of the object to him is his own private information. It is common knowledge however that this value, v is taken from a uniform distribution on the interval $[0, 1]$. We call the value of the object to the buyer - his type.

Moreover each player is characterized by a behavioral parameter. This parameter is a line of character for the player and is constant during the bargaining. We assume a given finite set of bargaining talent types for the seller and the buyer. We analyze here the case of two types of bargaining talent for the seller and two types for the buyer. Therefore $\beta^B \in \left\{\beta_l^B, \beta_h^B\right\}$ where $1 \leq \beta_l^B < \beta_h^B$ are integers and $\beta^S \in \left\{\beta_l^S, \beta_h^S\right\}$ where $1 \leq \beta_l^S < \beta_h^S$ are integers as well. The player knows his/her bargaining talent but not his/her opponent's talent. It is common knowledge that $\Pr\left(\beta^B = \beta_h^B\right) = \pi^B$ and $\Pr\left(\beta^S = \beta_h^S\right) = \pi^S$. These probabilities describe the current population state of bargaining talents.

Time is discrete. Assume that the players have bargaining talent types $\left\{\beta^B, \beta^S\right\}$ for the buyer and seller respectively then the bargaining ends no later than after $\Lambda\left(\beta^B, \beta^S\right) = \min\left\{\beta^B, \beta^S\right\}$ periods. Note that $\Lambda\left(\beta^B, \beta^S\right)$ is a random variable. In some of the cases we will discuss below this random variable is known to one of the players. For example, in the first case where $\beta_l^S < \beta_h^S < \beta_l^B < \beta_h^B$ we have $\Lambda\left(\beta^B, \beta^S\right) = \beta^S$ and therefore the seller, who knows her type, knows $\Lambda\left(\beta^B, \beta^S\right)$. From now in it will be more convenient to assume that each period is divided into two and therefore the number of periods is doubled. In each odd period t the seller can make an offer for a price in which the object will be sold. In each even period, the buyer can either accept the offer made in the previous period and they exchange the object at the agreed price or he can reject the offer and leave the bargaining (get out of the store) or turn down the offer and stay. In the latter situation it is understood that he is willing to listen to the next offer of the seller, whether there will be one. The periods' length is not relevant - how much time elapsed between the offer and the reply or between consecutive offers has no effect on the players utility. Moreover the length of the entire bargaining also does not effect the utility. We further assume that players are risk neutral. Therefore if the game between a seller and a buyer of type v ends with trade at a price p the seller's and buyer's payoffs are given by $U_S(p) = p$ and $U_B(v,p) = v - p$ respectively. If the game ends with no trade then the utilities are $U_S(p) = U_B(v,p) = 0$.

Since the acceptance of an offer or the rejection followed by leaving the game terminate the game, a relevant history is a series of turned down offers after which the buyer remained in the game. The seller moves in odd periods. When it is her time to move she can either make an offer or not. A non-terminal history h^N, of a not yet ended game of length N for $N = 2n$ is therefore a vector $h^N = (p_1, No\&Stay, p_2, No\&Stay, ..., p_n, No\&Stay)$ and for $N = 2n+1$ it is $h^N = (p_1, No\&Stay, p_2, No\&Stay, ..., p_n, No\&Stay, p_{n+1})$. We denote $h^0 = \dot{\varnothing}$. We denote the set of non-terminal histories by H, the set of non terminal histories of an even length by H_1 and the set of non terminal histories of an odd length by H_2. Then $H_1 \cup H_2 = H$.

A strategy for a seller of a bargaining talent type β^S specifies for every history h^{2n} such that $n < \beta^S$ and the game has not yet ended the offer she will make in the current period $2n+1$. After history of length $2n$ where $n = \beta^S$ no offer is made. We assume that the seller never stops making offers if the game has not yet ended and she has not made β^S offers yet. Therefore we do not consider strategies in which following a history h^{2n} for which $n < \beta^S$ and the game has not yet ended, the seller decides not to make an offer. These strategies are (weakly) dominated by strategies in which she remains in the game and makes an offer (e.g. a strategy in which she repeats the last offer she made weakly dominates the strategy of making no offers while her character still allows her to make them and the buyer is still in the game). We denote a strategy of a seller of a bargaining talent type β_i^S by $\sigma_i : H_1 \to \mathbb{R}$, $\sigma_i(h^{2n}) = p_{n+1}$ for $n = 0, 1, 2, .., \beta_i^S - 1$ and $i = l, h$.

A strategy for a buyer of a bargaining talent type β^B and type v specifies for every history h^{2n+1} whether he accepts the offer or rejects and stays or rejects and leave. We assume that the buyer never leaves the bargaining the game has not yet ended and he has not stayed for β^B offers yet. Again this strategy is weakly dominated by in strategy in which he stays instead of leaving and turns down all offers but stays until he can no longer stay (until the β^B offer was made) We denote a strategy of a buyer of a type v and of a bargaining talent type β_i^B by $\rho_{v,i} : H_2 \to \{Yes, No\&Stay\}$ for all histories $h^{2n+1} \in H_2$, such that $n = 0, 1, ..., \beta_i^B - 1$ and $\rho_{v,i} : H_2 \to \{Yes, No\&Leave\}$ for all histories $h^{2\beta_i^B+1} \in H_2$ of length $2\beta_i^B + 1$, where $i = l, h$ and $v \in [0,1]$. A terminal history for the game is a history that either ends when the buyer chooses Yes or when $N = 2 * \min\{\beta^B, \beta^S\}$. We consider only pure strategies. An outcome of the game is either an agreed price $p\left(\sigma_i, \rho_{v,j}\right)$ or an ending of the game with no trade.

3 Equilibria

We now turn to analyze the equilibria of the described game for the different cases. We denote the game in extensive form - $\Gamma\left(\left\{\beta_l^B, \beta_h^B, \beta_l^S, \beta_h^S\right\}\right)$. This game is fully characterized by the ordering of the four values $\left\{\beta_l^B, \beta_h^B, \beta_l^S, \beta_h^S\right\}$. In a Nash equilibrium of the bargaining game the seller and buyer chooses the strategy that is a best response to their opponent's strategy. However, a more suitable solution concept for our game is the sequential equilibrium concept. In a sequential equilibrium we need to specify the strategies $\left(\sigma_l, \sigma_h, \left\{\rho_{v,l}\right\}_{v\in[0,1]}, \left\{\rho_{v,h}\right\}_{v\in[0,1]}\right)$ and the beliefs of the players. A belief of the buyer $\mu^B : H \to [0,1]$ is the probability he assigns to the event $\beta^S = \beta_h^S$ after every history. Initially the belief of the buyer is given by $\mu^B\left(h^0\right) = \pi^S$. Moreover we assume that $\mu^B\left(h^{2n+1}, \cdot\right) = \mu^B\left(h^{2n+1}\right)$ for $n = 0, 1, ..., \beta^B - 1$. This last equation says that the buyer's belief can only change after an offer was made by the seller (his belief is independent of his actions). Moreover this belief is independent of the buyer's type (condition only on her still being in the bargaining) - the updating of belief is the same for all buyer's types and is only done based on the seller's offers.

A belief for the seller is given by both the probability she assigns to the event $\beta^B = \beta_h^B$ and two probability distribution functions F_l and F_h on $[0,1]$. Therefore we denote $\mu^S : H \to \mathbb{R} \times \Xi \times \Xi$ where Ξ is the set of all cumulative distribution functions on the interval $[0,1]$. The first distribution function F_l describes her belief on the distribution of types v for buyers with a bargaining talent type β_l^B and the second distribution function F_h describes her belief on the distribution of types v for buyers with a bargaining talent type β_h^B. Initially the belief of the seller is given by $\mu^S\left(h^0\right) = \left(\pi^B, F_l\left(x/h^0\right), F_h\left(x/h^0\right)\right) = \left(\pi^B, x, x\right)$ i.e. the uniform distribution. We again assume that $\mu^S\left(h^{2n}, \cdot\right) = \mu^S\left(h^{2n}\right)$ for $n = 0, 1, ...\beta^S - 1$. This last equation says that the seller's belief can only change after the buyer's response. Again this belief is independent of the seller's type.

In a sequential equilibrium the players' strategies are best responses not only at the beginning (as in the Nash equilibrium), but at any decision node (i.e. after any relevant history). For the seller, the test whether a strategy is the best response depends on her belief that the buyer is of type v and of a bargaining talent type β_h^B. For the buyer the test depends on his belief that the seller is of a bargaining talent type β_h^S. Therefore, a sequential equilibrium includes the method of updating the players beliefs. This updating should be consistent with the hypothesized equilibrium strategies. It satisfies Bayes' rule whenever is applies.

Definition 1 *A sequential equilibrium (Kreps and Wilson (1982)) for the game* $\Gamma\left(\left\{\beta_l^B,\beta_h^B,\beta_l^S,\beta_h^S\right\}\right)$ *is a pair: a profile* $\left(\sigma_l,\sigma_h,\left\{\rho_{v,l}\right\}_{v\in[0,1]},\left\{\rho_{v,h}\right\}_{v\in[0,1]}\right)$ *of strategies and* $\mu=\left(\mu^S,\mu^B\right)$ *- a belief system, such that after every history* h^N *in which it is the player's turn to move, each player's strategy is optimal given the other's strategy and his/her current beliefs about the other's valuation type and bargaining talent type and the beliefs are consistent with Bayes' Rule whenever possible.*

We can conclude several conclusions on the updating of beliefs. Since we only allow for pure strategies and we only have two possible bargaining talent types for the seller, an updating of the buyer's belief on the equilibrium path can only be done in one of two ways. The belief $\mu^B\left(h^{2n},\cdot\right)$ can either stay the same as $\mu^B\left(h^{2n}\right)$ (in case of a pooling equilibrium and a node in which both types β_l^S and β_h^S are supposed to choose the same action) or become certainty, i.e. zero or one in any other case.

Moreover, after any history h^{2n+1} there is a unique number $v_i\left(h^{2n+1}\right)\in[0,1]$ such that a buyer of a bargaining talent type i, $i\in\left\{\beta_l^B,\beta_h^B\right\}$ will accept the offer if his type v is above $v_i\left(h^{2n+1}\right)$ and will turn it down if his type v is less than $v_i\left(h^{2n+1}\right)$. This is a standard result and the reason is that buyer types who have a high valuation for the object have more to gain from trade and will therefore sometimes prefer to accept an offer rather then to take the risk of loosing the trade (in cases where they think the probability of a seller with a high bargaining talent who can offer them even lower offers is small). Since the distribution of types is continuous we get the claim. In general, the buyer's response can be a function of the entire history. However we narrow the discussion to strategies in which his response id dependent only on this marginal type $v_i\left(h^{2n+1}\right)$. We formalize this in the following stationarity condition we impose on the the buyer's strategies (as in Gul and Sonnenchein (1988)). This condition states that the response of the buyer will only depend on the marginal type $v_i\left(h^{2n+1}\right)$. Two histories h^{2n+1} and h^{2m+1} after which $v_i\left(h^{2n+1}\right)=v_i\left(h^{2m+1}\right)$ will induce the same response (between accept the offer or turn it down and stay) for all buyer types with the same bargaining talent type.

Condition 1 ***(Stationarity of Buyer's Strategy):*** *For each* $v\in[0,1]$, *and* $i\in\left\{\beta_l^B,\beta_h^B\right\}$, *if* $v_i\left(h^{2n+1}\right)=v_i\left(h^{2m+1}\right)$ *then* $\rho_{v,i}\left(h^{2n+1}\right)=\rho_{v,i}\left(h^{2m+1}\right)$

It follows that, after any relevant history, we can describe the buyer's strategy profile $\left\{\rho_{v,i}\right\}_{v\in[0,1]}$ using only $v_i\left(h^{2n+1}\right)$. Note that $v_i\left(h^{2n},p\right)$ is weakly decreasing with p as the offer increases more types will accept it.

After a turned down offer the seller will thus update her belief on the buyer's type always to a uniform distribution on some truncated interval of the form $[0,a]$. We can write $F_i\left(\cdot/h^{2n}\right) = a_i\left(h^{2n}\right)$ for $i \in \{l,h\}$ and understand that after the non-terminal history h^{2n} the seller believes that the buyer's type of a bargaining talent type β_i^B is distributed uniformly on the interval $[0,a_i\left(h^{2n}\right)]$. Therefore we denote $\mu^S\left(h^{2n}\right) = \left(\pi^B\left(h^{2n}\right), a_l\left(h^{2n}\right), a_h\left(h^{2n}\right)\right)$

We now impose some other restrictions on the strategies of the buyers and sellers in the game. We use the same monotonicity condition on the marginal type as in Gul and Sonnenchein (1988). This condition implies in particular that the possibility of additional high valuation buyers (of the same bargaining talent type) will not lead a low valuation buyer to lower his acceptance price. Moreover if the seller believes that the interval on which buyer's types are distributed is smaller then by making the same offer he ensures that all buyers who accept the offer when her belief is on a larger interval will still accept it and other buyer's types may also accept it. We will use this when proving Lemma 1.

Condition 2 ***(Monotonicity of the marginal type)****:* *For all* $p \in \mathbb{R}_+$, *and non terminal histories* h^{2n}, h^{2m} , *if* $a_i\left(h^{2n}\right) \leq a_i\left(h^{2m}\right)$ *then there exists* $q \geq p$ *such that* $v_i\left(h^{2n},p\right) = v_i\left(h^{2m},q\right)$

These conditions are sufficient for proving the following lemma:

Lemma 1 *The offers of the seller weakly decrease on any sequential equilibrium path that satisfies conditions (1) and (2).*

Proof. *In the appendix* ■

Finally we wish to impose conditions on off-the-equilibrium-path beliefs of the buyer. As it is usually the case our game has many sequential equilibria. Kreps and Wilson (1982) already stated that "the formation [of sequential equilibria] in terms of players' beliefs gives the analyst a tool for choosing among sequential equilibria" [p. 8841]. The obvious reason for that is that the equilibrium allows for freedom degrees in choosing the beliefs of the players off the equilibrium path. If a player observes a deviation - how will he/she update beliefs? This is left to the analyst to determine. Many refinements of the sequential equilibrium concept have been suggested e.g. the intuitive criterion (Cho and Kreps (1987)) or it's generalizations (Cho (1987), Banks and Sobel (1987) and McLennan (1985*)).*

We wish to keep our assumptions on off-the-equilibrium-path beliefs as simple as possible. We therefore restrict attention to off-the-equilibrium-path-beliefs that admit the following conditions. The first condition is in the spirit of assumption (B-2) in Rubinstein (1985). If the buyer observes an off-the-equilibrium-path offer in a pooling equilibrium (when he expects the same offer from both types of the seller), which is lower than the expected equilibrium offer then his belief stays the same as it was in the previous period (no updating in this case - by making lower offers the seller can not convince that she is of a low bargaining talent type). Moreover, if the buyer observes an off-the-equilibrium-path offer in a pooling equilibrium, which is higher than the expected equilibrium offer then the buyer updates his belief to 1 i.e. he is convinced that he is faced with a seller of a high bargaining talent. Formally,

Condition 3 ***(Updating after a deviation in a pooling equilibrium)****: Fix a sequential equilibrium* $\left(\left(\sigma_l, \sigma_h, \left\{\rho_{v,l}\right\}_{v\in[0,1]}, \left\{\rho_{v,h}\right\}_{v\in[0,1]}\right), \mu\right)$*. Then if* $\sigma_l\left(h^{2n}\right) = \sigma_h\left(h^{2n}\right)$ *and* $q < \sigma_l\left(h^{2n}\right)$ *then* $\mu^B\left(h^{2n}, q\right) = \mu^B\left(h^{2n}, \sigma_l\left(h^{2n}\right)\right)$*. If* $q > \sigma_l\left(h^{2n}\right)$ *then* $\mu^B\left(h^{2n}, q\right) = 1$

Finally we assume that if the buyer observes an off-the-equilibrium-path offer in a separating equilibrium, which is higher then the expected equilibrium offer from the low bargaining talent type seller then the buyer updates his belief to 1. If, on the other hand, the buyer observes an off-the-equilibrium-path offer in a separating equilibrium, which is lower then the expected equilibrium offer from the low bargaining talent type seller then the buyer updates his belief to 0.

Condition 4 ***(Updating after a deviation in a separating equilibrium)****: Fix a sequential equilibrium* $\left(\left(\sigma_l, \sigma_h, \left\{\rho_{v,l}\right\}_{v\in[0,1]}, \left\{\rho_{v,h}\right\}_{v\in[0,1]}\right), \mu\right)$*. Then if* $\sigma_l\left(h^{2n}\right) < \sigma_h\left(h^{2n}\right)$ *and* $q > \sigma_l\left(h^{2n}\right)$ *then* $\mu^B\left(h^{2n}, q\right) = 1$*. If* $q < \sigma_l\left(h^{2n}\right)$ *then* $\mu^B\left(h^{2n}, q\right) = 0$*.*

These restrictions on the beliefs allow us to identify both a pooling and a separating sequential equilibrium [3]

Before analyzing the different cases (for different ordering of $\left\{\beta_l^B, \beta_h^B, \beta_l^S, \beta_h^S\right\}$) in details we would like to point out features which are common to all the equilibria we find and define some of the notions we will use.

[3] Choosing these beliefs might seem arbitrary and the reader might ask him/her self whether we can support different types of equilibria with different beliefs. We hypothesize that the answer is positive - changing these beliefs can lead to different equilibria and can eliminate some of our equilibria. However, some restrictions must be put on off-the-equilibrium-path-beliefs in order to identify and describe the equilibria.

We first define the notion of a critical period. This is an odd period in which an offer is made and there exists a positive probability that this offer will be the last in the game - either since in the following period the buyer will have left the bargaining or since the seller will make no further offers. Each possible ordering of the different values $\{\beta_l^B, \beta_h^B, \beta_l^S, \beta_h^S\}$ gives rise to a different number of critical periods. This number is determined by when either all buyers leave the bargaining or by when all sellers stop making offers. For example for $\beta_l^B < \beta_h^B < \beta_l^S < \beta_h^S$ we have two critical periods - $n = 2\beta_l^B - 1$ and $m = 2\beta_h^B - 1$ since the game ends no later than at period $2\beta_h^B$ and only β_l^B is smaller than β_h^B. On the other hand, for $\beta_l^S < \beta_l^B < \beta_h^S < \beta_h^B$ we have three critical periods: $n = 2\beta_l^S - 1, m = 2\beta_l^B - 1$ and $k = 2\beta_h^S - 1$ since the game ends no later than at period $2\beta_h^S$ and two other values are smaller than β_h^S.

Formally, denote: $\kappa\left(\{\beta_l^B, \beta_h^B, \beta_l^S, \beta_h^S\}\right) = \min\{\beta_h^B, \beta_h^S\}$ then we have the following definition:

Definition 2 *A period $1 \leq n \leq 2\kappa\left(\{\beta_l^B, \beta_h^B, \beta_l^S, \beta_h^S\}\right)$ is a critical period if $n = 2t - 1$ for some $t \in \{\beta_l^B, \beta_h^B, \beta_l^S, \beta_h^S\}$*

This definition allows us to restrict attention to seller's strategy in which offers are changed only in critical periods and are repeated upon between critical offers. We define the following condition (recall that in an odd period $2n + 1$ the offer p_{n+1} is made and in even periods it is the buyer's turn to move):

Condition 5 *(**Repeated Offers between Critical Periods**): If $\left(\left(\sigma_l, \sigma_h, \{\rho_{v,l}\}_{v\in[0,1]}, \{\rho_{v,h}\}_{v\in[0,1]}\right), \mu\right)$ is a sequential equilibrium of $\Gamma\left(\{\beta_l^B, \beta_h^B, \beta_l^S, \beta_h^S\}\right)$, let n be a critical period (then n is odd) and $n+m$ the following critical period (if there exists one) then if $\sigma_i\left(h^{(n-1)}\right) = p$ and this offer is turned down by the buyer then $\sigma_i\left(h^{(n-1+2j)}\right) = p$ for every $1 \leq j \leq \frac{1}{2}m - 1$ and $i = h, l$ such that a seller of a bargaining talent type β_i^S is still in the game in period $n+m$. Moreover, if n is the first critical period then $\sigma_i\left(h^{2j}\right) = 1$ for every $0 \leq j \leq \frac{1}{2}(n-3)$.*

For example assume that $\beta_l^B = 2$ and $\beta_h^B = 5$ and $\beta_l^B < \beta_h^B < \beta_l^S < \beta_h^S$. Then the first critical period is period 3 (in which the second offer is made) and the second critical period is period 9 (in which the fifth offer is made) then this condition says that the offers the seller will make at period 1 will be $p = 1$, this is a "dummy offer" - the buyer will never accept it. We sometimes think of these first offers as just the written price - obviously it is the same for all buyers and the bargaining starts with that price. In periods $3, 5$ and 7 the offer will be identical and in period 9 she can make a different offer.

The reason for restriction attention to these strategies is that it is straightforward to show that any other strategy is weakly dominated by a strategy with this property. Since going through the game tree the seller's offers can only go down we know that on the sequential equilibrium path buyers will never accept an offer in a non critical period (they better wait for the next critical period - they know that the seller will surely continue to make offers until then). We cannot construct an equilibrium in which buyers accept an offer not in a critical period - if that's the case the seller will make an even lower offer in the next period but then those who accepted the offer will prefer to wait and so on. Only when there is a positive probability that the game will end buyers can accept an offer. Therefore the seller can replace any strategy in which offers go down between critical periods (or before the first critical period) with a strategy in which she repeats the offer she made in the last critical period until the next critical period arrives and then she can make a different offer. Therefore a change in the seller's offers can only come in a critical period. In periods that come between two consecutive critical periods the seller repeats the same offer. An immediate result of this condition is that we can describe an equilibrium strategy for the seller only by her offers at the critical periods (in which her type allows her to make an offer).

We therefore define a *simple sequential equilibrium*:

Definition 3 *A* ***simple*** *sequential equilibrium is a sequential equilibrium that satisfies conditions (1)-(5)*

Therefore we examine two of the cases and show that in a simple equilibrium we can get a series of descending offers. In the cases we do not examine in this paper the equilibria are either not interesting (sellers keep offering $\frac{1}{2}$ in all critical periods) or a trivial generalization of the cases we examine.

3.1 Sellers are less patient than buyers: $\beta_l^S < \beta_h^S < \beta_l^B < \beta_h^B$

In this case the buyer's bargaining talent type does not enter into equilibrium's considerations. All sellers would stop making offers before any buyer would leave the bargaining. We have only two critical periods: $2\beta_l^S - 1$ and $2\beta_h^S - 1$. The offers that are made by the seller in these periods are labeled $p^1 \equiv p_{\beta_l^S}$ and $p^2 \equiv p_{\beta_h^S}$.

We describe two equilibria for different ranges of the parameter π^S. The first is a pooling equilibrium which can only exist when the probability for a patient seller with a high value bargaining talent type - π^S is not too large. In this equilibrium both sellers types will make the same offer at the first critical period. The more patient seller imitates the less patient type.

Theorem 1 *For $0 < \pi^S < \frac{1}{2}$ the following is the unique simple sequential pooling equilibrium of $\Gamma\left(\beta_l^B < \beta_h^B < \beta_l^S < \beta_h^S\right)$:*

1. *Strategy for the seller of type $\beta_i^S, i = l, h$: if $1 \leq n < 2\beta_l^S - 1$, n odd, then $p_{\frac{1}{2}(n+1)} = 1$ and for $n = 2\beta_l^S - 1$, $p^1 = x^*$. For a seller of type β_h^S: if $2\beta_l^S - 1 < n < 2\beta_h^S - 1$, n odd, then $p_{\frac{1}{2}(n+1)} = p^1$ and for $n = 2\beta_h^S - 1$, if $\mu^S\left(h^{n-1}\right) = \left(\pi^B, a_l, a_h\right)$ then $p^2 = \frac{1}{2}\left(\pi^B a_h + \left(1 - \pi^B\right) a_l\right)$.*

2. *Belief for the seller: if $1 \leq n \leq 2\beta_l^S - 1$, n odd, then $\mu^S\left(h^{(n-1)}\right) = \left(\pi^B, 1, 1\right)$ and for $2\beta_l^S - 1 < n \leq 2\beta_h^S - 1$, n odd, if the game has not yet ended $\mu^S\left(h^{(n-1)}\right) = \left(\pi^B, \omega^*\left(p^1\right), \omega^*\left(p^1\right)\right)$.*

3. *Strategy for the buyer: For $i = l, h$ if $1 \leq n < 2\beta_l^S - 1$, n odd, then $v_i\left(h^n\right) = 1$. If $n = 2\beta_l^S - 1$ then if $p^1 \leq x^*$ then $v_i\left(h^n\right) = \omega^*\left(p^1\right)$ and otherwise $v_i\left(h^n\right) = 1$. If $2\beta_l^S - 1 < n < 2\beta_h^S - 1$, n odd, then $v_i\left(h^n\right) = 1$ and finally if $n = 2\beta_h^S - 1$ then $v_i\left(h^n\right) = p^2$.*

4. *Belief for the buyer: if $1 \leq n \leq 2\beta_l^S - 1$, n odd, then $\mu^B\left(h^n\right) = \pi^S$. If $n = 2\beta_l^S - 1$ then if $p^1 \leq x^*$ then $\mu^B\left(h^n\right) = \pi^S$ otherwise $\mu^B\left(h^n\right) = 1$. For $2\beta_l^S - 1 < n \leq 2\beta_h^S - 1$, n odd, $\mu^B\left(h^n\right) = 1$*

where $x^* = \frac{1}{2} - \frac{1}{4}\pi^S$ and $\omega^*(x) = \frac{2x}{\left(2 - \pi^S\right)}$
Proof. *In the Appendix* ■

On the equilibrium path both seller's types will offer x^* in the first critical period. All buyers with a valuation higher or equal to $\frac{1}{2}$ will accept it (note that if $p_{\beta_l^S} = x^*$ then $\omega^*(x^*) = \frac{1}{2}$ and is independent of π^S) and all others will reject it. Note that if the probability for a patient seller is very small (i.e. π^S goes to zero) the seller can exploit it and offer a higher price in the first critical period (closer to $\frac{1}{2}$ which is the optimal offer a seller can make in a take it or leave it game with buyers uniformly distributed on $[0, 1]$) while if this probability is high (goes to $\frac{1}{2}$) she will have to lower her price in order to convince the high valuation buyers to accept is.

In the second critical period, the seller of type β_h^S will make a lower offer of $\frac{1}{2}\omega^*(x^*) = \frac{1}{4}$ and all buyers of type $v \in [\frac{1}{4}, \frac{1}{2})$ will accept it. Therefore if the seller has a high bargaining talent and the buyer has a relatively low valuation we get a series of offers 1 and then x^* and then $\frac{1}{4}$ which can either end in trade or not.

Finally the expected payoff to a seller of a low bargaining talent type β_l^S in this equilibrium is $x^*(1 - \omega^*(x^*)) = \frac{1}{4} - \frac{1}{8}\pi^S$ while the expected payoff to a seller of a high bargaining talent type β_h^S in this equilibrium is higher: $x^*(1 - \omega^*(x^*)) + \frac{1}{16} = \frac{5}{16} - \frac{1}{8}\pi^S$.

The second equilibrium we describe is a separating equilibrium. The separation occurs in the critical periods. Here the more patient seller can not gain from convincing the buyer that he is less patient by imitating and therefore, even in the first critical period the offers are different from one another.

Theorem 2 *For* $0 < \pi^S < 1$ *the following is the unique simple sequential separating equilibrium of* $\Gamma\left(\beta_l^B < \beta_h^B < \beta_l^S < \beta_h^S\right)$*:*

1. *Strategy for the seller of type* β_l^S*: if* $1 \leq n < 2\beta_l^S - 1$*, n odd, then* $p_{\frac{1}{2}(n+1)} = 1$*, and for* $n = 2\beta_l^S - 1, p^1 = \frac{1}{3}$*. For a seller of type* β_h^S*: if* $1 \leq n < 2\beta_h^B - 1$*, n odd, then* $p_{\frac{1}{2}(n+1)} = 1$*, and for* $n = 2\beta_h^S - 1$*, if* $\mu^S\left(h^{n-1}\right) = \left(\pi^B, a_l, a_h\right)$ *then* $p^2 = \frac{1}{2}\left(\pi^B a_h + \left(1 - \pi^B\right) a_l\right)$*.*

2. *Belief for the seller: if* $1 \leq n \leq 2\beta_l^S - 1$*, n odd, then* $\mu^S\left(h^{(n-1)}\right) = \left(\pi^B, 1, 1\right)$*. If* $2\beta_l^S - 1 < n \leq 2\beta_h^S - 1$*, n odd, then* $\mu^S\left(h^{(n-1)}\right) = \left(\pi^B, p^1, p^1\right)$

3. *Strategy for the buyer: For* $i = l, h$ *if* $1 \leq n < 2\beta_l^S - 1$*, n odd, then* $v_i\left(h^n\right) = 1$*. If* $n = 2\beta_l^S - 1$ *then if* $p^1 \leq \frac{1}{3}$ *then* $v_i\left(h^n\right) = p^1$ *and otherwise* $v_i\left(h^n\right) = 1$*. If* $2\beta_l^S - 1 < n < 2\beta_h^S - 1$*, n odd, then* $v_i\left(h^n\right) = 1$ *and finally if* $n = 2\beta_h^S - 1$ *then* $v_i\left(h^n\right) = p^2$*.*

4. *Belief for the buyer: if* $1 \leq n < 2\beta_l^S - 1$*, n odd, then* $\mu^B\left(h^n\right) = \pi^S$*. If* $n = 2\beta_l^S - 1$ *then if* $p^1 \leq \frac{1}{3}$ *then* $\mu^B\left(h^n\right) = 0$ *otherwise* $\mu^B\left(h^n\right) = 1$*. For* $2\beta_l^S - 1 < n \leq 2\beta_h^S - 1$*, n odd,* $\mu^B\left(h^n\right) = 1$

Proof. In the appendix. ■

On the equilibrium path a seller with a low bargaining talent makes a low offer - $x = \frac{1}{3}$ while the seller with a high bargaining talent makes a high offer of $\frac{1}{2}$ in the relevant critical period. Note that the impatient seller of type β_l^S would have liked to make in the first critical period an offer of $\frac{1}{2}$ but since she knows the buyers will then believe she is not impatient she is forced to make a lower offer. Again we get that the expected payoff for the seller with the low bargaining talent $\left(\frac{2}{9}\right)$ is lower than the expected payoff for the seller with the high bargaining talent $\left(\frac{1}{4}\right)$.

In the following case we get similar results but with possibly more offers.

3.2 The most impatient buyer is the first to quit: $\beta_l^B < \beta_l^S < \beta_h^B < \beta_h^S$ or $\beta_l^B < \beta_l^S < \beta_h^S < \beta_h^B$

Here we have three critical periods. Again, we label the offers that are made by the seller in the critical periods $2\beta_l^B - 1$ and $2\beta_l^S - 1$ and $2\beta_h^B - 1$ or $2\beta_h^S - 1$ as $p^1 \equiv p_{\beta_l^B}$ and $p^2 \equiv p_{\beta_l^S}$ and $p^3 \equiv p_{\beta_h^B}$ or $p^3 \equiv p_{\beta_h^S}$ respectfully.

We again describe two equilibria for different ranges of the parameter π^S. The first is a pooling equilibrium. This equilibrium starts with an offer of 1 made by both seller's types. In the first critical period both seller's types make the same offer - $\frac{1}{2}$. Then the equilibrium continues exactly as in the previous subsection. Here again if the probability for a high bargaining talent type seller is not too high and if the seller is of a high bargaining talent type and the buyer has both a relatively low valuation and a high bargaining talent then we get a series of four decreasing offers: 1,$\frac{1}{2}$ and then $x^* = \frac{1}{2} - \frac{1}{4}\pi^S$ and then $\frac{1}{4}$. This is exactly the feature we expect to see in a real world bargaining situation between two such "strong" bargainers.

We describe the equilibrium for the case $\beta_l^B < \beta_l^S < \beta_h^B < \beta_h^S$ for the second case the equilibrium is the same except that we need to replace β_h^B with β_h^S everywhere (and some minor technical other replacements).

Theorem 3 *For $0 < \pi^S < \frac{1}{2}$ the following is the unique simple sequential pooling equilibrium of $\Gamma\left(\beta_l^B < \beta_l^S < \beta_h^B < \beta_h^S\right)$:*

1. *Strategy for the seller of type $\beta_i^S, i = l, h$: if $1 \leq n < 2\beta_l^B - 1$, n odd, then $p_{\frac{1}{2}(n+1)} = 1$, and for $n = 2\beta_l^B - 1$, $p^1 = \frac{1}{2}$. if $2\beta_l^B - 1 < n < 2\beta_l^S - 1$, n odd, then $p_{\frac{1}{2}(n+1)} = \frac{1}{2}$ and for $n = 2\beta_l^S - 1$, $p^2 = x^*$. For a seller of type β_h^S: if $2\beta_l^S - 1 < n < 2\beta_h^B - 1$, n odd, then $p_{\frac{1}{2}(n+1)} = x^*$ and for $n = 2\beta_h^B - 1$ if $\mu^S\left(h^{n-1}\right) = \left(\pi^B, a_l, a_h\right)$ then $p^3 = \frac{1}{2}\left(\pi^B a_h + \left(1 - \pi^B\right) a_l\right)$.*

2. *Belief for the seller: if $1 \leq n \leq 2\beta_l^B - 1$, n odd, then $\mu^S\left(h^{(n-1)}\right) = \left(\pi^B, 1, 1\right)$ .if $2\beta_l^B - 1 < n \leq 2\beta_l^S - 1$, n odd, then $\mu^S\left(h^{(n-1)}\right) = (1, 0, 1)$ and for $2\beta_l^S - 1 < n \leq 2\beta_h^B - 1$ - if $p^2 \leq x^*$ then $\mu^S\left(h^{(n-1)}\right) = (1, 0, \omega^*(p^2))$ otherwise $\mu^S\left(h^{(n-1)}\right) = (1, 0, 1)$.*

3. *Strategy for the buyer: For $i = l, h$ if $1 \leq n < 2\beta_l^B - 1$, n odd, then $v_i(h^n) = 1$. If $n = 2\beta_l^B - 1$ then $v_l(h^n) = p^1$ and $v_h(h^n) = 1$. If $2\beta_l^B - 1 < n < 2\beta_l^S - 1$, n odd, then $v_h(h^n) = 1$. If $n = 2\beta_l^S - 1$ then if $p^2 \leq x^*$ then $v_h(h^n) = \omega^*(p^2)$ and otherwise $v_h(h^n) = 1$. If $2\beta_l^S - 1 < n < 2\beta_h^B - 1$, n odd, then $v_h(h^n) = 1$ and finally if $n = 2\beta_h^B - 1$ then $v_h(h^n) = p^3$.*

4. *Belief for the buyer: if* $1 \leq n < 2\beta_l^B - 1$, *n odd, then* $\mu^B(h^n) = \pi^S$. *if* $n = 2\beta_l^B - 1$ *then if* $p^1 \leq \frac{1}{2}$ *then* $\mu^B(h^n) = \pi^S$ *otherwise* $\mu^B(h^n) = 1$ *if* $2\beta_l^B - 1 < n < 2\beta_l^S - 1$, *n odd, then* $\mu^B(h^n) = \mu^B(h^{n-2})$. *If* $n = 2\beta_l^S - 1$ *then if* $p^2 \leq x^*$ *then* $\mu^B(h^n) = \pi^S$ *otherwise* $\mu^B(h^n) = 1$. *For* $2\beta_l^S - 1 < n \leq 2\beta_h^B - 1$, *n odd,* $\mu^B(h^n) = 1$

where $x^* = \frac{1}{2} - \frac{1}{4}\pi^S$ *and* $\omega^*(x) = \frac{2x}{(2-\pi^S)}$

Proof. In the appendix ■

We also have one other equilibrium for this case which is a semi-separating equilibrium. In the first critical period both seller's type will make the same offer but in the second critical period they make different offers.

Theorem 4 *For* $0 < \pi^S < 1$ *the following is a simple sequential semi-separating equilibrium of* $\Gamma\left(\beta_l^B < \beta_l^S < \beta_h^B < \beta_h^S\right)$:

1. *Strategy for the seller of type* $\beta_i^S, i = l, h$: *if* $1 \leq n < 2\beta_l^B - 1$, *n odd, then* $p_{\frac{1}{2}(n+1)} = 1$. *If* $n = 2\beta_l^B - 1$, *then* $p^1 = \frac{1}{2}$. *If* $2\beta_l^B - 1 < n < 2\beta_l^S - 1$, *n odd, then* $p_{\frac{1}{2}(n+1)} = \frac{1}{2}$. *For type* β_l^S : *if* $n = 2\beta_l^S - 1$ *then* $p^2 = \frac{1}{3}$. *For type* β_h^B *if* $2\beta_l^S - 1 \leq n < 2\beta_h^B - 1$, *n odd, then* $p_{\frac{1}{2}(n+1)} = \frac{1}{2}$. *Finally for* $n = 2\beta_h^B - 1$ *if* $\mu^S(h^{n-1}) = \left(\pi^B, a_l, a_h\right)$ *then* $p^3 = \frac{1}{2}\left(\pi^B a_h + \left(1 - \pi^B\right) a_l\right)$.

2. *Belief for the seller of type* β_i^S : *if* $1 \leq n < 2\beta_l^B - 1$, *n odd, then* $\mu^S\left(h^{(n-1)}\right) = \left(\pi^B, 1, 1\right)$. *If* $2\beta_l^B - 1 < n \leq 2\beta_l^S - 1$, *n odd, then* $\mu^S\left(h^{(n-1)}\right) = (1, 0, 1)$. *For a seller of type* β_h^S : *if* $2\beta_l^S - 1 < n \leq 2\beta_h^B - 1$, *n odd, then* $\mu^S\left(h^{(n-1)}\right) = (1, 0, 1)$.

3. *Strategy for the buyer: For* $i = l, h$ *if* $1 \leq n < 2\beta_l^B - 1$, *n odd, then* $v_i(h^n) = 1$. *If* $n = 2\beta_l^S - 1$ *then* $v_l(h^n) = p^1$ *and* $v_h(h^n) = 1$. *For a buyer of type* β_h^B: *if* $2\beta_l^B - 1 < n < 2\beta_l^S - 1$, *n odd, then* $v_h(h^n) = 1$. *If* $n = 2\beta_l^S - 1$ *then if* $p^2 \leq \frac{1}{3}$ *then* $v_h(h^n) = p^1$ *and otherwise* $v_h(h^n) = 1$. *If* $2\beta_l^S - 1 < n < 2\beta_h^B - 1$, *n odd, then* $v_h(h^n) = 1$ *and finally if* $n = 2\beta_h^B - 1$ *then* $v_h(h^n) = p^3$.

4. *Belief for the buyer: if* $1 \leq n < 2\beta_l^B - 1$, *n odd, then* $\mu^B(h^n) = \pi^S$. *if* $n = 2\beta_l^B - 1$ *then if* $p^1 \leq \frac{1}{2}$ *then* $\mu^B(h^n) = \pi^S$ *otherwise* $\mu^B(h^n) = 1$ *if* $2\beta_l^B - 1 < n < 2\beta_l^S - 1$, *n odd, then* $\mu^B(h^n) = \mu^B(h^{n-2})$. *If* $n = 2\beta_l^S - 1$ *then if* $p^1 \leq \frac{1}{3}$ *then* $\mu^B(h^n) = 0$ *otherwise* $\mu^B(h^n) = 1$. *For* $2\beta_l^S - 1 < n \leq 2\beta_h^B - 1$, *n odd,* $\mu^B(h^n) = 1$

Proof. Follows the proof of theorem 2 and 3. ■

Note that in all the equilibria we described the more patient (or talented) players receive a higher expected payoff than the less patient players.

4 Generalizations

We discuss here some of the limitations of our assumptions. Moreover our model can be generalized in many ways and we present here our thoughts on the possible results in each such generalization. We hope to be able to achieve some of these results in the continuation research.

The first natural way to generalize our results will be to examine cases with more than two bargaining talent types for the seller and the buyer. The main features of our equilibria will continue to hold but we will get a longer series (as the number of bargaining talent types increases the number of critical periods can increase as well) of descending offers in the pooling equilibria.

We also assume in this paper independence between the type of the buyer and his bargaining talent type. In a future research we plan to relax this assumption. A more reasonable assumption could be that a "stronger" buyer i.e. a buyer with a low valuation, is more likely to be a more talented bargainer. Therefore we assume that the bargaining talent can be dependent on the value of the object to the buyer. In this case it is no longer reasonable to assume the buyer strategy's stationarity. We then get that the probability π^B i.e. the seller's belief regarding the type of the buyer, enters her equilibrium considerations - she changes her offers according to her belief. This suggests that we might expect to observe sellers making different offers for the same object to different individuals (again we can test this prediction) - for example if common wisdom says that women differ from man in their bargaining ability we would expect different offers to men and women. The initial written price is still the same but we hypothesize that we will be able to observe a gap in the offer made to women or men after they refuse to buy in the written price.

A related topic is our assumption of a uniform distribution of the buyer's type. The generalization to a general continuos distribution or to a discrete distribution is quite straightforward and does not change the nature of our results.

We also plan to examine the evolutionary stability of a population with different bargaining talent types. If one believes that evolution forces the distribution of such characters as a bargaining talent in the population and if one also believes that evolution had a long enough time by now to "activate" such an evolutionary process then we must be able to show that a population with different degrees of bargaining talent is indeed a stable one. For this to hold there must be a cost involved in being a high bargaining talent type player. For example we wish to study a model in which the cost of bargaining increases with the period.

We also would like to explore how will different attitude to risk might change the results. Risk averse buyers will tend to agree to an offer earlier to avoid the chance of losing the trade. We hypothesize that the seller will then be better off. Similarly if we bring back into the model time preferences and the buyer has in addition to his given bargaining talent a discount factor to time then the seller can surely exploit it and offer higher offers. On the other hand if the seller has a discount factor to time then she will want to reach an agreement sooner.

Finally we assumed here one sided incomplete information on the valuation type. The valuation for the seller is known and therefore it is also known for sure that gains from trade exist. We would like to examine a similar framework in which both bargainers type as well as their bargaining talent type are unknown to the second player. Moreover in this paper the seller was the only side to make offers. We wish to generalize the model to an alternating offers model.

5 Conclusions

This paper was conceived following observations we made in real markets. In these bargaining situations we observed, we noticed several patterns which could not be explained as equilibria of the given bargaining literature. Since we feel that the theory should make the effort of becoming closer to reality we set out to build a model with the characteristics we thought to be real and which give equilibrium behavior with the recognized patterns. Especially we constructed a model that does not treat individuals as homogeneous in their bargaining talent. We therefore describe here a model in which this talent is the number of offers the individual is capable of turning down, as a buyer, or of making as a seller, in any given bargaining situation. Individuals often say that they could have gotten a better price if they turned down the current price and asked for a discount. It turns out in this model, and also in our observations, that this is indeed the case - those who can stand the bargaining procedure for more offers have an advantage and will usually get better deals (either as buyers or as sellers). This is in contrast to most bargaining models in which a shorter horizon gives a player more bargaining power. Our model is a small step in the direction of describing real short termed, small valued, daily bargaining interactions. For it to become a useful tool in doing so it still needs to be generalized in many ways - some of which are described in the previous section.

6 References

A. Admatti and M. Perry (1987), "Strategic Delay in Bargaining" *Review of Economic Studies*, 54, 345-364.

L. Ausubel, P. Cramton, and R. Deneckere (2001), "Bargaining with Incomplete Information" Handbook of Game Theory, Vol. 3, R.J. Aumann and S. Hart, eds. Amsterdam: Elsevier Science B.V., Chapter 50.

J. Banks and J. Sobel (1987), "Equilibrium Selection in Signaling Games" *Econometrica*, 55, 647-661.

K. Binmore, A. Rubinstein, and A. Wolinsky (1986), "The Nash Bargaining Solution in Economic modelling" *Rand journal of Economics*, 17, 176-188.

K. Binmore, M. Osborne, and A. Rubinstein (1992), "Noncooperative Models of Bargaining" Handbook of Game Theory, Vol. 1, R.J. Aumann and S. Hart, eds. North Holland: Amsterdam, Chapter 7.

I. Cho (1987), "A Refinement of Sequential Equilibrium" *Econometrica*, 55, 1367-1389.

I. Cho and D. Kreps (1987), "Signaling Games and Stable Equilibria" *Quarterly Journal of Economics*, 102, 179-221.

P. Cramton (1992), "Strategic Delay in Bargaining with Two-Sided Uncertainty" *Review of Economic Studies*, 59, 205-225.

D. Fundenberg, D. Levine and J. Tirole (1985) "Infinite-Horizon Models of Bargaining with One-Sided Incomplete Information" Game Theoretic Models of Bargaining, Alvin Roth, ed. Cambridge, England: Cambridge University Press.

F. Gul, and H. Sonnenchein (1988), "On Delay in Bargaining with One-Sided Uncertainty" *Econometrica*, 56, 601-612.

J. Kennan, and R. Wilson (1993), "Bargaining with Private Information" Journal of Economic Literature, 31, 45-104.

D. Kreps and R. Wilson (1982), "Sequential Equilibria" *Econometrica*, 50, 863-894.

A. McLennan (1985), "Justifiable Beliefs in Sequential Equilibrium" *Econometrica*, 53, 889-904.

A. Muthoo (1999), "Bargaining Theory with Applications" Cambridge University Press.

J. Nash (1950), "The Bargaining Problem" *Econometrica*, 18, 155-162.

J. Nash (1953), "Two-Person Cooperative Games" *Econometrica*, 21, 128-140.

M. Osborne and A. Rubinstein (1990), "Bargaining and Markets", Academic Press, Inc.

M. Perry, and P. Reny (1993), "A Non-Cooperative Bargaining Model with Strategically Times Offers" *Journal of Economic Theory*, 59, 50-77.

A. Rubinstein (1982), "Perfect Equilibrium in a Bargaining Model" *Econometrica*, 50, 97-100.

A. Rubinstein (1985), "A Bargaining Model with Incomplete Information About Time Preferences" *Econometrica*, 53, 1151-1172.

P. Schweinzer (2007), "Sequential bargaining with pure common values" mimeo.

7 Appendix

Proof of Lemma 1

Let $\left(\left(\sigma_l, \sigma_h, \left\{\rho_{v,l}\right\}_{v\in[0,1]}, \left\{\rho_{v,h}\right\}_{v\in[0,1]}\right), \mu\right)$ be a sequential equilibrium and assume in contradiction that for some $i \in \{l, h\}$, $\sigma_i\left(h^{2n}\right) < \sigma_i\left(h^{2n+2}\right)$ where $h^{2n+2} = \left(h^{2n}, \sigma_i\left(h^{2n}\right), No\&Stay\right)$ i.e. that after the response of the buyer the strategy of the seller tells her to strictly increase her offer. We now show that by making the offer $\sigma_i\left(h^{2n+2}\right)$ two periods earlier the seller would have been better off. We show that the seller prefers to deviate to a strategy that is similar to σ_i except that it replaces $\sigma_i\left(h^{2n}\right)$ by $\sigma_i\left(h^{2n+2}\right)$. Assume therefore that we look at an alternative history: $\tilde{h}^{2n+1} = \left(h^{2n}, \sigma_i\left(h^{2n+2}\right)\right)$. Assume that the buyer's type is v and his bargaining talent type is β_j^B then after the history $h^{2n+1} = \left(h^{2n}, \sigma_i\left(h^{2n}\right)\right)$ he will accept the offer iff $v < v_j\left(h^{2n+1}\right)$. Then at period $2n+4$ after the history $h^{2n+3} = \left(h^{2n}, \sigma_i\left(h^{2n}\right), No\&Stay, \sigma_i\left(h^{2n+2}\right)\right)$ he will accept the offer iff $v < v_j\left(h^{2n+3}\right)$. If the history was $\tilde{h}^{2n+1} = \left(h^{2n}, \sigma_i\left(h^{2n+2}\right)\right)$ then at period $2n+2$ the buyer will accept the offer iff $v < v_j\left(\tilde{h}^{2n+1}\right)$ and after the history $\tilde{h}^{2n+3} = \left(h^{2n}, \sigma_i\left(h^{2n+2}\right), No\&Stay, \sigma_i\left(h^{2n+2}\right)\right)$ he will accept the offer iff $v < v_j\left(\tilde{h}^{2n+3}\right)$. Now, first we know that $v_j\left(h^{2n+1}\right) \geq v_j\left(\tilde{h}^{2n+1}\right)$ since $\sigma_i\left(h^{2n}\right) < \sigma_i\left(h^{2n+2}\right)$ and $v_j\left(h^{2n}, p\right)$ is decreasing with the offer. Moreover $v_j\left(h^{2n+3}\right) \geq v_j\left(\tilde{h}^{2n+3}\right)$ since $a_j\left(h^{2n+2}\right) \geq a_i\left(\tilde{h}^{2n+2}\right)$ and we have the monotonicity condition that insures that in such a situation all buyer's types who accepted the offer $\sigma_i\left(h^{2n+2}\right)$ when the history was h^{2n+3} will also accept it when the history is $\tilde{h}^{2n+3}$. We can repeat this argument to show that in all following periods we have $v_j\left(h^{2n+k}\right) \geq v_j\left(\tilde{h}^{2n+k}\right)$ where all seller's offers in both histories are the same except for the offer at period $2n+1$. Therefore the seller prefers this strategy - a contradiction.

Proof of Theorem 1

We first prove that what is described in the theorem is indeed a sequential equilibrium. Given the seller's strategy and the buyer's belief we wish to prove that the buyer's strategy is a best response. As was noted in the text in any simple sequential equilibrium the buyer will never accept an offer that is made in a non-critical period. Moreover, we wish to examine a buyer of type $v \geq x^*$. Given a first critical period's offer of $p \leq x^*$ if a buyer of type v accepts the offer his payoff is $v - p$ for sure while if he turns it down and waits for the second critical period's offer his payoff is $\pi^S\left(v - \frac{1}{2}\omega^*(p)\right)$ (he will not trade if the seller turned out to be impatient - and that happens with probability $1-\pi^S$ and with the remaining probability he will get from the more patient seller an offer of $\frac{1}{2}\omega^*(p)$). Now solving for the indifferent type

$$v - p = \pi^S \left(v - \frac{1}{2}\omega^*(p) \right)$$

we indeed get $v_i\left(h^{2n+1}\right) = \omega^*(p) = \frac{2p}{(2-\pi^S)}$. Therefore all buyers type $v \geq \omega^*(p)$ (note that $\omega^*(p) > p$) are indeed better off by accepting the first critical period's offer of p. If $p = x^*$ then $v_i\left(h^{2n+1}\right) = \omega^*(x^*) = \frac{1}{2}$. However, if the first critical period's offer is $p > x^*$ then the buyer believes he is faced with a high bargaining talent type seller and therefore he is better off turning down the offer and waiting for the second critical period's offer. When the second critical period's offer is made the buyer knows he is faced with a high bargaining talent type seller and if he turns down the offer his payoff will be zero. Therefore he accepts the second critical period's offer iff $v \geq p$.

Now a seller of a bargaining talent type β_l^S will not deviate. If she acts upon the equilibrium strategy her payoff is: $x^*(1-\omega^*(x^*)) = \frac{1}{8}\left(2-\pi^S\right)$. If she deviates to a price $y < x^*$ at the first critical period's offer then by deviating she cannot convince the buyer that she is impatient (due to off-the-equilibrium-path-beliefs). Therefore her payoff will be $y(1-\omega^*(y)) = y\left(1-\frac{2y}{(2-\pi^S)}\right)$ but for all $0 \leq y < x^*$

$$y\left(1 - \frac{2y}{(2-\pi^S)}\right) < \frac{1}{8}\left(2-\pi^S\right)$$

therefore she would not like to offer a different price $y < x^*$ (the price x^* is the one that maximizes $x\left(1-\frac{2x}{(2-\pi^S)}\right)$). If she deviates and makes an offer $y > x^*$ then the buyer is convinced that he is faced with a patient seller and turns down the offer. Obviously then she does not want to deviate.

A seller of a bargaining talent type β_h^S will not deviate either. If she acts upon the equilibrium strategy her payoff will be $x^*(1-\omega^*(x^*)) + \frac{1}{4}(\omega^*(x^*))^2 = \frac{1}{8}\left(2-\pi^S\right) + \frac{1}{16}$ (the first term is her expected payoff in the first critical period and the second in the second critical period - her belief is $\left(\pi^B, \omega^*(x^*), \omega^*(x^*)\right)$ and she will make an offer of $\frac{1}{2}\omega^*(x^*)$ and all buyers in the interval $\left[\frac{1}{2}\omega^*(x^*), \omega^*(x^*)\right]$ will accept it). If she deviates and makes in the first critical period an offer $y < x^*$ then her payoff will be $y(1-\omega^*(y)) + \frac{1}{4}(\omega^*(y))^2 = y\left(1-\frac{2y}{(2-\pi^S)}\right) + \frac{1}{4}\left(\frac{2y}{(2-\pi^S)}\right)^2$ but for all $0 \leq y < x^*$ we get

$$y\left(1-\frac{2y}{(2-\pi^S)}\right) + \frac{1}{4}\left(\frac{2y}{(2-\pi^S)}\right)^2 < x^*\left(1-\frac{2x^*}{(2-\pi^S)}\right) + \frac{1}{4}\left(\frac{2x^*}{(2-\pi^S)}\right)^2$$

therefore she would not like to offer a price $y < x^*$. If she deviates and makes an offer $y > x^*$ then the buyer is convinced that he is faced with a patient seller and turns down the offer in the first critical period. In the equilibrium of the continuation subgame that follows such a deviation, her belief is that all buyers are still in the game and therefore she offers $p = \frac{1}{2}$ in the second critical period and her expected payoff is $\frac{1}{4}$. Now for all $0 < \pi^S < 0.5$ we have

$$\frac{1}{4} < x^* \left(1 - \frac{2x^*}{(2-\pi^S)}\right) + \frac{1}{4}\left(\frac{2x^*}{(2-\pi^S)}\right)^2$$

Finally, given her belief, she would not want to deviate in the second critical period - she is making the offer that maximizes her expected payoff given her belief. We therefore conclude that this is indeed a sequential equilibrium. It is also obvious that conditions (1)-(5) hold (we defined the strategies according to these conditions). For uniqueness note first that in such an equilibrium the buyer's strategy can not change. His belief is given by the definition of a simple equilibrium and he chooses a best response to a price strategy of the seller. The more patient seller on the other hand will not change her strategy either, given the strategy of the β_l^S seller of making an offer x^* in the first critical period - it is a best response to imitate this offer and then offer $\frac{1}{2}\omega^*(x^*)$ as long as $\frac{1}{4} < x^*\left(1 - \frac{2x^*}{(2-\pi^S)}\right) + \frac{1}{4}\left(\frac{2x^*}{(2-\pi^S)}\right)^2$ or equivalently $\frac{1}{2}\frac{\left(2-\pi^S\right)}{(3-2\pi^S)} < x^* < 1 - \frac{1}{2}\pi^S$. Therefore there can only be a pooling equilibrium as long as $\frac{1}{2}\frac{\left(2-\pi^S\right)}{(3-2\pi^S)} < x^* < 1 - \frac{1}{2}\pi^S$. Finally, the impatient seller chooses an offer that maximizes her expected payoff: $\max_x x\left(1 - \omega^*(x)\right) = \max_x x\left(1 - \frac{2x}{(2-\pi^S)}\right) \Rightarrow x^* = \frac{1}{2} - \frac{1}{4}\pi^S$ and indeed, for $0 < \pi^S < \frac{1}{2}$ we have $\frac{1}{2}\frac{\left(2-\pi^S\right)}{(3-2\pi^S)} < x^*$ while $\frac{1}{2} - \frac{1}{4}\pi^S < 1 - \frac{1}{2}\pi^S$ for all π^S.■

Proof of Theorem 2

We first prove that what is described in the theorem is indeed a sequential equilibrium. Given the seller's strategy and the buyer's belief we wish to prove that the buyer's strategy is a best response. We look at a buyer of type v. Given a first critical period's offer of $p_{\beta_l^S} \leq \frac{1}{3}$ the buyer is convinced he is faced with the type β_l^S and then he accepts the offer iff $v \geq p_{\beta_l^S}$. Otherwise he is convinced that he is faced with the type β_h^S and turns down the offer (in this case he will accept the offer in the second critical period iff it is smaller or equal to his type). When the second critical period's offer is made the buyer knows he is faced with a high bargaining talent type seller and if he turns down the offer his payoff will be zero. Therefore he accepts an offer iff $v \geq p$.

Now a seller of a bargaining talent type β_l^S will not deviate. If she acts upon the equilibrium strategy her payoff is: $\frac{1}{3}\left(1 - \frac{1}{3}\right) = \frac{2}{9}$. If she deviates to a price $y < \frac{1}{3}$ at the first critical offer then by deviating she convinces the buyer that she is impatient and her payoff will be $y(1-y)$ but for all $0 \leq y < \frac{1}{3}$ we have $y(1-y) < \frac{2}{9}$. If she deviates and makes an offer $y > \frac{1}{3}$ then the buyer is convinced that he is faced with a patient seller and turns down the offer. Obviously then she does not want to deviate.

A seller of a bargaining talent type β_h^S will not deviate either. If she acts upon the equilibrium strategy her payoff will be $\frac{1}{2}\left(1-\frac{1}{2}\right) = \frac{1}{4}$. If she deviates and makes in the first critical period an offer $y \leq \frac{1}{3}$ then the buyer is convinced he is faced with a β_l^S and therefore her payoff will be $y(1-y) + \frac{1}{4}y^2$ (she will make an offer of $\frac{1}{2}y$ in the second critical offer) but for all $0 \leq y \leq \frac{1}{3}$ we have $y(1-y) + \frac{1}{4}y^2 \leq \frac{1}{4}$. If she deviates and makes an offer $y > \frac{1}{3}$ then the buyer is convinced that he is faced with a patient seller and turns down the offer in the first critical period. Therefore she does not gain from such a deviation. Obviously then we have a simple sequential separating equilibrium.

Finally, for uniqueness note that again the buyer's strategy will not change, given his beliefs he will either accept an offer that is lower than his valuation in the first critical period (when the offer p is lower or equal to the impatient seller's equilibrium offer) or will turn down any offer in the first critical period and accept an offer in the second critical period if it is lower than his type. Moreover, a patient seller would not want to deviate to an offer lower or equal to the impatient seller's equilibrium offer as long as this offer is lower or equal to $\frac{1}{3}$ therefore the impatient seller chooses her best response $x^* = \frac{1}{3}$.■

Proof of Theorem 3

In the first critical period $2\beta_l^B - 1$ the seller knows that only buyers of a bargaining talent type β_l^B might accept her offer (all other buyers will turn the offer down and stay in the bargaining). Therefore the best she can do, given the buyer's strategy is offer $\frac{1}{2}$ (this is the offer that maximizes $\max_x x(1-x)$). Moreover, the buyer's strategy is indeed a best response since β_h^B is better off turning down the offer in the first critical period and waiting for the second or even third critical offer. A buyer β_l^B on the other hand can do no better than accepting the offer iff it is below his valuation. We only need to make sure that a seller would not want to deviate in the first critical offer. If a seller of a bargaining talent type β_l^S deviates and makes an offer $y < \frac{1}{2}$ in the first critical period and then an offer of x in the second, then her expected payoff is $\left(1-\pi^B\right) y(1-y) + x(1-\omega^*(x))$ since by making such an offer she does not convince the buyer that she is impatient and therefore $\omega^*(x)$ does not change. We conclude that she will want to offer $y = \frac{1}{2}$. The same computations for β_h^B lead us to the conclusion that he will not want to deviate as well. After the period β_l^B the game is identical to the game in subsection 3.1.■