Higher Education Funding – A Portfolio of Loans

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Abstract

The key contribution of this article is introducing the advantages of a portfolio into higher education funding in order to develop efficient funding schemes. In higher education funding, credit market loans (CMLs) lead to underinvestment while income-contingent student loans (ICLs) produce overinvestment. The research introduces a 'portfolio regime' (PR) that allows students to combine CMLs and ICLs. The model assumes that agents privately invest in higher education after receiving a noisy signal about their future incomes. The article compares a PR with a 'competition regime' (CR) a-la Eckwert and Zilcha that allows students to choose one type of loan but forbids a portfolio. The key insight is that implementation of a PR may improve the efficiency of investment in higher education and the social welfare. Nevertheless, the PR does not maximize the social welfare because of adverse selection into the ICLs program.

Keywords: Human capital accumulation; Education policy; Adverse selection; Higher education; Income contingent loans.

JEL classification: I21; I22; I23; I24; I28; D31; H31

I would like to thank my Ph.D advisor, Itzhak Zilcha, for invaluable advice and guidance.

I. Introduction

The article introduces the common knowledge of portfolio into higher education funding. The main insights are that a portfolio of certain loans may improve the efficiency of the investment process and the social welfare. Though, it does not maximize the social welfare. I start with background on higher education funding. Then, I discuss the most common types of student loans, which leads to my contribution – a portfolio of student loans.

The motivation relates to the evidence on the importance of higher education and the shift to private funding. Abundant empirical evidence highlights the important role of higher education in generating personal incomes and promoting the economic development of countries (Barro, 1998; Bassanini and Scarpenta, 2001; Restuccia and Urrutia, 2004). Correspondingly, investment in education has increased substantially in OECD countries during the second half of the twentieth century (Greenaway and Haynes, 2003; Checchi, 2006). As a result, budgetary pressures have recently led some European countries to shift away from public funding of higher education (through various forms of income support transfers) towards private funding (based on student loans). A structural transformation towards private higher education funding requires a cautious treatment of inherent market failures.

The two existing types of student loans suffer from market failures. The first, Credit market loans (CMLs), cause underinvestment in higher education because of credit constraints, inadequate risk-sharing and insufficient risk diversification. First, financial institutions avoid providing loans to disadvantaged students or provide loans with less favorable terms. The reason is that their uncertain future incomes are considered insufficient collateral against the loans (see Galor and Zeira, 1993). Even students themselves partly grasp their future ability. Thus, without a governmental intervention, disadvantaged talented students may not obtain higher education. Even if the government guarantees access to CMLs, other factors still cause underinvestment in higher education. CMLs provide inadequate risk sharing: While future incomes are uncertain, the repayment is fixed. However, ex-ante, risk averse students prefer a lower repayment in case of bad luck and a higher repayment in case of prosperity. Income-contingent repayment provides insurance against the uncertainty in future incomes. Furthermore, the fixed repayment damages not only students but also investors. Investors who buy shares of the student loans have insufficient risk diversification. These market failures have led the literature to conclude that in the presence of CMLs, students under-invest in their higher education.

To overcome the inherent underinvestment in higher education, several countries provide a second type of student loans: Income-contingent loans (ICLs). The unique feature of ICLs – income-contingent repayment – has advantages and disadvantages. Besides their positive implications on inequality measures, ICLs operate as an ex-ante risk sharing and diversification mechanism. The reason is cross-subsidization: Agents with a higher income realization (discovered after the completion of higher education), have a larger payback. As a result, high income individuals subsidize low income ones. The first to mention the potential advantages of ICLs to students and investors is Milton Friedman (1962). He recommends ICLs because they allow students to forego a fraction of their future income flow in order to finance their higher education investment. They gain risk sharing and insurance against uncertainty in future incomes. Investors can buy ICLs and diversify their risks over students with different income prospects. The diversification reduces the interest rate on ICLs, and thus increases their attractiveness. Another advantage of ICLs is breaking even. The government can design the ICLs program to break even without governmental funds. The advantages of ICLs program have spurred an increasing interest in the literature and implementation by several countries, e.g., Australia, New Zeland, Chile, Sweden and the UK. Chapman (2006) describes the experience in Australia, the first country to implement ICLs in 1989 (see also Barr and Crawford, 1998; Nerlove, 1975; Lleras, 2004; Woodhall, 1988). However, Zilcha and Eckwert (2011) reveal the disadvantage of ICLs. Because of the cross subsidization, even individuals with negative net returns acquire higher education. Therefore, while funding higher education merely with CMLs leads to underinvestment, pure ICLs cause the opposite – overinvestment in higher education. The inefficiency in the two types of loans calls for improving the funding schemes for higher education.

To proceed one step further towards efficiency in higher education funding, I introduce a 'portfolio regime' (PR). A PR allows students to combine CMLs and ICLs. A PR is a general case of a competition regime (CR) proposed by Zilcha and Eckwert

(2011). Their CR allows students to choose one type of loan: a CML or an ICL but *forbids* a combination of them. In contrast, A PR does not constrain students to one market only. Students can choose any loan combination with non-negative shares of ICLs and CMLs. Therefore, a PR provides more financial flexibility than a CR. Moreover, it is more realistic, because no country forces students to choose one type of loan. To investigate the implications of a PR, I develop a theory of educational production with uncertainty in future incomes and full access to private higher education funding. Risk averse agents invest in their higher education after receiving a noisy ability signal. The source of heterogeneity is agents' abilities which are imperfectly correlated with their signals[1]. In this framework, I characterize the optimal loan decisions of students and derive key insights on social welfare.

First, I characterize which students choose CMLs only, ICLs only, and a portfolio of loans. The results suggest that intermediate ability agents (according to their signal) choose a portfolio, whereas the rest of the students finance their higher education via one market only. Furthermore, agents with better signals or higher wealth prefer larger shares of CMLs than others under CRRA or CARA utility functions. This pattern reverses under a quadratic utility function.

The main insights on social welfare are the following: a PR may lead to a higher social welfare than a CR. Though, a PR does not maximize the social welfare. Implementing a PR, some students enjoy the financial flexibility and choose a portfolio of loans. Clearly, they are better off than under a CR. However, their shift into a portfolio may deteriorate the borrowing terms of ICLs participants. Nevertheless, I prove that a PR Pareto dominates a CR in the common case where CMLs co-exist alongside ICLs in both regimes. A PR improves the efficiency of the investment process and leads to a lower income inequality[2]. Though introducing a PR into higher education funding basically improves the social welfare, it is not first best.

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[1] To simplify the analysis, I focus on one source of heterogeneity and ignore other influences, e.g., family background.

2 An exception is the case where all students choose ICLs only under a CR. In this case, a CR leads to higher social welfare than a PR.

The PR does not maximize the social welfare because of an externality in students' behavior. The externality originates from students' believes that their entrance into the ICLs program has a negligible effect on all ICLs participants. However, in fact, an augmented ICL share of high signal students benefits all ICLs participants. The reason is the cross-subsidization – high income individuals subsidize low income ones, thereby their participation improves the ICLs borrowing terms. The externality causes adverse selection into the ICLs program. High signal students under-invest in ICLs. The adverse selection further worsens the borrowing terms of ICLs participants and reduces the attractiveness of the program. As a result, high signal students further depart to the credit market, which pushes the financing costs of the disadvantaged even higher, increases the income inequality and reduces the social welfare. Thus, a PR improves the social welfare relative to a CR, though it is not maximized.

Section II introduces the PR model. Section III characterizes the optimal loan decision in each signal group and conducts a comparative static analysis. Additionally, I derive a closed-form solution for a quadratic utility function. Section 0 provides a social welfare analysis. Section V compares a PR with a CR a-la Eckwert and Zilcha and section 0 concludes. **All proofs are relegated to the Appendix.**

II. The Model

This section depicts the PR model. I start by describing the timeline and the human capital formation. Afterwards, I introduce the unique higher education funding scheme, a PR. Then, I present the individual behavior and production, and define a welfare index. After describing the ingredients of the model, I define PR Equilibrium.

A. Timeline

Consider an Overlapping-Generations model. The lifetime of agents consists of two periods: the youth period and the working period. In the youth period, agents acquire public education and higher education. All children obtain public education (K-12). The public education is compulsory, equal and free. Then, agents decide whether to take a loan and acquire higher education. After acquiring education, the working period arrives. In the working period, they earn a labor income based on their human capital, repay the student loan and consume the rest of their income. The two periods complete the lifetime of an agent[3].

B. Human capital formation

In this section, I describe the information on abilities and how abilities transform into human capital and incomes. The information on abilities changes at three points in life: birth, the completion of public education, and the completion of higher education. At birth, a continuum of agents $[0,1]$ is randomly endowed with innate ability $\tilde{a}^i \in \left[a^1, a^2\right] \subset \mathbb{R}_+$. The realization of agent i's ability, a^i, is unknown. After the completion of compulsory public education, agents receive an ability signal. The signal has a simple interpretation – a result of matriculation tests or high school achievements *partly* correlated with the actual ability. That is, larger signals are 'good news', because they forecast higher expected realization of ability and income. Therefore, I assume that signals and abilities satisfy the Monotone Likelihood Ratio Property (MLRP) (see Milgrom (1981))[4]. Additionally, ability signals $y \in \left[y^1, y^2\right] \subset \mathbb{R}_+$ are publicly observable and I denote their distribution by $\nu(y)$. At this point, the information on each agent is the ability signal y. Therefore, I denote all agents with signal y as signal group y. Thus, from this agent's perspective, ability is a realization of a random variable $\tilde{a}_y$ with an expectation of $\bar{a}_y = E\left[\tilde{a}_y\right]$. Yet, there is no *aggregate* uncertainty in the economy, because the ability distribution of each signal group is known. To simplify the analysis and without loss of generality, I assume

Assumption 1: $\tilde{a}_y = y + \tilde{\varepsilon}$, *and* $\tilde{\varepsilon} \sim \left(0, \sigma^2\right)$. ∘

Therefore, the signal reflects the mean ability within signal group y, i.e., $\bar{a}_y = E\left[\tilde{a}_y\right] = y$, and the variance, σ^2, measures the signal quality. If the variance is

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[3] The results remain in an extended model with an additional retirement period.

[4] The assumption of MLRP suggests that the ability distribution in a higher signal group 'First Degree Stochastically Dominates' the ability distribution in a lower signal group, i.e., $y' > y \Rightarrow \tilde{a}_{y'} \succ_1 \tilde{a}_y$.

zero, there is no uncertainty. However, I assume that the actual ability is *partly* correlated with the signal. That is, the variance is positive. After the completion of higher education, the realization of ability fully reveals and determines labor income. Thus, the information on abilities changes at three points in life.

After describing the information on abilities, I explain how they convert to human capital and incomes. After receiving the signal, agents choose their investment in higher education, $I \in \{0,1\}$. For simplicity, the investment choice is binary, i.e., they choose whether to invest in higher education ($I = 1$) or not ($I = 0$). Accordingly, the random human capital in signal group y is

$$(1) \qquad \tilde{h}_y = \begin{Bmatrix} A & , if \ I = 0 \\ A + \tilde{a}_y , & if \ I = 1 \end{Bmatrix}.$$

where A is the basic level of human capital after the completion of compulsory public education; and $A + \tilde{a}_y$ is the random level of human capital of agents in signal group y in case they invest in higher education (ex-ante). Assume that each individual inelastically supplies l units of labor. Without loss of generality, let l=1. Incomes are based on the human capital. The *random* labor income, $\omega\tilde{h}_y$, equals the human capital in equation (1), multiplied by the wage rate for an effective unit of human capital, ω. Recall that the human capital and incomes are random until the completion of higher education. Then, in the working period, there is no uncertainty. Labor incomes are based on the *realization* of ability, a^i, which is already known. After describing the human capital and income formation, I introduce the PR.

C. Higher education funding

The key novelty of a PR is allowing students to use the basic tool of capital markets – a portfolio. I start with standard assumptions on student loans. Then, I depict the two types of loans that compose the portfolio. Afterwards, I introduce the PR. I assume that the costs of higher education are normalized to '1'. Accordingly, each student receives a loan of 1 unit and repays the loan in the working period. The government monitors the repayment through the tax system, thereby there is no tax evasion. Now, the portfolio of loans consists of the two most common student loans, CMLs and ICLs, which differ in their repayment obligation. The payback of CMLs is the interest rate $R = 1 + r$. The interest rate is exogenously given by the gross international interest rate (see the following discussion in section E). In contrast, the payback of ICLs, $R\frac{a^i}{\bar{a}}$, depends on the realization of ability, a^i, and on $\bar{a}$, a plug number that breaks even student loans (see a discussion in the sequel). Accordingly, the payback is increasing as ability rises. Moreover, high income ICLs participants, with $a^i > \bar{a}$, cross-subsidize the remaining participants. That is, their repayment is larger than the interest rate, whereas all other participants' repayment is lower than the interest rate. Note that while $R\frac{a^i}{\bar{a}}$ is the *actual* ICL repayment in the working period, the repayment after the completion of public education, $R\frac{\tilde{a}_y}{\bar{a}}$, is the realization of a random variable with an expectation of $R\frac{\bar{a}_y}{\bar{a}}$ for each signal group y. After presenting the two loans that compose the portfolio, I introduce the PR.

In a PR, students simply mix the two loans I presented. That is, after the completion of public education, each agent in signal group y decides whether to finance his higher education entirely through an ICL or a CML, or combine them. Denote by $\theta_y \in [0,1]$ the CML share, and by $1-\theta_y$ the ICL share in his portfolio. Thus, the random payback of each agent in signal group y is a weighted average of the two, and the weights are the shares of each loan in the portfolio

(2) $$\theta_y R + (1-\theta_y) R\frac{\tilde{a}_y}{\bar{a}}.$$

Recall that the government determines $\bar{a}$ to break even the ICLs program. Thus, the government equates the ex-post mean repayment *across all signal groups* and the interest rate, i.e.,

(3) $$E\left[\theta_y R + (1-\theta_y) R\frac{\bar{a}_y}{\bar{a}}\right] = R$$

Recall that the interest rate is exogenously given. Using $E\left[\theta_y R\right] = RE\left[\theta_y\right]$ and equality (3), $\bar{a}$ satisfies

$$(4) \qquad \bar{a} = \frac{E\left[\left(1-\theta_y\right)\bar{a}_y\right]}{E\left[1-\theta_y\right]} = E\left[\bar{a}_y \middle| \theta_y < 1\right] - \frac{cov\left[\bar{a}_y, \theta_y\right]}{E\left[1-\theta_y\right]}$$

The first equality suggests that $\bar{a}$ is a *weighted* mean ability of ICL participants ($\theta_y < 1$). The second equality implies that the calculation of $\bar{a}$ takes into account the selection of students into the ICLs program: Suppose that $cov\left[\bar{a}_y, \theta_y\right]$ is positive. That is, signal groups with higher expected ability choose larger shares of CMLs than lower signal groups. In this case, $\bar{a}$ is lower than the mean ability of ICL participants, i.e., $\bar{a} < E\left[\bar{a}_y \middle| \theta_y < 1\right]$. The intuition is clear: When high signal groups – the subsidiaries – discard from the ICLs program, $\bar{a}$ declines, thereby the expected payback, $R\frac{\bar{a}_y}{\bar{a}}$, increases for all ICLs participants. In contrast, if $cov\left[\bar{a}_y, \theta_y\right]$ is negative, then high signal groups prefer larger shares of ICLs. Their augmented participation in the ICLs program reduces the financing costs for all ICLs participants. After presenting the borrowing terms in a PR and how it breaks even, I turn to the optimal choice of individuals.

D. Individual behavior

In this section, I describe the optimal loan decision of students, i.e., how they choose their optimal mixture of ICLs and CMLs. Then, I verify that there exists a set of students who select a portfolio of the two loans. Agents gain utility from consumption in the working period. The random consumption of agents with signal y equals their labor income net of their repayment obligation (2) given their human capital (1), i.e.,

$$(5) \qquad \tilde{c}_y = \begin{cases} A\omega & , \quad if \ I = 0 \\ \underbrace{\left(A + \tilde{a}_y\right)\omega}_{income} - \left[\underbrace{\overbrace{\theta_y R}^{CML} + \overbrace{\left(1-\theta_y\right)R\frac{\tilde{a}_y}{\bar{a}}}^{ICL}}_{repayment\ obligation}\right], & if \ I = 1 \end{cases}.$$

I assume that investment in higher education is profitable.

Assumption 2: $\bar{a}\omega > R$. ∘

Assumption 2 assures that at least one student has positive net expected returns from financing higher education with ICLs only, i.e., $\bar{a}_y\left(\omega - \frac{R}{\bar{a}}\right) > 0$. However, as a result, investment in higher education is beneficial for *all* agents, though they may differ in the mixture of ICLs and CMLs they prefer[5]. The VNM utility function $u(\tilde{c}_y)$: $\mathbb{R}_+ \to \mathbb{R}$ is twice differentiable, strictly increasing and concave. Each agent with signal y chooses the optimal share of CML, θ_y, by maximizing expected utility from consumption, (5),

$$(6) \qquad \underset{\theta_y}{Max}\left\{E\left[u\left(\tilde{c}_y\right)\right]\right\}.$$

Definition 1 *in the Appendix denotes standard CRRA, CARA and quadratic utility functions that I use in the sequel.*

After characterizing the optimization problem, I verify that the portfolio set exists. That is, there exists a set of students who prefer a portfolio $\theta_y \in (0,1)$ of the two loans rather than CMLs only or ICLs only. The first step defines three signal groups. Let

$$(7) \qquad \tilde{c}_{y,0} = \tilde{c}_y\left(\theta_y = 0\right) = A\omega + \tilde{a}_y\left(\omega - \frac{R}{\bar{a}}\right)$$

and

$$(8) \qquad \tilde{c}_{y,1} = \tilde{c}_y\left(\theta_y = 1\right) = A\omega + \tilde{a}_y\omega - R$$

denote the random consumption of an agent in signal group y if he finances his higher education through ICLs only or CMLs only, respectively. The third signal group, $\hat{y}$, denotes agents who are indifferent between ICLs only and CMLs only.

[5] This undesirable feature of ICLs that all agents invest in higher education is usually prevented by access restrictions to higher education. Note that if I restrict access to agents with positive returns, the results are qualitatively similar.

Definition 2: *The cutoff signal* $\hat{y}$ *between ICLs only and CMLs only satisfies*[6]

(9) $$E\left[u\left(\tilde{c}_{\hat{y},0}\right)\right] = E\left[u\left(\tilde{c}_{\hat{y},1}\right)\right] > u\left(A\omega\right). \circ$$

The second step asserts that agents in the cutoff signal group belong to the portfolio set. Thus, the portfolio set exists if, as a sufficient condition, the cutoff signal $\hat{y}$ lies within the signals' distribution[7]. Furthermore, continuity implies that signal groups in a sufficiently small neighborhood around $\hat{y}$ also prefer to combine the two loans.

Proposition 1: *Assume that* $\hat{y} \in \left(y^1, y^2\right)$. *Then, the portfolio set* $\theta_y \in \left(0,1\right)$ *is not empty.* ◦

The portfolio set lies between the following cutoffs,

Definition 3: *The portfolio cutoff signals* y' *and* y'' *are indifferent between a portfolio and ICLs (CMLs) only, respectively*[8],

(10) $$E\left[u\left(\tilde{c}_{y',\theta}\right)\right] = E\left[u\left(\tilde{c}_{y',0}\right)\right] \text{ and}$$

$$E\left[u\left(\tilde{c}_{y'',\theta}\right)\right] = E\left[u\left(\tilde{c}_{y'',1}\right)\right]. \circ$$

Now, after presenting how the PR works, it is left to describe the production sector and the welfare index. Then, I define the equilibrium under the PR.

[6] Assumption 2 ensures that the ICL program does not break down. Thus, the inequality holds. Note that there can be more than one cutoff signal, as I demonstrate in the sequel.

[7] Note that in CR equilibrium, the assumption that $\hat{y} \in \left(y^1, y^2\right)$ assures that the sets of CMLs only and ICLs only are not empty.

[8] Note that agents in signal groups y' and y'' strictly differ from $\hat{y}$ because they strictly prefer ICLs (CMLs) only upon CMLs (ICLs) only, respectively. Combining the concavity of the utility function and equalities (10) obtains:

$$E\left[u\left(\tilde{c}_{y',\theta}\right)\right] = E\left[u\left(\tilde{c}_{y',0}\right)\right] > \theta E\left[u\left(\tilde{c}_{y',0}\right)\right] + \left(1-\theta\right) E\left[u\left(\tilde{c}_{y',1}\right)\right]. \text{ Thus, } E\left[u\left(\tilde{c}_{y',0}\right)\right] > E\left[u\left(\tilde{c}_{y',1}\right)\right],$$

$$E\left[u\left(\tilde{c}_{y'',\theta}\right)\right] = E\left[u\left(\tilde{c}_{y'',1}\right)\right] > \theta E\left[u\left(\tilde{c}_{y'',0}\right)\right] + \left(1-\theta\right) E\left[u\left(\tilde{c}_{y'',1}\right)\right]. \text{ Thus, } E\left[u\left(\tilde{c}_{y'',1}\right)\right] > E\left[u\left(\tilde{c}_{y'',0}\right)\right].$$

E. Production

I use standard simplifying assumptions on the production sector. Recall that because of assumption 2, all agents invest in higher education. Accordingly, given the distribution of abilities, the stock of human capital (1) equals

$$(11) \qquad H = \int_{y^1}^{y^2} \bar{h}_y \nu(y) dy = A + E[\tilde{a}].$$

Using the stock of human capital and the stock of physical capital, competitive firms produce one consumption good. The production function $F(K,H)$ exhibits constant returns to scale with positive and decreasing marginal returns, $F_K > 0$, $F_H > 0$, $F_{KK} < 0$, $F_{HH} < 0$. I assume a small country and international mobility of physical capital. This assumption is consistent with the empirical observation that globalization has promoted international mobility of physical capital much more than that of labor. Therefore, the interest rate, $R = 1 + r$, is exogenously given by the gross international interest rate. Physical capital fully depreciates in the production process. Maximization of profits yields the stocks of physical capital and human capital.

$$(12) \qquad \begin{aligned} F_K\left(\frac{K}{H},1\right) &= R \\ F_L\left(\frac{K}{H},1\right) &= \omega \end{aligned}$$

That is, firms hire physical capital and human capital until their marginal product equals the interest rate and the wage rate, respectively.

F. Welfare index

In this section, I define the social planner's welfare index and its main features. The social welfare index equals

$$(13) \qquad W = \int_{y^1}^{y^2} \upsilon(\bar{c}_y) \nu(y) dy$$

where $\bar{c}_y = E[\tilde{c}_y]$

and

Assumption 3: $\upsilon : \mathbb{R}_+ \to \mathbb{R}$ *is twice differentiable, strictly increasing and concave.* ◦

The welfare function aggregates observable data – the mean consumption in each signal group given the distribution of signals. The concavity of the social planner's welfare index reflects inequality aversion. That is, social welfare rises either if the mean consumption increases in a particular signal group, or if the income inequality declines across signal groups. I shall use the index in the subsequent welfare analysis. After introducing the ingredients of the model, the following section defines a PR equilibrium.

G. A PR Equilibrium

Now, I define the equilibrium, and then explain how to solve it.

Definition 4: *Given the international gross interest rate,* $R = 1 + r$, *the distribution of abilities and the distribution of signals, PR equilibrium consists of a vector* $(\omega, K, H) \in R_+^3$, *a share of CMLs,* θ_y, *for each signal group , such that*

i. *The stock of human capital,* H, *satisfies (11).*

ii. *The wage rate and physical capital satisfy (12).*

iii. *The share of CMLs,* θ_y, *maximizes the expected utility in signal group* y, *(6).*

iv. *The ICLs program breaks even by* $\bar{a}$ *according* to *equality (4).* ◦

Given the distribution of abilities, the stock of human capital is determined by equation (11). The first equality in equation (12) implies that the exogenous interest rate, R, uniquely determines the ratio $\frac{K}{H}$, thus K; Then, the wage rate, ω, is uniquely determined by substituting $\frac{K}{H}$ in the second equality; Then, given the interest rate, the wage rate, the signal and $\bar{a}$, each agent chooses the share of CMLs, θ_y, which completes the equilibrium. The following section analyzes the optimal loan decision of each signal group, and the rest of the article conducts a welfare analysis and compares a PR with a CR.

III. Optimal loan decisions

This section first characterizes the trade off between the two types of loans; Second, I analyze low signal groups and high signal groups separately. Third, I analyze a specific case – a quadratic utility function.

A. The trade off between ICLs and CMLs

The optimal loan decision involves a trade off, which is common in financial markets. That is, a trade-off between risk and expected returns (loan repayment, in this case). Comparing ICLs and CMLs, ICLs reduce uncertainty in future incomes but increase expected repayment of high signal groups – both because of the cross-subsidization. Deriving the expected utility (6) by θ_y obtains[9]

$$(14) \qquad \frac{\partial E\left[u\left(\tilde{c}_y\right)\right]}{\partial \theta_y} = \frac{R}{\bar{a}} E\left[\left(\tilde{a}_y - \bar{a}\right) u'\left(\tilde{c}_y\right)\right] = \frac{R}{\bar{a}}\left(\left(\bar{a}_y - \bar{a}\right) E\left[u'\left(\tilde{c}_y\right)\right] + cov\left(\bar{a}_y, u'\left(\bar{c}_y\right)\right)\right)$$

$$\frac{\partial E\left[u\left(\tilde{c}_y\right)\right]}{\partial \theta_y} = \frac{R}{\bar{a}} E\left[\left(\tilde{a}_y - \bar{a}\right) u'\left(\tilde{c}_y\right)\right] = \frac{R}{\bar{a}}\left(\left(\bar{a}_y - \bar{a}\right) E\left[u'\left(\tilde{c}_y\right)\right] + cov\left(\bar{a}_y, u'\left(\bar{c}_y\right)\right)\right)$$

A careful inspection of the first order condition recognizes the trade off between risk and expected returns. The first component reflects the desire to reduce expected repayment. Its sign depends on the signal's magnitude, $\bar{a}_y - \bar{a}$. Recall that high signal groups, with $\bar{a}_y > \bar{a}$, are expected to cross-subsidize low signal ones in the ICLs program. Therefore, their ICLs expected repayment is larger than the interest rate, whereas the remaining participants enjoy a lower expected repayment. Thus, if the only concern of students had been to reduce their expected payback, then high signal students would have preferred CMLs only, whereas low signal students would have preferred ICLs only[10]. However, students are risk averse.

The second component of the first order condition (14) reveals the desire to mitigate uncertainty in future incomes. Large absolute values of $cov\left(\bar{a}_y, u'\left(\bar{c}_y\right)\right)$ suggest that the expected consumption strongly relates to the signal (see Lemma 1 in the Appendix)[11]. In this case, ICLs are a risk-sharing tool, because they reduce the

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[9] Note that $\theta_y = ArgMax\left[E\left(u\left(\tilde{c}_y\right)\right)\right]$ is unique because the expected utility is strictly concave and the random consumption (5) is linear with respect to θ_y.

[10] As a result, if agents are risk neutral, then $\bar{a} \to 0$ and the equilibrium collapses to CMLs only for all y. Thus, risk aversion is necessary to avoid this trivial solution in the PR as well as the CR.

[11] The $cov\left(\bar{a}_y, u'\left(\bar{c}_y\right)\right)$ is negative because the signal is positively correlated to the expected consumption according to MLRP and agents are risk averse. Note that an ICL has a riskier payback. Thus, it is the 'safer' loan with regards to uncertainty in future income.

dispersion of ex-post incomes. Thus, the optimal mixture of loans for each student balances the trade off between risk and expected returns.

Note that the first order condition (14) reveals *externalities* in students' behavior. Students take $\bar{a}$ as given. Thus, they assume that their decision to enter the ICLs program has a negligible effect on the ICLs expected repayment, $R\frac{\bar{a}_y}{\bar{a}}$. However, in fact, ICLs participation of high (low) signal groups improves (worsens) the borrowing terms. The externalities imply that high (low) signal students under (over)-invest in ICLs. I discuss the welfare implications of these externalities in sections 0-V. After understanding the trade off between ICLs and CMLs, the following sections analyze low signal groups and high signal groups separately.

B. Low signal groups

The optimal loan decision of agents with low income prospects, i.e., $\bar{a}_y \le \bar{a}$, is quite trivial. They prefer to fund their higher education entirely through the ICLs program.

Proposition 2: *Suppose* y *satisfies* $\bar{a}_y \le \bar{a}$. *Then,* $\theta_y = 0$. ◦

They choose 'ICLs only' because of two reasons. First, the cross subsidization provides them relatively improved borrowing terms. That is, their expected repayment obligation, $\frac{R\bar{a}_y}{\bar{a}}$, is lower than that of CMLs (the interest rate). Second, they gain insurance against the uncertainty in future incomes. These reasons lead them to prefer ICLs only[12]. In contrast, for high signal groups the optimal loan decision is more complicated, as the following section explains.

C. High signal groups

In this section, I focus on the optimal loan decision of high signal groups, with $\bar{a}_y > \bar{a}$, and discuss two arguments: First, high signal groups may choose a portfolio of the

[12] Note that according to the first order condition (14), low signal groups actually prefer a negative share of CMLs, but are restricted to $\theta_y \in [0,1]$. Even signal group $y = \bar{a}$, with identical expected repayment under CMLs and ICLs, prefers 'ICLs only' because of the risk aversion. Its random consumption with a CML is a Mean Preserving Spread of its random consumption with an ICL.

two loans. Second, with CRRA or CARA utility functions, as the signal rises, they use increasingly larger shares of CML. First, a subset of high signal students chooses a portfolio of the two loans.

Corollary 1: *Assume that* $\hat{y} \in \left[\underline{y}, \overline{y}\right]$. *Then, the portfolio set* $\theta_y \in (0,1)$ *exhibits* $\overline{a}_y > \overline{a}$. ◦

Recall that the portfolio set exists and low signal groups prefer ICLs only (see propositions 1 and 2). Therefore, the portfolio set consists of high signal groups[13]. It is not surprising, because high signal groups face a trade-off between ICLs and CMLs. ICLs provide them insurance against the uncertainty in future incomes and at the same time less favorable borrowing terms than CMLs. The balance of these contradicting incentives may produce a portfolio, CMLs only or ICLs only, depending on their signal, risk aversion and wealth.

Now, using comparative static I analyze the effect of the signal on the optimal loan decision of high signal groups. An increase in the signal augments the expected income, which has two effects: First, a higher expected income implies worse ICLs borrowing terms. That is, the expected ICLs repayment, $\frac{\overline{a}_y}{\overline{a}} R$, rises. As a result, ICLs becomes less attractive[14]. Second, the Absolute Risk Aversion (ARA) alters. Assuming a CRRA utility function, wealthier agents are *less* risk averse (see definition 1). As a result, they are willing to tolerate a more risky income in order to raise its expectancy. Combining the two effects, as the signal rises, CRRA students use increasingly larger shares of CMLs to finance their higher education. The result further holds for CARA utility function, in which ARA does not depend on wealth.

Proposition 3: U*nder CRRA or CARA utility functions,* θ_y *is an increasing function of* y. ◦

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[13] It is easy to verify that $\overline{a}_y > \overline{a}$ is a necessary condition for an interior solution in equation (14).

[14] Note that the price effect of a larger signal consists of a substitution effect (ICL become more expensive) and a negative income effect. As income declines, agents may wish to increase the share of ICL. However, according to Proposition 2 in Landsberger and Meillison (1989), the substitution effect overcomes the income effect (see p. 208-209 there).

The following section illustrates the results so far.

D. Illustration of the optimal loan decisions

This section summarizes the optimal loan decisions of all signal groups with CRRA or CARA utility functions. Illustration 1 demonstrates the CML share under a PR (the completed line) and a CR a-la Zilcha and Eckwert (2011) (the dashed line). Note that a CR is a private case of a PR, where agents are not allowed to choose a portfolio.

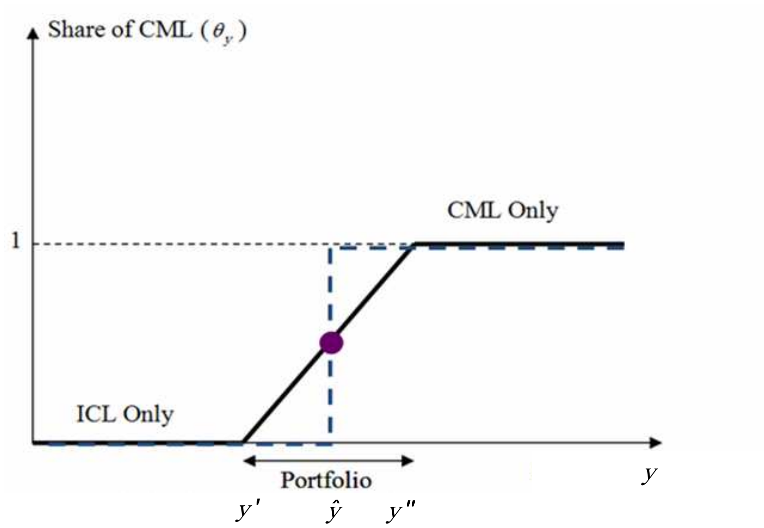

Illustration 1. CMLs share with a CRRA or A CARA utility function. The X-axis denotes the expected ability given the signal and the Y-axis denotes the CMLs share $\theta_y \in [0,1]$. $\theta_y = 0$ if the funding scheme consists of ICLs only and $\theta_y = 1$ if the agent prefers funding entirely through CML. The completed line represents PR; and the dashed line denotes CR. To simplify the illustration, I draw θ_y as a straight line. y' denotes the cutoff signal between a portfolio and ICLs only; y'' denotes the cutoff signal between a portfolio and CMLs only (see definition 3).

Low signal groups, with $\bar{a}_y \le \bar{a}$, finance their higher education entirely through the ICLs program (recall propositions 2 and 5). In contrast, for high signal groups, with $\bar{a}_y > \bar{a}$, selecting ICLs only has a cost in terms of expected consumption. As a result, a set of these students prefer a portfolio (recall corollary 1), including indifferent agents between ICLs only and CMLs only (recall proposition 1). Therefore, the portfolio set has intermediate signals. However, agents with extreme signals select one market only. While students with CRRA or CARA utility functions use increasingly larger shares of CMLs as the signal rises (recall proposition 3), the following section suggests that this pattern does not hold for quadratic utility functions.

E. A quadratic utility function: A closed-form solution

In this section, I assume that the utility function is quadratic with the range $u'(\tilde{c}_y) = \alpha - \beta \tilde{c}_y \geq 0$ (see definition 1). In this case, I obtain a closed-form solution. Using the closed-form solution, I reveal the concavity of the CML share and analyze its determinants. Afterwards, I discuss existence. Rearranging the random consumption $\tilde{c}_y$ (5) and substituting $u'(\tilde{c}_y) = \alpha - \beta \tilde{c}_y$ in the first order condition (14), an interior solution satisfies:

$$\frac{\partial E\left[u(\tilde{c}_y)\right]}{\partial \theta_y} = \frac{R}{\bar{a}} E\left[\left(\tilde{a}_y - \bar{a}\right)\left(\alpha - \beta\left(\left(A + \tilde{a}_y\right)\omega - R\frac{\tilde{a}_y}{\bar{a}} + \frac{R}{\bar{a}}\theta_y\left(\tilde{a}_y - \bar{a}\right)\right)\right)\right]$$

$$(15) \qquad = kE\left[\tilde{a}_y - \bar{a}\right] - \left(\omega - \frac{R}{\bar{a}}\left(1 - \theta_y\right)\right) E\left[\left(\tilde{a}_y - \bar{a}\right)^2\right]$$

$$= 0$$

where $k = \frac{\alpha}{\beta} - \left(\left(A + \bar{a}\right)\omega - R\right)$ is positive[15].

Substituting $E\left[\left(\tilde{a}_y - \bar{a}\right)^2\right] = \left[E\left(\tilde{a}_y - \bar{a}\right)\right]^2 + \sigma^2$ in equality (15), I solve the CML share.

$$(16) \qquad \theta_y = \frac{k\bar{a}\left(y - \bar{a}\right)}{R\left(\left(y - \bar{a}\right)^2 + \sigma^2\right)} - \left(\frac{\bar{a}\omega - R}{R}\right)^{16}.$$

Equation (16) reveals that the CML share is *concave* because of two contradicting effects. First, a larger signal forecasts a higher expected income, thereby a larger ICLs expected repayment. As a result, agents prefer to increase the share of CML. Second, in a quadratic utility function, wealthier agents are more risk averse. Thus, they are

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[15] See Lemma 2 on the Appendix; I obtain the second equality in (15) using:
$E\left[\tilde{a}_y\left(\tilde{a}_y - \bar{a}\right)\right] = E\left[\left(\tilde{a}_y - \bar{a}\right)^2\right] + \bar{a}E\left[\tilde{a}_y - \bar{a}\right]$.

[16] $E\left[\left(\tilde{a}_y - \bar{a}\right)^2\right] = E\left[\tilde{a}_y^2\right] - 2\bar{a}E\left[\tilde{a}_y\right] + \bar{a}^2$. Using assumption 1, $E\left[\tilde{a}_y^2\right] = E\left[\left(\bar{a}_y + \varepsilon\right)^2\right] = \bar{a}_y^2 + \sigma^2$;
Note that it is easy to confirm from equation (16) that the results in proposition 2 and corollary 1 hold. That is, low signal groups, with $y \leq \bar{a}$, participate solely in the ICL-program, whereas high signal students may choose a portfolio of the two loans.

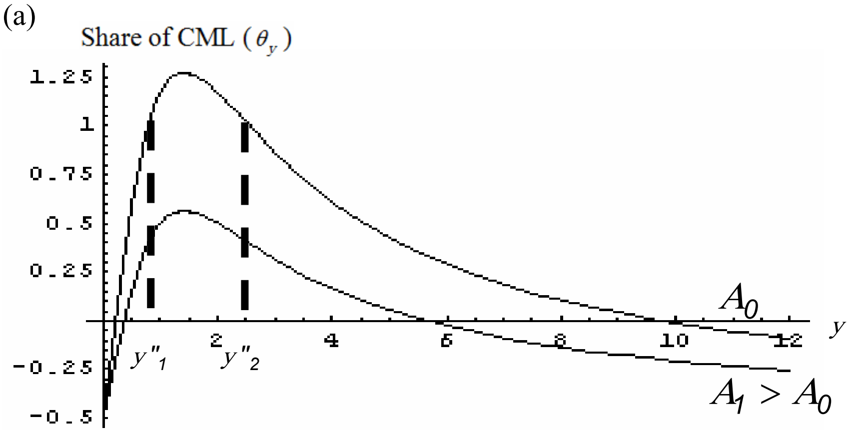
willing to forego more consumption to acquire ICLs as a risk-sharing tool. Therefore, these two contradicting effects generate a *concave* CML share.

Proposition 4

θ_y is a concave function of y. ◦

Illustration 2 summarizes the optimal loan decisions of all signal groups.

(a)



(b)

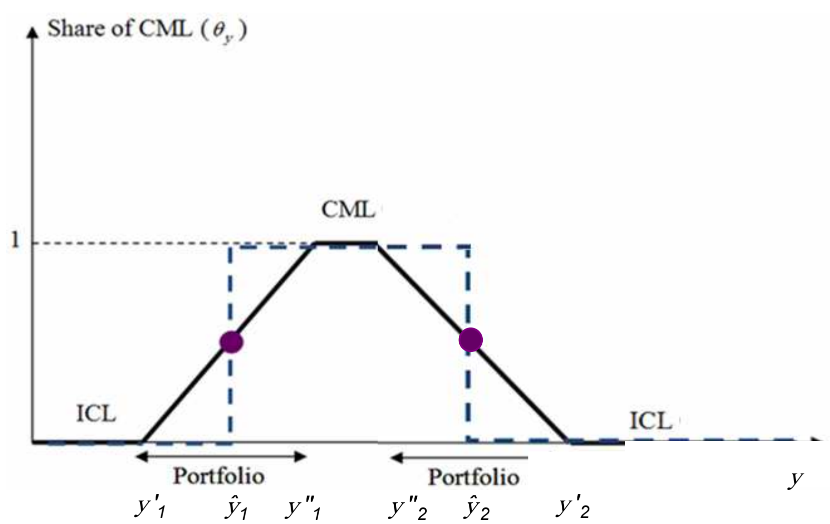


Illustration 2. CMLs share under a quadratic utility function. The X-axis denotes the signal y. $y'_{1,2}$ are the cutoff signals between a portfolio and ICLs only; $y''_{1,2}$ are the cutoff signals between a portfolio and CMLs only (recall definition 3). (a) Two illustrations of equation (16). The lower illustration demonstrates a higher basic level of human capital, i.e., $A_1 > A_0$ (recall equation (1)). (b) An illustration of the CML share under the constraint that $\theta_y \in [0,1]$. The completed line represents PR; and the dashed line denotes CR. To simplify the illustration, I draw θ_y as a straight line.

According to illustration 2 and proposition 5, agents with sufficiently low signals (lower than $\bar{a} + \sqrt{\sigma^2}$) behave qualitatively similar to the cases of CRRA and CARA utility functions (recall proposition 3). That is, they increase their CML share as the signal ascends. Intermediate signal agents, with $y \in [y''_1, y''_2]$, prefer CMLs only. However, for sufficiently large signals the opposite occurs. The increasing ARA becomes the dominant factor. That is, as the signal rises, agents reduce the CMLs share because they become more risk averse. The highest signal groups, with $y \geq y'_2$, participate solely in the ICLs program. Thus, the CML share is concave.

Besides the concavity, equation (16) reveals the main determinants of the optimal loan decision: ARA, wealth and signal quality. First, a set of parameters affect the ARA. Recall that assuming a quadratic utility function, agents are more risk averse if β/α rises or if they are more affluent, i.e., A or ω grow (recall definition 1). Higher ARA results in a larger ICL share. The second reason to increase insurance through ICLs is income uncertainty. Recall that the variance of the ability distribution of each signal group, σ^2, measures the signal quality. Because the variance is positive, the actual ability is *partly* correlated to the signal (recall assumption 1). If the variance increases, then the signal becomes less informative, thereby the uncertainty in future incomes grows. To diminish the increasing uncertainty, high signal groups increase their ICL share.

Corollary 2: *θ_y is decreasing in β/α, A, ω and σ^2.* ◦

For example, illustration 2(a) demonstrates that as agents become more risk averse, they increase their participation in the ICLs program. Ceteris paribus, I increase the basic level of human capital, i.e., $A_1 > A_0$, which increases incomes, and thus the ARA. As a result, the 'ICLs only' set expands at the expense of the portfolio set, which in turn pushes the 'CMLs only' set. Recall that a qualitatively similar results occur if β/α, ω, or σ^2 rise. That is, higher ARA, wealth or signal quality reduce CML shares.

Moreover, corollary 2 has implications regarding the existence of the three sets: portfolio, 'CMLs only' and 'ICLs only'. Corollary 2 implies that a sufficiently high ARA, wealth, and σ^2 generate an equilibrium with all students choosing ICLs only.

Proposition 5: *If* $\sigma^2 \geq \left(\frac{k\bar{a}}{2(\bar{a}\omega - R)} \right)^2$*, then* $\theta_y = 0$ *for all y.* ◦

To avoid this trivial solution and have a portfolio set, I add a necessary assumption that the ARA, wealth, and σ^2 are sufficiently low.

Assumption 4: $\sigma^2 < \left(\frac{k\bar{a}}{2(\bar{a}\omega - R)} \right)^2$. ◦

A more binding assumption is necessary to ensure existence of the 'CMLs only' set.

Assumption 5: $\sigma^2 \leq \left(\frac{k}{2\omega} \right)^2$. ◦

Corollary 3 summarizes the conditions for existence of the three sets: portfolio, CMLs only and ICLs only.

Corollary 3:

(a) The 'ICLs only' set is not empty.

(b) If assumption 4 holds and $y^2 > y'_1$*, then the portfolio set is not empty.*

(c) If assumption 5 holds and $y^2 > y''_1$*, then the 'CMLs only' set is not empty.* ◦

For example, in illustration 2(a), when the basic level of human capital equals A_0, the three sets exist. In this case, assumption 5 holds and the upper bound of the ability distribution, y^2, is sufficiently large. However, an increase in the basic level of human capital, $A_1 > A_0$, eliminates the 'CMLs only' set completely. In this case, assumption 5 does not hold. After analyzing the optimal loan decisions, the rest of the article examines the social welfare implications of the PR and compares it to the CR.

IV. Social welfare

In this section, I demonstrate that the PR does not maximize the social welfare (13). I describe the externalities in students' behavior and their two implications: overinvestment in higher education and adverse selection into the ICLs program. The source for inefficiencies in higher education funding is externalities in students' behavior. Students believe that their decision whether to join the ICLs program or not has a negligible effect on all ICL participants. However, in fact, an increase in the ICL share of high signal groups reduces the expected repayment obligation of *all* ICLs participants, $R\frac{\bar{a}_y}{\bar{a}}$ (because $\bar{a}$ rises, recall equation (4)). In contrast, an augmented ICLs participation of low signal groups, who are expected to be subsidized by the program, damages the borrowing terms of *all* ICL participants. Because students do not internalize their influence on the borrowing terms, high signal students under-invest in ICLs and low signal students over-invest in ICLs relative to the social optimum. Furthermore, low signal students, spending high signal groups' money, invest in higher education even if the expected net returns is negative. As a result, the ICL program is adversely selected towards low signal groups. Therefore, the terms of loan repayment become worse for all ICLs participants, which further reduces the attractiveness of the program. Consequently, high signal students further depart to the credit market to lower their repayment obligation. This effect pushes the financing costs of low signal groups even higher, which further reduces the social welfare and increases the spread of incomes.

Proposition 6

(a) Aggregate investment in education is suboptimally high.

(b) Investment of high signal groups in ICLs is suboptimally low. ◦

Thus, PR does not maximize the social welfare. Illustration 3 demonstrates that the portfolio cutoff signals do not coincide with their socially optimal levels. The arrows indicate the direction for improving social welfare. The social planner prefers to increase the ICLs participation of high signal groups.

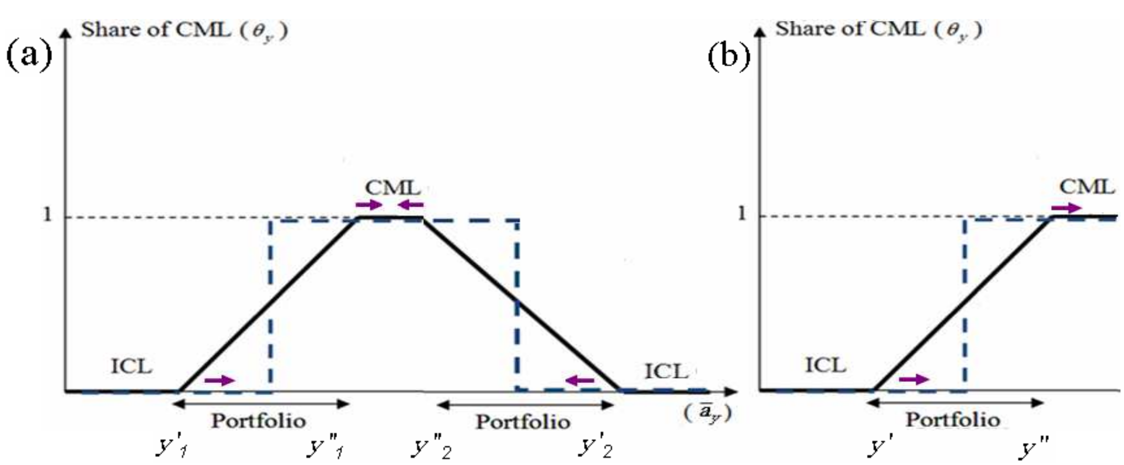


Illustration 3 CMLs share under (a) a quadratic utility function and – (b) a CRRA or a CARA utility function. The arrows indicate the direction for improving social welfare.

While a PR does not maximize the social welfare, the following section demonstrates that in most cases a PR Pareto dominates a CR.

V. Portfolio versus Competition

This section compares between a PR and a CR. First, I discuss the effect of implementing a PR on each signal group. Second, I prove that in most cases a PR Pareto dominates a CR. Suppose that a CR exists and now the government establishes a PR. That is, agents are not restricted to one market only anymore. What are the implications on each signal group? Some students enjoy this financial flexibility and decide to combine the two loans. They are clearly better off, because they *voluntarily* shift to a portfolio. That is, their preferences reveal that they prefer a portfolio rather than borrowing in one market only. The rest of the students do not change their loan decision and still finance their higher education through one market only[17]. Signal groups who remain entirely in the credit market enjoy the same borrowing terms. Thus, they are indifferent between the two regimes. Therefore, a shift to a PR does not damage the 'CMLs only' set and the portfolio set.

Now, it is left to analyze the effect on signal groups who remain entirely in the ICLs program. Though they continue choosing ICLs only, they may be worse off. The shift of other students to a portfolio may deteriorate their borrowing terms. That is, their expected payback, $R\frac{\bar{a}_y}{\bar{a}}$, may rise (if $\bar{a}$ declines). Illustration 4 demonstrates the effect of the portfolio set on ICLs participants under CRRA or CARA utility functions (A similar illustration is derived for a quadratic utility function). For this purpose, I divide the portfolio set into two sub-sets. The right set consists of high signal groups who partially enter the ICLs program i.e., their θ_y declines. In contrast, the left set includes high signal groups who partially exit the ICLs program, i.e., their θ_y increases (recall corollary 1). Consequently, while the right triangle improves the borrowing terms of ICLs participants, the left triangle worsens them.

[17] Note that they must choose the same market because of continuity.

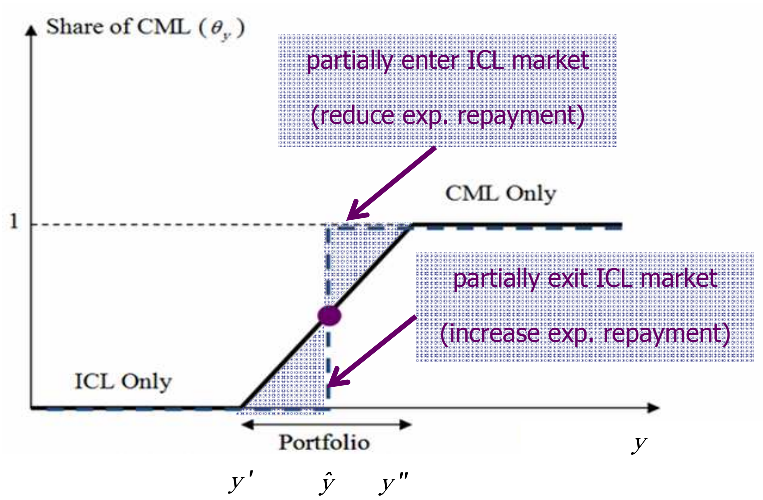


Illustration 4. CMLs share under CRRA or CARA functions. The completed line represents PR; and the dashed line denotes CR. Allowing agents to combine ICLs and CMLs, the right (left) triangle denotes individuals who partially enter (exit) the ICLs program. To simplify the illustration, I draw θ_y as a straight line. A similar illustration is derived for a quadratic utility function.

Because of the offsetting effects, detecting whether ICLs participants are better off requires a formal proof.

Proposition 7

Suppose that the utility function is CRRA or CARA or quadratic.

Then,

(a) if $\hat{y} \in \left(y^1, y^2\right)$*, then a PR Pareto dominates a CR.*

(b) if $y^2 \in \left(y', \hat{y}\right)$*, then a CR leads to higher social welfare than a PR.* ◦

Proposition 7 compares the PR and the CR in two cases. The common case, demonstrated in illustration 4, assumes that CMLs co-exist alongside ICLs, i.e., $\hat{y} \in \left(y^1, y^2\right)$ (recall proposition 1). In this case, the PR Pareto dominates the CR because ICLs participants are better off as well as the portfolio set, whereas signal groups with CMLs only are not damaged. The income inequality declines, because the spread between incomes in high signal groups and incomes in low signal groups shrinks.

The only exception to this result occurs if under a CR all students choose ICLs only. In this case, a CR leads to higher social welfare than the PR.

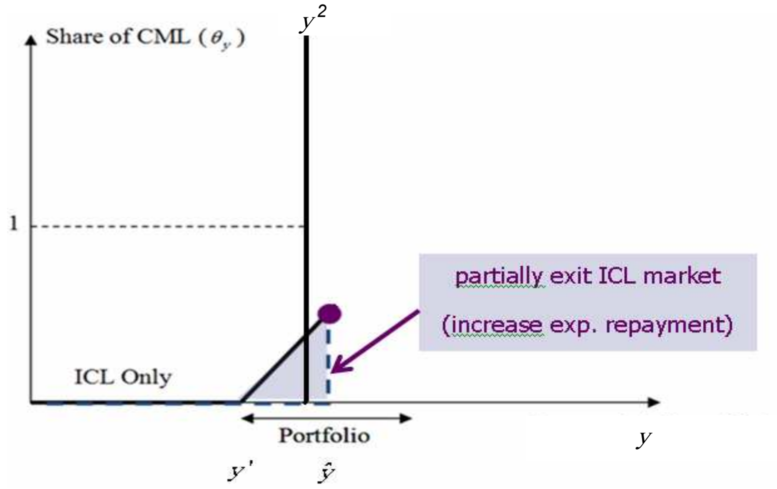


Illustration 5. CMLs share for $y^2 \in (y', \hat{y})$. The completed line represents PR; and the dashed line denotes CR.. Allowing agents to combine ICLs and CMLs, the triangle denotes individuals who partially exit the ICLs program. To simplify the illustration, I draw θ_y as a straight line.

It is easy to see that shifting to a PR, high signal agents partially exit the ICLs program. Consequently, ICLs participants are damaged because their borrowing terms deteriorate. Then, adverse selection occurs and the income distribution becomes less equal. This is the only exception to the Pareto dominance of the PR on the CR.

VI. Conclusion

I introduce the basic tool of financial markets – a portfolio – into student loans. I characterize the optimal loan decisions of students and then derive key insights on social welfare. Characterizing the optimal loan decisions, I find that the portfolio set consists of intermediate ability agents. Agents with better signals or higher wealth prefer larger shares of CMLs than others under CRRA or CARA utility functions. This pattern reverses under a quadratic utility function. Note that similar results can be generated using additional assumptions instead of specific utility functions.

Second, I derive key insights on social welfare. In most cases, a PR leads to higher social welfare relative to a CR. Nevertheless, the PR is still inefficient. The selection of students between the two types of loans involves externalities. Individuals do not consider their effect on the borrowing terms of all ICLs participants. As a result, aggregate investment in education is sub-optimally high. Moreover, there is adverse selection of students into the ICLs program, which damages the borrowing terms in the ICL program. Therefore, it is socially beneficial to increase the ICL share of high signal groups at the expense of low signal groups.

In future research, I plan to analyze policies and circumstances that may alleviate the externalities. One suggestion is a government policy that restricts access to higher education similar to Eckwert and Zilcha (2011). Another suggestion is subsidizing ICLs. Another key factor is the quality of signals. A larger uncertainty may encourage high signal groups to increase their ICL share. These circumstances may lead to higher social welfare.

VII. Appendix

Definition 1:

a. *The utility function* $u\left(\tilde{c}\right)=\frac{\tilde{c}^{1-\gamma}}{1-\gamma}$ *exhibits constant relative risk aversion (CRRA).*

The Absolute Risk Aversion (ARA), $-\frac{u''}{u'}=\frac{\gamma}{\tilde{c}}$, is decreasing as the income rises.

b. *The utility function* $u\left(\tilde{c}\right)=1-e^{-\lambda\tilde{c}}$ *exhibits constant absolute risk aversion (CARA).* The ARA equals λ.

c. *The utility function* $u\left(\tilde{c}\right)=\alpha\tilde{c}-\frac{1}{2}\beta\tilde{c}^{2}$ *is quadratic.*

The ARA, $\frac{\beta}{\alpha-\beta\tilde{c}}$, is increasing as the income rises. Because the utility function is strictly increasing and concave, $\alpha>0, \quad \beta>0$, and $\frac{\alpha}{\beta}>\tilde{c}_y, \quad \forall\tilde{c}_y$.∘

Proof of proposition 1:

Take

$$(17) \qquad \tilde{c}_{y,\lambda}=\lambda\tilde{c}_{y,1}+\left(1-\lambda\right)\tilde{c}_{y,0}\text{, for all } \lambda\in\left(0,1\right).$$

Then, substituting $\tilde{c}_{y,0}$, $\tilde{c}_{y,1}$ in equation (17), I obtain

$$\tilde{c}_{y,\lambda}=\tilde{c}_y\left(\theta_y=\lambda\right)=A\omega+\tilde{a}_y\omega-\lambda R-\left(1-\lambda\right)\frac{\tilde{a}_y}{\bar{a}}R, \quad \text{for all } \lambda\in\left(0,1\right).$$

Now, because the utility function is concave, it satisfies the following:

$$E\left[u\left(\tilde{c}_{y,\lambda}\right)\right]>\lambda E\left[u\left(\tilde{c}_{y,0}\right)\right]+\left(1-\lambda\right)E\left[u\left(\tilde{c}_{y,1}\right)\right].$$

This inequality can be rewritten as

$$(18)\qquad \begin{aligned}&E\left[u\left(\tilde{c}_{y,\lambda}\right)\right]>\lambda\left(E\left[u\left(\tilde{c}_{y,0}\right)\right]-E\left[u\left(\tilde{c}_{y,1}\right)\right]\right)+E\left[u\left(\tilde{c}_{y,1}\right)\right] \text{ or}\\ &E\left[u\left(\tilde{c}_{y,\lambda}\right)\right]>(1-\lambda)\left(E\left[u\left(\tilde{c}_{y,1}\right)\right]-E\left[u\left(\tilde{c}_{y,0}\right)\right]\right)+E\left[u\left(\tilde{c}_{y,0}\right)\right]\end{aligned}$$

Now, recall that the cutoff signal $\hat{y}$ satisfies equation (9). Substituting $E\left[u\left(\tilde{c}_{\hat{y},0}\right)\right]=E\left[u\left(\tilde{c}_{\hat{y},1}\right)\right]$ in equation (18) implies that $E\left[u\left(\tilde{c}_{\hat{y},\lambda}\right)\right]>E\left[u\left(\tilde{c}_{\hat{y},1}\right)\right]$ and $E\left[u\left(\tilde{c}_{\hat{y},\lambda}\right)\right]>E\left[u\left(\tilde{c}_{\hat{y},0}\right)\right]$ for all $\lambda\in(0,1)$. ∘

Lemma 1:
$cov\left(\bar{a}_y, u'\left(\bar{c}_y\right)\right)$ is negative, and it becomes more negative if θ_y rises.

Proof: The relation between the expected consumption and the signal $\partial\bar{c}_y/\partial\bar{a}_y=\omega-\frac{(1-\theta_y)R}{\bar{a}}$ is positive (recall assumption 2). The $cov\left(\bar{a}_y, u'\left(\bar{c}_y\right)\right)$ is negative because agents are risk averse. An increase in θ_y amplifies the relation between the signal and the expected consumption, $\partial^2\bar{c}_y/\partial\bar{a}_y\partial\theta_y=\frac{R}{\bar{a}}>0$. ∘

Proof of proposition 2:
The first term in equation (14) is negative or zero because $\bar{a}_y\le\bar{a}$. The second term is negative according to Lemma 1 in the Appendix. ∘

Lemma 2: Suppose that the utility function is quadratic and y satisfies $y>\bar{a}$. *Then,* $k>0$.

Proof: The expected consumption (see equation (5)) is given by

$$(19)\qquad \begin{aligned}E\left(\tilde{c}_y\right)&=A\omega+E\left(\tilde{a}_y\right)\left(\omega-\left(1-\theta_y\right)\frac{R}{\bar{a}}\right)-\theta_y R\\ &>A\omega+\bar{a}\left(\omega-\left(1-\theta_y\right)\frac{R}{\bar{a}}\right)-\theta_y R\\ &=\left(A+\bar{a}\right)\omega-R\end{aligned}.$$

The inequality derives from $y>\bar{a}$ and assumption 2. Recall that the utility function is strictly increasing in $\tilde{c}_y$, i.e., $\frac{\alpha}{\beta}>\tilde{c}_y,\quad\forall\tilde{c}_y$. In particular,

$$(20)\qquad \frac{\alpha}{\beta}>E\left(\tilde{c}_y\right)$$

Combining equations (19) and (20), $\frac{\alpha}{\beta}>\left(A+\bar{a}\right)\omega-R$. That is, $k>0$ (recall that k is defined in equation (21)). ∘

Proof of proposition 4:
The cutoff signals between a portfolio and ICL (CML) only are $y'_{1,2}-\bar{a}=\frac{A'}{2C}\pm\sqrt{\left(\frac{A'}{2C}\right)^2-B}$ and

$y''_{1,2}-\bar{a} = \frac{A'}{2(C+1)} \pm \sqrt{\left(\frac{A'}{2(C+1)}\right)^2 - \sigma^2}$, respectively (recall definition 3). θ_y, given by equation (16), is positive if $-C(y-\bar{a})^2 + A'(y-\bar{a}) - C\sigma^2 > 0$. Assumption 4 ensures that there are two positive roots to this quadratic equation, given by $y'_1-\bar{a} < y'_2-\bar{a}$. θ_y is concave because $C > 0$. Thus, θ_y assumes positive values between the roots. The maximum level of θ_y is obtained by $\frac{\partial \theta_y}{\partial(y-\bar{a})} = \frac{-A\left((y-\bar{a})^2 - \sigma^2\right)}{\left((y-\bar{a})^2+\sigma^2\right)^2} = 0$. That is, $y-\bar{a} = \pm\sqrt{\sigma^2}$. ∘

Proof of proposition 5:

To ease the presentation, I define $A' = \frac{k\bar{a}}{R}$ and $C = \frac{\bar{a}\omega - R}{R}$. Substitute *A'* and *C* in equation (16) and rearranging, θ_y is non-positive for all *y* if $-C(y-\bar{a})^2 + A'(y-\bar{a}) - (\sigma^2)C \le 0$. There is a single root to this quadratic equation or no root if $A'^2 \le 4\sigma^2 C^2$. Its concavity follows from $C > 0$. ∘

Proof of corollary 3:

(a)(b) see proof of proposition 5.

(c) θ_y, given by equation (16), equals '1' (recall that it cannot exceed 1) if $-(C+1)(y-\bar{a})^2 + A'(y-\bar{a}) - \sigma^2(C+1) \ge 0$. There is a single root or two positive roots to this quadratic equation, given by $y''_{1,2}-\bar{a}$, if $A'^2 \ge 4\sigma^2(C+1)^2$. $\theta_y = 1$ between the roots because θ_y is concave.

Proof of proposition 6:

(a) The returns of an agent in signal group y from acquiring higher education, $\bar{a}_y\omega$, is the expected excess income above his income as a high school graduate. The cost of higher education is normalized to '1', which in future values equals R. Thus, the expected net returns of signal group y is $\bar{a}_y\omega - R$.

Accordingly, it is efficient that only students with signals higher than $y_e = \frac{R}{\omega}$, will acquire higher education. However, according to assumption 2, investment in higher education through ICL is beneficial for *all* students. As a result, all students acquire higher education. ∘

(b) Suppose that the utility function exhibits CRRA or CARA. First, I prove that $\frac{\partial W(y')}{\partial y'} > 0$. Then, I prove that $\frac{\partial W(y'')}{\partial y''} > 0$. Aggregation of the mean consumption (5) in each signal group yields

$$\int_{\underline{y}}^{\overline{y}} \bar{c}_y v(y)dy = \int_{\underline{y}}^{y'} \left(A\omega + \bar{a}_y \left(\omega - \frac{R}{\bar{a}} \right) \right) v(y)dy + \int_{y'}^{y''} \left(A\omega + \bar{a}_y \left(\omega - \frac{(1-\theta_y)R}{\bar{a}} \right) - \theta_y R \right) v(y)dy + \int_{y''}^{\overline{y}} \left(A\omega + \bar{a}_y\omega - R \right) v(y)dy$$

$$(21) \quad = \int_{\underline{y}}^{\overline{y}} \left(A\omega + \bar{a}_y\omega - R \right) v(y)dy + R\left(\int_{\underline{y}}^{y'} \left(1 - \frac{\bar{a}_y}{\bar{a}} \right) v(y)dy + \int_{y'}^{y''} \left(1 - \frac{(1-\theta_y)\bar{a}_y}{\bar{a}} - \theta_y \right) v(y)dy \right)$$

$$= \int_{\underline{y}}^{\overline{y}} \left(A\omega + \bar{a}_y\omega - R \right) v(y)dy + \frac{R}{\bar{a}} \left(\underbrace{\bar{a} \int_{\underline{y}}^{y''} (1-\theta_y) v(y)dy - \int_{\underline{y}}^{y''} \bar{a}_y (1-\theta_y) v(y)dy}_{=0 \ (4)} \right)$$

I obtain the last two equalities because signal groups $\left[\underline{y}, y'\right]$, choose ICL only, i.e., $\theta_y = 0$, and because the ICL program breaks even according to equality (4). Note that equality (4) equals $\bar{a} = \int_{\underline{y}}^{y''} (1-\theta_y)\bar{a}_y v(y)dy \Big/ \int_{\underline{y}}^{y''} (1-\theta_y) v(y)dy$ in the case of CRRA or CARA utility functions because signal groups with $(y'', \overline{y})$ choose CMLs only ($\theta_y = 1$). Since the RHS of equality (21) is independent of y', differentiation with respect to y' obtains

$$(22) \quad \frac{R(\partial \bar{a}/\partial y')}{\bar{a}^2} \left(\int_{\underline{y}}^{y'} \bar{a}_y v(y)dy + \int_{y'}^{y''} \left((1-\theta_y)\bar{a}_y \right) v(y)dy \right) = v(y')\left(\bar{c}_{y',\theta} - \bar{c}_{y',0} \right).$$

Now, increasing y' changes the size of the relevant sets, and increases $\bar{a}$, which damages the borrowing terms for ICL participants. Differentiating the welfare function,
$W = \int_{\underline{y}}^{y'} \upsilon\left(\bar{c}_{y,0}\right) v(y)dy + \int_{y'}^{y''} \upsilon\left(\bar{c}_{y,\theta}\right) v(y)dy + \int_{y''}^{\overline{y}} \upsilon\left(\bar{c}_{y,1}\right) v(y)dy$
using equations (7) and (8), yields

$$\frac{\partial W}{\partial y'} = v(y')\left(\upsilon\left(\bar{c}_{y',0}\right) - \upsilon\left(\bar{c}_{y',\theta}\right) \right) +$$
$$\frac{R(\partial \bar{a}/\partial y')}{\bar{a}^2} \left(\int_{\underline{y}}^{y'} \left(\partial\upsilon\left(\bar{c}_{y,0}\right) / \partial \bar{c}_{y,0} \right) \bar{a}_y v(y)dy + \int_{y'}^{y''} \left(\partial\upsilon\left(\bar{c}_{y,\theta}\right) / \partial \bar{c}_{y,\theta} \right) \left((1-\theta_y)\bar{a}_y \right) v(y)dy \right)$$
$$> v(y')\left(\upsilon\left(\bar{c}_{y',0}\right) - \upsilon\left(\bar{c}_{y',\theta}\right) \right) +$$
$$\left(\partial\upsilon\left(\bar{c}_{y'',\theta}\right) / \partial \bar{c}_{y'',\theta} \right) \frac{R(\partial \bar{a}/\partial y')}{\bar{a}^2} \left(\int_{\underline{y}}^{y'} \bar{a}_y v(y)dy + \int_{y'}^{y''} \left((1-\theta_y)\bar{a}_y \right) v(y)dy \right)$$
$$\overset{(22)}{=} v(y')\left(\upsilon\left(\bar{c}_{y',0}\right) - \upsilon\left(\bar{c}_{y',\theta}\right) \right) + \left(\partial\upsilon\left(\bar{c}_{y'',\theta}\right) / \partial \bar{c}_{y'',\theta} \right) v(y')\left(\bar{c}_{y',\theta} - \bar{c}_{y',0} \right)$$
$$> 0$$

I assume that the Frontier Theorem holds. That is, I ignore feedback effects of $\bar{a}$ on the optimal choices of θ_y, because $\frac{\partial W(y')}{\partial \theta_y} \frac{\partial \theta_y}{\partial \bar{a}} \frac{\partial \bar{a}}{\partial y'} = 0$. The first and second inequalities follow from MLRP ($\bar{c}_y$ is strictly increasing in y, see assumption 1) and the concavity of $\upsilon(\cdot)$ (see assumption 3). Recall that the cutoff signal y' is indifferent between ICL only and a portfolio (recall definition 3). Thus, because of the risk aversion, $\bar{c}_{y',\theta} > \bar{c}_{y',0}$.

Second, I prove that $\frac{\partial W(y'')}{\partial y''} > 0$. Differentiation of equality (21) with respect to y'' obtains

$$(23) \quad \frac{R(\partial \bar{a}/\partial y'')}{\bar{a}^2}\left(\int_{y'}^{y''}\left((1-\theta_y)\bar{a}_y\right)v(y)dy\right) = v(y'')\left(\bar{c}_{y'',1}-\bar{c}_{y'',\theta}\right).$$

Differentiating the welfare function (13)

$$W = \int_{\underline{y}}^{y'} \upsilon\left(\bar{c}_{y,0}\right)v(y)dy + \int_{y'}^{y''} \upsilon\left(\bar{c}_{y,\theta}\right)v(y)dy + \int_{y''}^{\bar{y}} \upsilon\left(\bar{c}_{y,1}\right)v(y)dy$$

yields

$$\frac{\partial W}{\partial y''} = v(y'')\left(\upsilon\left(\bar{c}_{y'',\theta}\right)-\upsilon\left(\bar{c}_{y'',1}\right)\right)+$$
$$\frac{R(\partial \bar{a}/\partial y'')}{\bar{a}^2}\left(\int_{y'}^{y''}\left(\partial\upsilon\left(\bar{c}_{y,\theta}\right)/\partial\bar{c}_{y,\theta}\right)\left((1-\theta_y)\bar{a}_y\right)v(y)dy\right)$$
$$> v(y'')\left(\upsilon\left(\bar{c}_{y'',\theta}\right)-\upsilon\left(\bar{c}_{y'',1}\right)\right)+$$
$$\left(\partial\upsilon\left(\bar{c}_{y'',\theta}\right)/\partial\bar{c}_{y'',\theta}\right)\frac{R(\partial \bar{a}/\partial y'')}{\bar{a}^2}\left(\int_{y'}^{y''}\left((1-\theta_y)\bar{a}_y\right)v(y)dy\right)$$
$$\overset{(23)}{=} v(y'')\left(\upsilon\left(\bar{c}_{y'',\theta}\right)-\upsilon\left(\bar{c}_{y'',1}\right)\right)+\left(\partial\upsilon\left(\bar{c}_{y'',\theta}\right)/\partial\bar{c}_{y'',\theta}\right)v(y'')\left(\bar{c}_{y'',1}-\bar{c}_{y'',\theta}\right)$$
$$> 0$$

The explanation is similar to the previous proof of $\frac{\partial W}{\partial y'} > 0$. If the utility function is quadratic, then $\frac{\partial W(y'_1)}{\partial y'_1} > 0$, $\frac{\partial W(y''_1)}{\partial y''_1} > 0$, and $\frac{\partial W(y'_2)}{\partial y'_2} < 0$, $\frac{\partial W(y''_2)}{\partial y''_2} < 0$. The proof is similar with the necessary changes. The proof is available from the author upon request.

Proof of proposition 7:
(a) To prove that the PR Pareto dominates the CR, I verify that $\bar{a}$ increases, which improves the borrowing terms of ICL (see discussion in section V). Shifting from a CR to a PR, the change in $\bar{a}$ (4) is measured by

$$(24) \quad \begin{aligned} \Delta\bar{a} &= \frac{E^{PR}\left[(1-\theta_y)\bar{a}_y\right]}{E^{PR}\left[1-\theta_y\right]} - E^{CR}\left[\bar{a}_y \middle| \theta_y = 0\right] \\ &> E^{PR}\left[(1-\theta_y)\bar{a}_y\right] - E^{CR}\left[\bar{a}_y \middle| \theta_y = 0\right] \end{aligned}$$

The equality derives because in a CR, $\bar{a} = E\left[\bar{a}_y \middle| \theta_y = 0\right]$. The inequality derives because $E^{PR}\left[1-\theta_y\right] < 1$.

First, suppose that the utility function exhibits CRRA or CARA with $\bar{y} > y''$. Then, θ_y is an increasing function of y (recall proposition 3). Therefore, Shifting from a CR to a PR, agents $[y', \hat{y}]$ partially exit the ICL program, whereas agents $[\hat{y}, y'']$ partially enter the ICL program (recall definition 3), i.e.,

$$\begin{aligned}
\Delta\bar{a} &> E_{\hat{y}}^{y''}\left[\left(1-\theta_y\right)\bar{a}_y\right]-E_{y'}^{\hat{y}}\left[\theta_y\bar{a}_y\right] \\
&= E_{\hat{y}}^{y''}\left[\bar{a}_y\right]-E_{y'}^{y''}\left[\theta_y\bar{a}_y\right] \\
&> 0
\end{aligned}$$

The inequality derives because MLRP implies that $E_{\hat{y}}^{y''}\left[\bar{a}_y\right] > E_{y'}^{y''}\left[\bar{a}_y\right]$ and $\theta_y < 1$. Now, if $\bar{y} < y''$, then the set $\left[\hat{y}, y''\right]$ becomes $\left[\hat{y}, \bar{y}\right]$ and the proof is identical.

Second, assume that the utility function is quadratic. If $\bar{y} \le \bar{a} + \sqrt{\sigma^2\tilde{\varepsilon}}$, θ_y is an increasing function of y (see proposition 4). Therefore, the proof is similar to part(a) where y'_1, $\hat{y}_1$ and y''_1 correspond to y', $\hat{y}$ and y''. If $\sqrt{\sigma^2\tilde{\varepsilon}} + \bar{a} < \bar{y} \le \hat{y}_2$, an additional set $\left[y''_2, \bar{y}\right]$ partially enters the ICL program and further increases $\bar{a}$. Thus, $\Delta\bar{a}$ is even larger. Now, suppose that $\bar{y} > y'_2$. Then, shifting from a CR to a PR, agents $\left[y'_1, \hat{y}_1\right]$ and $\left[\hat{y}_2, y'_2\right]$ partially exit the ICL program and reduce $\bar{a}$, whereas agents $\left[\hat{y}_1, y''_1\right]$ and $\left[y''_2, \hat{y}_2\right]$ partially enter the ICL program and increase $\bar{a}$. Using equation (24),

$$\begin{aligned}
\Delta\bar{a} &> E_{\hat{y}_1}^{y''_1}\left[\left(1-\theta_y\right)\bar{a}_y\right]+E_{y''_2}^{\hat{y}_2}\left[\left(1-\theta_y\right)\bar{a}_y\right]-\left(E_{y'_1}^{\hat{y}_1}\left[\theta_y\bar{a}_y\right]+E_{\hat{y}_2}^{y'_2}\left[\theta_y\bar{a}_y\right]\right) \\
&= E_{\hat{y}_1}^{y''_1}\left[\bar{a}_y\right]+E_{y''_2}^{\hat{y}_2}\left[\bar{a}_y\right]-\left(E_{y'_1}^{y''_1}\left[\theta_y\bar{a}_y\right]+E_{y''_2}^{y'_2}\left[\theta_y\bar{a}_y\right]\right) \\
&= E_{\hat{y}_1}^{\hat{y}_2}\left[\bar{a}_y\right]-E_{y'_1}^{y'_2}\left[\theta_y\bar{a}_y\right] \\
&\rightarrow E_{\hat{y}_1}^{\hat{y}_2}\left[\bar{a}_y\right] \\
&> 0
\end{aligned}$$

I obtain the second equality by adding and subtracting $E_{y''_1}^{y''_2}\left[\bar{a}_y\right]$ (recall that signal group $\left[y''_1, y''_2\right]$ chooses CMLs only, therefore $E_{y''_1}^{y''_2}\left[\bar{a}_y\right] = E_{y''_1}^{y''_2}\left[\theta_y\bar{a}_y\right]$). To derive the last inequality, I use assumption 1 and integration by parts to obtain that

$$\begin{aligned}
E_{y'_1}^{y'_2}\left[\theta_y\bar{a}_y\right] &= \int_{y'_1}^{y'_2}\left(\theta_y y\right)v\left(y\right)dy \\
&= \left[y\left(\frac{\theta_y^{\,2}}{2\left(\partial\theta_y/\partial y\right)}\right)\right]_{y'_1}^{y'_2} - \int_{y'_1}^{y'_2}\left(\frac{\theta_y^{\,2}}{2\left(\partial\theta_y/\partial y\right)}\right)v\left(y\right)dy \\
&= \left[y\left(\frac{\theta_y^{\,2}}{2\left(\partial\theta_y/\partial y\right)}\right)-\frac{\theta_y^{\,3}}{6\left(\partial\theta_y/\partial y\right)^2}\right]_{y'_1}^{y'_2} \longrightarrow 0
\end{aligned}$$

Recall from proposition 4 that $\partial\theta_{y'_1}/\partial y > 0$, $\partial\theta_{y'_2}/\partial y < 0$, $\theta_{y'_{1,2}} = 0$. Note that if $\hat{y}_2 < \bar{y} < y'_2$, then the set $\left[\hat{y}_2, y'_2\right]$ that partially exits the ICL program reduces to $\left[\hat{y}_2, \bar{y}\right]$. In this case, $\Delta\bar{a}$ is even larger.

(b) Recall that in both equilibria all individuals invest in higher education. Therefore, aggregate consumption is identical in both equilibria, i.e.,

$$(1) \qquad E\left[\bar{c}_y^{PR}\right] = E\left[\bar{c}_y^{CR}\right]$$

where consumption in signal group y is defined in equation (5). Because $y^2 \in (y', \hat{y})$, under a CR all agents choose ICL only. Implementing a PR, agents $[y', y^2]$ partially exit the ICL program. Therefore, it is easily verified that

$$(2) \qquad \overline{c}_y^{CR} \overset{(\leq)}{>} \overline{c}_y^{PR}, \quad if\ \overline{a}_y \overset{(\geq)}{<} \overline{a}^{CR}$$

Equations (1) and (2) together imply that $\overline{c}_y^{PR}$ is a mean-preserving spread of $\overline{c}_y^{CR}$, from which I conclude

$$W^{CR} = \int_{\underline{y}}^{\overline{y}} \upsilon\left(\overline{c}_y^{CR}\right) v(y)\, dy > \int_{\underline{y}}^{\overline{y}} \upsilon\left(\overline{c}_y^{PR}\right) v(y)\, dy = W^{PR}.$$

Thus, CR dominates PR in welfare terms.

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