Scholarly Influence

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Abstract

We introduce a new class of measures of scholarly influence, which we term *step-based indices*. This class includes the prominent h-index, the publication count, and the $i10$-index. We show that the class of step-based indices is characterized by three axioms, consistency with worse scientists, consistency with better scientists, and full range. We also introduce a new index, the junior/senior-index, which combines the best features of the h-index with those of the $i10$-index.

1 Introduction

In recent years there has been a significant growth in the use of influence measures: indices used to measure the influence of scientists in terms of citations of their work. The most prominent example of an influence measure is that of the h-index, developed by a physicist but now used in a wide range of scientific disciplines. (Hirsch, 2005) We introduce new properties that, in our view, influence measures should satisfy. Surprisingly, these properties characterize a tractable class, which we term *step-based indices*. This class includes the aforementioned h-index as well as several other prominent measures.

Influence measures rank scientists in terms of two factors: the number of publications produced by a scientist, and the number of citations that each publication has received. A simple influence measure is the citation count: the total number of citations to all of the scientist's publications. Another influence measure is the popular h-index: the largest number h such that the scientist has at least h publications, each with at least h citations. Google has recently introduced a new influence measure, the $i10$-index, which is the number of the scientist's publications that have at

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least ten citations each.[1] To illustrate these measures, consider a scientist with four publications, which have ten, five, four, and two citations respectively. This scientist would have a citation count of twenty-one, an h-index of three, and an $i10$-index of one.

Before we begin to describe our properties, we need to introduce the concept of objective comparisons. In many cases, it is difficult to find purely objective criteria which we can use to compare scientists. For example, suppose that one scientist, Alice, has two publications, each with six citations, while another scientist, Bob, has three publications, each with three citations. Which scientist is more influential? Different methods may lead to different results. According to the citation count, it is Alice, with twelve total citations; according to the h-index, it is Bob, with an h-index of three.

There is one clear case, however, in which one scientist is *objectively at least as influential* as another scientist. This is when every publication of the latter scientist can be matched with a publication of the former scientist with at least as many citations. For example, consider a third scientist, Carol, who has two publications, each with three citations. Objectively, Alice is at least as influential as Carol, because Alice's top two publications have at least three citations each. Along these lines, we may say that Carol is *objectively at most as influential* as Alice. Similarly, we may say that Bob is objectively at least as influential as Carol, because his top two publications also have at least three citations each.[2]

In figure 1, scientists are identified with a graph which shows the number of citations that each of their publications have. Thus Alice's two publications have six citations each (shown in figure 1(a)), while Bob's three publications have three citations each (shown in figure 1(b)). In the figure, it is easy to see that both Alice and Bob are at least as influential as Carol (shown in figure 1(c)).

Furthermore, one can see that Carol is the most influential scientist who is objectively at most as influential as both Alice and Bob. If Carol were to publish an additional paper, we would not be able to make an objective comparison between her and Alice, because Carol would have more publications (three) than Alice (two), but Alice would have more citations. Similarly, if one of Carol's publications were to receive an additional citation, we would not be able to make an objective comparison between her and Bob, because Carol would have a publication with four citations (while Bob's most-cited publication has three citations), but would have fewer publications (two, while Bob has three).

So there is a sense in which we can make objective comparisons, but it only takes

[1] See http://scholar.google.com. The internet application "Google Scholar Citations" calculates these three measures for users of their service.

[2] Implicitly, we assume that all citations should be counted equally. This assumption is open to controversy; for example, some might argue that citations from highly-cited journals should count for more than citations from lesser-cited journals. While we have chosen to give equal weight to all citations for reasons of simplicity and consistency with the literature, our results would apply in the more general case. We discuss this issue at length in section 3.1, below.

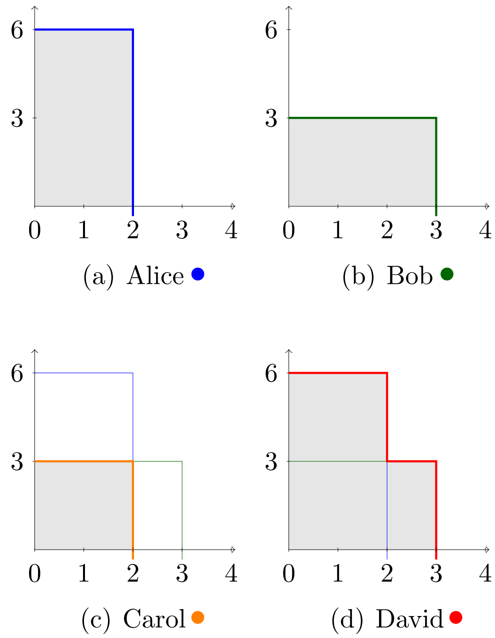

Figure 1: Four Scientists.

us so far. Some scientists (e.g. Alice and Bob) are not objectively comparable. The point of an influence measure is to enable us to rank scientists when objective comparisons are not possible. That is, the measure should allow us to compare Alice and Bob. This is both a positive feature of influence measures (satisfying a need to compare scientists) but also a source of complaint (because one can always find a ground for criticism).

Although all influence measures are open to criticism, this does not mean that all are equally problematic. Some have more flaws than others; certain defects are relevant only in select applications and not in general. For this reason, a natural way to study these measures is to characterize them in terms of axioms, or properties that they satisfy. We introduce two such axioms on influence measures here; we believe that they are entirely new to this literature.

To describe our first axiom, consider the three scientists we have already described, Alice, Bob, and Carol. Recall that both Alice and Bob are objectively at least as influential as Carol. Because the influence measure should reflect the objective ranking, it should not rank Carol strictly ahead of Alice or Bob. However, objectively speaking, Carol is the most influential scientist who is at most as influential as Alice and Bob. So Carol should not be too far behind them — if she gets one extra citation or one more publication, we can no longer make an objective comparison between her and both Alice and Bob. As a result, we maintain, Carol should be considered equivalent to the weaker of the two. If Alice is ranked ahead of Bob, then Bob is equivalent to Carol; similarly, if Bob is ranked ahead of Alice, then Alice is equivalent to Carol. We can turn this into an axiom: for any two scientists, the one deemed less influential shall be considered equally influential as the best possible scientist who would be objectively less influential than both.[3] We call this property *consistency with worse scientists*.

To describe our second axiom, consider a fourth scientist, David, who has three publications, two with six citations and one with three (shown in figure 1(d)). David is objectively at least as influential as Alice, because Alice has only two publications, with six citations each. Similarly, David is objectively at least as influential as Bob, because while Bob has three publications, they only have three citations each. Consequently, the influence measure should not rank either Alice or Bob strictly ahead of David. However, objectively speaking, David is the least influential scientist who is at least as influential as Alice and Bob. So, for reasons analogous to those we described in the case of Carol, David should not be too far ahead of Alice and Bob — if he had one fewer citation (or one fewer publication), he would no longer be objectively at least as influential as both. As a result, we maintain, David should be considered equivalent to the stronger of the two. If Alice is ranked ahead of Bob, then Alice is equivalent to David; similarly, if Bob is ranked ahead of Alice, then Bob is equivalent to David. We can turn this into an axiom: for any two scientists, the one deemed more influential shall be considered equally influential as the worst possible scientist who would be objectively more influential than both. We call this property *consistency with better scientists*.

We use one additional axiom, *full range*, which states that every score is theoretically achievable. That is, for every non-negative integer, there is a set of publications and citations which will result in that score. It requires the worst possible scientist to have a score of zero, the next best scientist to have a score of one, and so on. It disallows the case where all scores are even. This axiom is technical and guarantees us a meaningful cardinal structure, and does not affect the corresponding ordinal ranking of the scientists.

Our main result is that the three axioms — consistency with worse scientists, consistency with better scientists, and full range — characterize the set of influence measures that we call *step-based indices*. We can describe step-based indices in the following way. A *step* is a pair of two numbers, x and y, and describes a scientist with x publications with y citations each. For example, Alice can be described by the step $(2, 6)$ because she has two publications with six citations each. Similarly Bob and Carol can be described by the steps $(3, 3)$ and $(2, 3)$, respectively. David, however, can not be described by a step, as there is no pair of numbers which describes him in this way. Because David has two publications with six citations each, we may say that he *achieves* the step $(2, 6)$ but this does not describe him because he also has a third paper with three citations. Similarly, because David has three papers with at least three citations, we may say that he achieves the step $(3, 3)$, but again this does

[3]Implicitly, we assume that every combination of papers and citations is possible; for more on this assumption see section 3.2, below.

not describe him because two of those papers have six citations.

A step-based index is one for which there is an increasing set of steps, one associated with each integer, so that a scientist's score is determined by the best step she achieves. A scientist who achieves none of the steps receives a score of zero. For example, the aforementioned h-index is a step-based index defined by steps of the form (x, x), so that a scientist receives a score of five if she has five publications with five citations each, but does not have six publications with six citations each. The $i10$-index is also a step-based index defined by steps of the form $(x, 10)$ so that a scientist receives a score of five if he has five publications with ten citations each, but does not have six publications with ten citations each. The citation count is not a step-based index; while one must achieve either of the steps $(1, 2)$ or $(2, 1)$ to receive a citation count of two, neither is necessary.

Four step-based indices are pictured in Figure 2: the h-index, the $i10$-index, the maximum-index, and the publication count. In addition, three hypothetical scholars are represented by lines showing the number of citations of their n-th most cited publications. For example, scholar A has twelve publications, three of which have six citations each, three of which have five, three with three, and three with one. Scholar C, on the other hand, has eighteen publications, one with seventeen citations, one with sixteen citations, and so on. We wish to draw attention to a few features of this model. First, scholar C is objectively more influential than scholar A, but we can not make an objective comparison between either of them and scholar B. Thus no index should rank scholar A ahead of scholar C. Second, step-based indices are determined according to the maximal step which each scholar achieves. Thus according to the h-index, scholar C is the most productive, with a score of nine, followed by scholar A with a score of five, and with scholar B in last place with a score of four. Under both the $i10$-index (scores of zero, three, and eight, respectively) and the publication count (scores of twelve, sixteen, and eighteen, respectively), scholar B is ranked ahead of scholar A but behind scholar C. The maximum-index, which looks at the number of citations received by the scientist's most-cited paper, ranks scholar B (with nineteen citations) ahead of scholar C (with seventeen citations).

One particularly interesting step-based index is the junior/senior-index. This index follows the h-index up to a score of 10, and then follows the $i10$-index for higher scores. The intuition is simple: it is rare for junior scholars to have a paper with 10 citations, and so the h-index is more useful at distinguishing between them. For more senior scholars, the $i10$-index may be a more useful metric; a paper with ten citations may be considered influential enough to be counted. Of course there is nothing special about the number ten; in general what is important is that the step-path be concave.

Several prominent influence measures are not step-based. The g-index (Egghe, 2006) is the maximum number n such that the scientist has a n publications with an average of n citations each. The w_1-index[4] (Woeginger, 2008b) is the maximum

[4]There are several indices named the w-index; to distinguish we refer to the index by Woeginger

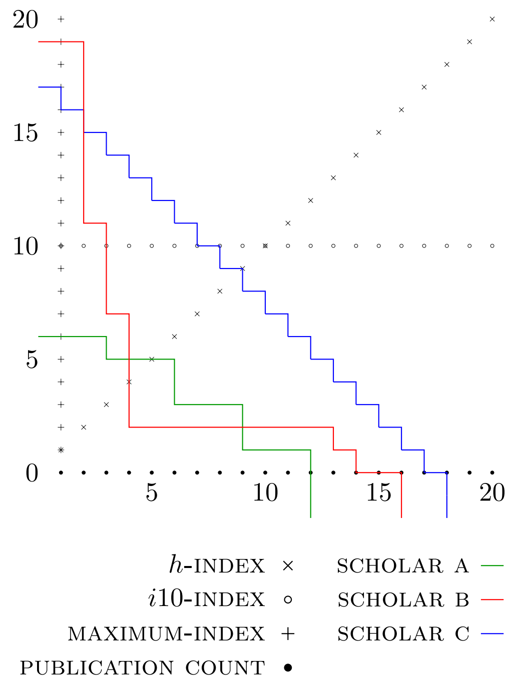

Figure 2: Step-based indices.

number n such that the scientist has at least n publications where the most cited publication has at least n publications and the j-th most cited publication has at least $n + 1 - j$ citations.[5] The citation count is the total number of the scientists' citations. Citations per paper is the total number of the scientist's citations divided by the total number of that scientist's publications. The citation count has a simple graphical representation: it is the area under the curve as shown in Figure 3.[6]

Step-based indices are nice for several reasons. First they are very tractable;

(2008b) as w_1 and the index by Wu (2009) as w_2.

[5]The w_1-index satisfies the consistency with worse scientists and full range axioms but not the consistency with better scientists axiom. These first two axioms themselves characterize an interesting class, which we call *minimal scientist indices*. These rules can be described as follows. First, there is a set of reference scientists, each one objectively more influential than the last. For example, the first reference scientist may have one publication with one citation, while the second might have two publications, with two and one citations, respectively. Because they are objectively comparable, we can order them in a line. A scientist is judged by comparison to these reference scientists, according to the following method. Each reference scientist induces a equivalence class — the set of scientists who are objectively as influential as the reference scientist but not objectively as influential as the next. Because we can order the reference scientists in a line, the equivalence classes can themselves be turned into a ranking. One scientist is more influential than another if the former is in a higher-ranked class; two scientists are equally influential if they are in the same class. A scientist who is objectively as influential as the third reference scientist, but not as objectively as influential as the fourth, receives a score of three. A scientist who is not as objectively as influential as any of the reference scientists receives a score of zero. Clearly, every step-based index is a minimal scientist

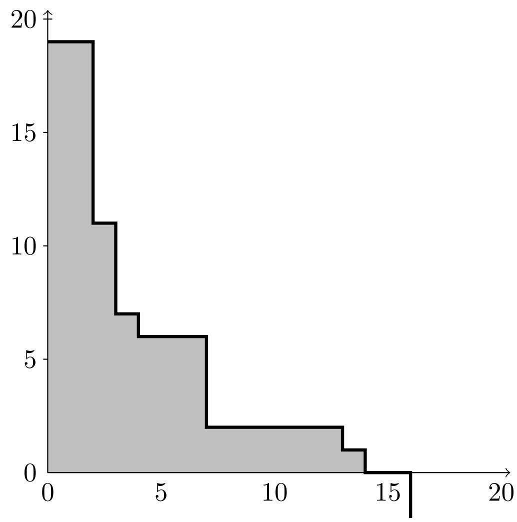

Figure 3: The Citation Count.

they can be understood and computed with very little effort. Second, the class of step-based indices is very flexible. Different step-based indices prioritize citations and publications differently, and a variety of step-based indices are available to suit different needs. They can be used a benchmark model in determining the productivity or influence of a scientist.

However, like all citation-based influence measures, the step-based indices are not intended to be an overall summary of a scientist, but rather a simple "ballpark" measure of how productive a scientist is at some given point. Presumably in tenure or promotion cases, the value of a step-based index (or of any other citation measure) should be used in accommodation with something else. The finer details of a scientist's career would obviously be important in evaluating a scientists' productivity.

Like all measures, step-based indices are not without their flaws. One particularly egregious and relevant problem is what we call the "renaming paradox." Suppose that a scientist changes the title of their paper in response to comments by a referee. The paper had already been cited under an original title, and then receives more citations under the new title. A nice property of an index would require that whether we consider the paper as two distinct papers, or as one paper, would not matter. It is easy to see that all step-based indices violate this property. Indeed, the only indices satisfying this property are those that depend only on the total number of citations.

The renaming paradox has another implication. An article may be cited by both versions of a renamed paper. As a consequence, these citations will be double-counted. This problem will exist primarily in scientific fields in which extensive use is made of pre-print archives such as arXiv, RePEc, and SSRN.

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index, but there exist minimal scientist indices (such as that w_1-index) which are not step-based.

[6]The g-index, the citation count, and citations per paper all satisfy the full range axiom but violate both of the consistency axioms.

This work differs from previous axiomatic studies of influence measures in two ways. First, most prior works have studied specific measures instead of general classes of measures. These include characterizations of the h-index (Woeginger, 2008b,c; Marchant, 2009a; Quesada, 2009, 2010, 2011), the g-index (Woeginger, 2008b,a; Quesada, 2011), generalized g-indices (Woeginger, 2009), the w_1-index (Woeginger, 2008b,c), the maximal-index (Woeginger, 2008b; Marchant, 2009a), the family of i-indices (such as the i10-index) (Marchant, 2009a), the publication count (Marchant, 2009a), and the citation count (Marchant, 2009a). Second, while we use a cardinal framework for simplicity, our characterization can be converted into an ordinal environment by replacing the full range axiom with one which requires that there is no best scientist. Every ordinal ranking method which satisfies this axiom, completeness, transitivity, and ordinal versions of our consistency axioms can be cardinally represented by a step-based index.[7] We note two exceptions. First, the class of generalized Kosmulski-indices (Deineko and Woeginger, 2009) is closely related to that of the step-based indices; all generalized Kosmulski-indices are step-based but the converse is not true. The maximal-index, for example, is step-based but is not a generalized Kosmulski-index. In contrast to our approach, the axioms in the characterization in Deineko and Woeginger (2009) correspond directly to the cardinal structure of the rule. The second exception is the work of Marchant (2009b) who characterizes a class of scoring rules based on ordinal principles.

The consistency with better scientists and consistency with worse scientists axioms are mathematically equivalent to the lattice-theoretic concepts of the join and meet-homomorphism. (Davey and Priestley, 2002) Variants of these axioms were first introduced in economics by Kreps (1979) and have since been studied in a variety of economic environments. (Hougaard and Keiding, 1998; Christensen, Hougaard, and Keiding, 1999; Miller, 2008; Chambers and Miller, 2011; Dimitrov, Marchant, and Mishra, 2012; Leclerc, 2011; Leclerc and Monjardet, 2013; Chambers and Miller, forthcoming).

2 The model

A *scientist* consists of a function $\mathbf{s} : \mathbb{N} \to \mathbb{Z}_+ \cup \{\varnothing\}$. The quantity $\mathbf{s}(n)$ represents the total number of citations of the scientist's n-th most cited paper. A value $\mathbf{s}(n) = \varnothing$ indicates that the scientist has not written n papers. The function $\mathbf{s}$ must be nonincreasing, there must exist n for which $\mathbf{s}(n) = \varnothing$, and if $\mathbf{s}(n) = \varnothing$ and $n' > n$, then $\mathbf{s}(n') = \varnothing$. The total number of papers the scientist has authored is therefore given by $|\mathbf{s}| \equiv \max\{n : \mathbf{s}(n) \neq \varnothing\}$. The set of all scientists is written $\mathcal{S}$.

An *influence measure* is a function $I : \mathcal{S} \to \mathbb{Z}_+$ which assigns to each scientist a non-negative integer which represents the influence of the scientist's work amongst the community of scientists.

[7] Details are available upon request.

For $z \in \mathbb{Z}_+$, we define $z > \varnothing$. As a consequence, $\min\{z, \varnothing\} = \varnothing$ and $\max\{z, \varnothing\} = z$ whenever $z \in \mathbb{Z}_+$. We say that scientist $\mathbf{s}$ is *objectively at least as influential* as $\mathbf{s}'$ if $\mathbf{s}(n) \geq \mathbf{s}'(n)$ everywhere. We write $\mathbf{s} \geq \mathbf{s}'$ in this case. Using this definition, we can define two operations on scientists, $\vee$ and $\wedge$. By $\mathbf{s} \vee \mathbf{s}'$ we denote the objectively least influential scientist which is objectively at least as influential as each of $\mathbf{s}$ and $\mathbf{s}'$; that is, for each $n \in \mathbb{N}$, $(\mathbf{s} \vee \mathbf{s}')(n) = \max\{\mathbf{s}(n), \mathbf{s}'(n)\}$. Similarly, by $\mathbf{s} \wedge \mathbf{s}'$ we denote the objectively most influential scientist which is objectively at most as influential as each of $\mathbf{s}$ and $\mathbf{s}'$, that is, for each $n \in \mathbb{N}$, $(\mathbf{s} \wedge \mathbf{s}')(n) = \min\{\mathbf{s}(n), \mathbf{s}'(n)\}$.

We define the following three axioms on influence measures:

Consistency with better scientists: For all scientists $\mathbf{s}, \mathbf{s}'$, if $I(\mathbf{s}) \geq I(\mathbf{s}')$, then $I(\mathbf{s} \vee \mathbf{s}') = I(\mathbf{s})$.

Consistency with worse scientists: For all scientists $\mathbf{s}, \mathbf{s}'$, if $I(\mathbf{s}) \geq I(\mathbf{s}')$, then $I(\mathbf{s} \wedge \mathbf{s}') = I(\mathbf{s}')$.

Full range: For all $z \in \mathbb{Z}_+$, there exists a scientist $\mathbf{s}$ for which $I(\mathbf{s}) = z$.

These axioms are described at length in the introduction.

An *step-path* is a function $P : \mathbb{N} \to \mathbb{N} \times \mathbb{Z}_+$ such that $n > n'$ implies that $P(n) \geq P(n')$ and $P(n) \neq P(n')$. For a scientist $\mathbf{s}$ let $Z^{\mathbf{s}} \equiv \{n \in \mathbb{N} : P_2(n) \leq \mathbf{s}(P_1(n))\}$. A step-path *generates* an influence measure given by:

$$I^P(\mathbf{s}) = \begin{cases} 0 & , \textit{if } Z^{\mathbf{s}} = \varnothing \\ \sup Z^{\mathbf{s}} & , \textit{if } Z^{\mathbf{s}} \neq \varnothing \end{cases}$$

The class of influence measures generated by step-paths are termed the *step-based indices*. Several step-based indices are described in Table 1. Examples of influence measures which are not step-based indices are given in Table 2.

Table 1: Step-based indices

Name	$I(\mathbf{s})$ **[Influence Measure]**	$P(n)$ **[Step-path]**
h-index (Hirsch, 2005)	$\max\{z : \mathbf{s}(z) \geq z\}$	(n, n)
$h(2)$-index (Kosmulski, 2006)	$\max\{z : \mathbf{s}(z) \geq z^2\}$	(n, n^2)
$i10$-index	$\max\{z : \mathbf{s}(z) \geq 10\}$	$(n, 10)$
Publication Count	$\lvert\mathbf{s}\rvert$	$(n, 0)$
maximum-index	$\max\{\mathbf{s}(1), 0\}$	$(1, n)$
w_2-index (Wu, 2009)	$\max\{z : \mathbf{s}(z) \geq 10z\}$	$(n, 10n)$
junior/senior-index	$\max\{z : \mathbf{s}(z) \geq \min\{z, 10\}\}$	$(n, \min\{n, 10\})$

Table 2: Influence measures which are not step-based

Name	$I(\mathbf{s})$ **[Influence Measure]**
g-index (Egghe, 2006)	$\max\{z : \mathbf{s}(z) \in \mathbb{Z}_+ \text{ and } \sum_{i=1}^{z} \mathbf{s}(i) \geq z^2\}$
Citation Count	$\sum_{i=1}^{\lvert\mathbf{s}\rvert} \mathbf{s}(i)$
Citations per Paper	$\frac{1}{\lvert\mathbf{s}\rvert} \sum_{i=1}^{\lvert\mathbf{s}\rvert} \mathbf{s}(i)$
w_1-index	$\max\{z : \mathbf{s}(i) \geq z + 1 - i \text{ for all } i \leq z\}$

Our main result is that an influence measure is a step-based index if and only if it satisfies consistency with better scientists, consistency with worse scientists, and full range.

Theorem 1. *An influence measure is a step-based index if and only if it satisfies consistency with better scientists, consistency with worse scientists, and full range.*

3 Discussion

3.1 Unequal treatment of citations

In this note we have implicitly assumed that all citations should be counted equally. This assumption is open to criticism on the grounds that not all journals are equal—some are considered to be more "prestigious" or "influential" according to certain criteria, such as the difficulty of acceptance or the extent to which papers in the journals are cited. Arguably, a citation from a more heavily cited journal should be a greater signal of influence than a citation from a less cited journal. (However, a citation in a more heavily cited journal is more likely to lead to additional citations, so one should be careful not to over count the influence of an article. The extent to which a citation is likely to lead to other citations may depend on citation practices within the relevant scientific community.)

To adjust the model to account for journal differences, we simply need to let $\mathbf{s}(n)$ be the weighted number of citations, where the weights are taken from a cardinal measure of journal influence such as the impact factor or the PageRank method characterized by Palacios-Huerta and Volij (2004). In this case, our basic characterization of step-based influences measures will still hold; however, if the journal weights are not integers, then it will be necessary to replace the set of non-negative integers $\mathbb{Z}_+$ with a set that includes all possible scores.[8] Citations may also be weighted by other means; for example, citations might count more if they come from papers which were heavily cited or which cited relatively few papers.

[8] As long as the set of journals is finite, it will be sufficient to use any countable set that is order-isomorphic to $\mathbb{N}$.

Weighting citations may affect the strategic incentives of the authors and journal editors in several ways. First, if all citations are counted equally, authors will have an increased strategic incentive to write papers that they expect to be heavily cited in relatively uninfluential journals. If citations are weighted according to a cardinal journal influence measure such as the impact factor, scholars within a research field may have an incentive to boost citations per paper within the field. Journal editors may pressure authors to increase the length of their reference lists.

To counteract this latter problem, and to make the citation measure comparable across disciplines, one may weigh citations of an article according to the journal in which the original paper was published. For example, a paper published in a law journal may expect to be cited more heavily than a paper published in an economics journal because of a difference in citation practices within the two academic communities. One might normalize the citation count by the average number of citations contained within articles published in the journal.[9]

It is possible to measure the influence of scientists (or of individual articles, or of journals) using the PageRank method characterized by Palacios-Huerta and Volij (2004). One advantage of using a step-based index instead of the PageRank method is that it is the latter method is difficult to compute in the case of authors. While PageRank may be a reasonable method to estimate the influence of journals, the use of the method to estimate author influence involves much more data—specifically, the number of times that each author has cited by every other author. Step-based indices, by contrast, are easily computable; an author who wishes to compute her score only needs to know the number of citations that her papers have received.

3.2 The domain of scientists

We assume that every possible scientist may exist; i.e. every portfolio of papers and citations is theoretically achievable. This assumption may be limited. It is possible, for example, to place an upper bound on the set of possible scientists; however, this would mean that there would be a maximum possible score that a scientist could possibly achieve. Our proof does not depend upon the possibility that a scientist could exist who is far better than any other scientist, but only upon the possibility that a scientist could be objectively as influential as all existing scientists.

4 Conclusion

We have introduced a new class of influence measures, which we term the step-based indices. This class includes the popular h-index as well as several other interesting measures, such as Google's $i10$-index, the publication count, and the maximum-index.

[9]This intuition motivates Property 1 in Palacios-Huerta and Volij (2004).

Step-based indices are characterized by three axioms, consistency with better scientists, consistency with worse scientists, and full range.

In the setting which we study, the only relevant information about a scientist is the set of papers that he or she has published. However, it is important to note that the scientist in our model is not simply a collection of publications. It is a collection of publications without reference to their titles, subject matter, or the names of the journals in which they are published. What is relevant is only the number of other papers which cite them. This assumption may seem rather limiting. However, in a certain sense, this other information should be irrelevant in determining the *influence*, as opposed to the importance, of the scientist.

All influence measures are subject to the problem of strategic behavior, in the sense that scientists may try to meet the index. For example, the maximum-index could discourage scholars from writing potentially valuable papers for a smaller audience. The publication count, on the other hand, could encourage scholars to write papers that will never be read. The use of citation measures generally (and the citation count specifically) increases authors' incentives to self-cite. A natural problem would be to ask which measures lead to less distortion of incentives. We speculate that the h-index may do well in this regard.

5 Appendix

Proof of Theorem . That any step-based index satisfies the three axioms is immediate. We shall prove the converse direction. Let I satisfy the three axioms. Each of the two consistency axioms directly implies that I is *monotonic:* if $\mathbf{s} \leq \mathbf{s}'$, then $I(\mathbf{s}) \leq I(\mathbf{s}')$.

We claim that for every $z \in \mathbb{Z}_+$, there is $\mathbf{s}_z \in \mathcal{S}$ such that $z \leq I(\mathbf{s}')$ if and only if $\mathbf{s}_z \leq \mathbf{s}'$. To see this, let $z \in \mathbb{Z}_+$ and let $\mathbf{s}^* \in \mathcal{S}$ be such that $z = I(\mathbf{s}^*)$. The full-range axiom implies that $\mathbf{s}^*$ must exist. Now, consider the collection $\mathcal{S}_z \equiv \{\mathbf{s} \in \mathcal{S} : \mathbf{s} \leq \mathbf{s}^* \text{ and } z = I(\mathbf{s})\}$. Clearly, $\mathcal{S}_z$ is nonempty as $\mathbf{s}^* \in \mathcal{S}_z$. Because $|\mathcal{S}_z|$ is finite, a repeated application of consistency with worse scientists implies that $I\left(\bigwedge_{\mathbf{s}\in\mathcal{S}_z} \mathbf{s}\right) = z$. Define $\mathbf{s}_z = \bigwedge_{\mathbf{s}\in\mathcal{S}_z} \mathbf{s}$. Because I is monotonic, for all $\mathbf{s} \in \mathcal{S}$, if $\mathbf{s}_z \leq \mathbf{s}$ then $z \leq I(\mathbf{s})$. On the other hand, suppose that $z \leq I(\mathbf{s})$. It follows from consistency with worse scientists that $z = I(\mathbf{s}^* \wedge \mathbf{s})$, and therefore that $\mathbf{s}^* \wedge \mathbf{s} \in \mathcal{S}_z$. As a consequence, $\mathbf{s}_z \leq \mathbf{s}^* \wedge \mathbf{s} \leq \mathbf{s}$.

Next, consider the scientist with no publications $\mathbf{s}^\varnothing$, defined by $\mathbf{s}^\varnothing(n) = \varnothing$ for all $n \in \mathbb{N}$. By the full range axiom there exists $\mathbf{s}_0$ for which $I(\mathbf{s}_0) = 0$. Because $I(\mathbf{s}^\varnothing) \leq I(\mathbf{s}_0) = 0$, it follows that $\mathbf{s}^\varnothing = \mathbf{s}_0$. The full range axiom also implies that for all $n \in \mathbb{N}$, $\mathbf{s}_n \neq \mathbf{s}_0$, which implies that $\mathbf{s}_n(1) \geq 0$.

We claim that for all $n \in \mathbb{N}$ there exist $P_1(n) \in \mathbb{N}$ and $P_2(n) \in \mathbb{Z}_+$ for which $\mathbf{s}_n(w) = P_2(n)$ if $w \leq P_1(n)$ and $\mathbf{s}_n(w) = \varnothing$ otherwise. To see this, suppose false. Define, for a point $(x, y) \in \mathbb{N} \times \mathbb{Z}_+$, the function $\mathbf{s}_{(x,y)}(w) = y$ if $w \leq x$ and $\mathbf{s}_{(x,y)}(w) = \varnothing$ otherwise. Let $n \in \mathbb{N}$. Because $\mathbf{s}_n(1) \geq 0$ the finite set $\{(x, y) : \mathbf{s}_n(x) \geq y\} \neq \varnothing$ and therefore we can define $\mathbf{s}_n = \bigvee_{\{(x,y):\mathbf{s}_n(x)\geq y\}} \mathbf{s}_{(x,y)}$. Furthermore, since we assume that there is no (x, y) for which $\mathbf{s}_n = \mathbf{s}_{(x,y)}$, it follows that if $\mathbf{s}_n(x) \geq y$, then $I(\mathbf{s}_{(x,y)}) < n$. As a consequence, a repeated application of the consistency with better scientists axiom implies that $I(\mathbf{s}_n) = I\left(\bigvee_{(x,y):\mathbf{s}_n(x)\geq y} \mathbf{s}_{(x,y)}\right) < n$, a contradiction. This allows us to define $P(n) = (P_1(n), P_2(n))$.

Now, let $n, n' \in \mathbb{N}$ such that $n \leq n'$. Because $n' = I(\mathbf{s}_{n'})$ it follows that $n \leq I(\mathbf{s}_{n'})$ and therefore that $\mathbf{s}_n \leq \mathbf{s}_{n'}$. But the definition of $P(n)$ it follows that $P(n) \leq P(n')$. The full range assumption implies that if $n < n'$ then $P(n) \neq P(n')$. □

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