Political Lobbying in a Recurring Environment

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Abstract

This paper develops a dynamic model of the labor market, in which the employed workers, organized in a special interest group (SIG), can lobby decision makers for changes in labor market policies. The recurring nature of the political process, modeled as a lobbying effort per electoral cycle, means that the SIG has to take into consideration its future lobbying efforts when deciding on its optimal lobbying decision now. Lobbying for a very high compensation level has a negative side effect of reducing the hiring probability for employed workers who loses their jobs, and is not optimal as a permanent strategy. However, the model shows that it is optimal for the SIG to employ a "step strategy" in which they lobby for a high level of benefits at first, and then reduces their demands. This dynamic path allows the SIG to change the payment schedule by "pulling" some of the wages to earlier periods, benefiting workers who are currently employed at the expense of the unemployed workers and the economy as a whole. More than that, the distortion, in terms of aggregate productivity, is higher when the elections are more frequent. This mechanism cannot be studied in a non-dynamic labor market model that only allows for permanent decisions by the model agents.

1 Introduction

The classic Diamond-Mortensen-Pissarides (DMP) model (for example at Pissarides (2000))of the labor market features a set of agents, acting in a free-entry environment. Entrepreneurs are taking optimal actions regarding vacancy opening while the workers are always looking for a job. The wage setting is done through a "Nash bargaining" mechanism, which splits the surplus from a matched worker-position according to some exogenous parameter, the "bargaining power". This parameter is supposed to captures the details of an actual dynamic negotiation, which the model abstracts from, and allows closing the model with this static split (Binmore, Rubinstein and Wolinsky (1986)). Models that use the "Nash bargaining" wage negotiation usually estimate the value based on aggregate data regarding productivity and wages, or use a value that is accepted in the literature. I am extending the model by allowing the employed workers to form the "employed workers special interest group" (SIG) in order to influence the bargaining power using political lobbying. This extension is motivated by the fact that the bargaining power is used to abstract market details, which (among others) captures legal and institutional details of the economy that are set in the real world by the political process.

The reasons for choosing to model only the employed workers, and not the firms, as strategic agents are the following. First, modeling only the firms is not very interesting. As the firms do not have any scrap value, the firm does not take into consideration its value in case of a separation, and thus will always want the lowest possible bargaining power for the employed workers. Second, in practice it is harder for firms to organize effectively because (a) free entry will cause higher profits to erode by new entrants[1] and (b) it is harder for firms to avoid the free rider problem and force everyone to pay for the lobbying costs. That said, this is a potential future enhancement.

Given some cost function for the lobbying process, assuming a one-time lobbying effort by the SIG reduces the SIG problem to a static optimization problem (taking into account the steady state to steady state dynamics). However, as it is not realistically to assume a political result that holds forever, I am considering a dynamic environment in which a new policy maker is voted into power every n periods (an "electoral cycle"). Once a new policy maker holds power, the SIG can lobby for a change in the bargaining power that will be in effect for this coming cycle. Given the forward looking nature of the agents, the optimal lobbying policy of the SIG has to take into account the expectations of the agents regarding the SIG future actions. Consequently, the SIG problem turns into a strategic repeated game, where an equilibrium strategy of the SIG has to be optimal given the market's expectations.

I am solving the repeated game analytically with the simplified assumption that the cost for every possible lobbying effort is a constant. Given this assumption, I am showing that the optimal policy for the SIG is a "step" policy, featuring a one-time high level of bargaining power followed by an infinite series of a constant, lower level of bargaining power. This step policy allows the SIG to capture a higher value for the employed workers than the best stationary policy (i.e. a single, constant level for all periods) they can employ. The reason is not due to the higher wages that the first electoral cycle (with the high level of bargaining power) provides. Indeed if that was the case it was optimal for the SIG to

[1] Indeed in sectors like agriculture, where free entry is naturally weaker, SIG representing the sector interests are better organized and are able to achieve higher rents through the political process.

stay in this high level forever. Rather, the higher value results from a big one-time payoff, in the form of a single-period high wage, one period before the SIG reduces the bargaining power to a lower level (i.e. in the last period of the first electoral cycle). This results highlights a general point regarding the dynamic labor market environment: “pulling forward” at least part of the wages, even if it reduces the total discounted sum of payments, is optimal for the workers who are currently employed. This is because conditional on being employed now, current employees have a higher probability of being employed in an earlier period than being employed in a later period. Thus, it is optimal for them to alter the payment schedule, even at the expense of future employees (who are currently unemployed) and the economy as a whole. Obviously such a phenomenon cannot be studied in a model that does not allow a dynamic strategy.

To show that the result is not restricted to the simple environment in which I prove it analytically, I am showing numerically that the result holds with two extensions:

a. A lobbying cost based on the special interest group (SIG) model of Grossman and Helpman (2001) where the cost is higher the bigger is the change in the bargaining power requested (compared to the economy wide optimal level)
b. An overlapping generations (OLG) economy populated by m generations of workers, where the most senior workers are determining the SIG policy, conditional on a member’s vote.

The paper is organized as follows: in section 2 I describe the model. In section 3 I discuss the repeated game that arises from the need of the SIG to lobby repeatedly in an economy populated by forward looking agents. In section 4 I describe and prove the optimal path of the SIG. in section 5 I describe how the optimal path can be supported as an equilibrium path. Sections 6 and 7 describe the two extensions of the model and section 8 concludes.

1.1 Bargaining power as the policy tool

In this paper I model a channel through which the SIG can influence labor market policies. The value for the workers, and all other aggregate values, are determined based on the wages workers receive when they are employed, and the probability of re-hiring determined by the market tightness and the unemployment level. The benefits of influencing the politically-determined properties are thus entirely driven by the change they impose on the labor market outcomes of wages, unemployment and vacancies. The cost of lobbying, based on the Grossman-Helpman model, is also due to the changes of the labor market outcome, which determines the social value.

I am using the bargaining power as the channel through which political lobbying influences the labor market. The bargaining power is an existing property of the simple DMP model, and is usually treated as an exogenous parameter. The static bargaining game solved in DMP models can be viewed as an approximation of a dynamic strategic game of negotiations with alternating offers, as described in Binmore, Rubinstein and Wolinsky (1986). In the dynamic game the outcome is decided according to properties governing the negotiations process, as well as the income streams available to the parties if the negotiation breaks down permanently. The negotiation process parameters include, among others, the relative patience of the participants, the exact negotiation procedure, the income streams accruing to the participants during the negotiation and the actual cost of breaking up the negotiations and starting to look for a new match. In the static approximation, only the "outside option", or the "credible threat" of each participant is explicitly used in the solution. All other properties, focusing on the negotiation process itself, are bundled together in the "bargaining power" parameter[2].

The bargaining power is not, of course, a property that can be directly decided upon by a political decree. However, many policies that influence the bargaining outcome between employees and firms can be part of the politically - influenced environment. The level of unionization affects the ability of employees to sustain an income stream during the negotiation period using a strike fund. Legally-determined rules govern how easy it is for a firm to use temporary workers to sustain production during a strike. Legally-determined firing rules and firing costs determine how credible is the firm's threat to cut the negotiations and look for other workers, etc. Note that while some of the properties governing the negotiations outcome are about agent's ability to sustain long negotiations or about breakup threats, in the reduce form that I am using the negotiations are always instantaneous and successful, as usually done in the DMP literature.

In this paper, following the standard DMP literature, I do not model the dynamic bargaining game explicitly. However, it is very reasonable to believe that politically determined properties of the bargaining environment have a strong impact on the outcome, through the channels I described. As such, the worker's SIG have the incentive to influence these parameters through the political process and reap the rewards with a better outcome of the negotiation process.

2 The model

In this section I develop a model of the labor market with political lobbying, where the SIG lobby policy makers to set a specific bargaining power level. I consider here the simplest case of homogenous infinitely lived workers. The model features a continuum of workers with measure 1, all of them infinitely lived. The workers are homogenous except for the fact that in each period, each worker is either employed or unemployed.

Lobbying happens periodically, every time a new policy maker takes power (an "electoral cycle"). As all the agents are forward looking, the SIG must consider, when choosing a bargaining power level to lobby for, its own future decisions, and also the expectations by other agents in the economy regarding its future decisions. I show that this transforms the SIG's problem not to a dynamic optimization problem, but to a strategic game in which the SIG plays against its future decision and the market's expectations. This lobbying game has multiple equilibria, both stationary and non-stationary.

There is a continuum of workers, with measure 1, in the economy. Each worker is either employed or unemployed, and unemployed workers are looking for a job. There are many firms, each employing a single worker. Entrepreneurs can post vacancies in order to create a new firm in a free-entry

[2] Hall and Milgrom (2008) take a different approach as they do not consider the threat to break the negotiation a credible threat.

environment. Vacant jobs and unemployed workers are randomly matched each period according to an aggregate matching function $M(u, v)$. The probability that a vacant job is filled is:

$$\frac{M(u_t, v_t)}{v_t} = M\left({}^{u_t}/_{v_t}, 1\right) = M\left({}^{1}/_{\theta_t}, 1\right) \equiv q(\theta_t)$$

where $\theta \equiv {}^{v}/_{u}$ is the market tightness. The probability that an unemployed worker finds a job is:

$$\frac{M(u_t, v_t)}{u_t} = \theta_t \frac{M(u_t, v_t)}{v_t} = \theta_t q(\theta_t)$$

I am assuming the regular assumptions on the matching function, namely that the matching function is CRS, the probability of hiring is decreasing in the market tightness $\left(\frac{\partial q(\theta_t)}{\partial \theta_t} < 0\right)$ and the probability of finding a job is increasing in the market tightness $\left(\frac{\partial \theta_t q(\theta_t)}{\partial \theta_t} > 0\right)$.

Matches are separated with an exogenous probability σ in each period. The transition of the unemployment rate u is:

$$u_{t+1} = \left(1 - \theta_t q(\theta_t)\right)u_t + \sigma(1 - u_t) \tag{2.1}$$

There are no savings in the economy and workers consume all of their income. The value of being employed is:

$$W_t = w_t + \beta[\sigma U_{t+1} + (1 - \sigma)W_{t+1}] \tag{2.2}$$

where w_t is the wage in period t. I am assuming that the utility from the wage is equal to the wage.

The value of being an unemployed worker is:

$$U_t = b + \beta\left[\left(1 - \theta_t q(\theta_t)\right)U_{t+1} + \theta_t q(\theta_t)W_{t+1}\right] \tag{2.3}$$

where b is the exogenous utility derived from unemployment (home production, leisure, etc.)[3].

In order to create a firm, an entrepreneur must post a vacancy with a per-period cost of ξ and the vacancy will be filled with the probability $q(\theta_t)$, which the entrepreneur sees as exogenous. The value of a posted vacancy is:

$$V_t = -\xi + \beta\left[\left(1 - q(\theta_t)\right)V_{t+1} + q(\theta_t)J_{t+1}\right]$$

As if the vacancy is indeed filled then the job will start producing in the next period. In equilibrium, assuming free entry for firms, entrepreneurs post new vacancies until there is no expected profit to be made. With $V_t = 0$, we get the more useful form:

$$J_{t+1} = \frac{\xi}{\beta q(\theta_t)} \tag{2.4}$$

[3] I am not assuming that b represents unemployment benefits as the model has no taxes to pay for them.

The value of a filled job, given the wage w_t is:

$$J_t = p - w_t + \beta[\sigma V_{t+1} + (1-\sigma)J_{t+1}]$$

With $V_t = 0$, we get the more useful form:

$$J_t = p - w_t + \beta(1-\sigma)J_{t+1} \quad (2.5)$$

where p is the match productivity which is assumed to be constant over time.

Assuming that the wage is set through generalized Nash bargaining, the first order condition for the bargaining problem is:

$$W_t - U_t = \gamma_t(J_t + W_t - U_t) \quad (2.6)$$

where $\gamma_t \in (0,1)$ is the bargaining power of the employed workers at period t.

2.1 The Employed Workers SIG

The SIG's problem is to decide whether to lobby the policy maker for a specific bargaining power level. I assume here that up to a certain period, period 0, there was no lobbying in the economy. When there is no lobbying, the policy maker sets the bargaining power at some steady state level[4]. At a certain period, designated period 1 (which is also the beginning of electoral cycle 1) , the employed workers figure out that they can lobby, and they gather to decide if and to which extent to lobby. If they decide to lobby, the change in the bargaining power takes effect immediately. Any successful lobbying changes the bargaining power for the entire planning period of the current policy maker, which is called an "electoral cycle". Each electoral cycle lasts n periods. If n is infinite the understanding is that the planning horizon of the policy maker is infinite. In this case the lobbying change will last forever and the optimization problem of the SIG is reduced to a static optimization problem. Similarly, if $n = 1$, then the SIG can lobby for a change every period.

The SIG problem is to maximize the value of an employed worker, so for every period t in which they can lobby, they are maximizing:

$$W_t = \max_{\gamma}\{w_t(\gamma) + \beta[\sigma U_{t+1}(\gamma) + (1-\sigma)W_{t+1}(\gamma)]\}$$

Given the expectations for future actions (as explained below).

2.2 The Dynamics of the DMP model

The model outlines a series of periodic changes in the bargaining power of the employed workers, so it is worthwhile to describe the dynamics of the model under such periodic shocks.

[4] For my analysis here the level of the bargaining power before the worker's first lobbying effort is not important. I will discuss this level in the section about lobbying costs

To get some intuition consider a case in which the bargaining power level for electoral cycle 1 is γ^1, and the level for all subsequent electoral cycles is γ^2. The level change in period $(n+1)$ is known in advance. Everyone assumes that there will never be another such change.

As it is well known, at period $(n+1)$ the values of all the relevant model variables (W, U, J, w, θ) jump immediately to the steady state level associated with γ^2. At period n, one period before the change, several variables are already in their new steady state level. It is clear from (2.3) and (2.4), that the market tightness and the value of being unemployed adjust in advance to the steady state levels:

$$q(\theta_n) = \frac{\xi}{\beta J_{n+1}} \equiv q(\theta_{n+1})$$

$$U_n = b + \beta\big[\big(1 - \theta_n q(\theta_n)\big)U_{n+1} + \theta_n q(\theta_n) W_{n+1}\big] \equiv U_{n+1}$$

And more generally, if the bargaining power is not constant starting electoral cycle 2, they do not depend on the first cycle level γ^1.The reason is that the current period utility flow of the unemployed is exogenous, while entrepreneurs, when considering whether to invest the cost needed to create a vacancy, are looking at next period value of a filled job, which is the first period in which they could start producing. It is also clear from (2.2) and (2.5) that the sum of the values of the employed workers and the firm is also adjusted in advance to the steady state level:

$$W_n + J_n = p + \beta[\sigma U_{n+1} + (1-\sigma)W_{n+1} + (1-\sigma)J_{n+1}] \equiv W_{n+1} + J_{n+1}$$

which means that the surplus $S_n \equiv W_n + J_n - U_n$ also adjusts one period before the shock such that $S_n = S_{n+1}$. The value of an employed worker, W_n, however, is not adjusted before the shock:

$$W_n = \gamma^1(W_n + J_n - U_n) + U_n = \gamma^1 S_{n+1} + U_{n+1}$$

and in the new steady state:

$$W_{n+1} = \gamma^2 S_{n+1} + U_{n+1}$$

Together we get that:

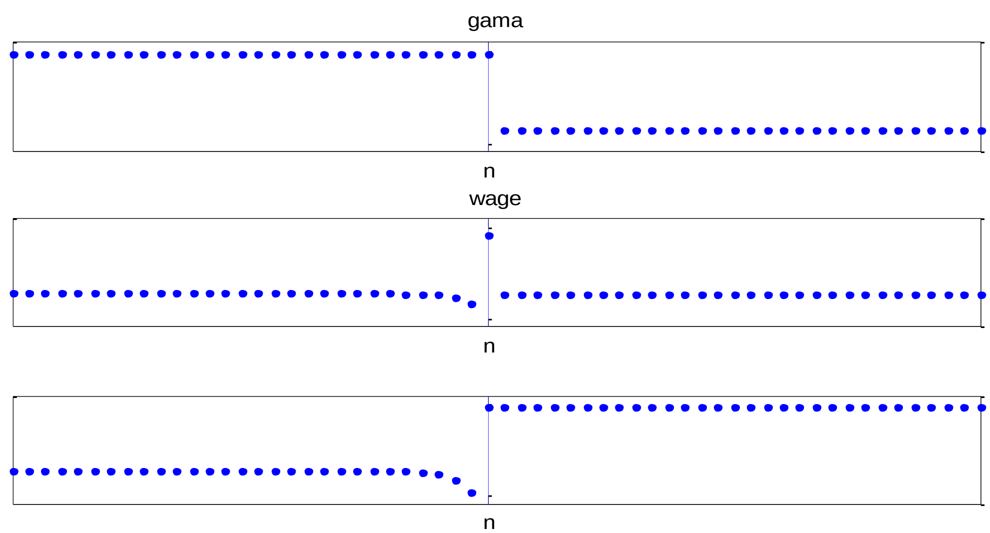
$$W_n = S_{n+1}(\gamma^1 - \gamma^2) + W_{n+1}$$

so that the value of the employed worker, one period before the shock, is higher than the new steady state level if the bargaining power is going to drop $(\gamma^1 > \gamma^2)$, and lower than the new steady state level if it is going to rise. From (2.2) we can see that:

$$W_n - w_n = \beta[\sigma U_{n+1} + (1-\sigma)W_{n+1}] \equiv W_{n+1} - w_{n+1}$$

so that the increase in the value of an employed worker is fully due to a wage increase in the same amount in the period before the shock[5]. The intuition is simple. The value of the employed worker is a discounted sum of the expected per-period utility flows. Starting from the next period ($n+1$), the worker will receive a lower wage due to the lower bargaining power, and the firm will get a higher share of the surplus. But this period, the bargaining power is still high, representing a high share of the discounting surplus. A high wage this period is required in order to keep the surplus share high for he employed worker this period, given the lower wage starting from next period. Another way to look at it is form the firm's perspective. A lower bargaining power (and the accompanying lower wage) starting next period makes it more valuable to the firm to keep that match, and thus the firm is "willing" to pay a higher wage. Of course, the opposite happens when the bargaining power is about to increase. The workers will get a higher wage (and higher share of surplus) starting from next period, but the surplus this period, which includes this period's wage and the discounted surplus starting next period, is still low, so a lower wage this period is required.

Figure 1 – values for γ, wage and θ before and during a shock



As all the model agents are forward looking, the model is solved backwards from period $n+1$ – the period in which the bargaining power is changed. It is easy to show that all model variables converge backwards towards the steady state values associated with γ^1, although with different rates of convergence. As all the model variables can adjust instantaneously (other than the unemployment level which does not affect other variables), previous values of the bargaining power have no effect on current levels. Figure 1 shows the values of the market tightness and the wage from this one-time reduction in the bargaining power γ. The wage is very high one period before the bargaining power change (period n), but it was falling till two periods before the reduction (up to period $n-1$). The

[5] I am assuming in the example, for simplicity, that next period values are steady state values (i.e. there will not be any additional shock in the future). More generally, W_{t+1}depends on all future levels of the bargaining power, but not on period t level γ^1

market tightness θ is already in its new level in period n, but it also was falling till two periods before the reduction. Appendix A.4 shows this formally.

The reason that the wage is high only one period before the bargaining power change is that the model that I use, as is standard in the DMP literature, includes re-negotiation every period and thus allows all the required adjustment from the second electoral cycle to fall on the last period of the first electoral cycle. In this case, the wage and the market tightness will be a bit lower than the steady state before period n, due to the need to pay higher wages in period n. While this result is specific to the DMP setting I am using, the general result is not. If I am limiting the ability to renegotiate every period and forces a negotiation every electoral cycle, the wage will be higher than the steady state level for the entire period, but of course not as high as in the last period if renegotiation every period is allowed.

3 The Repeated Lobbying Game

I consider a case in which lobbying happens periodically, with a new policy maker in power after each electoral cycle. The SIG needs to decide about the lobbying effort (or lack thereof) after each election and the lobbied bargaining power will then be in effect for the entire following electoral cycle, until the next election. For simplicity I assume that the policy maker cares about all future periods and not just the electoral cycle she is in power.

The problem facing the SIG is to choose the level of bargaining power that will provide the workers with the highest value, assuming that they cannot commit in advance to the full path of bargaining power levels. The SIG takes into account that it will need to re-choose at the beginning of each electoral cycle. The SIG is the only agent in the model that chooses strategically, but this does not make the decision problem a simple dynamic optimization problem. The reason is that all the agents in the model are forward looking and are basing their actions on their expectations of the relevant future values. Specifically, expectations are needed as new vacancies are created proportionally to the (expected) value of a filled position next period, and the surplus is split using current values of the employed, unemployed and firms, which are a function of (expected) future values. I assume here that expectations for future SIG actions, once established, are common to all the agents in the economy.

Once all the agents in the economy hold expectations regarding the future lobbying sequence of the SIG, for any possible history, the SIG can solve the problem as a recursive optimization problem. But how the expectations established? As there is no aggregate uncertainty in the economy, it can be seen as if the SIG decides up front on the entire sequence of future lobbying levels, and communicates its strategy to all relevant agents. The strategy includes both the “on path” sequence, which is the lobbying sequence that the SIG wants to follow, and the “off path” (or “punishment”) sequence that will be taken following each possible deviation. The SIG communicates the strategy to all the agents in the economy, and the strategy becomes the set of expectations for everyone. If the strategy is consistent, in the sense that assuming that the agents expect the strategy to be carried out (after any possible history) it is still optimal for the SIG to follow, then the strategy is an equilibrium strategy.

3.1.1 Formal definition

- Let $A^t = \{a^1, a^2, \dots a^t\}$ be a history in electoral cycle $t+1$, consists of a sequence of lobbying decisions for the first t electoral cycles.
- Let $H = \{\emptyset\} \cup (\cup_{t=1}^{\infty} A^t)$ be the set of all possible histories. Define $A^0 = \{\emptyset\}$
- Let Γ be the set of possible lobbying levels, such that for every period t the lobbying level is $\gamma^t \in \Gamma$.
- Let $L: H \rightarrow \Gamma$ be a strategy that defines the lobbying levels following any possible history A^t, both on path and off path.
- Let $E(A^t)$ be the expectations of the economy agents regarding the future path of bargaining power choices starting at cycle $t+1$, given the history A^t. The expectations state the actions the SIG will take given any possible path that starts with A^t.

An $equilibrium\ strategy$ L of the game G is a strategy of the SIG : $H \rightarrow \Gamma$, such that for any possible history (A^t), if the expectations of all the model agents from the SIG future behavior are equal to $L(A^t)$, the optimal strategy for the SIG, starting at period $t+1$, is $L(A^t)$.

The sequence actually chosen in equilibrium by the employed workers starting from the first electoral cycle is called the $equilibrium\ sequence$.

3.1.2 Finding an Equilibrium

Assume that L_p (here p stands for "punishment") is an equilibrium strategy of the lobbying game G, and that the equilibrium sequence S_p, if indeed chosen by the SIG, yields them a first-period value of W_p. By definition of L_p being equilibrium, any other sequence yields a lower value. If at a certain electoral cycle $t+1$ in the lobbying game the history is (A^t) and the expectations $E(A^t)$ are that the SIG will employ the strategy L_p, the SIG optimally choose from that point the sequence S_p and the value it receives is W_p .

It is possible to support a sequence of lobbying efforts $S = \{s_1, s_2, \dots\}$, as long as for any node $s_t \in S$, which yields the value for employed workers W_t at the beginning of electoral cycle t, there is an equilibrium L_p such that $W_p < W_t$. If this is indeed the case, the equilibrium strategy L is defined as follows:

- As long as the history is equal to the desired sequence S, keep choosing bargaining power levels according to sequence S. If at a certain cycle $t+1$ the history A^t is for the first time not according to the sequence S, choose optimally according to L_p.

Lemma 1: $L$is an equilibrium of the lobbying game

Proof: by construction, as long as the SIG is on the equilibrium path, it continues to choose according to the equilibrium path. If the SIG deviates at a node t, the expectations are that it will play according to L_p starting from node $t+1$. By construction of L_p being an equilibrium, if the SIG deviates it is optimal for it to deviate to the strategy L_p and W_p is the highest value it can achieve. As we assumed that for every electoral cycle t , $W_p < W_t$, it is optimal for the SIG to choose according to the equilibrium sequence as long as it is on the equilibrium sequence ■

Example: A strategy that is not an equilibrium: consider again the example described in section (2.X) on the dynamics of the DMP model. Assume that the SIG wants to support a path of constant level of bargaining power γ^2 that yields the first-period value of W^2 (or a path that ends with a constant level of bargaining). Also assume that the path $\{\gamma^1, \gamma^2, \gamma^2, \gamma^2 \ldots\}$ yields a value of $W^{1,2}$, which is higher than W^2. In this case, the strategy "always lobby for γ^2" is not an equilibrium strategy. The reason is that the workers have a profitable deviation, to lobby for γ^1 once. If the workers deviate by lobbying once for γ^1, they do not suffer the loss in wage one period before the deviation as the deviation is not expected by the other agents. As everyone expects the workers to go back to γ^2 for the next electoral cycle, at the period of the deviation the workers receive a value of $W^{1,2}$ which makes the deviation profitable. Note that the example does not mean that the workers cannot support a path consisting of a constant bargaining power γ^2. It does mean that in order to support such a path, the expectations after the deviation should provide the workers with a value lower than W^2.

4 The Optimal Strategy

In this section I characterize the optimal strategy of the SIG. First I show what the best path for the SIG is, assuming that it can pre-commit to the entire path of lobbying decisions. Then, I show under which conditions this optimal path can be supported as an equilibrium strategy and how.

As is well known since Hosios (1990), the optimal path for the social planner interested in maximizing the total output of the economy (net of investment in new vacancies) is to hold the bargaining power constant at a level equal to the elasticity of the matching function. Specifically, the Hosios type social planner maximizes:

$$\sum_{t=0}^{\infty} \beta^t \{u_t b + (1-u_t) w_t + (1-u_t)(p - w_t) - \xi v_t\}$$

s.t. $u_0, \;\; u_{t+1} = (1-\lambda^w) u_t + \sigma(1-u_t)$

and the optimal constant bargaining power is $\gamma^* = \eta(\theta) = -\frac{\theta q'(\theta)}{q(\theta)}$[6].

4.1 The Optimal Static Path for the SIG

First I consider the optimal path for the SIG in case it has to choose the level of bargaining power only once (or alternatively, in case that the length of the electoral cycle, n, is infinite). If the SIG is setting a certain level of bargaining power at period 1, that will hold forever, all model variables, including the value of the employed workers, will immediately jump to the new steady state. In practice, the SIG is maximizing the steady state level of $W(\gamma)$.

[6] Note that while γ^* maximizes the target function of the dynamic path, it does not, in general, maximizes the total output in the steady state. This is the case only if $\beta = 1$.

Proposition 1: the steady state value of the employed worker as a function of the bargaining power $W(\gamma)$ is a singled picked function of the bargaining power. The maximum, which is the best static bargaining power level γ^S for the SIG, is higher than γ^*.

Proof: In Appendix B

Figure 2: Steady state values of employed and unemployed workers

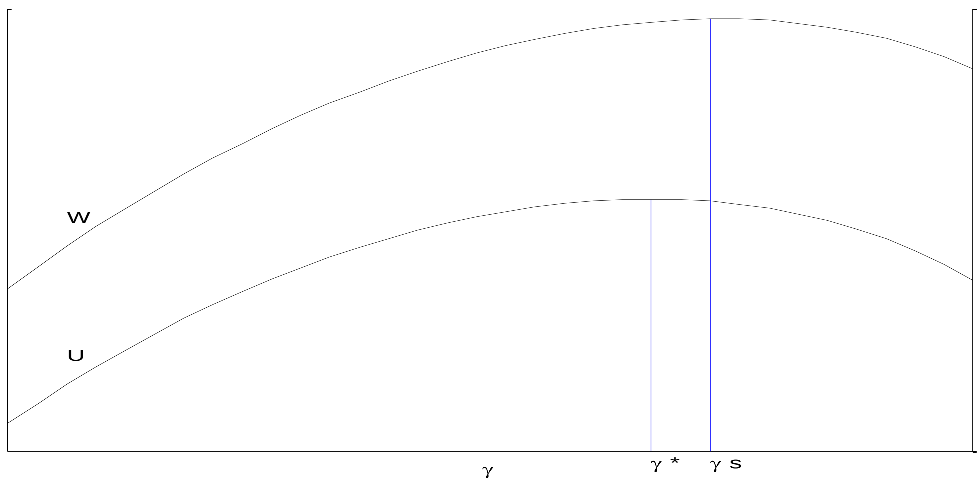


Figure 2 illustrates Proposition 1, showing the bargaining power levels in which the steady state values of the employed and unemployed workers is attained. As is well known, the steady state level of the unemployed workers is attained at the Hosios policy maker's optimal level [7]. A higher bargaining power is beneficial for the employed workers, who are gaining a higher wage immediately while suffering from the tighter labor market only when they are separated and looking for a job again. However, internalizing this cost limits the extent to which they want to increase their bargaining power. A detailed discussion of the employed worker's optimal static path can be found in Lifschitz (2015).

4.2 Characterizing the Optimal Dynamic Path

In this section I am characterizing the optimal (potentially dynamic) path for the SIG representing the employed worker. Note that for now I am assuming that the SIG can pre-commit to the path, abstracting from the need to support the path as an equilibrium strategy. In section (5) I will show under which conditions this optimal path can indeed be supported as an equilibrium strategy.

Proposition 2*:* The optimal path for the SIG, maximizing the value in the first period W_1, is step function with a high level of bargaining power in the first electoral cycle designated γ^b, followed by a constant path at the socially optimal bargaining power level γ^*.

Proof: The value of employed worker in the last period of the first electoral cycle, W_n, can be written based on (2.6) as $W_n = \gamma_1(W_n + J_n) + (1 - \gamma_1)U_n$, where γ_1 is the bargaining power in the first

[7] See for example Ljungqvist and Sargent (2004) pp 880.

electoral cycle[8]. I already showed in section (2.3) that the values of θ_n, U_n and $(W_n + J_n)$ do not depend on γ_1, only on next period's values which are determined by the rest of the bargaining power sequence $\{\gamma_2, \gamma_3, \gamma_4, \dots\}$.

The sequence of lobbying levels starting from γ_2 that maximizes both $(W_n + J_n)$ and U_n (if there is such a sequence that maximizes both at the same time), also maximizes W_n, for any given level of γ_1. Proposition 3 states that indeed there is such a path:

Proposition 3: The static sequence $\{\gamma_2 = \gamma^*, \gamma_3 = \gamma^*, \gamma_4 = \gamma^*, \dots\}$ maximizes both $(W_n + J_n)$ and U_n.

I am proving Proposition 3 by showing that iterating through the bargaining power levels γ_t starting from γ_2, for each case in which $\gamma_t \neq \gamma^*$ changing its value to γ^* increases $(W_n + J_n)$ and U_n. Full proof is in Appendix B.

Proposition 3 shows that for any given γ_1, the static sequence $\{\gamma_2 = \gamma^*, \gamma_3 = \gamma^*, , \dots\}$ maximizes $(W_n + J_n)$ and U_n and thus also maximizes W_n. Note also the analogy to Hosios (1990) - proposition 3 shows that the dynamic problem of the unemployed workers is equivalent to the dynamic problem of the central planner. While it is well known that the static problem of the unemployed (i.e. the best steady state value of U) is maximized at γ^*, as far as I know the dynamic result is a new result in the literature.

Proposition 4 concludes the proof that the path $\{\gamma^b, \gamma^*, \gamma^*, \dots\}$ maximizes W_1 for some γ^b:

Proposition 4: for a given γ_1, the sequence $\{\gamma^*, \gamma^*, \gamma^*, \dots\}$ starting from the second electoral period that maximizes $W_n + J_n$ and U_n, and thus maximizes W_n, also maximizes W_1.

The intuition behind the result that the optimal path has a single period of high bargaining power and then a static path (a "step" structure) is the following (I also show in the next section that $\gamma^b > \gamma^s > \gamma^*$). An employed worker in period 1 has a higher probability of still being employed in an early period than in a later period. Hence, it is beneficial for the employed workers to change the payment schedule and "pull" at least some of the wages to an earlier period. The way to do it is to set a high bargaining power level for the first electoral cycle and to move it down to the level where the total value of the employed workers and the firms is the highest. The high wage that the employed workers receive one period before the end of the first electoral cycle (period n) "compensates" them for the low wage that they will receive starting the next period, but of course benefits only the ones who are actually employed in period n. This schedule change is at the expense of workers who are unemployed at period 1, and the economy as a whole. Note that the higher value of the workers is not due to the higher wage in the first electoral cycle (before period n), as this higher wage is accompanied by a lower market tightness which reduces the probability for the period 1 employed workers to still be employed in period n. The higher bargaining value is a "necessary evil" employed just to rip the single-period high wage one period before the end of the electoral cycle.

[8] I am always using the subscript of bargaining power to represent the electoral cycle rather than the period (like for all other variables) as the bargaining power can only be changed once every electoral cycle.

4.3 The optimal level of γ^b

The previous section showed that for every bargaining power in the first electoral cycle γ_1, it is optimal for the SIG to move to a constant level of bargaining power γ^*, starting from the second electoral cycle. It follows that the optimal path for the SIG is a sequence of the step structure $\{\gamma^b, \gamma^*, \gamma^*, \dots\}$. In this section I discuss the considerations that affect the optimal level of the first period γ^b.

Lemma 2: $\gamma^b \geq \gamma^*$

Proof: appendix B

The intuition for Lemma 2 is simple. For a given $\gamma_1 < \gamma^*$, it is indeed optimal to move to γ^* starting in the second electoral cycle, but the value in the period before the move, W_n, is increasing in $\gamma_1 (W_n = \gamma_1 S^* + U^*)$, and is lower than the value W^* that can be achieved by setting $\gamma_1 = \gamma^*$. Also, in this case W_n is higher then the steady state value associated with γ_1, $W_{ss}^{\gamma_1}$, and thus it converges lower, and will be below W^* $\forall$ $1 \leq t \leq n$.

For all possible values of $\gamma_1 \geq \gamma^*$, it is a simple numerical calculation to calculate backwards the value in the first period $W_1^{\gamma_1}$ and determine the best value γ^b. However, it is clear that γ^b is not only above γ^*, but also above γ^s, the best static value for the employed workers.

To understand why consider what does it mean that γ^s is the best static level. When choosing γ^s, there is a tradeoff. Higher levels of the bargaining power provide higher wages, but only for periods in which the worker is actually employed. For periods where the worker is unemployed, higher levels of bargaining power provide lower probability of re-hiring. Conditional on being employed in the first period, the worker has a higher chance of being employed in period t, the lower is t. For later periods, and as t approaches infinity, the probability of being employed, conditional on being employed in the first period, converges down to the steady state level of employment. So, if it is optimal for the first-period employed worker to move to γ^s when the planning horizon is infinite, it is surely optimal to move to higher level for shorter planning horizons, as the higher wage will outweight the lower re-hiring probability. Now, consider the optimal level for the first electoral cycle, given that we already determined there is a move to γ^* starting from the second electoral cycle. The payoff consists of the payoff from the first $(n-1)$ period in the first electoral cycle, the one-time "bonus" is period n, and the payoff starting from the second electoral cycle, which is the same regardless of γ^b. As it would have been optimal for the employed worker to choose a level of bargaining power higher than γ^s for the first electoral cycle even without considering the higher bonus that the higher level will provide at period n, it is obvious that $\gamma^b > \gamma^s$. Numerically, for $\gamma^* \leq \gamma_1 < \gamma^s$, $W_n < W_n^{\gamma^s}$, and it also converges to a lower steady state, so there is no n for which $\gamma^b < \gamma^s$.

For $\gamma_1 \geq \gamma^s$, there is a tradeoff. The higher is γ_1, the higher is the one time wage bump in period n and the value W_n, but the probability that a worker employed in period 1 is employed in period n, are lower.

The higher is n, the value from the wage in period n is discounted with a higher factor and the probability of being employed in period n, conditional on being employed in period 1, is lower. Thus, the optimal level of the employed workers in period 1 is decreasing is n. At the limits, if $n = 1$ it is optimal to move as high as possible, and as $n \to \infty$, the optimal value converges to γ^s.

Figure 3 shows a typical chart of employee value W_t in the first electoral cycle ($t \leq n$), with a static path of γ^* from the second electoral cycle. In the figure $\gamma^* < \gamma^a < \gamma^s < \gamma^b < \gamma^c$. W_n is always higher for higher levels of first cycle bargaining power. For first cycle level of γ^*, the line representing W_t is flat, as the value is always in its steady state level denoted as W^*. In the figure this is the solid line. For levels below γ^* $W_n < W^*$ and it converges to a lower level, so the value in the first electoral cycle is always below W^*, as shown in Lemma 2, so such values are omitted here for clarity. The blue line represents γ^s. As this is the highest steady state value for the SIG, it converges to the highest level. For levels above γ^* but below γ^s (represented by γ^a), W_n is lower than for γ^s, and it also converges to a lower level, so these values can never be optimal. For values above γ^s, represented by γ^b and γ^c, W_n is higher than for γ^s, but they converge to a lower steady state. For higher levels of bargaining power, W_n is higher but it also falls faster. This is why a lower level of bargaining power (but still higher than γ^s) is optimal for longer electoral cycles (larger values of n).

Figure 3 - Employee Value per period for various levels of first cycle γ

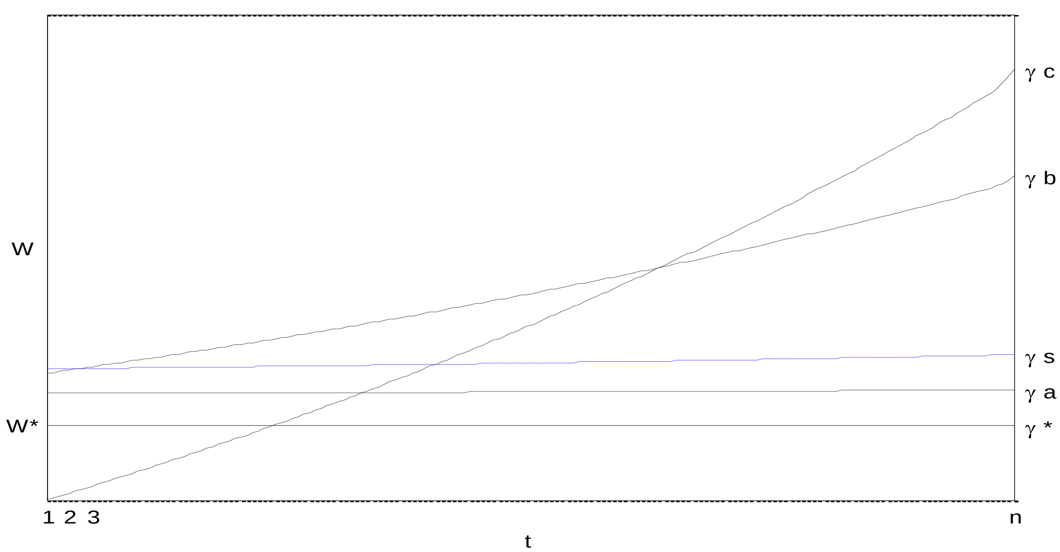
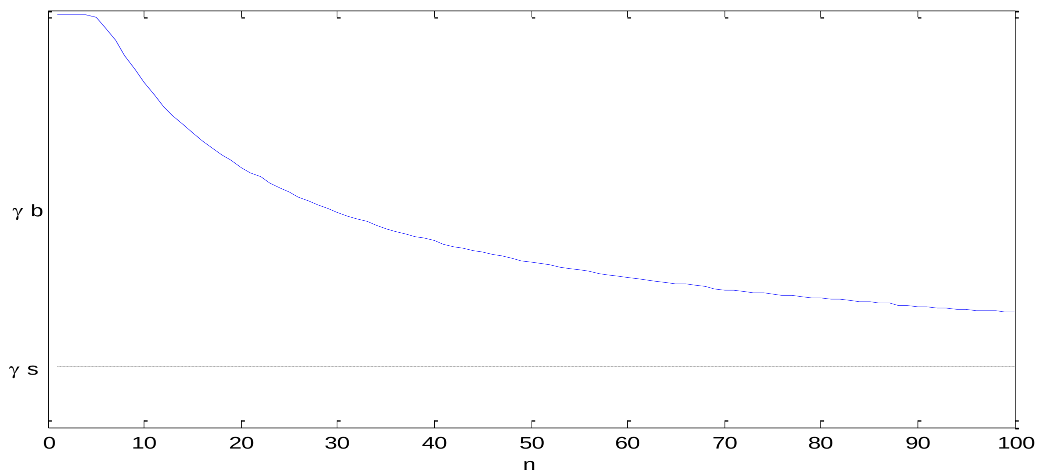


Figure 4 shows how the optimal bargaining power in the first electoral cycle γ^b is falling with n and converging towards γ^s.

Figure 4 – optimal bargaining power γ^b falls with electoral cycle length n



4.4 The Welfare Loss

We have seen that the optimal strategy of the SIG depends on the electoral cycle length n. It is interesting to look at the total welfare loss due to the lobbying effort of the SIG. I am measuring the welfare, based on Hosios (1990) as the discounted sum of total production, minus vacancy creation costs, starting from period 1. The policy maker, given no lobbying effort, chooses the constant path $\gamma = \eta(\theta) = \gamma^*$, which maximizes total welfare, and given any other lobbying path, the welfare will be lower.

Figure 5 – Welfare loss depending on the electoral cycle length, as a % of optimal welfare

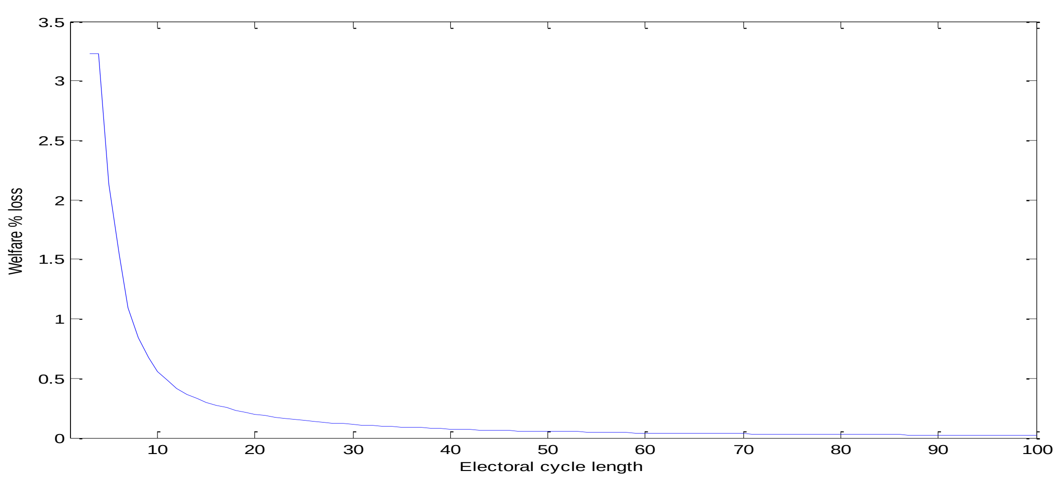


Figure 5 shows the welfare loss, as a percentage of the optimal welfare, due to the lobbying effort, depending on the electoral cycle length. As can be seen, the welfare loss is higher, the shorter is the electoral cycle. Shortening the electoral cycle has two effects. The first effect is that the SIG is choosing a higher level of bargaining, as they will need to suffer the too-high level for a shorter time, causing a larger distortion per period. The second effect is that the distortion of the first electoral cycle is shorter. Numerically the first effect is always stronger, as increasing the bargaining power has a convex effect on the distortion, making it larger and larger the further the bargaining power is from its optimal level. Obviously, reducing the electoral cycle length reduces the number of periods with distortion only linearly. As can be seen in figure 5, for very short electoral cycles (I use a monthly calibration period), the welfare loss can be very high, up to more than 3% of total discounted sum of output, but the distortion falls very quickly. The distortion is bounded from below, even for very infrequent elections, by the welfare loss from a permanent move to γ^s, which is the optimal strategy for the SIG when the electoral cycle length approaches infinity.

5 Supporting the Optimal Path as an Equilibrium Strategy

In order for the optimal path $\{\gamma^b, \gamma^*,\ \gamma^*, \dots\}$ to be supported as an equilibrium strategy, it must be optimal for the SIG to follow the path given the expectations of all the agents in the economy. As we have seen, the expectations that the SIG is declaring at period 1 include both the optimal path and the behavior off-path for all possible histories.

Assume that L_p ('p' for punishment) is an equilibrium strategy (containing both the on-path and off-path expectations) yielding the sequence S_p and providing the period-1 value of W_p for the SIG. Being an equilibrium strategy, by definition it is optimal for the SIG to choose the sequence S_p when they are faced with the expectations of this strategy. Now consider the strategy L:

1. As long as the followed sequence was according to the optimal sequence $\{\gamma^b, \gamma^*,\ \gamma^*, \dots\}$, continue choosing according to the optimal sequence.
2. Once there was a deviation, continue according to the strategy L_p

By construction, if the SIG do deviate, they will deviate to the sequence S_p, as this is the most profitable deviation, and will receive the value W_p at the time of deviation. It follows that in order for L to be an equilibrium strategy, W_p has to be lower than the value provided at any electoral period on the optimal path. We have already proved that the value on the optimal path in period 1 is the highest possible value for the employed workers, and the value at any subsequent electoral period is the steady state value associated with $\gamma = \gamma^*$ which we mark as $W_{ss}^{\gamma^*}$.

How can we find an equilibrium strategy that yields a value lower than $W_{ss}^{\gamma^*}$? Consider the following Bellman equation:

$$W(\gamma) = \max_{\gamma'}\{u(\gamma') + \beta^n W(\gamma')\} \tag{5.1}$$

Where $W(\gamma)$ is the value of the employed workers, at a period in which they are choosing the next electoral cycle bargaining power γ', given that the last cycle bargaining power was γ. $u(\gamma')$ is the total utility received by the employed workers in the next electoral cycle, which is the expected discounted sum of their income over n periods – the wage w in periods of employment and the unemployment utility b in periods of unemployment. The expected stream is of course influenced by the wage but also by the re-hiring probability once an employed worker becomes unemployed, according to the market tightness. In general this utility can depend on both γ and γ', but in our case it only depends on γ' as previous cycle bargaining power does not affect current cycle wages or market tightness[9]. $W(\gamma')$ is the value received in the next electoral cycle and it is discounted by β^n as each electoral cycle is n periods long.

Consider the solution for this bellman equation, with the derived policy function $\gamma' = g(\gamma)$. What will such a solution mean in the context of the employed worker problem? $\gamma' = g(\gamma)$ is the optimal choice for the SIG, assuming that they will choose optimally again for the next electoral cycle (i.e. according to the function g), and implicitly assuming that everyone expects them to choose that way. It follows that if there is a solution for the Bellman equation, and the policy function $g(\cdot)$ represents an equilibrium strategy.

Lemma 3: There is a solution to the Bellman equation (5.1). More than that, the policy function is a constant function, $g(\gamma) = \gamma^p$

Proof: the Bellman equation is a contraction mapping and thus it has a solution. Note that the only state variable γ does not affect the utility from the current cycle $u(\gamma')$ nor next cycle value $W(\gamma')$. It follows that, for any given next level values $W(\cdot)$, the same γ' will maximize the RHS of the equation for all possible levels of current γ, and thus the solution must be a constant policy function. ■

Lemma 4: The strategy L_p in which the SIG choose the bargaining power $\gamma = \gamma^p$ after any history (on path and off path) is an equilibrium strategy.

Proof: if the policy function $g(\gamma) = \gamma^p$ is the solution of the Bellman equation (5.1), then by construction if everyone assumes that the SIG will always choose γ^p in future electoral cycles, it is optimal for the SIG to choose γ^p for the current electoral cycle. ■

γ^p is the level of bargaining power that is optimal for the SIG to choose, if the expectations of all the economy agents is that γ^p will be chosen in all future periods. We already saw that γ^p cannot be equal to γ^*, as it is profitable to move for one period to γ^b and then move back to γ^*. For the same reason it cannot be below γ^*, as in this case the move up to γ^b is even more profitable. γ^p also has to be above γ^s for the following reason. γ^p is a level where it is not profitable to deviate (either to a higher or a lower level). When you deviate to a higher level for one electoral cycle, the payoff includes the payoff of the electoral cycle in which there is a higher level of bargaining power, and the high wage in the last period before the return to γ^p. In order for the deviation not to be profitable, it has to be that the payoff for the electoral cycle is lower, as the wage in the last period is always higher. As moving (a bit) up from a level below γ^s increases the electoral cycle payoff, it has to be that $\gamma^p > \gamma^s$.

It is also clear the γ^p is decreasing in the electoral cycle length n. To see this assume that for a certain cycle length n, $g(\gamma) = \gamma^p$ is an equilibrium. This means that deviating to a higher level is not profitable, as the lower payoff from the first $(n-1)$ periods will more than offset the gain from the last period increase in wages. Assume by contradiction that for a shorter cycle (smaller n), $(\gamma) = \gamma^p$ is also an

[9] I am considering later as an extension a case where the utility is affected by previous cycle bargaining power through the unemployment level, when I consider costly lobbing based on the Grossman-Helpman model.

equilibrium. Now, a deviation to a higher level will have the same gain from the last period wage increase (as the move down from the higher level to γ^p is the same), but payoff form the $(n-1)$ periods before that is less negative, as n is smaller. This means that at the margin, a higher level of constant path is required in order to be an equilibrium for a lower n.

Given that the solution is a constant policy function $g(\gamma)=\gamma^p$, the value for the employed workers from this equilibrium strategy is the steady state value associated with γ^p, $W_{ss}^{\gamma_p}$. Based on that, we can now construct the optimal strategy for the SIG.

SIG optimal strategy: If $W_{ss}^{\gamma_p} \leq W_{ss}^{\gamma^*}$, the optimal strategy for the SIG is the following:

1. As long as there is no deviation, choose according to the optimal path $\{\gamma^b, \gamma^*, \gamma^*, \ldots\}$
2. Once there was a deviation, always choose γ^p.

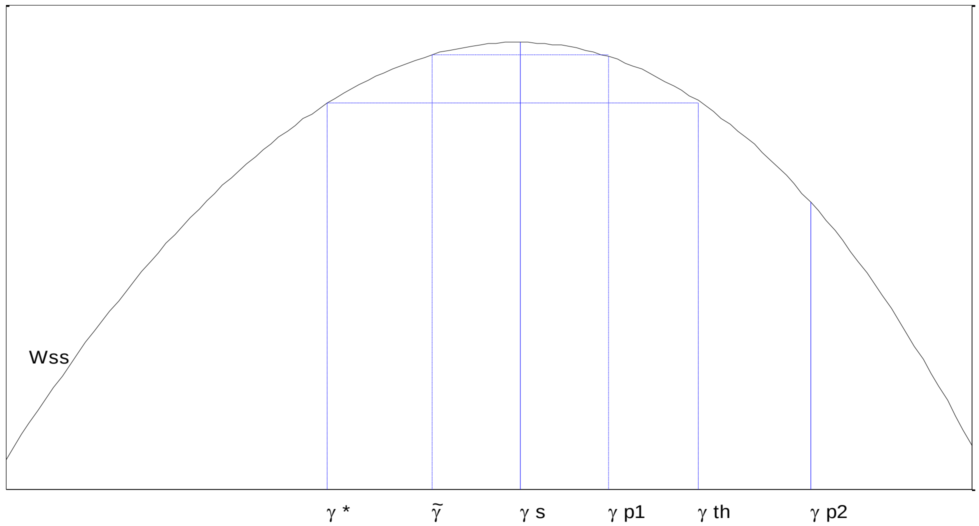
What can the workers do if $W_{ss}^{\gamma_p} > W_{ss}^{\gamma^*}$? In that case we saw that it is still the case that $\gamma_p > \gamma_s$. If $W_{ss}^{\gamma_p} > W_{ss}^{\gamma^*}$ this means that there is a level of bargaining power $\tilde{\gamma}$, such that $\gamma^* < \tilde{\gamma} < \gamma_s$ and $W_{ss}^{\gamma_p} = W_{ss}^{\tilde{\gamma}}$. The strategy in this case is:

1. As long as there is no deviation, choose according to $\{\gamma^b, \tilde{\gamma}, \tilde{\gamma}, \ldots\}$
2. If there was a deviation, always choose γ^p.

It is interesting to consider the resemblance to other repeated games results. In a classical repeated game, a high enough discount factor is required to support some equilibria, in order to make sure that the future punishment due to deviation is high enough even after discounting to offset the benefit from deviation that is accrued today. As the SIG can only lobby once every electoral cycle, the "effective" discount rate by which the SIG discounts the future is β^n, so when electoral cycles are shorter the discount rate is higher. Shorter electoral cycles result in higher levels of γ^p, which means that If the discount rate is high enough (shorter electoral cycles), $W_{ss}^{\gamma_p} < W_{ss}^{\gamma^*}$ and it is possible to support the best optimal bargaining power γ^b. For larger values of n, the effective discounting factor is not large enough and the SIG will only be able support lower values. At the limit, with n approaching infinity, there is no point in giving any consideration for the next period, so this is the equivalent of being totally impatient. In this case the SIG will only be able to support the static equilibrium and will receive a value of $W_{ss}^{\gamma_s}$.

Figure 6 shows the result graphically. The curve represents the steady state value of the employed workers for various levels of the bargaining power. $\gamma_{th} > \gamma_s$ is the bargaining power level above γ_s that provides the same value as γ^*. As long as $\gamma_p > \gamma_{th}$ (represented by γ_{p2} in the figure), $W_{ss}^{\gamma_p} \leq W_{ss}^{\gamma^*}$ and the optimal path can be supported. If $\gamma_p < \gamma_{th}$ (represented by γ_{p1} in the figure), $W_{ss}^{\gamma_p} > W_{ss}^{\gamma^*}$ and the optimal path cannot be supported as a deviation to γ_{p1} will be profitable. In this case the lowest bargaining power that can be supported starting form the second electoral cycle is $\tilde{\gamma}$.

Figure 6 - Various levels of γ_p and the resulting optimal policy



6 Extension - Lobbying Costs based on Grossman-Helpman

In this section I am relaxing the assumption that the lobbying costs do not matter for the employed worker's decision. This assumption means that that lobbying costs are either zero or constant regardless of the lobbying level[10]. In order to relax the assumption, I am adding lobbying costs based on the Grossman-Helpman (2002) Special Interest Groups model. in the SIG model, the policy maker cares about social welfare and about cash transfers from the lobbying party. in order to ensure the required level of lobbying, the SIG need to compensate the policy maker for the loss in social welfare. As the loss of social welfare is higher (and convex) the further the requested lobbying is from the socially optimum level, it is expected that adding lobbying cost will lower the level of the optimal bargaining power that the SIG will seek. I am showing numerically that adding the lobbying costs based on the Grossman-Helpman model does not change the result qualitatively, and the optimal strategy for the employed worker's is still a step function, with a level of first-period lobbying lower than the no-cost benchmark.

6.1 The Model

The model is very similar to the benchmark model, I am focusing here on the relevant differences only. There are no savings in the economy and workers consume all of their income. The value of being employed is:

$$W_t = w_t + c_t + \beta[\sigma U_{t+1} + (1-\sigma)W_{t+1}] \tag{6.2}$$

where c_t the mandatory union fee, used for covering lobbying costs

[10] Of course if the constant lobbying costs are too high, there would be no lobbying at all. I assume that this is not the case.

The rest of the basic DMP equations for the values of an unemployed worker, posting a vacancy, a filled position and the FOC of the bargaining are the same, listed here for completeness:

$$U_t = b + \beta\left[\left(1-\theta_t q(\theta_t)\right)U_{t+1} + \theta_t q(\theta_t) W_{t+}\right] \tag{6.3}$$

$$V_t = -\xi + \beta\left[\left(1-q(\theta_t)\right)V_{t+1} + q(\theta_t) J_{t+1}\right] \tag{6.4}$$

$$J_t = p - w_t + \beta[\sigma V_{t+1} + (1-\sigma)J_{t+1}] \tag{6.5}$$

$$W_t - U_t = \gamma_t(J_t + W_t - U_t) \tag{6.6}$$

6.1.1 The Policy Maker

The policy maker can influence the economy by enacting labor market policies that will determine the bargaining power of the employed workers. This is the only channel in which the policy maker can influence the economy, as I do not consider here a central planner that can dictate behavior to workers, firms or entrepreneurs. The policy maker is interested in the discounted sum of weighted average of the total surplus of all the agents in the economy and the political contribution. The per-period total surplus of all agents, net of vacancy cost, is defined, as in Hosios (1990) as:

$$g(u,v) = ub + (1-u)w + (1-u)(p-w) - \xi v = p - u(p-b) - \xi v$$

Even though the policy maker is replaced every electoral cycle, she considers the social welfare to be the infinite discounted sum, such that the utility function of the policy maker is:

$$G = \lambda\left\{\sum_{1}^{\infty} \beta^{t-1} g(u_t, v_t)\right\} + (1-\lambda)C_1$$

Where C_1 is the political contribution that she receives at period 1 and $\lambda \in [0,1]$ is the weight that the policy maker puts on the economy's welfare. Note that although the policy maker is replaced every electoral cycle, when considering the social welfare she is discounting the entire future. Also note that the analysis with zero lobbying cost can be seen as a special case of this model, with $\lambda = 0$. As we know from Hosios, regardless of the initial unemployment level, setting $\gamma = \gamma^*$, the elasticity of the matching function, maximizes social welfare. Thus, I assume that if there is no lobbying, the policy maker sets $\gamma = \gamma^*$.

Denote $\hat{G}(u_t)$ as the utility that the policy maker achieves if there is no lobbying in the economy:

$$\hat{G}(u_t) = \max_{\gamma} \lambda \sum_{j=0}^{\infty} \beta^j g\left(u_{t+j}, v_{t+j}\right) = \lambda \sum_{j=0}^{\infty} \beta^j g\left(u_{t+j}, v_{t+j}\right) | \, \gamma_{t+j} = \gamma^* \; \forall j$$

6.1.2 The Employed Workers

The employed worker's problem is to decide whether to lobby the policy maker for a change in the bargaining power level, and to which extent. Like before, I assume here that up to a certain period, there was no lobbying in the economy, and the economy was in the steady state. At a certain period, the employed workers figured out that they can lobby, and they gather to decide if and to which extent to lobby. If they decide to lobby, the change in the bargaining power takes effect immediately.

The SIG problem is to maximize the value of an employed worker:

$$W_t(u) = \max_{\gamma}\{w_t(\gamma) - c_t(\gamma) + \beta[\sigma U_{t+1}(\gamma) + (1-\sigma)W_{t+1}(\gamma)]\}$$

Subject to the constraint $G \geq \hat{G}$, where u, the unemployment level, is the only state variable, and $c_t(\gamma)$ is the per period fee that each employed worker pays to cover lobbying costs.

Assuming that the contribution schedule provides the policy maker exactly the same utility as she can get without the contribution, the policy maker must be paid:

$$C(\gamma) = \frac{\hat{G}}{1-\lambda} - \frac{\lambda}{1-\lambda}\left[\sum_0^\infty \beta^t g(u_t, v_t)\right] \tag{6.7}$$

The entire lobbying cost is due upon lobbying, in period 1. I assume that the cost is paid by the workers throughout the electoral cycle and is spread evenly across periods, so the per period fee, $c(\gamma)$, satisfies:

$$C(\gamma) = \sum_{t=1}^{n} \beta^{t-1}(1-u_t)\, c(\gamma)$$

$$c(\gamma) = {C(\gamma)}\Big/{\sum_{t=1}^{n} \beta^{t-1}(1-u_t)} \tag{6.8}$$

Note that there is some level of double payment by the employed workers, as if there is an electoral cycle in the future for which $\gamma \neq \gamma^*$, all previous electoral cycles will have to pay for the deviation from the optimal path. I do not consider that a problem here as (a) this might be just a property of reality that you need to pay to multiple administrations. It seems reasonable to assume that in a situation where you want to buy some permanent (or long term) deviation from the optimal social state, you will need to pay more when there are more interested parties (or to put it otherwise, when the elections are more frequent), and (b) this can be compensated by the level of λ and I am reporting the results for all levels of $\lambda \in [0,1]$. Also note that this assumption is crucial if I want to ensure that the payments are always positive, as if the target function of the policy maker is not the entire optimal path (for example, only the discounted sum over the n periods of her electoral cycle), there can be a path that is more beneficial to the policy maker (during the first electoral cycle) than the value from the first electoral cycle of the optimal path.

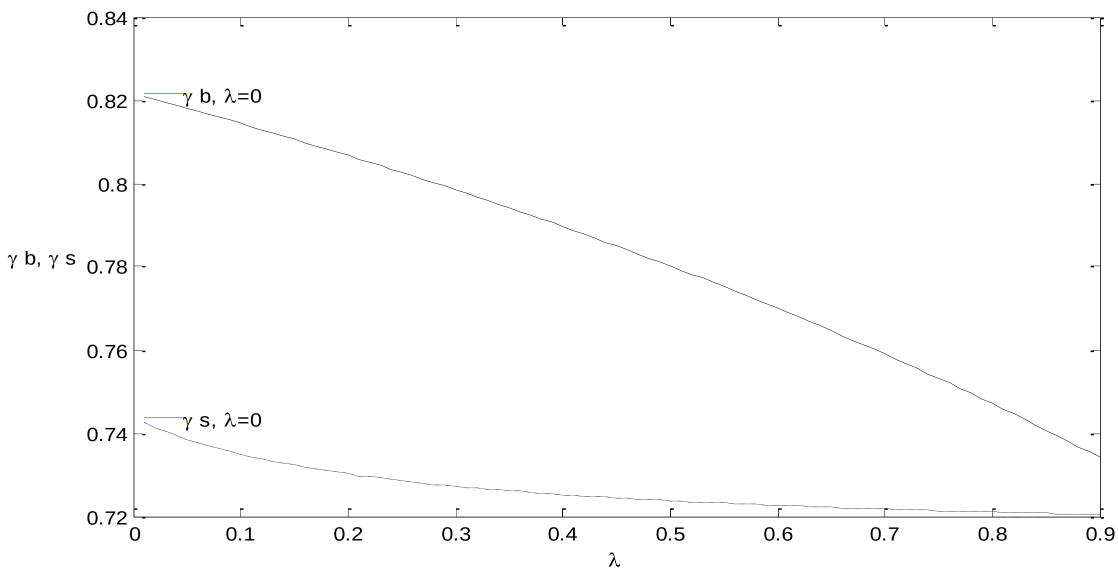
6.2 Calibration

I am using the following calibration based on Shimer (2005), for a period of one month: $\beta = 0.996$ for a 5% yearly interest rate, elasticity of matching function $= 0.72$, separation rate $\sigma = 0.034$, productivity is normalized to $p = 1$, cost of vacancy $\xi = 0.211$ to calibrate $\theta = 1$ for the optimal level, $b = 0.4$ to calibrate 40% of the wage and matching function efficiency $\chi = 0.45$ so a worker finds a job with 0.45 probability per month when $\theta = 1$. The electoral cycle length is set to $n = 48$, or 4 years.

6.3 Results

I am solving this extended model numerically, using the calibration in section (6.2). Appendix C has the exact details of the solution algorithm. The solution shows that the optimal path for SIG is indeed still a "step" policy, with a single electoral cycle of high bargaining power followed by a constant path of $\gamma = \gamma^*$. As can be expected, for higher values of λ, where the cost of deviating is higher, the optimal level of first-cycle bargaining power is lower. The higher is λ, the closer is the optimal γ_1 to γ^*. Figure 7 shows the optimal level of first cycle bargaining power for different values of λ (the solid black line. As can be seen that level is falling, and it is below that optimal level when there is no cost for lobbying. The dotted black line is the best static path for the SIG, also falling with λ and below the non-lobbying level.

Figure 7 – optimal bargaining power γ^b, γ^s falls with λ



7 Extension – OLG Model with finitely-lived agents

In my baseline model, all the economy agents are infinitely lived and homogenous. In such a case there is never a conflict of interest between the employed workers themselves, and when the employed workers pursue a strategy, which has an infinite horizon by definition, they internalize the impact of the strategy on their future self. In order to check if these assumptions are crucial for my results, I am building an OLG model with finitely-lived agents. In this OLG model all the employees have a final planning horizon, and the planning horizon varies between employees as some of them are closer to retirement then others. A conflict of interest arises between the senior and junior employees.

The OLG model with repeated lobbying is very similar to the baseline model with repeated lobbying. The only strategic agents are the senior workers who are offering the level of bargaining power to lobby for each electoral cycle. As we have seen in the description of the game for the baseline model, the senior workers offer, in the first time that the vote is taking place, an entire strategy to be carried out, following each possible history (including histories not on the equilibrium sequence). The difference between the game in the OLG case and in the baseline case is that in the OLG case there is another relevant agent – the median voter[11]. The expectations of the median voter, like all other agents, are derived from the strategy that the senior workers offer in the first electoral cycle. This means that there is an additional constraint on the strategy, which is that it always must be (weakly) better for the median voter to vote for the lobbying offer, then to vote against it.

In the repeated lobbying scenario, it is natural to define the senior workers as workers whose value depends on the current electoral cycle results only. This leaves workers with less than 47 periods left in the workforce.

An $equilibrium$ of the lobbying game of the OLG model is a strategy of the senior employed workers $L(u_0): H \rightarrow \Gamma$, such that for any possible history (A^t, u_t), if the expectations $E(A^t, u_t)$ are equal to $L(A^t, u_t)$, the optimal sequence for the union is $L(A^t, u_t)$, and it is optimal for the median voter to vote for the offer presented by the senior workers.

Another difference in the OLG version of the repeated lobbying game is in the "punishment" equilibrium. In the baseline model, the punishment equilibrium has expectations for a high level of bargaining power. Moving up to this high level provides a low value to the workers due both to the one time low wage just before the move, and the low value of the high level itself. However, it is optimal for the workers to move there, given the expectations, in order to suffer the low wage before the move up only once. In the OLG model, there is no way to punish the senior workers if they deviate and offer a different level to lobby for, as by construction they do not care cycles beyond the closest one. However, it is possible to punish the median voter. The punishment equilibrium will include expectations for a high level of bargaining power, which is better for the senior workers and thus optimal for them to offer, but bad to the median voter. Once this becomes the expected sequence, the median voter will be forced to vote for the punishment equilibrium given the expectations, in order not to get the low wage associated with the expectations of moving the level up multiple times. This threat of punishment will keep the median voter voting for the expected sequence.

7.1 The Model

There is a measure 1 of employees, divided into m overlapping generations of equal measure, with each generation being in the workforce for m periods. All the workers are homogenous in terms of their productivity and bargaining power, and the difference between their various value functions and their value to their employer only results from the remaining time they have left in the workforce.

There is a single matching market, and firms cannot distinguish between the prospective worker's cohorts beforehand. The actual matchings' are split proportionally between unemployed from all cohorts. The probability of a firm to find a worker is:

$$\frac{M(u,v)}{v} = M\left({}^{u}/_{v}, 1\right) = M\left({}^{1}/_{\theta}, 1\right) \equiv q(\theta)$$

[11] I am assuming here that the value from strategy that the senior workers will offer is increasing for each subsequent generation, and thus I limit the discussion on the vote to the value of the median voter only, as all older generations will vote for any offer that the median voter votes for. This assumption is easily verifiable for any specific strategy.

Where $\theta \equiv {}^{v}/_{u}$ is the market tightness and u is the aggregate number of unemployed in the economy. The probability that an unemployed person will find a job, equal to all unemployed workers, is:

$$\frac{M(u,v)}{u} = \frac{\theta M(u,v)}{v} = \theta q(\theta)$$

The number of unemployed workers includes unemployed that are joining the workforce in the current period (i.e. in the youngest cohort), as I am assuming that new workers do not necessarily have to spend their first period as unemployed and can be matched immediately. Unemployed from the last cohort m have no chance for a matching as they will already be retired next period.

Matches are separated with probability σ per period. The unemployment level for first generation workers is:

$$u_{1,t+1} = \frac{1}{m}\left(1 - \theta_t q(\theta_t)\right) \tag{7.1}$$

As the measure of each generation is $\frac{1}{m}$, where $u_{1,t+1}$ represents the number of unemployed workers from cohort #1 in period $t+1$. The number of employed workers from the youngest generation is:

$$l_{1,t+1} = \frac{1}{m} - u_{1,t+1} = \frac{1}{m} - \frac{1}{m}\left(1 - \theta_t q(\theta_t)\right) = \frac{1}{m}\theta_t q(\theta_t) \tag{7.2}$$

And for each subsequent generation:

$$u_{i+1,t+1} = \left(\frac{1}{m} - \theta_t q(\theta_t)\right) u_{i,t} + \sigma l_{i,t} \tag{7.3}$$

$$l_{i+1,t+1} = \frac{1}{m} - u_{i+1,t+1} \tag{7.4}$$

Note that for a given steady state level θ, the model will converge to the benchmark model level of unemployment when m grows to infinity. It will diverge above that level for lower levels of m as the first generation has a high unemployment level.

Without savings in the economy, employed workers consume all of their income, so the value of an employed worker from generation i in period t, denoted $W_{i,t}$, is:

$$W_{i,t} = w_{i,t} + \beta\left[\sigma U_{i+1,t+1} + (1-\sigma) W_{i+1,t+1}\right] \tag{7.5}$$

Where all the values for period $(m+1)$ are 0 as workers retire after period m and firms are closed if their employed worker retires.

The value of an unemployed worker is:

$$U_{i,t} = b + \beta\left[\left(1 - \theta_t q(\theta_t)\right) U_{i+1,t+1} + \theta_t q(\theta_t) W_{i+1,t+1}\right] \tag{7.6}$$

The value of a posted vacancy is:

$$V_t = -\xi + \beta\big[\big(1 - q(\theta_t)\big)V_{t+1} + q(\theta_t)E(J_{t+1})\big] \tag{7.7}$$

where $E(J_{t+1})$ is the expected value from a filled position next period for newly formed positions, given that each unemployed worker has an equal chance of landing a job, and taking into account the amount of unemployed in each generation, such that:

$$E(J_{t+1}) = E\left(\frac{1}{\sum_{i=1}^{m} u_{i-1,t}} \sum_{i=1}^{m} u_{i-1,t} J_{i,t+1}\right)$$

where $\sum_{i=1}^{m} u_{i-1,t}$ is the total number of unemployed workers who are looking for a job (as unemployed workers from the last generation are not looking anymore) and $u_0 \equiv 1/_m$.

The value of a position filled by an employee from generation i is:

$$J_{i,t} = p - w_{i,t} + \beta\big[\sigma V_{t+1} + (1-\sigma)J_{i+1,t+1}\big] \tag{7.8}$$

Assuming standard generalized Nash bargaining, FOC for the bargaining problem is:

$$W_{i,t} - U_{i,t} = \gamma_t\big(J_{i,t} + W_{i,t} - U_{i,t}\big) \tag{7.9}$$

7.2 The Employed Worker's SIG

The SIG is deciding at the beginning of a certain period if they want to lobby. If they decide to lobby, the change in the bargaining power takes effect immediately. I am assuming that the senior workers can suggest the bargaining power level for a union vote, which must receive at least 50% of the votes.

The value for an employee from generation i in the period of the decision t is:

$$W_t(i,\gamma) = w_{i,t}(\gamma) + \beta\big[\sigma U_{i+1,t+1}(\gamma) + (1-\sigma)W_{i+1,t+1}(\gamma)\big]$$

With this setting, each generation has its preferred lobbying sequence. This is the sequence that if executed will provide the employees from this given generation that highest first-period value (of course, assuming as before that the sequence can be supported as an equilibrium). Also, each generation knows what its default value is – that value in case no lobbying is carried out and the policy maker keeps setting the socially-optimal bargaining level. As such, only lobbying sequences that provide at least half of first-period employees a value no lower than their default value can win a SIG-wide vote.

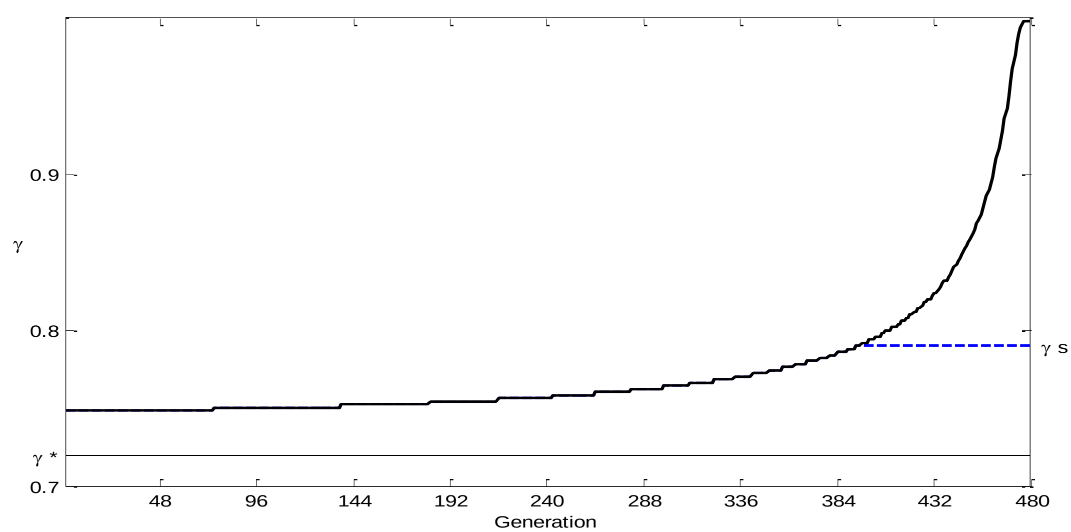
7.3 Results

I am solving this extended model numerically with the exact algorithm is in Appendix D. Before analyzing the complete dynamic results, it is helpful to look at the optimal static path for each generation. The meaning of a "static path" here is the same as in the benchmark model, where only a single change can be done to the bargaining power of the employed workers, and this change will hold forever. The full dynamic transition from the previous steady state to the new steady state is calculated, which allows finding the best static policy for each generation.

7.3.1 The Optimal Static Path

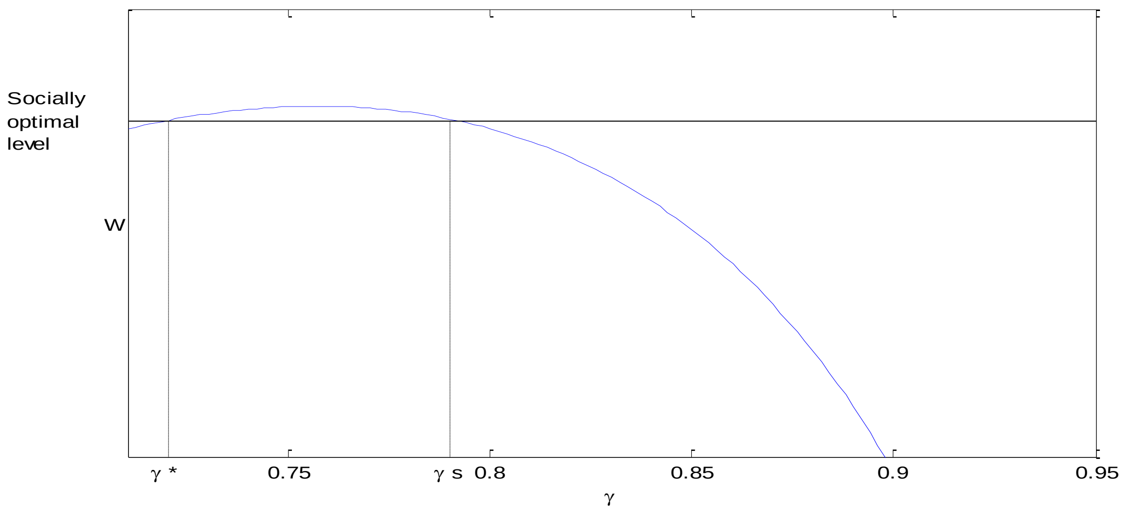
Figure 8 shows the best static path per generation, not considering the need to win a majority vote (in solid black) – the "unconditional path", and also the best static path per generation that can win a majority approval (in dotted blue) – the "conditional path". Even the youngest generation would like to increase the bargaining power, at least somewhat, above the socially optimal level γ^* (shown as a horizontal line). The optimal level for the youngest generation is numerically almost the same as the optimal level of the infinitely lived employees in the benchmark case (using the same calibration), as they have a very long planning horizon. As workers get older, and conditional on them being employed, they start to internalize less and less the impact of an increase in the bargaining power on the unemployed workers, as their expected unemployment duration (as a ratio of time left in the workforce) shrinks. Their optimal static level increases, slowly at the beginning and faster towards retirement. The most senior generation, of course, does not put any weight on the value of being unemployed and would like to raise the bargaining power as high as possible.

Figure 8 - Best Static Path per Generation



Higher levels of bargaining power, while beneficial for old workers, hurt younger workers with a longer planning horizon. This is why the very high levels of bargaining power cannot win a majority approval as they will provide a lower value to the median employee than the alternative, which is to stay with the socially optimal level. As can be seen in figure 8, all employees in their last electoral cycle in the workforce (generations 433-480) and most of the employees with one additional cycle left (generations 385-432) cannot achieve their optimal level and will have to settle for a lower value, that provides the median employee with at least the same value as the default option, denoted γ^s. To show the point better, figure 9 shows the median worker value from all possible levels of bargaining power. The peak of the hump represents the optimal static level for the median voter, while the horizontal line represents the value that the median voter can receive from the socially optimal level, which will be the case if the vote fails. As the value from the static levels of bargaining power falls very fast when the level increases, we get the result that the seniors workers cannot achieve a level even close to their desired level with a static path.

Figure 9 – Median Voter value for different static levels of bargaining power



The core problem for the senior workers in this case is that with a static path, they cannot compensate the median worker for the high level of bargaining power that they want, even though being close to exiting the workforce, they only need the high level for a single electoral cycle, before they retire.

7.3.2 The Optimal Dynamic Path

With the possible use of a dynamic path, the senior workers have more options. First consider the unconditional dynamic optimal path for a senior worker, regardless of the need to win an approval vote. As the senior worker will not be in the workforce in the next electoral cycle, and not even on the last period of the current electoral cycle, it might seem as if she will only care about the current period bargaining level and will be indifferent about future levels. This is almost the case. Recall that for the benchmark case, I have proved analytically that if the bargaining power level is falling towards the socially optimal level in the next electoral cycle, the market tightness, while higher after the change, will be lower than the steady state level associated with the value of the first cycle for the periods preceding the period just before the change (see Figure 1). Of course the opposite is true if the change increases the bargaining power level – low wage in the period before the change, but higher wage and market tightness in all preceding periods. This is also true for the OLG case. The consequence of this is that for all workers senior enough not to care about the wage one period before the end of the electoral cycle (and all periods after that, of course), it is optimal to have a path with the first cycle level equals to their static optimal level, following by an increase of the bargaining power level for as high as possible after that. This is due to the fact that the higher market tightness and wages during their remaining periods in the workforce both increases their value. This increase is very small numerically, but exists none the less. Extrapolating for the rest of the workers, it is clear that the unconditional dynamic path for all the employed workers, is to move to a high level of bargaining power for the first electoral cycle, move down to the socially optimal level for as long as they are in the workforce, and shoot up to the highest possible level starting the first electoral cycle for which they are already retired.

Figure 10 – first and second electoral cycle optimal levels

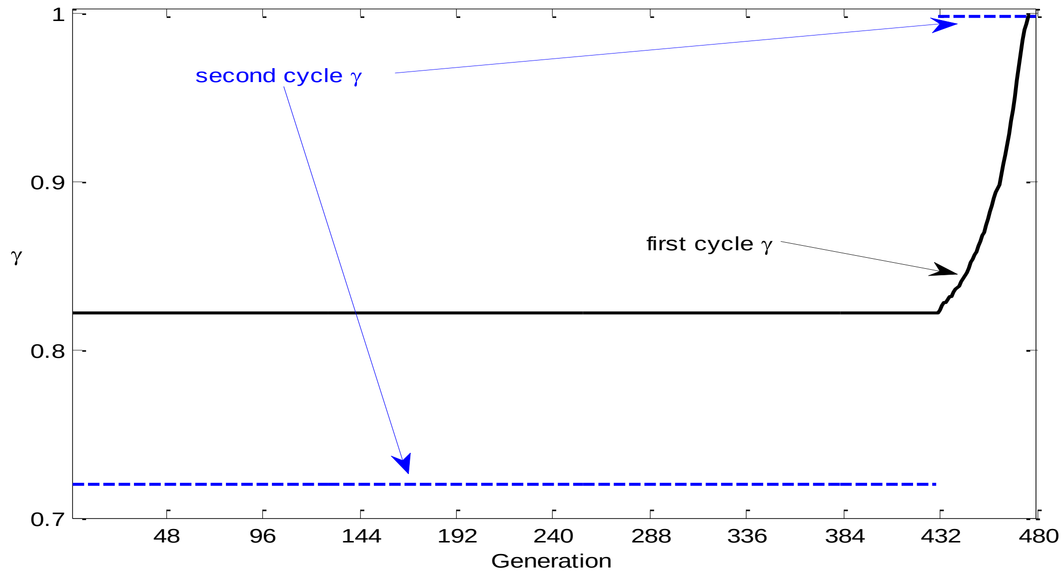


Figure 10 shows this result graphically. The solid black line shows the optimal bargaining power level in the first electoral cycle and the dashed blue line the optimal level in the second electoral cycle. All employees with one full cycle left in the labor force have the same first and second cycle optimal level, with the familiar pattern of a high level and a drop to the socially optimal level. Employees that cannot benefit from the high wage that such a path provides at the end of the first electoral cycle (the most senior employees) will just choose their optimal static level for the first cycle, and the highest possible level afterwards.

7.3.3 The Optimal Dynamic Path with Majority Vote

I now consider the optimal dynamic path that can win a majority vote. The most senior employees, who can offer a lobbying level, only care about results in the coming electoral cycle. They cannot benefit by definition from a high wage during the last period of the electoral cycle. However, they can use the promise of such a high wage, resulting from a reduction in the bargaining power, to get an approving vote for a high first period level. This is indeed their optimal policy given the approval constrain.

Changing the bargaining power higher between the first and the second electoral cycle will result in a very low wage one period before the change. For that reason, the senior workers cannot get an approval vote for their unconditional optimal path described in the previous section. However, they can achieve a first level bargaining power higher than the best static path, by offering a path with a high level in the first electoral cycle and a reduction in the second electoral cycle. The high wage in the last period of the first electoral cycle compensates the median voter for the low value derived from the rest of the first electoral cycle.

Figure 11 – first and second electoral cycle optimal levels

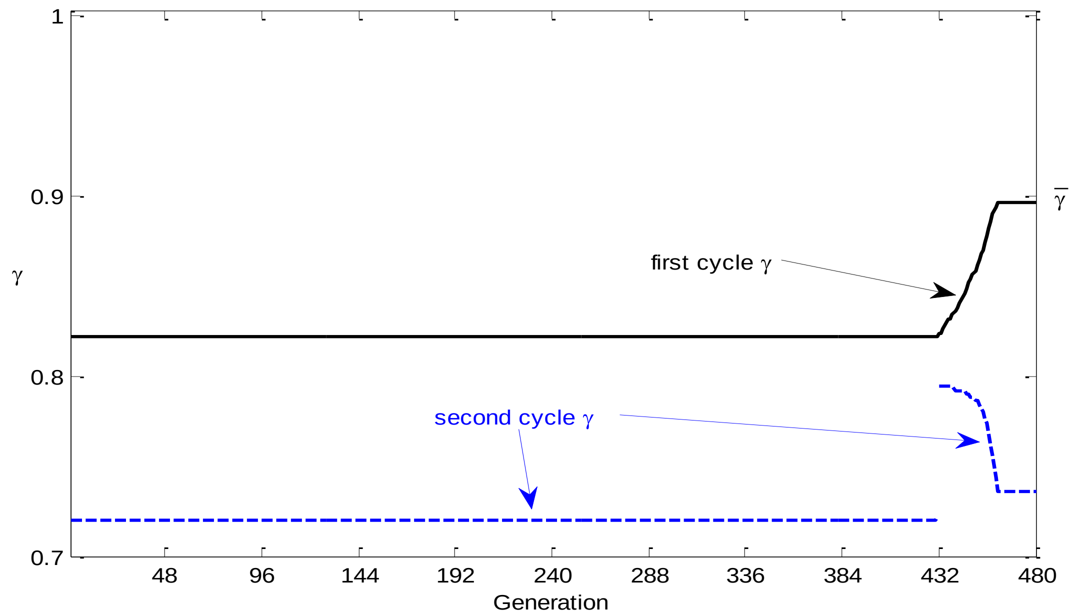


Figure 11 shows the optimal path that win a majority approval for each generation. All the generations young enough to enjoy a wage increase in the last period of the first electoral cycle are utilizing the standard path for that, with a high first period bargaining power level following by a drop to the socially optimal level in the second cycle. The most senior employees would like an even higher level during their remaining time in the workforce, as shown in figure 9. In order to compensate the median voter for the higher level, the need to offer a path with a drop in the second electoral cycle, although they will not be in the workforce to enjoy it themselves. As shown in Figure 9, the senior workers actually want the highest possible second-cycle level. Hence, they offer the highest second cycle level that still leaves the median voter indifferent to the no-lobbying option. As can be seen in Figure 10, the higher you want the first cycle level to be, the lower you need to drop in the second cycle in order to compensate the median voter. Above a certain first cycle level, designated $\bar{\gamma}$ in Figure 10, even a drop to the socially optimal level cannot leave the median voter with a high enough value to approve the path. Hence, $\bar{\gamma}$ is the highest level that can win a majority vote I am deliberately refraining from discussing the actual level chosen by the senior employees as the point of the extension is to show that the path will still be a "step" path.

7.4 Supporting the Equilibrium Path

Like in the baseline model, in order to support the required path we need to define the expectations in case the path is not followed. The path here can be deviated in two ways. (a) the senior workers can offer a bargaining power level different than the expected one, and (b) the voters can reject the offer. In order to prevent a deviation from happening, we need to define the off-path, or "punishment" equilibrium. This punishment equilibrium must have the following properties:

1. Given the expectations, it is optimal for the senior workers to continue offering along the punishment path.

2. Given the expectations, it is optimal for the median voter (and all voters more senior than the median voter) to vote for the punishment path.
3. The punishment path provides a lower value for the median voter (and all voters less senior than the median voter) than continuing on the original equilibrium path, such that they will reject any deviation by the senior workers.

Like in the benchmark case, I am looking for a stationary punishment equilibrium. Using the same technique as before, I am looking for a constant bargaining power level γ^p. γ^p is such that given the expectations for a continuing bargaining level of γ^p in every electoral cycle after the current one, it is optimal for the senior workers to choose γ^p for this electoral cycle. Regardless of the choice mechanism within the senior workers, this level is easily found numerically. As the senior workers value is derived almost exclusively from current electoral cycle bargaining power, this level will be very close to their optimal static level. Thus, it is easy to verify that:

1. Given the expectations that the senior workers will continue to offer γ^p, it is best for the median workers to vote for it, rather than deviate for the much lower socially optimal level for one electoral cycle and suffer a very low wage one period before the end of the current electoral cycle due to the expected rise in the bargaining power back to γ^p.
2. This static path provides the median worker with a lower value than the original equilibrium path.

8 Conclusion

The paper describes and analyzes a dynamic model of the labor market where employees can organize in a SIG and influence their bargaining power through a political mechanism. The paper shows that considering a fully dynamic model induces a qualitatively different optimal policy for the SIG than from a stationary environment. The SIG can pull forward some of the wages by first employing a high bargaining power level, not optimal by itself, and then reducing the level back to the socially optimal level. Such a mechanism can only be studied in a fully dynamic environment, where all agents take into effect their expectations for future actions. The paper shows that the result, analytically proven for infinitely-lived employees with no lobbying costs, holds with the addition of lobbying costs and an OLG models of the employees. The distortion caused by the SIG in order to maximize their value, in terms of aggregate productivity, is higher when elections are more frequent.

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10 Appendix A: The dynamic evolution of the market tightness θ

10.1 Deriving the Dynamic Equation

In this appendix I derive the dynamic evolution of the market tightness (θ_t). In a dynamic environment where the bargaining power changes, the set of dynamic equations of the DMP model can be solved into a single dynamic equation for the market tightness only:

From (2.4), (2.6), and using $W - U = \gamma S$ and $J = (1-\gamma)S$:

$$q(\theta_t) = \frac{\xi}{\beta J_{t+1}} = \frac{\xi}{(1-\gamma_{t+1})\beta S_{t+1}} \tag{A.1}$$

From (2.6) and (A.1):

$$W_{t+1} - U_{t+1} = \frac{\gamma_{t+1}}{1-\gamma_{t+1}} J_{t+1} = \frac{\gamma_{t+1}}{1-\gamma_{t+1}} \frac{\xi}{\beta q(\theta_t)} \tag{A.2}$$

From (2.2), (2.3) and (A.2):

$$U_t = b + \beta U_{t+1} + \beta\theta_t q(\theta_t)(W_{t+1} - U_{t+1}) = b + \beta U_{t+1} + \xi\theta_t \frac{\gamma_{t+1}}{1-\gamma_{t+1}} \tag{A.3}$$

$$W_t = w_t + \beta W_{t+1} - \beta\sigma(W_{t+1} - U_{t+1}) = w_t + \beta W_{t+1} - \sigma \frac{\gamma_{t+1}}{1-\gamma_{t+1}} \xi\theta_t \tag{A.4}$$

From (2.2), (2.5) and (A.2):

$$W_t + J_t = p + \beta[W_{t+1} + J_{t+1} - \sigma S_{t+1}] \tag{A.5}$$

From (A.3) and (A.5):

$$S_t = W_t + J_t - U_t = p - b + \beta(1-\sigma)S_{t+1} - \xi\theta_t \frac{\gamma_{t+1}}{1-\gamma_{t+1}} \tag{A.6}$$

From (2.2), (A.3), (2.5) and (2.4), Into (2.6):

$$\begin{aligned} w_t + \beta[\sigma U_{t+1} + (1-\sigma)W_{t+1}] - b - \beta U_{t+1} - \frac{\gamma_{t+1}}{1-\gamma_{t+1}}\xi\theta_t \\ = \gamma_t \Big\{ w_t + \beta[\sigma U_{t+1} + (1-\sigma)W_{t+1}] - b - \beta U_{t+1} - \frac{\gamma_{t+1}}{1-\gamma_{t+1}}\xi\theta_t + p - w_t \\ + \beta\left[(1-\sigma)\frac{\xi}{\beta q(\theta_t)}\right]\Big\} \end{aligned}$$

$$w_t + \beta(1-\sigma)(W_{t+1} - U_{t+1}) - b - \frac{\gamma_{t+1}}{1-\gamma_{t+1}}\xi\theta_t$$
$$= \gamma_t \left\{\beta(1-\sigma)(W_{t+1} - U_{t+1}) - b - \frac{\gamma_{t+1}}{1-\gamma_{t+1}}\xi\theta_t + p + \beta(1-\sigma)\frac{\xi}{\beta q(\theta_t)}\right\}$$

Using (A.2):

$$w_t + (1-\sigma)\frac{\gamma_{t+1}}{1-\gamma_{t+1}}\frac{\xi}{q(\theta_t)} - b - \frac{\gamma_{t+1}}{1-\gamma_{t+1}}\xi\theta_t$$
$$= \gamma_t \left\{(1-\sigma)\frac{\gamma_{t+1}}{1-\gamma_{t+1}}\frac{\xi}{q(\theta_t)} - b - \frac{\gamma_{t+1}}{1-\gamma_{t+1}}\xi\theta_t + p + (1-\sigma)\frac{\xi}{q(\theta_t)}\right\}$$

$$w_t + (1-\sigma)\frac{\gamma_{t+1}}{1-\gamma_{t+1}}\frac{\xi}{q(\theta_t)} - b - \frac{\gamma_{t+1}}{1-\gamma_{t+1}}\xi\theta_t = \gamma_t \left\{(1-\sigma)\frac{1}{1-\gamma_{t+1}}\frac{\xi}{q(\theta_t)} - b - \frac{\gamma_{t+1}}{1-\gamma_{t+1}}\xi\theta_t + p\right\}$$

$$w_t = b(1-\gamma_t) + \gamma_t p + \frac{\gamma_{t+1}}{1-\gamma_{t+1}}[\xi\theta_t - \gamma_t\xi\theta_t] + (1-\sigma)\frac{\gamma_t - \gamma_{t+1}}{1-\gamma_{t+1}}\frac{\xi}{q(\theta_t)}$$

$$w_t = b(1-\gamma_t) + \gamma_t p + \gamma_{t+1}\xi\theta_t\frac{1-\gamma_t}{1-\gamma_{t+1}} + (1-\sigma)\frac{\gamma_t - \gamma_{t+1}}{1-\gamma_{t+1}}\frac{\xi}{q(\theta_t)}$$

And if $\gamma_{t+1} = \gamma_{t+2} = \gamma$ (i.e we are before the two last periods of the electoral cycle) we get the wage equation:

$$w_{t+1} = b(1-\gamma) + \gamma p + \gamma\xi\theta_{t+1} \qquad \text{(A.7)}$$

From (2.4) and (2.5) we get:

$$\frac{\xi}{\beta q(\theta_t)} = p - w_{t+1} + \frac{(1-\sigma)\xi}{q(\theta_{t+1})}$$

And inserting the expression for w_{t+1}, We get the dynamic evolution of the market tightness[12]

$$\frac{\xi}{\beta q(\theta_t)} = (p-b)(1-\gamma) - \gamma\xi\theta_{t+1} + \frac{(1-\sigma)\xi}{q(\theta_{t+1})} \qquad \text{(A.8)}$$

Given the forward looking nature of the model, I am using the θ evolution equation to solve the model backward, i.e. derive θ_t given the bargaining power and θ_{t+1}.

[12] Note that the familiar steady state equation of θ can be derived from the steady state version of (A.8):

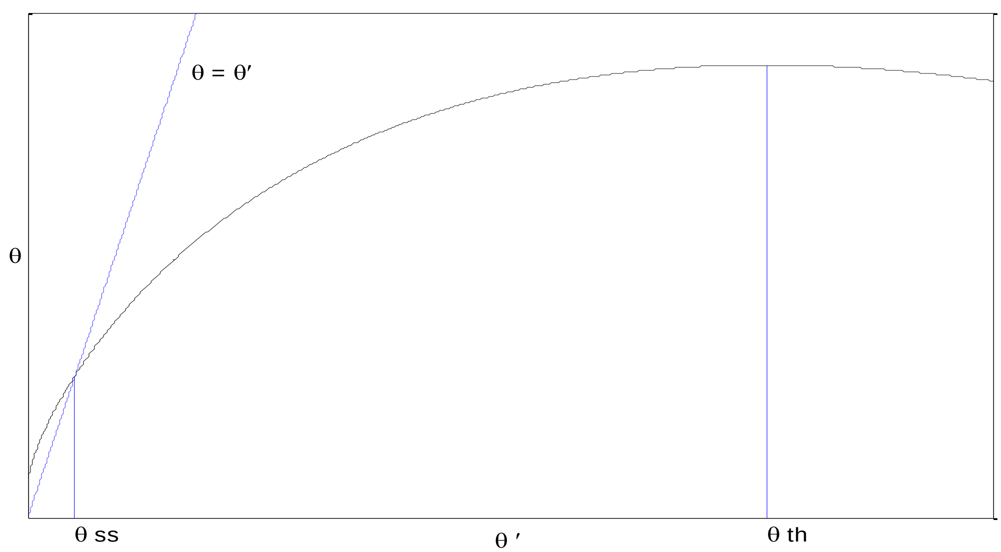
$$\frac{\xi}{q(\theta)}\left(\frac{1}{\beta} - (1-\sigma)\right) = (p-b)(1-\gamma) - \gamma\xi\theta$$

10.2 Understanding the Dynamic Equation

The dynamic evolution equation (A.8) implicitly define the function $\theta = prev(\theta')$. Figure 12 shows this function with θ on the vertical axis and θ' - the dependent variable – on the horizontal axis. The function is increasing at start and then decreasing. In order to have a stable steady state, it has to be that the peek of the function (where $\frac{\partial\theta_t}{\partial\theta_{t+1}} = 0$, denoted θ^{th}) is above the steady state, otherwise there is no convergence towards the steady state. I am assuming here that this is indeed the case for all $\gamma \in (0,1)$[13]. More than that, I am assuming that for all plausible paths, it is always the case that $\theta_t \leq \theta^{th}$. While the non-linear nature of the DMP equations allows for such cases, which also implies a non-monotonic response functions for θ_t. But it seems that this is more a mathematical than an economic possibility. In practice this is not a string restriction, forbidding only the most extremes values for the bargaining power, very close to 0 or 1.

Given that assumption, for each γ there is a threshold $\theta^{th} > \theta^{ss}$ for which $\frac{\partial\theta_t}{\partial\theta_{t+1}} = 0|_{\theta_{t+1}=\theta^{th}}$. As $prev(\theta^{ss}) = \theta^{ss}$, θ increases towards θ^{ss} if it is lower and decreases towards θ^{ss}if it is higher (but lower or equal θ^{th}). More than that, as for all $\theta_{t+1} > \theta^{th}$, $prev(\theta_{t+1}) < prev(\theta^{th})$, than if $\gamma_2, \dots, \gamma_{t+2} = \gamma$ than $\theta_1, \dots, \theta_t$ are all lower than $prev(\theta^{th})$ and $\frac{\partial\theta_t}{\partial\theta_{t+1}} > 0$.

Figure 12 – dynamic evolution of the market tightness $\theta = prev(\theta')$



10.3 Deriving θ^{th}

Derive θ_t according to θ_{t+1}:

$$\frac{\partial LHS}{\partial\theta_{t+1}} = -\frac{\xi}{\beta q^2(\theta_t)} \frac{\partial q(\theta_t)}{\partial\theta_{t+1}}$$

[13] This is easily verifiable for all reasonable calibrations

$$\frac{\partial RHS}{\partial \theta_{t+1}} = -\gamma\xi + (1-\sigma)\xi\left[\frac{-1}{q^2(\theta_{t+1})}\frac{\partial q(\theta_{t+1})}{\partial \theta_{t+1}}\right]$$

Combining:

$$\frac{\partial q(\theta_t)}{\partial \theta_{t+1}} = -\beta q^2(\theta_t)\left[-\gamma - \frac{(1-\sigma)}{q^2(\theta_{t+1})}\frac{\partial q(\theta_{t+1})}{\partial \theta_{t+1}}\right] \quad \text{(A.8)}$$

From the definition of M, q we know that:

$$\frac{\partial q(\theta_t)}{\partial \theta_t} = -\frac{\eta(\theta)q(\theta_t)}{\theta_t} \quad \text{(A.9)}$$

Inserting Into (A.8)

$$\frac{\partial q(\theta_t)}{\partial \theta_{t+1}} = -\beta q^2(\theta_t)\left[-\gamma + \frac{(1-\sigma)}{q^2(\theta_{t+1})}\frac{\eta(\theta)q(\theta_{t+1})}{\theta_{t+1}}\right] = \beta q^2(\theta_t)\left[\gamma - \frac{(1-\sigma)}{q(\theta_{t+1})}\frac{\eta(\theta)}{\theta_{t+1}}\right]$$

Using the chain rule:

$$\frac{\partial q(\theta_t)}{\partial \theta_{t+1}} = \frac{\partial q(\theta_t)}{\partial \theta_t}\frac{\partial \theta_t}{\partial \theta_{t+1}} \quad \text{(A.10)}$$

$$\frac{\partial \theta_t}{\partial \theta_{t+1}} = \frac{\partial q(\theta_t)}{\partial \theta_{t+1}}\Big/\frac{\partial q(\theta_t)}{\partial \theta_t} = \beta q^2(\theta_t)\left[\gamma - \frac{(1-\sigma)}{q(\theta_{t+1})}\frac{\eta(\theta)}{\theta_{t+1}}\right]\Big/\left(-\frac{\eta(\theta)q(\theta_t)}{\theta_t}\right)$$

$$\frac{\partial \theta_t}{\partial \theta_{t+1}} = \frac{\beta q(\theta_t)\theta_t}{\eta(\theta)}\left[\frac{(1-\sigma)}{q(\theta_{t+1})}\frac{\eta(\theta)}{\theta_{t+1}} - \gamma\right]$$

The FOC:

$$\frac{\partial \theta_t}{\partial \theta_{t+1}} = 0 \rightarrow \frac{(1-\sigma)}{q(\theta_{t+1})}\frac{\eta(\theta)}{\theta_{t+1}} - \gamma -= 0 \rightarrow \theta^{th}q\left(\theta^{th}\right) = \frac{(1-\sigma)\eta(\theta)}{\gamma}$$

And If $\theta_{t+1}$is smaller than the threshold we get $\frac{\partial \theta_t}{\partial \theta_{t+1}} > 0$

10.4 The evolution of the wage and the market tightness before a bargaining power reduction

Claim in end of (2.5): in case of a bargaining power γ reduction towards the socially optimal level:

- The market tightness θ is already in its new level one period before the reduction (period t), but it is falling till two periods before the reduction (till period $t-1$)
- The wage is very high one period before the reduction, but it is falling till two periods before the reduction.

Proof: We already saw that one period before the reduction in γ, the wage is high and θ is in its new level. We already saw that the surplus S is in it new level such that $S_t = S_{t+1}$. In Proposition 1, it is proved (Appendix B.1) that in the steady state S has its minimal value in the socially optimal level of bargaining power. As $J_t = (1-\gamma^1)S_t$ it has to be that J_t is smaller than the steady state level of J associated with γ^1. And as $J_t = \frac{\xi}{\beta q(\theta_{t-1})}$, it also has to be that θ_{t-1} is smaller than the steady state value of θ associated with γ^1. And based on the evolution of θ described in Appendix A.2, θ, if solved backwards, is moving up towards its steady state level, or in other words falling till two periods before the reduction.

From the wage equation (A.7) it is easy to see that if θ is below its steady state level and falling, so is the wage w■

Appendix B: Proofs

10.5 Proof of Proposition 1

Proposition 1: the steady state value of the employed worker W as a function of the bargaining power γ is a singled picked function of the bargaining power. The maximum, which is the best static bargaining power level γ^S for the SIG, is higher than γ^*.

Proof: The steady state versions for equations (2.2),(2.3),(2.4),(2.5) and (2.6) are:

$$W = \frac{w+\beta\sigma U}{1-\beta(1-\sigma)} \quad \text{(B.1)}$$

$$U = \frac{b+\beta\theta q(\theta)W}{1-\beta\big(1-\theta q(\theta)\big)} \quad \text{(B.2)}$$

$$J = \frac{\xi}{\beta q(\theta)} \quad \text{(B.3)}$$

$$J = \frac{p-w}{1-\beta(1-\sigma)} \quad \text{(B.4)}$$

$$W - U = \gamma(J + W - U) \quad \text{(B.5)}$$

Step 1: in the steady state, both J and θ are decreasing in regards to γ.

Proof: from (B.4):

$\frac{\partial J}{\partial w} < 0.$

From (B.3):

$\frac{\partial q(\theta)}{\partial J} < 0$ and hence $\frac{\partial q(\theta)}{\partial w} > 0$, and according to our assumption on the matching function, this means that $\frac{\partial \theta}{\partial w} < 0$. As in the steady state $w = b + \gamma(p-b) + \xi\theta$ or $w - \xi\theta = b + \gamma(p-b)$, Increasing γ has to increase the LHS, which can happen only if w is increased and θ decreased, so $\frac{\partial w}{\partial \gamma} > 0$, $\frac{\partial \theta}{\partial \gamma} < 0$ and also $\frac{\partial J}{\partial \gamma} < 0$ ■

Step 2: in the steady state, U and $W + J$ are maximized and S is minimized at $\gamma^* = \eta(\theta)$.

Proof: Form (B.1) and (B.4):

$$W + J = p + \beta[\sigma U + (1 - \sigma)W + (1 - \sigma)J] = p + \beta(1 - \sigma)(W + J) + \beta\sigma u$$

$$W + J = \frac{p + \beta\sigma U}{1 - \beta(1 - \sigma)}$$

And:

$$S = W + J - U = \frac{p+\beta\sigma U}{1-\beta(1-\sigma)} - U = \frac{p+U\big(\beta\sigma-1+\beta(1-\sigma)\big)}{1-\beta(1-\sigma)} = \frac{p-U(1-\beta)}{1-\beta(1-\sigma)}$$

From (B.2):

$$U(1 - \beta) = b + \beta\theta q(\theta)(W - U)$$

Form (B.5) and (B.3)

$$W - U = \frac{\gamma}{1 - \gamma}J = \frac{\gamma}{1 - \gamma}\frac{\xi}{\beta q(\theta)}$$

Combining with we get:

$$U(1 - \beta) = b + \frac{\gamma}{1-\gamma}\theta\xi$$

Plugging into the expression for S:

$$S\big(1 - \beta(1 - \sigma)\big) = p - b + \frac{\gamma}{1 - \gamma}\theta\xi = p - b + \theta\xi - \frac{1}{1 - \gamma}\theta\xi$$

As:

$$\frac{\xi}{\beta q(\theta)} = J = (1 - \gamma)S$$

$$\frac{\theta\xi}{(1 - \gamma)} = \beta\theta q(\theta)S$$

Plugging back and defining $\left(r = \frac{1}{\beta} - 1\right)$:

$$S\big(1 - \beta(1 - \sigma)\big) = p - b + \theta\xi - \beta\theta q(\theta)S$$

$$S = \frac{p - b + \theta\xi}{\big(1 - \beta(1 - \sigma) + \beta\theta q(\theta)\big)}$$

$$S = \frac{p - b + \theta\xi}{\big(r + \sigma + \theta q(\theta)\big)}$$

Deriving according to θ to get the FOC:

$$\frac{\partial S}{\partial \theta}=\frac{\xi\big(r+\sigma+\theta q(\theta)\big)-\big(q(\theta)+\theta q'(\theta)\big)(p-b+\theta\xi)}{\big(r+\sigma+\theta q(\theta)\big)^2}$$

The elasticity of the matching function in respect to unemployment is $(\theta)=-\frac{\theta q'(\theta)}{q(\theta)}$:

$$\frac{\partial S}{\partial \theta}=\frac{\xi\big(r+\sigma+\theta q(\theta)\big)-q(\theta)\big(1-\eta(\theta)\big)(p-b+\theta\xi)}{\big(r+\sigma+\theta q(\theta)\big)^2} \tag{B.6}$$

To determine if $\frac{\partial S}{\partial \theta}$ is increasing or decreasing:

$$\xi\big(r+\sigma+\theta q(\theta)\big)-q(\theta)\big(1-\eta(\theta)\big)(p-b+\theta\xi)=$$

$$\xi(r+\sigma)+\xi\theta q(\theta)-q(\theta)(p-b)-\xi\theta q(\theta)+\eta(\theta)(p-b+\theta\xi)=$$

$$\xi(r+\sigma)-q(\theta)(p-b)+\eta(\theta)(p-b+\theta\xi)$$

Which is increasing when θ is increasing, so the FOC will determine a minimum, with a single peak.

To determine when $\frac{\partial S}{\partial \theta}=0$, from (B.6):

$$(p-b+\theta\xi)\big(1-\eta(\theta)\big)=\frac{\xi\big(r+\sigma+\theta q(\theta)\big)}{q(\theta)}$$

$$\big(1-\eta(\theta)\big)(p-b)=\frac{\xi\big(r+\sigma+\theta q(\theta)\big)}{q(\theta)}-\theta\xi\big(1-\eta(\theta)\big)$$

$$\big(1-\eta(\theta)\big)(p-b)=\xi\,\frac{\big(r+\sigma+\theta q(\theta)\big)-\theta q(\theta)\big(1-\eta(\theta)\big)}{q(\theta)}=\xi\,\frac{r+\sigma+\theta q(\theta)\eta(\theta)}{q(\theta)} \tag{B.7}$$

In a steady state equilibrium, solving for gives the familiar:

$$(p-b)(1-\gamma)=\frac{(r+\sigma)\xi}{q(\theta)}+\gamma\xi\theta=\xi\,\frac{r+\sigma+\gamma\theta q(\theta)}{q(\theta)} \tag{B.8}$$

And comparing these two conditions (B.6) and (B.7) we find the minimum of S is obtained when $\gamma=\eta(\theta)$, the “socially optimal” level of bargaining power. As we saw:

$$S=\frac{p-U(1-\beta)}{1-\beta(1-\sigma)}$$

The maximum of U is obtained at the same point of the minimum of S, and is also single peaked, and as:

$$W+J=\frac{p+\beta\sigma U}{1-\beta(1-\sigma)}$$

The maximum of $W+J$ is obtained at the same point of the maximum of U ■

Step 3: the steady state value of the employed worker W as a function of the bargaining power γ is singled picked function of the bargaining power. The maximum, which is the best static bargaining power level γ^S for the SIG, is higher than γ^*.

Proof: according to Step 2, for $\gamma < \eta(\theta)$, $\frac{\partial U}{\partial \gamma} > 0$, and according to Step 1, $\frac{\partial J}{\partial \gamma} < 0$. As:

$$W + J = \frac{p + \beta\sigma U}{1 - \beta(1 - \sigma)}$$

$$W = \frac{p + \beta\sigma U}{1 - \beta(1 - \sigma)} - J$$

For $\gamma < \eta(\theta)$, $\frac{\partial W}{\partial \gamma} > 0$.

As for $\gamma = \eta(\theta)$, $\frac{\partial U}{\partial \gamma} = 0$, for $\gamma = \eta(\theta)$ it is still true that $\frac{\partial W}{\partial \gamma} > 0$, so the maximum of W is obtained at $\gamma > \eta(\theta)$■

10.6 Proof of Proposition 3

Proposition 3: The static sequence $\{\gamma_2 = \gamma^*, \gamma_3 = \gamma^*, \gamma_4 = \gamma^*, \dots\}$ maximizes both $(W_n + J_n)$ and U_n.

Proof:

Lemma 5: (a) if at period $t + 1$, $\gamma_{t+1} \neq \gamma^*$, setting $\gamma_{t+1} = \gamma^*$ increases U_t and (weakly) increases $W_t + J_t$. (b) If $\gamma_{t+1} = \gamma^*$ and $\gamma_{t+2} \neq \gamma^*$, setting $\gamma_{t+2} = \gamma^*$ increases U_t and $W_t + J_t$.

Lemma 6: if $\gamma_{t+1}, \gamma_{t+2}, \dots, \gamma_{t+n} = \gamma^*$, $n \geq 2$, but $\gamma_{t+n+1} \neq \gamma^*$, setting $\gamma_{t+n+1} = \gamma^*$ increases U_t and $(W_t + J_t)$.

Assume that $\gamma_2 \neq \gamma^*$. By Lemma 5 and Lemma 6, setting $\gamma_2 = \gamma^*$ increases $W_n + J_n$ and U_n. (note that setting $\gamma_2 = \gamma^*$ here means the entire second electoral cycle, i.e. periods $n + 1$ to $2n$). Then, if $\gamma_3 \neq \gamma^*$setting $\gamma_3 = \gamma^*$ increases $W_n + J_n$ and U_n, etc. Thus, the static sequence $\{\gamma^*, \gamma^*, \gamma^*, \dots\}$ maximizes both $W_n + J_n$ and U_n ■

Proof of Lemma5:

Step 1 – proof of (a):

θ_t and U_t do not depend on γ_t from (2.3) and (2.4). Also $(W_t + J_t)$ do not depend on γ_t from (A.5). Also, S_t does not depend on γ_t as $S_t = (W_t + J_t) - U_t$. In order to show that setting $\gamma_{t+1} = \gamma^*$ increases $W_t + J_t$ and U_t, first show how θ_t depends on γ_{t+1}:

Using the chain rule (A.9):

$$\frac{\partial \theta_t}{\partial \gamma_{t+1}} = \frac{\partial q(\theta_t)}{\partial \gamma_{t+1}} \Big/ \frac{\partial q(\theta_t)}{\partial \theta_t}$$

From (2.4):

$$\frac{\partial q(\theta_t)}{\partial \gamma_{t+1}} = -\frac{\xi}{\beta}\frac{\frac{\partial J_{t+1}}{\partial \gamma_{t+1}}}{(J_{t+1})^2}$$

Combining, and using (A.9):

$$\frac{\partial \theta_t}{\partial \gamma_{t+1}} = -\frac{\xi}{\beta}\frac{\frac{\partial J_{t+1}}{\partial \gamma_{t+1}}}{(J_{t+1})^2} \cdot -\frac{\theta_t}{\eta(\theta)q(\theta_t)} = \frac{\xi}{\beta}\frac{\frac{\partial J_{t+1}}{\partial \gamma_{t+1}}}{J_{t+1}}\frac{q(\theta_t)\beta}{\xi}\frac{\theta_t}{\eta(\theta)q(\theta_t)} = \frac{\frac{\partial J_{t+1}}{\partial \gamma_{t+1}}}{(1-\gamma_{t+1})S_{t+1}}\frac{\theta_t}{\eta(\theta)}$$

As S_{t+1} do not depend on γ_{t+1} according to stage 1:

$$\frac{\partial J_{t+1}}{\partial \gamma_{t+1}} = \frac{\partial(1-\gamma_{t+1})S_{t+1}}{\partial \gamma_{t+1}} = -S_{t+1}$$

And combining we get:

$$\frac{\partial \theta_t}{\partial \gamma_{t+1}} = \frac{-1}{(1-\gamma_{t+1})}\frac{\theta_t}{\eta(\theta)} \qquad \text{(B.9)}$$

Using (A.3) and as U_{t+1} does not depend on γ_{t+1}:

$$\frac{\partial U_t}{\partial \gamma_{t+1}} = \xi\frac{\partial\left(\theta_t\frac{\gamma_{t+1}}{1-\gamma_{t+1}}\right)}{\partial \gamma_{t+1}} = \xi\frac{\partial \theta_t}{\partial \gamma_{t+1}}\frac{\gamma_{t+1}}{1-\gamma_{t+1}} + \xi\theta_t\frac{1-\gamma_{t+1}+\gamma_{t+1}}{(1-\gamma_{t+1})^2}$$
$$= \frac{-\xi}{(1-\gamma_{t+1})}\frac{\theta_t}{\eta(\theta)}\frac{\gamma_{t+1}}{1-\gamma_{t+1}} + \theta_t\frac{\xi}{(1-\gamma_{t+1})^2} = \frac{\xi\theta_t}{(1-\gamma_{t+1})^2}\left(1-\frac{\gamma_{t+1}}{\eta(\theta)}\right)$$

And as $\frac{\partial U_t}{\partial \gamma_{t+1}} > 0$ when $\gamma_{t+1} < \eta(\theta)$ and $\frac{\partial U_t}{\partial \gamma_{t+1}} < 0$ when $\gamma_{t+1} > \eta(\theta)$ it follows that U_t is maximized when $\gamma_{t+1} = \eta(\theta) = \gamma^*$

As $W_t + J_t = p + \beta[W_{t+1} + J_{t+1} - \sigma S_{t+1}]$, $W_t + J_t$ doesn't change with γ_{t+1}, and as $S_t = (W_t + J_t - U_t)$, S_t is minimized when $\gamma_{t+1} = \eta(\theta) = \gamma^*$.

Step 2 – proof of (b):

according to step 1, setting $\gamma_{t+2} = \gamma^*$ decreases S_{t+1} and doesn't change $(W_{t+1} + J_{t+1})$. As:

$$W_t + J_t = p + \beta[W_{t+1} + J_{t+1} - \sigma S_{t+1}]$$

$(W_t + J_t)$ increases. Using (A.3) and iterating one period forward:

$$U_t = b(1+\beta) + \xi\left[\theta_t\frac{\gamma_{t+1}}{1-\gamma_{t+1}} + \beta\theta_{t+1}\frac{\gamma_{t+2}}{1-\gamma_{t+2}}\right] + \beta^2 U_{t+2} \qquad \text{(B.10)}$$

Using the chain rule:

$$\frac{\partial q(\theta_t)}{\partial \gamma_{t+2}} = \frac{\partial q(\theta_t)}{\partial \theta_t}\frac{\partial \theta_t}{\partial \gamma_{t+2}}$$

$$\frac{\partial \theta_t}{\partial \gamma_{t+2}} = \frac{\partial q(\theta_t)}{\partial \gamma_{t+2}} / \frac{\partial q(\theta_t)}{\partial \theta_t} \tag{B.11}$$

Using (2.4) we get:

$$\frac{\partial q(\theta_t)}{\partial \gamma_{t+2}} = -\frac{\xi}{\beta}\frac{\frac{\partial J_{t+1}}{\partial \gamma_{t+2}}}{(J_{t+1})^2} \tag{B.12}$$

Inserting (B.12) and (A.9) into (B.11):

$$\frac{\partial \theta_t}{\partial \gamma_{t+2}} = \frac{\xi}{\beta}\frac{\frac{\partial J_{t+1}}{\partial \gamma_{t+2}}}{(J_{t+1})^2}\frac{\theta_t}{\eta(\theta)q(\theta_t)} = \frac{\xi}{\beta}\frac{\frac{\partial J_{t+1}}{\partial \gamma_{t+2}}}{J_{t+1}}\frac{q(\theta_t)\beta}{\xi}\frac{\theta_t}{\eta(\theta)q(\theta_t)} = \frac{\frac{\partial J_{t+1}}{\partial \gamma_{t+2}}}{J_{t+1}}\frac{\theta_t}{\eta(\theta)} \tag{B.13}$$

Using (A.6)

$$\frac{\partial J_{t+1}}{\partial \gamma_{t+2}} = \frac{\partial (1-\gamma_{t+1})S_{t+1}}{\partial \gamma_{t+2}} = (1-\gamma_{t+1})\frac{\partial S_{t+1}}{\partial \gamma_{t+2}}$$

$$\frac{\partial J_{t+1}}{\partial \gamma_{t+2}} = (1-\gamma_{t+1})\frac{\partial\left(p - b + \beta(1-\sigma)S_{t+2} - \xi\theta_{t+1}\frac{\gamma_{t+2}}{1-\gamma_{t+2}}\right)}{\partial \gamma_{t+2}}$$

$$\frac{\partial J_{t+1}}{\partial \gamma_{t+2}} = -(1-\gamma_{t+1})\xi\frac{\partial\left(\theta_{t+1}\frac{\gamma_{t+2}}{1-\gamma_{t+2}}\right)}{\partial \gamma_{t+2}}$$

$$\frac{\partial J_{t+1}}{\partial \gamma_{t+2}} = \frac{-(1-\gamma_{t+1})\xi\theta_{t+1}}{(1-\gamma_{t+2})^2}\left(1 - \frac{\gamma_{t+2}}{\eta(\theta)}\right)$$

And inserting back into (B.13), we get:

$$\frac{\partial \theta_t}{\partial \gamma_{t+2}} = \frac{-(1-\gamma_{t+1})\xi\theta_{t+1}}{(1-\gamma_{t+2})^2 J_{t+1}}\left(1 - \frac{\gamma_{t+2}}{\eta(\theta)}\right)\frac{\theta_t}{\eta(\theta)} = \frac{-\xi\theta_{t+1}}{(1-\gamma_{t+2})^2 S_{t+1}}\left(1 - \frac{\gamma_{t+2}}{\eta(\theta)}\right)\frac{\theta_t}{\eta(\theta)}$$

$$\frac{\partial \theta_t}{\partial \gamma_{t+2}} = \frac{-(1-\gamma_{t+1})\xi\theta_{t+1}}{(1-\gamma_{t+2})^2 J_{t+1}}\left(1 - \frac{\gamma_{t+2}}{\eta(\theta)}\right)\frac{\theta_t}{\eta(\theta)} = \frac{-(1-\gamma_{t+1})\theta_{t+1}\beta q(\theta_t)}{(1-\gamma_{t+2})^2}\left(1 - \frac{\gamma_{t+2}}{\eta(\theta)}\right)\frac{\theta_t}{\eta(\theta)}$$

And Hence:

$$\frac{\partial\left(\theta_t\frac{\gamma_{t+1}}{1-\gamma_{t+1}}\right)}{\partial \gamma_{t+2}} = -\frac{\gamma_{t+1}}{1-\gamma_{t+1}}\frac{\xi\theta_{t+1}}{(1-\gamma_{t+2})^2 S_{t+1}}\left(1 - \frac{\gamma_{t+2}}{\eta(\theta)}\right)\frac{\theta_t}{\eta(\theta)} \tag{B.14}$$

$$\frac{\partial\left(\theta_t\frac{\gamma_{t+1}}{1-\gamma_{t+1}}\right)}{\partial \gamma_{t+2}} = -\gamma_{t+1}\frac{\theta_{t+1}\beta q(\theta_t)}{(1-\gamma_{t+2})^2}\left(1 - \frac{\gamma_{t+2}}{\eta(\theta)}\right)\frac{\theta_t}{\eta(\theta)}$$

As already seen in step 2:

$$\frac{\partial\left(\beta\theta_{t+1}\frac{\gamma_{t+2}}{1-\gamma_{t+2}}\right)}{\partial\gamma_{t+2}} = \frac{\beta\theta_{t+1}}{(1-\gamma_{t+2})^2}\left(1-\frac{\gamma_{t+2}}{\eta(\theta)}\right) \tag{B.15}$$

Deriving (B.10) and using (B.14), (B.15) and the assumption that $\gamma_{t+1} = \gamma^* = \eta(\theta)$:

$$\frac{\partial U_1}{\partial\gamma_{t+2}} = \xi\frac{\partial\left(\theta_t\frac{\gamma_{t+1}}{1-\gamma_{t+1}} + \beta\theta_{t+1}\frac{\gamma_{t+2}}{1-\gamma_{t+2}}\right)}{\partial\gamma_{t+2}}$$

$$\frac{\partial U_1}{\partial\gamma_{t+2}} = -\xi\gamma_{t+1}\frac{\theta_{t+1}\beta q(\theta_t)}{(1-\gamma_{t+2})^2}\left(1-\frac{\gamma_{t+2}}{\eta(\theta)}\right)\frac{\theta_t}{\eta(\theta)} + \xi\frac{\beta\theta_{t+1}}{(1-\gamma_{t+2})^2}\left(1-\frac{\gamma_{t+2}}{\eta(\theta)}\right)$$

$$\frac{\partial U_1}{\partial\gamma_{t+2}} = \frac{\xi\beta\theta_{t+1}}{(1-\gamma_{t+2})^2}\left(1-\frac{\gamma_{t+2}}{\eta(\theta)}\right)\left(1-\gamma_{t+1}\frac{q(\theta_t)\theta_t}{\eta(\theta)}\right)$$

$$\frac{\partial U_1}{\partial\gamma_{t+2}} = \frac{\xi\beta\theta_{t+1}}{(1-\gamma_{t+2})^2}\left(1-\frac{\gamma_{t+2}}{\eta(\theta)}\right)(1-q(\theta_t)\theta)$$

Note that $\theta_t q(\theta_t)$ is the probability that an unemployed worker finds a job in period t. While sometimes the DMP model is calibrated for a period that represents a substantial length of time, in which case the "probability" to find a job is the total number of matches divided by the average number of unemployed, there must be a "fundamental", shorter, period in which the probability must be smaller or equal to 1. For this shorter period $\gamma_{t+2} = \eta(\theta)$ is the only point in which $\frac{\partial U_t}{\partial\gamma_{t+2}} = 0$. And as $\frac{\xi\beta\theta_{t+1}}{(1-\gamma_{t+2})^2}\left(1-\theta_t q(\theta_t)\right) \geq 0$ this has to be a maximum, proving that setting $\gamma_{t+2} = \gamma^*$ increases U_t.

Proof of Lemma 6: Assume now that $\gamma_{t+1}, .., \gamma_{t+n} = \eta(\theta), n \geq 2$. We saw in Lemma 5 that setting $\gamma_{t+n+1} = \eta(\theta)$ increases U_{t+n} and $(W_{t+n} + J_{t+n})$. As it also decreases S_{t+n} it decreases θ_{t+n-1}. There are 3 options to what happened to S_{t+n-1}.

Option 1: S_{t+n-1} decreased: this means that θ_{t+n-2} also decreased. If a decrease in θ_{t+n-1} is accompanied with a decrease in θ_{t+n-2}, this means that θ_{t+n-1} is within the region for which $\frac{\partial\theta}{\partial\theta'} > 0$, and that $\theta_{t+1}, .. \theta_{t+n-2}$ all decrease. This means that $S_{t+1}, .. S_{t+n-2}$ all decreased. As for all t:

$$W_t + J_t = p + \beta[W_{t+1} + J_{t+1} - \sigma S_{t+1}]$$

if $W_{t+n-1} + J_{t+n-1}$ increased and S_{t+n-1} decreased $W_{t+n-2} + J_{t+n-2}$ also increased, and $W_{t+1} + J_{t+1}, .., W_{t+n-2} + J_{t+n-2}$ all increased. As:

$$S_t = W_t + J_t - U_t$$

This means that $U_{t+1}, .., U_{t+n-2}$ also increased.

Option 2: S_{t+n-1} didn't change. In this case θ_{t+n-2} didn't change and also all previous θ till θ_{t+1} and all previous S as well. As $W_{t+n-1} + J_{t+n-1}$ increase, the same argument as in Option 1 applies and $W_{t+1} + J_{t+1}, .., W_{t+n-2} + J_{t+n-2}$ and $U_{t+1}, .., U_{t+n-2}$ all increased.

Option 3: S_{t+n-1} increased. If a decrease in θ_{t+n-1} is accompanied with an increase in θ_{t+n-2}, this means that θ_{t+n-1} is above the point for which $\frac{\partial\theta}{\partial\theta'} > 0$. However, $\theta_{t\mp n-2}$ still has to be below that point (as explained in appendix A.2). So, if θ_{t+n-2} increased, all $\theta_{t+1},..,\theta_{t+n-2}$ increased also all $S_{t+1},..S_{t+n-2}$ increased. Now as:

$$U_{t+n-2} = b + \beta U_{t+n-1} + \xi\theta_{t+n-2}\frac{\gamma_{t+n-1}}{1-\gamma_{t+n-1}}$$

And U_{t+n-1} increased, U_{t+n-2} also increased and using the same argument backwards all $U_{t+1},..U_{t+n-2}$ increased. As

$W_t + J_t = S_t + U_t$ all $W_{t+1} + J_{t+1},..,W_{t+n-2} + J_{t+n-2}$ increased.■

10.7 Proof of Proposition 4

Proposition **4:** for a given γ_1, the sequence $SQ^* = \{\gamma_1, \gamma^*, \gamma^*, \gamma^*, ...\}$ that maximizes $W_n + J_n$ and U_n, and thus maximizes W_n, also maximizes W_1.

Proof: Consider an alternative sequence SQ'. We saw in Proposition 2 that a move form SQ' to SQ^* increases $W_n + J_n$ and U_n. There are 2 options as to what happened to S_{n-1}, S_n:

Option 1: S_n increased. In this case θ_{n-1} also increased from the move to SQ^* as a higher surplus means higher market tightness one period earlier. For SQ^*, $S_n = S_{\gamma^*}^{ss}$, the surplus associated with the steady state level of γ^*. If S_n increased when the sequence changed, this means that for both SQ^* and SQ' S_n is at or below $S_{\gamma_1}^{ss}$[14]. In this case, θ_{n-1} is below $\theta_{\gamma_1}^{ss}$ for both sequences, and according to the assumption in Appendix A, in the range for which $\frac{\partial\theta}{\partial\theta'} > 0$. This means that if the move to SQ^* increased θ_{n-1}, it also increased all previous levels of θ_t, and consequently al previous levels of S_t, $1 \leq t \leq n-1$.

As the move increased U_n and increased θ_{n-1}

$$U_{n-1} = b + \beta U_n + \xi\theta_n\frac{\gamma_1}{1-\gamma_1}$$

U_{n-1} also increased. Working backwards, all levels of U_t, $1 \leq t \leq n-1$ increased.

As $W_t + J_t = S_t + U_t$, all levels of $W_t + J_t$, $1 \leq t \leq n-1$ increased.

Option 2: S_n decreased. If S_n, based on our monotonicity assumption (section A.2), S_{n-1} also decreased. In this case both θ_{n-1} and θ_{n-2}, decreased. As θ_{n-2} has to be in the range for which $\frac{\partial\theta}{\partial\theta'} > 0$, this means that θ_t, $1 \leq t \leq n-1$, decreased, and consequently S_t, $1 \leq t \leq n-1$, decreased.

As the move increased $W_n + J_n$ and decreased S_n, and as

$$W_{n-1} + J_{n-1} = p + \beta[W_n + J_n - \sigma S_n]$$

[14] This is due to the fact the steady state level of the surplus is minimized for γ^*, as shown in step 2 of proposition 1

$(W_{n-1} + J_{n-1})$ also increased. Working backwards, all levels of $W_t + J_t$, $1 \le t \le n-1$ increased.

As $U_t = W_t + J_t - S_t$, all levels of U, $1 \le t \le n-1$ increased.

And to conclude:

We saw that for both options, all levels of U_t,$W_t + J_t$, $1 \le t \le n-1$ increased, and as

$W_t = \gamma_1(W_t + J_t) + (1-\gamma_1)U_t$

We conclude that all levels of W_t, $1 \le t \le n-1$ increased and hence the path that maximized W_n also maximized W_1 ■

10.8 Proof of Lemma 2

Lemma 2: $\gamma^b \ge \gamma^*$

Proof: Consider the path $SQ' = \{\gamma_1, \gamma^*, \gamma^*, \gamma^*, \ldots\}, \gamma_1 < \gamma^*$. In order to prove that $\gamma^b \ge \gamma^*$, I will prove that for $\forall t, 1 \le t \le n-1$ it has to be that the value for the employed workers W_t' derived from the path SQ' is lower from W_t^* derived from the path $SQ^* = \{\gamma^*, \gamma^*, \gamma^*, \gamma^*, \ldots\}$. Of course as SQ* is a static path, for $\forall t, W_t^* = W^*$.

We already saw that at the last period of the first electoral cycle, period n, S_n', U_n' and $(W_n' + J_n')$are already at the new steady state level, which in this case is equal to S^*, the steady state level associated with γ^* (see A.6). From proposition 1 we know that the steady state surplus is minimized at $\gamma = \gamma^*$, so S_n' is below its steady state level. as $J_n' = (1-\gamma_1)S_n'$, J_t' is also below its steady state level, and from (2.4) θ_n' is also below its steady state level. From the dynamics of θ in section 10.2, θ_t' is converging (backwards) toward its steady state level such that $\theta_t' < \theta_{t-1}'\ \forall t, 1 \le t \le n$, and fro (2.4) J_t' and S_t' do so as well. As

$W_t + J_t = p + \beta[W_{t+1} + J_{t+1} - \sigma S_{t+1}]$

It has to be that $(W_{n-1}' + J_{n-1}') = (W_n' + J_n')$, but $S_t' > S_{t-1}'$, $(W_t' + J_t')$ would be converging (backwards) lower such that $(W_t' + J_t') > (W_{t-1}' + J_{t-1}')$. but if $(W_t' + J_t')$ are converging (backwards) lower and J_t' is converging (backward) higher, it has to be that W_t' is converging (backward) lower. And as in period n, $W_n' = \gamma_1 S_n' - U_n' < W^*$, it has to be that $W_t' < W^*\ \forall t, 1 \le t \le n$ ■

11 Appendix C: solving the dynamic path with lobbying costs

Numerically solving a lobbying path when there are no lobbying cost is straight forward. Assume the lobbying path $\{\gamma_1, \gamma_2, \ldots, \gamma_m, \gamma_m, \gamma_m, \ldots\}$ where I assume that at a certain point m the path is steady. Solving the path includes solving for the steady state of $\gamma = \gamma_m$ and then solving backwards using the model dynamic equations.

Solving the path with lobbying cost is more complex and requires the following stages:

1. Find the value for the policy maker in case there is no lobbying, the policy maker sets the bargaining power to $\gamma = \gamma^*$, and the unemployment level is u_t. Assuming the unemployment level will reach (close enough to) the steady state level u_{ss} after m periods, this value is equal to:

$$\hat{G}(u_t) = \lambda \sum_{j=0}^{m} \beta^j g(u_{t+j}, v_{t+j}) + \frac{\beta^{m+1}}{1-\beta} g(u_{ss}, v_{ss})$$

Where:

$$u_{t+j} = \left(1 - \theta_{ss} q(\theta_{ss}) u_{t+j-1}\right) + \sigma\left(1 - u_{t+j-1}\right) \forall 1 < j \leq m$$

$$v_{t+j} = \theta_{ss} u_{t+j}$$

As the wage and the market tightness jump immediately to the steady state level.

2. Solve (i.e. find θ_m, c_m) for the steady state of $\gamma = \gamma_m$. The steady state includes a per-period lobbying cost of c_m. Solving the steady state equations of the model in the standard way for θ_m yields the following equation:

$$p - (b + c_m)(1 - \gamma_m) - \gamma_m(p + \xi\theta_m) = \frac{(r+\sigma)\xi}{q(\theta_m)} \qquad \text{(C.1)}$$

And the second equation is from requiring that the cost is indeed the required cost:

$$c_m \sum_{t=1}^{n} \beta^{t-1}(1 - u_m) = \frac{\hat{G}(u_m)}{1-\lambda} - \frac{\lambda}{1-\lambda}\left[\sum_0^\infty \beta^t g(u_m, v_m)\right] \qquad \text{(C.2)}$$

Where u_m, v_m are the levels associated with the steady state values of θ_m, c_m:

$$u_m = \frac{\sigma}{\sigma + \theta_m q(\theta_m)}$$

$$v_m = u_m \theta_m$$

$$g(u_m, v_m) = p - u_m(p - b) - \xi v_m$$

The LHS represents the cost paid to the policy maker for a given steady state electoral cycle and the RHS represents the amount required to compensate the policy maker. Equations (C.1) and (C.2) can be solved numerically for (θ_m, c_m).

3. Solve the path $\{\gamma_1, \gamma_2, \ldots, \gamma_m\}$. Unlike the no-cost case, we cannot assume that the steady state levels associated with γ_m will be achieved immediately in electoral cycle m, as this will be the case only if the initial unemployment level in electoral cycle m is already at the steady state level. Assume that it takes l cycles of constant $\gamma = \gamma_m$ for the steady state levels of the unemployment (and thus cost) to be (close enough to) the steady state levels[15]. Now, I need to solve the path including $m + l$ electoral periods out of the steady state - $\{\gamma_1, \gamma_2, \ldots, \gamma_m, \gamma_{m+1}, \ldots, \gamma_{m+l}\}$, $\gamma_{m+i} = \gamma_m \ \forall i > 0$, assuming that afterwards a steady state is achieved. The unknowns here are the series of $m + l$ per-period costs.
4. As I can solve the steady state values (θ_{ss}, c_{ss}) given $\gamma_{ss} = \gamma_m$, and given $\{c_1, c_2, \ldots, c_{m+l}\}$ I can solve backwards to find the path for all levels of θ, the algorithm, is

a. Solve for the steady state (θ_{ss}, c_{ss}) using the two equations (C.1) and (C.2)

[15] I find l numerically by trying increasing number of electoral cycles until the last one is close enough to the steady state.

b. Solve for $\{c_1, c_2, \ldots, c_{m+l}\}$ using the $(m+l)$ equations:

$$\frac{\lambda}{1-\lambda}\left[\hat{G}(u_{ni+1}) - G(u_{n(i-1)+1})\right] = \sum_{t=1}^{n} \beta^{t-1}\left(1 - u_{n(i-1)+1}\right) c_i \quad i = 1,2,\ldots,m+l$$

Which states that the loss for the policy maker (LHS) is what is paid to her (RHS), and:

- $\hat{G}(u_{ni+1})$ is the level of utility that the policy maker gets when there is no lobbying and the unemployment level is u_{ni+1}.
- θ_t is solved backwards using the model's dynamic equation. Specifically I used:

$$J' = \frac{1-\gamma'}{\gamma'} W\_\min\_U'$$

$$q(\theta) = \frac{\xi}{\beta J'}$$

$$\theta = \left(\frac{\chi}{q(\theta)}\right)^{1/\eta}$$

$$W\_\min\_U = \gamma\left(p - b - c + \beta\left(1 - \sigma - \theta q(\theta) + (1-\sigma)\frac{1-\gamma'}{\gamma'}\right) W\_\min\_U'\right)$$

- u_t is solved forward, given u_1 and θ_t:

$$u_{t+1} = (1 - \theta_t q(\theta_t) u_t) + \sigma(1 - u_t)$$

- $G\left(u_{n(i-1)+1}\right)$ is given by:

$$G\left(u_{n(i-1)+1}\right) = \sum_{t=n(i-1)+1}^{n(m+l)} \beta^{t-n(i-1)-1}\left(p - u_t(p-b) - \xi u_t \theta_t\right) + \frac{\beta^{n(m+l)-n(i-1)}}{1-\beta}\left(p - u_{ss}(p-b) - \xi u_{ss}\theta_{ss}\right)$$

Once I have the ability to solve a given lobbying path, and derive the employed workers value, I can run a matrix of all possible paths (assuming that after some constant m the path is static) and find the best path.

In order to verify that there is an appropriate "punishment" path, I need to find a self-sustainable equilibrium strategy with an employed worker's value that is below the policy maker optimal path (which is also the path starting from the second electoral cycle). Finding a self-sustainable equilibrium strategy is to find a bargaining power level γ^p, which satisfies:

$$\gamma^p = arg \max_{\gamma_1}\{W(1) | \gamma_2, \gamma_3, \ldots = \gamma^p\}$$

Which means finding γ^p such that that $\gamma_1 = \gamma^p$ is the best choice for the SIG given that everyone believes that the path starting from the second electoral cycle is always γ^p. I have verified that there is always such punishment equilibrium.

12 Appendix D: Solving the Dynamic OLG Model

The first step is to solve the steady state given γ, which means finding the market tightness θ_{ss}. From (7.5), (7.6), (7.8) and (7.9) I get the surplus for the last generation m and the value of a firm:

$$S_m = p - b$$

$$J_m = (1-\gamma)(p-b)$$

And now, from (7.5) and (7.6):

$$W_m = w_m = p - J_m$$

$$U_m = b$$

Given the dynamic equations and a guess for θ_{ss} I calculate the values for all subsequent generations. From (7.1), (7.2), (7.3), (7.4) and a guess for θ_{ss} I calculate the employment and unemployment levels for each generation. What is now left is to numerically find the proper θ_{ss} that will clear the vacancy equation:

$$\xi = \beta q(\theta_{ss}) \left(\frac{1}{\sum_{i=1}^{m} u_{i-1}} \sum_{i=1}^{m} u_{i-1} J_i \right)$$

Given the steady state solution, I can now solve a path $\{\gamma_1, \gamma_2, \dots, \gamma_l, \gamma_{ss}\}$, given an initial unemployment level, which means finding the market tightness θ_t for the total of $n * (l+1)$ periods of eh path, and assuming (and verifying numerically) that the market tightness is (close enough) to its steady state value at the last period. Again, given a guess for the vector θ_t and the levels of the bargaining power I can solve using the dynamic equations, going backwards from the steady state level. Also given the guess for θ_t and an initial level of unemployment, I can solve forward to get the entire unemployment path. What is now left is to numerically find the proper vector θ_t that clears the vacancy equation in each period:

$$\xi = \beta q(\theta_t) \left(\frac{1}{\sum_{i=1}^{m} u_{i-1,t}} \sum_{i=1}^{m} u_{i-1,t} J_{i,t+1} \right)$$