# Patents and Pools in Pyramidal Innovation Structures[1]

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## Abstract

We study sequential innovation in two pyramidal structures. In both, patent pools are socially desirable. In a regular pyramid, patent pools are stable. In an inverse pyramid, patent pools are not stable when following a standard formation process. We offer a more elaborate formation protocol that allows for the creation and stability of the largest pool possible. We also examine welfare implications of patent protection in both structures. In the regular pyramid, as it expands, patent protection increases the likelihood of innovation. In the inverse pyramid patent protection always decreases the likelihood of innovation.

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**JEL Codes:** D43, D45, L13, L24, O31

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# 1. Introduction

John Wager's group at Oregon State University developed the world’s first transparent transistor in 2003 (See http://oregonstate.edu/ua/ncs/archives/2003/mar/osu-engineers-create-worlds-first-transparent-transistor). John Wager and his team developed the world’s first simple integrated circuit that will also be transparent in 2005 (See http://engineering.oregonstate.edu/news/ar/2005/transparentelec.html). These in turn formed the "basis" for innovations in diverse fields such as solar cell technology, displays manufacturing and others.

Robert N. Hall, working at General Electric filed his patent for a semiconductor laser on October 24, 1962, (See "Remembering the Laser Diode" (2012)). This invention was again the source of several innovations along a wide spectrum of areas such as, fiber-optic cables, DVD's, scanning devices and others.

A different pattern of innovative activity is observed in the creation of flash memory drives. There several innovations combined to produce a viable solution for conveniently storing and accessing large amounts of data. It started with the inventions of the flash memory chips and the USB. Fujio Masuoka leading a team of engineers invented the flash memory chip, filing a patent for EEPROM (electrically erasable, programmable read-only memory) in 1981. In 1994 Ajay V. Bhatt, Bala Sudarshan Cadambi, Jeff Morriss, Shaun Knoll and Shelagh Callahan created the first USB (universal serial bus) port. This brought about the first flash memory drives known as memory sticks (by Sony in October 1998) and disk on key (by M-Systems in 1999).

The above two patterns can also be discerned in other areas. Consider for example the medical research community. The Human Genome Project entailing a host of new technologies and approaches to map and identify specific genes, led to innovations in diverse areas. In biotechnology, sequencing of the DNA helped in devising new treatments and medications. In the energy industry, it improved the use of fossil-based resources. In environmental protection, studying the genomes of several bacteria led to better methods of environmental rejuvenation and waste disposal.

The efforts to find a viable cure for AIDS exhibited the opposite pattern where innovations addressing different facets of the disease led to partial solutions for different stages of it and different symptoms experienced by the patients. These efforts culminated in the AIDS cocktail. It combined several drugs, each addressing a specific concern such as, revitalizing immunologic functions, suppressing viral load and improving quality of life. It is of interest to note that this research was accompanied by a call to expand patent pools. Médecins Sans Frontières called for strengthening Medicines Patent Pools to make it easier for new medications to arrive at the market place.

Such examples are prevalent in most areas of sequential innovation and indeed, are often associated with the formation of patent pools. Sequential innovation occurs when current innovation(s) depends on the successful realization of a previous innovation(s). Patent pools form when several patentees agree to jointly determine the price of licenses, granting the right to use their patents.

In this paper we analyze sequential innovation and patent pools within the two innovation structures that characterized the previous examples. We describe them as a "*regular pyramid*" and an "*inverse pyramid*" structures. In a regular pyramid the number of firms in each period expands, with each current period successful innovation giving rise to several subsequent possible innovations in the next period. In an inverse pyramid the number of firms in each period shrinks with only one firm in the final stage. The firms active at each stage solve more and more problems focusing the “research question” and leading to a smaller set of relevant innovations in the next period.

In both structures we consider first the question whether or not a patent pool will form[4] (that is whether a patent pool arrangement is stable) and its impact on the innovative activity followed by studying the effect of patent protection on innovative activity. We also examine, in the case of a regular pyramid, the welfare implications of the various protection measures and compare them to the benchmark of a central planner wishing to maximize social welfare.

Our main findings regarding pool formation and patent protection are the following:

- As in previous studies we show that the creation of a pool increases the probability of innovation and the innovators' profits in each period.

[4] We consider the case where at most a single pool might form.

- In the regular pyramid structure, a pool can be form endogenously using a "natural" protocol. However, this protocol fails to lead to the formation of a pool in the inverse pyramid structure, and we provide a novel and more elaborate protocol supporting, in equilibrium, the creation of the largest possible pool.
- In the regular pyramid structure patent protection increases the probabilities to innovate as the pyramid expands, while in the inverse pyramid structure patent protection leads to reduced innovation probabilities for any pyramid size.
- Finally, comparing the outcomes generated in the regular pyramid structure to the "first-best" scenario achieved by a central designer. We see that the absence of patent protection leads to under investment in R&D and introducing patent protection is more effective when a pool forms. Yet, both with and without a pool, there are circumstances under which patent protection will lead to a further welfare decrease.

Policy implications thus differ for the two pyramid structures. In a regular pyramid, policy makers can rely on a "natural" offer and reject protocol to support the creation of a pool. In an inverse pyramid structure, they need to intervene by increasing the bargaining power of later rather than earlier innovators leading to a more elaborate offer and reject protocol. In an inverse pyramid the introduction of patent protection reduces the likelihood of innovation, whereas, in a regular pyramid, it might lead to higher innovation probabilities when the pyramid is not too narrow.

The paper proceeds as follows: Section 2 discusses the related literature on sequential innovation, patent pools and patent protection. Section 3 introduces and analyzes the regular pyramid structure. Section 4 analyzes the inverse pyramid structure. Section 5 concludes and raises directions for further research.

# 2. Related Literature

Patent protection for sequential innovations has been extensively discussed in previous literature. In a two-period model of sequential innovation, Green and Scotchmer

(1995) and Scotchmer (1991) examined the effect of patent policy on the sharing of profits among the first and second-generation innovators and the implications regarding patent length and breadth. Scotchmer (1996) analyzed how patent protection offered to second-generation innovators affected the division of profits between them and the first innovator. It was shown that denying such protection increased the profit of the first innovator and might be optimal.

Boldrin and Levine (2005) found that the intellectual property rights monopoly created by patent protection is exasperated in the presence of sequential innovation. The accumulation of patentees with claims upon the profits of future innovations reduced the incentives to innovate. Bessen and Maskin (2009) showed that in an infinite period model of sequential innovation the introduction of patents reduces welfare. This was in contrast to their finding in a static one-period model. Llanes and Trento (2012) obtained the same result even when accounting for the ex-ante incentive to innovate which is positively affected by the expectations of future revenues in the form of license fees or royalties in the presence of patent protection.  They showed that the formation of a patent pool alleviates the problem. Hopenhayn et al. (2006) analyzed a rich model of sequential innovation with private information and designed an optimal patent menu.

The analysis of patent pools started with one-period innovations. Shapiro (2001) and Tirole (2015) analyzed the effect of patent pools in the presence of patent thickets. Patent pools were shown to be socially desirable in the case of perfectly complementary patents. On the other hand, when patents are perfect substitutes they are socially detrimental as they prevent competition and lead to higher license fees. Lerner and Tirole (2004) extended these results to the full spectrum from perfect substitutability to perfect complementary of patents. Furthermore, it was shown that if there is compulsory independent licensing, only welfare enhancing pools will emerge in equilibrium. Boutin (2016) pointed out that when there are more than two patent holders, there might be other equilibria giving rise to welfare reducing pools. She constructed a modified mechanism imposing caps on revenue generated through independent licensing to rule out such equilibria. Brenner (2009) showed welfare reducing pools can emerge when it is possible to form partial pools (Lerner and Tirole (2004) considered only complete pools consisting of all available patents). He proceeded to suggest a pool formation protocol which leads to the formation of welfare improving pools only.

Quint (2014) draws a distinction between essential and nonessential patents. Considering a set of final products, a patent is called essential if it is necessary to use it in the manufacturing of any of the products. Previous literature considered the case where all patents are inessential, as was the case in Lerner and Tirole (2004), or all are essential as in Llanes and Trento (2012). Quint (2014) derived a payoff structure for any partition of the patents into pools and showed that even under compulsory independent licensing requirements welfare decreasing pools may form.

Aoki and Nagaoka (2004) followed a different line of research focusing on the incentive to form a pool. They highlighted the free-rider problem that occurs when an innovator outside the pool licenses its innovation to firms that also rely on the pool patents. The bargaining power of such an innovator often makes it impossible for a pool to form.

Llanes and Trento (2012) considered a linear structure of innovations and examined the incentives to innovate and welfare under three distinct regimes, patent protection, no patents and patent pools. They also constructed a reward-payment mechanism, leading to an optimal outcome, where an innovator is first rewarded for a successful innovation and is taxed later on when subsequent innovations are realized.

An interesting perspective regarding several of the above findings is provided by Heller (1998) and Heller and Eisenberg (1998). These works analyzed the "tragedy of the anticommons". In contrast to the "tragedy of the commons" where many agents have the right to freely use a certain resource, the "anticommons" case describes a scenario where many agents have the right to deny access to a certain resource. The "tragedy of the anticommons" leads to suboptimal utilization of the resource. In the innovation literature this tragedy surfaces as several patent owners have the right to prevent further innovations relying on one of their patents.

Baron and Pohlman (2013) analyzed the composition of firms cooperating via standard arrangements (bearing several similarities to pool arrangements). Their findings indicate that firms are more likely to form alliances with those conducting research on substitute technologies or patents than with those pursuing complementary research. This might be due to savings created by avoiding duplicate research but might lead to welfare reductions as predicted by standard theory. Galasso and Schankerman (2015), examined the problems posed by patent protection in a sequential innovation setting. They looked at the effect of court decisions revoking patent rights, and found that removing patent protection has a larger effect on subsequent innovative activity in the cases where the patent blocks several subsequent innovations.

# 3. The Regular Pyramid Model

We now procced to describe the innovative environment we analyze[5]. We consider a 3-period pyramidal structure of innovations. There is a single firm in period *1*, called firm 1, there are *s* firms in period *2* called firm *2,…,s+1*, and *n·s* firms in period *3*, where firms $i_1$,..,$i_n$ denote the n period 3 firms relying on the innovation of firm i with i=2,…,s+1. Each of the firms has to decide whether or not to undertake R&D. Furthermore, engaging in R&D activity in period t=2, 3 is possible only if the t-1 period (parent) firm has innovated. In each period every firm, if the preceding firm was successful, decides whether or not to invest in R&D. The first period firm (there is no one preceding it) takes into account both current revenues due to the innovation (realized in period 1) as well as future revenues in periods 2 and 3 due to the possible payments that subsequent firms relying on her innovation must pay it. The period 2 firms take into account current costs (payment to the parent firm) and revenues (realized in period 2) and future revenues due to payments made by period 3 firms. The period 3 firms again take into account current costs and current revenues. All firms wish to maximize their expected profit.

The innovative environment is introduced more formally as follows:

The direct cost of undertaking R&D, denoted by ε, is known and common to all firms. The revenue due to the innovation itself is uniformly distributed over [0,1], and its realization, denoted by v, is privately observed by the innovating firm[6]. We let $p(1,i_j)$ denote the payment made by firm $i_j$ in period 3 to the period 1 firm, and $p(i, i_j)$ be the payment made by firm $i_j$ to firm i of period 2 (its parent) where i=1,…,n and j=1,…,s. Future payment periods are discounted by β. $\pi_k(1)$ denotes the future revenues of a firm 1 from period k

[5] This innovation technology is similar to the one used in Llanes and Trento (2012).
[6] This value is appropriated by the innovating firm even when there is no patent protection.

(k=2,3) onwards, $\pi_3(i)$ denotes the future revenues of firm i in period 2 from period 3 with i=2,…,s+1. Each firm decided to innovate, if it can, whenever the realization of the innovation value is greater than or equal to the cost of innovation incorporating both license fees and the direct cost. The innovation threshold implied by the cost determines probability to innovate respectively denoted by, $q_1$, $q_2(i)$ and $q_3(i_j)$ , the probability that the period 1 firm, firm i in period 2 and firm $i_j$ in period 3 undertake R&D.

The strategic decisions of the firms depend on the scope of protection offered to innovators. We analyze and compare four possible scenarios: patents, patent pools, no patents and finally having a central planner organize the innovative activity.

## 3.1 Patents

Patent protection allows the firms to charge license fees for any subsequent use of their innovations. These license fees are their sole source of income in the periods following the period where innovation occurred. License fees are set simultaneously in the beginning of each period leading to the following extensive form game, which we refer to as the License Setting Game (LSG). At each period k, previous period firms that have innovated decide on the license fees and period k firms decide, having observed their innovation value, whether or not to innovate. We assume firms wish to maximize their discounted expected profits and examine the resulting subgame perfect equilibrium (SGPE) outcome.

Starting at period 3, let $m \leq s$ denote the number of firms that undertook R&D in period 2. Hence, there are m+1 firms that have to decide upon the license fees that $m \cdot n$ period 3 firms will pay for the right to use their innovations. A period 3 firm, after having observed the license fees as well as its innovation value decides whether or not to undertake R&D. The firms in each period are symmetric and the resulting equilibrium strategies and outcome are symmetric as well. To simplify the notation we, whenever possible, omit the firm index, keeping only the period index. Thus, we replace $p(1,i_j)$ by $p_{13}$ , $p(i, i_j)$ by $p_{23}$, $\pi_3(i)$ by $\pi_{23}$ , $\pi_3(1)$ by $\pi_{13}$ and $q_3(i_j)$ by $q_3$ $\forall i = 1, \dots, m, j = 1, \dots, n$ . We also replace $p(1,i)$ by $p_{12}$ , $\pi_2(1)$ by $\pi_{12}$ and $q_2(i)$ by $q_2$ $\forall i = 2, \dots, s + 1$ .

The following proposition describes the equilibrium strategies and outcome of the LSG in the innovative environment we analyze.

*Proposition 1.*

a. *In period 3 the subgame perfect equilibrium strategies and outcome of the LSG are given by:* $p_{13} = p_{23} = \frac{1-\epsilon}{3}$ *yielding:* $q_3 = \frac{1-\epsilon}{3}$ , $\pi_{13} = \frac{1}{9}mn(1-\epsilon)^2$ , $\pi_{23} = \frac{1}{9}n(1-\epsilon)^2$

b. *In period 2 the subgame perfect equilibrium strategies*[7] *and outcome of the LSG are given by:*

   i. $p_{12} = \frac{1-\epsilon}{2}$ , $q_2 = \frac{1}{18}\big(9 + 2n\beta(1-\epsilon)\big)(1-\epsilon)$ *and* $\pi_{12} = \frac{1}{324}s(9 + 2\beta n(1-\epsilon))^2(1-\epsilon)^2$ *for* $n < \frac{9(1+\epsilon)}{2\beta(1-\epsilon)^2}$, *(the interior solution*[8]*).*

   ii. $p_{12} = \frac{n\beta(1-\epsilon)^2}{9} - \epsilon$ , $q_2 = 1$ *and* $\pi_{12} = \frac{1}{9}s(2n\beta(1-\epsilon)^2 - 9\epsilon)$ *for* $n \geq \frac{9(1+\epsilon)}{2\beta(1-\epsilon)^2}$ *(the corner solution)*

c. *In period 1 the innovation threshold of undertaking R&D yields the following probabilities:*

   i. $q_1 = min\{\frac{1}{324}\left(324 + s\beta\big(9 + 2n\beta(1-\epsilon)\big)^2(1-\epsilon)\right)(1-\epsilon), 1\}$ *for the interior solution.*

   ii. $q_1 = min\{\frac{1}{9}(9 + s\beta(2n\beta(-1+\epsilon)^2 - 9\epsilon) - 9\epsilon), 1\}$ *for the corner solution*

   The proof is in Appendix A.1.

So far we have ignored the incentives for and consequences of coordination among the various innovators. Cooperation leading to such coordination is often done within patent pools which we address in the next section.

[7] A firm's strategy consists of setting an innovation threshold and license fees to be charged. Since the innovation threshold uniquely determines the probability to innovate and vice versa, we specify in the strategy description the probability to innovate rather than explicitly state the threshold.

[8] In an interior (corner) solution $0<q_2<1$ ($q_2=1$).

## 3.2 Patent Pools

A common coordination device both in theory and in practice is a patent pool arrangement. Pools have emerged in several industries, some examples are the sewing machine industry in the 19th century, aircraft industry in the 20th century and MPEG video and DVD standards in the late 1990s. We consider a patent pool as defined in Lerner and Tirole (2004). The firms comprising the pool jointly decide the license fee for freely using any patent in the pool. In contrast to Lerner and Tirole (2004), we explicitly consider the dynamic creation of the pool and its stability. The equilibrium analysis carried out in the previous section is now augmented by the pool formation process with the new strategic decision of whether or not to join and how to share the pool's profits.

As in Lerner and Tirole (2004) we distinguish between two scenarios. The first is the standard case where firms in the pool cannot sell individual licenses. The second is, the independent licenses scenario, where each firm in the pool can charge and collect an independent license fee from any firm that wishes to use its innovation, this in addition to receiving its part in the revenues generated by the pool.

### 3.2.1 The Standard Case

The possible formation of a pool takes place at the end of period 2. Note that the pyramid structure implies forming a pool consisting just of period 2 firms cannot increase the aggregate profits of the pool members. Hence, the period 1 firm must be a member in any patent pool that forms. We make the simplifying assumption that the period 1 firm has all the bargaining power[9].

We assume the formation protocol is the following: The period 1 firm, sequentially approaches period 2 patentees (those who chose to undertake R&D), offering them the option to join the pool. The option consists of an invitation to join the pool together with a promise to receive a part of the pool's profits. The patentee approached can accept or reject. If it accepts, it joins the pool, if it rejects, it stays out of the pool. This process ends when

[9] This assumption simplifies the computations but is inconsequential with respect to the conclusions regarding the formation of the pool.

all the patentees have been approached and yields a partition of the patentees into two sets, those who are in the pool (together with the period 1 firm) and those who are not. The game that unfolds with respect to setting license fees is similar to the one analyzed in the previous section with the pool acting as a single player.

Period 3 firms face the same decision problem as in the previous section. At the start of period 3 the patentees (period 2 firms who chose to innovate) decide whether or not to enter a pool and license fees are determined in a LSG between the pool and the firms outside the pool. At period 2, each firm after observing its innovation value, decides whether or not to undertake R&D. The period 1 firm decides whether or not to undertake R&D and sets the license fees charged to period 2 firms in case it chooses to innovate.

Determining a full solution leading to the equilibrium values of probabilities to innovate and firms' profits in each period is computationally cumbersome. However, to examine the pool formation and its stability, it suffices to obtain the equilibrium outcomes in period 3, which we now proceed to determine.

The formation of the pool takes place in the beginning of period 3. We let z denote the number of firms in the pool. Again we simplify the notation and omit the firm index since the solution will be symmetric. Let $Q_{z3}$, $q_{z3}$ respectively denote the probability of undertaking R&D for a firm relying on a period 2 innovation that belongs, does not belong, to the pool when there are z firms in the pool. $P_{z3}$, $p_{z3}$ respectively denote the license fee paid by a period 3 firm using a patent in, not in, the pool. $\Pi_{z3}$, $\pi_{z3}$ are defined as pool revenue and the revenue of a patentee that does not belong to the pool, respectively.

We again assume firms wish to maximize their expected profits and analyze the resulting SGPE outcome.

*Claim 1:*

i. *The revenues of a pool containing z firms and a firm outside the pool are* $\Pi_{z3} = \frac{mn(1-m-z)^2(1-\epsilon)^2}{(1-3m-z)^2}$, *and* $\pi_{z3} = \frac{m^2n(1-\epsilon)^2}{(1-3m-z)^2}$ *respectively.*

*ii.* *The payoff functions of a pool containing z firms and a firm outside the pool are* $P_{z3} = \frac{(m+z-1)(1-\epsilon)}{-1+3m+z}$ *and* $p_{z3} = \frac{m(1-\epsilon)}{-1+3m+z}$ *respectively.*

*iii.* *The probabilities that a period 3 firm with both "parents" in the pool and a period 3 firm with only one "parent" in the pool will undertake R&D are given by:* $Q_{z3} = \frac{2m(1-\epsilon)}{-1+3m+z}$ *and* $q_{z3} = \frac{m(1-\epsilon)}{-1+3m+z}$ *respectively.*

The proof is in Appendix A.2

*Proposition 2:*

*In any SGPE, a pool consisting of period 1 firm and all period 2 firms, which innovated, is formed.*

*Proof:*

Differentiating the payoff functions derived in Claim 1, we see that as the number of firms in the pool increases the profit of the pool increases and the profit of a firm outside the pool decreases. The increase in the pool's revenues is due to the "double marginalization" phenomena[10].

In light of the pool formation protocol where the period 1 firm has all the bargaining power it must be that in any SGPE:

(i) The period 1 firm offers a firm outside the pool the profits it would obtain, were it to reject the offer.

(ii) A firm outside the pool accepts any offer great than or equal to the profit it obtains when it decides not to join the pool.

[10] The prices charged by competing firms producing complementary goods exceed the prices charged for these goods when the firms merge to form a single monopoly.

In light of that the offer made to a period 2 firm when there are z patents in the pool is given by $\pi_{z3}$. To see whether it pays the period 1 firm to make this offer we evaluate the expression

$$\Pi_{z+1,3} - \Pi_{z3} - \pi_{z3} = \frac{m^2 n(3m^2 - z(4-3z) - 2m(4-5z))(1-\epsilon)^2}{(1-3m-z)^2(3m+z)^2}$$

This expression is positive for $1 \leq z \leq m+1$. Hence, in any SGPE equilibrium, the largest pool possible is formed at the beginning of period 3. Also note that the pool is stable, the fact $\pi_{z3}$ decreases in z implies no firm in the pool can gain by deciding to exit the pool. Also, the assumption we made regarding the pool formation protocol whereby a firm that rejected an offer to join, does not receive any further offers is without loss of generality. Since, no firm can gain by rejecting an offer anticipating to get a better one at a later stage. ■

Since the patent pool that forms consists of all the patents in period 1 and 2 we can now simplify the notation and use:

$\Pi_3 = \Pi_{m+1,3}$ ; $P_3 = P_{m+1,3}$ ; $Q_3 = Q_{m+1,3}$

It is also easy to see that the creation of a pool in the beginning of period 3 increases both the probability that a period 3 firm will engage in R&D and its profits in such a case.

In summary, we have shown that in the regular pyramid model a patent pool forms endogenously in period 3. In the next Section we study the independent licenses case which raises several new issues and also allows for a full analytic solution, yielding equilibrium levels of innovation probabilities and profits for all periods.

### 3.2.2 The Independent Licenses Scenario

The analysis so far has been carried out under the assumption that the use of a patent in the pool is possible only when buying access to the whole pool. We now consider the case where firm 1 after forming the pool may offer independent licenses for any patent belonging to the pool[11]. Note that due to the "double marginalization" the period 1 firm

[11] The independent licensing feature appeared in Lerner and Tirole (2004) where its purpose was to screen out welfare reducing pools.

and period 2 firms in the pool will not sell individual licenses to period 3 firms relying on their innovations. As before the formation of the pool takes place in the beginning of period 3 and z denotes the number of firms in the pool. $\widehat{Q_{z3}}$, $\widehat{q_{z3}}$ respectively denote the probability of undertaking R&D for a firm relying on period 2 innovation that belongs, does not belong, to the pool when there are z firms in the pool. $\widehat{P_{z3}}$, $\widehat{\pi_{z3}}$ are similarly defined as in the previous section. $\widehat{\Pi_{z3}}$, now denotes the revenue of the period 1 firm from selling access to the pool as well as independent licenses to firms outside the pool. Furthermore, $\widehat{p^1_{z3}}$ , $\widehat{p^2_{z3}}$ denote the independent license fees charged by the period 1 firm and a period 2 firm not belonging to the pool when there are z firms in the pool.

We now proceed to analyze the equilibrium in the game starting at the beginning of period 3 with z-1 period 2 firms in the pool and m-z+1 outside the pool (again looking for a symmetric equilibrium). The period 1 firm has to choose two license fees, $\widehat{p^1_{z3}}$, an independent license fee for period 3 firms that do not rely on period 2 patents belonging to the pool and $\widehat{P_{z3}}$ , a pool license. Each of the period 2 firms not in the pool has to choose $\widehat{p^2_{z3}}$ , a license for its patent.

*Claim 2:*

1. *The revenues of a pool containing z firms and a firm outside the pool are* $\widehat{\Pi_{z3}} = \frac{1}{36}n(4m+5z-5)(1-\epsilon)^2$ *and* $\widehat{\pi_{z3}} = \frac{1}{9}n(1-\epsilon)^2 = \pi_{23}$ *respectively*
2. *The payoff functions of the pool and the firm are given by* $\widehat{P_{z3}} = \frac{1-\epsilon}{2}$ *and* $\widehat{p^1_{z3}} = \widehat{p^2_{z3}} = \frac{1-\epsilon}{3}$.
3. *The probabilities that a period 3 firm with both "parents" in the pool and a period 3 firm with only one "parent" in the pool will undertake R&D are given by:* $\widehat{Q_{z3}} = \frac{1-\epsilon}{2}$ *and* $\widehat{q_{z3}} = \frac{1-\epsilon}{3}$ *respectively.*

The proof is in Appendix A.3

*Proposition 3:*

*In any SGPE, a pool consisting of all period 1 and 2 firms, who innovated, is formed.*

Proof:

The revenues derived in Claim 2 show that the pool's profit is increasing in the number of firms in the pool.

Similar to before it must be that in any SGPE:

(i) The period 1 firm offers the firm outside the pool the profits it would obtain, were it to reject the offer.

(ii) The firm outside the pool accepts any offer great than or equal to the profit it obtains when it decides not to join the pool.

In light of that the offer made to a period 2 firm when there are z patents in the pool is given by $\widehat{\pi^{0}_{z,3}}$. Since $\widehat{\Pi_{z+1,3}} - \widehat{\Pi_{z3}} - \widehat{\pi_{z3}} = \frac{1}{36}n(1-\epsilon)^2 > 0$, this must be the offer made by firm 1 in any SGPE.

Hence, in any SGPE, the largest pool possible forms at the beginning of period 3. Also note that since $\widehat{\pi_{z3}}$ is constant, the pool is stable, that is, no firm in the pool can gain by deciding to exit the pool. ■

Hence, in equilibrium the pool formed consists of all period 1 and 2 patents. Every period 2 patent receives the same payment for joining the pool, equal to its profits in the case where there is no pool. Thus, the possibility to issue independent licenses increased the bargaining power of period 2 firms, even though no independent licensing occurs in equilibrium. Also, note that pool payoff and probability of innovation are the same as in Proposition 3. The only thing that changes is the profit sharing within the members of the pool in favor of the period 2 firms.

Note that the profit of firm 1 declines once it cannot commit not to offer independent licenses. This may seem surprising at first since it seems that even when having to offer licenses it could set the independent license equal to the pool license. However, this choice is not part of a Nash equilibrium of the LSG in period 3. The Nash equilibrium outcome of this game indeed yields lower profits. Furthermore, note that the profit of a period 2 firm outside the pool is larger than in the case of no independent licensing. This is partially due to the fact that when joining the pool, a period 2 firm gives up the right of issuing independent licenses. Finally, in contrast to the previous case we stress that the profit of a firm 2 outside the pool is independent of the pool size.

The next step is to analyze the firms' behavior in periods 1 and 2. We show the probability that a period i firm innovates, denoted by $Q_i$, increases following the formation of the pool

*Proposition 4:*

*A pool increases weakly the probability to innovate at each of the three periods for each firm.* $(q_i \leq Q_i \; for \; any \; i)$

The proof is in Appendix A.4

Since the probability of innovation increases in profits we also obtain that the formation of a pool increases firm profits as well.

### 3.3 No Patents

Without patent protection, the probability that a firm innovates given that its "parent" innovation occurred, is constant and equals 1-ε, since in the absence of protection there are no license fees to pay or receive. However, the probability of innovation in a given period must also incorporate the probability that all innovations leading up to this period have been realized. Hence the probability of any period i innovation occurring, with no patent protection is $(1-\varepsilon)^i$.

The effect of patent protection on the innovation probabilities as a whole is not obvious. License fees that need to be paid reduce the innovation probability for the later generation firms while increasing the probability for earlier generation firms. However, comparing the probabilities for innovation in periods 2 and 3 across the two regimes of protection or no protection we see that protection is more justified as n and s increase.

*Proposition 5*

*Introduction of patent protection increases (decreases) the probability of innovation in period 3 relative to the no patent regime when s and n are large (small) and* $\epsilon$ *is close enough to 1 (0).*

The proof is in Appendix A.5

### 3.4 Social welfare

Aggregate social welfare is determined by the innovation thresholds set by the various firms, since the license fees paid to early innovators cancel out. Denoting by $v_i$ the innovation threshold's value in period i, aggregate social welfare, denoted by W, is given by:

$$W(v_1, v_2, v_3) = \int_{v_1}^{1} (t - \epsilon + \beta \cdot s \int_{v_2}^{1} (u - \epsilon + \beta \cdot n \int_{v_3}^{1} (v - \epsilon)\, dv)\, du)\, dt$$

The last integral is the welfare gain in period 3 with threshold $v_3$. The second to last integral is the welfare gain in period 2, taking into account the discounted period 3 welfare gains, with threshold $v_2$ and so on. Substituting the thresholds obtained we can provide a welfare ranking of the pool and no pool cases. The ranking is unequivocal in the interior case and yields the following proposition:

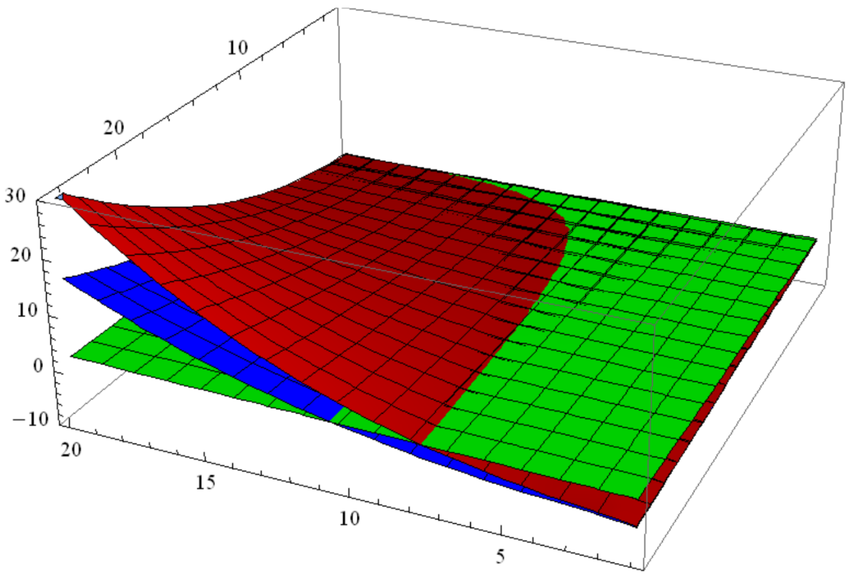
*Proposition 6:*

*The introduction of a patent pool improves welfare relative to the no pool case for the interior solution.*

The proof is in Appendix A.6

The cases where some innovation probabilities are 1, are more cumbersome computationally and at times require the addition of some constraints on the parameters and are detailed in the appendix following the proof of the proposition.

However, note that the introduction of patents and/or patent pools can either increase or decrease social welfare compared to the no patents regime. This is clearly shown by the following figure which is drawn for $\epsilon = 0.7$ and $\beta = 0.95$. n and s are measured along the horizontal axis and W is given by the vertical axis. The red graph represents the pool scenario, the blue graph the patent without a pool scenario and the green graph represents the no patent scenario.[12]




We now examine the decisions a central planner would take in order to maximize social welfare. The central planner decides to innovate as long as the social benefit exceeds the social cost. The social benefit equals the value of the innovation itself plus the value of future innovations made possible by it. Note, as before, that since the central planner is interested in maximizing total welfare she does not take into account the license fees transferred between different innovators. Thus, the social cost is the direct R&D cost given by the realization of $\epsilon$.

The social planner determines the innovation thresholds, denoted by $v_i$, for each period i. That is each firm is instructed to innovate if the realization of its innovation value exceeds the threshold.

To determine the socially optimal innovation thresholds for interior solutions in each period we start from period 3. In period 3, the final period, the central planner

[12] It can also be shown that as $\epsilon$ decreases the set of (n,s) pairs for which patents are welfare increasing, expands.

maximizes social welfare by innovating when the R&D cost is lower than the commercial value. Thus we obtain $\widetilde{v_3} = \epsilon$. Hence, in period 2 the social value of innovation will be: $v + \beta n \cdot \int_{\epsilon}^{1} (t - \epsilon)\, dt$. Therefore, we obtain $\widetilde{v_2} = \epsilon - \beta n \cdot \frac{(1-\epsilon)^2}{2}$ . Finally, in period 1, the social value of innovation is given by: $v + \beta s \cdot \int_{\epsilon - \beta n \cdot \frac{(1-\epsilon)^2}{2}}^{1} \left( t - \epsilon + \beta n \cdot \frac{(1-\epsilon)^2}{2} \right) dt$. Therefore, we get:

$$\widetilde{v_1} = \epsilon - \frac{\beta s}{2} \cdot \left(1 - \epsilon + \beta n \cdot \frac{(1-\epsilon)^2}{2}\right)^2 = \epsilon - \frac{1}{8} s\beta(2 + n\beta(1-\epsilon))^2 (1-\epsilon)^2.$$

We note that the thresholds set by a central planner are not higher than the thresholds set by the firms when patents are not available. Since the firms take into account only the value of the invention itself rather than the aggregate social value.

# 4. The Inverse Pyramid Model

We again consider a 3-period pyramidal structure of innovation with the same per-firm innovation technology. However, the pyramid is “inverted”. There is a single firm in period 3, it has s parent firms in period 2, and each one of those s firms has n parent firms in period 1. Each firm relies on the innovations of its parent firms and can engage in R&D activity only if all its parent firms have successfully innovated. All firms maximize their expected profit and each firm takes into account both current and future revenues when reaching its innovation and licensing decisions. In this setting we just analyze the behavior of the firms in period 3. A full equilibrium analysis is computationally complex as the profits of each firm depend on the innovation activities of both, firms in the same period and subsequent periods.

We analyze the strategic decisions of all firms in the beginning of period 3. We show that a patent pool, when considered independently of the pool formation process is not stable. This is not surprising as we deal here with essential patents as in Aoki and Nagaoka (2004), Llanes and Trento (2012) and Quint (2014). However we use the inverse pyramid structure to suggest a pool formation process that augmented by the LSG may lead to the formation of a patent pool consisting of all ns firms.

At period 3 each of the s firms in period 2 decides upon $p^{k}_{23}$ where k=1,…s, and each of the ns firms from period 1 decides upon $p^{k}_{13}$ where k=1,…,ns. Innovation in period 3 is possible only if all the s potential firms of period 2 have innovated. All the (n+1)s firms from periods 1 and 2 face the same decision problem. As before in equilibrium they all set the same license fee and denoting it by $p_3$, the probability of innovation in the third period by $q_3$ and profit of each firm (from periods 1 and 2) in period 3 by $\pi_3$ we obtain: $p_3 = \frac{1-\epsilon}{1+s+sn}$ , $q_3 = \frac{1-\epsilon}{1+s+sn}$. $\pi_3 = \frac{(1-\epsilon)^2}{(1+s+sn)^2}$

We now show that patent pools in this environment are not stable. This is due to the fact that each firm exiting the pool is just as "powerful" as the rest of the pool, since all patents are essential for creating new innovations. Each firm leaving the pool is "free riding" on the pool, since the pool's existence increases the probability of innovation in the third period which increases its expected profits.

To show that a pool in the beginning of period 3 is unstable we first consider the profits of a pool formed by all the innovators. These profits are given by: $\Pi_3 = Q_3 \cdot P_3$ When $Q_3$ is the probability to innovate after a pool has been established, and $P_3$ is the pool's license fee. As before $Q_3 = 1 - \epsilon - P_3$ and thus we obtain $P_3 = Q_3 = \frac{1-\epsilon}{2}$ and $\Pi_3 = \frac{1}{4}(1-\epsilon)^2$ note that in the inverse pyramid structure double marginalization actually turns into multi marginalization. Hence, as before, the pool increases the probabilities to innovate, decreases the license fees and increases the firms' profit in each period.

To analyze the decision to leave the pool we now examine the profits of a pool after z-1 firms have left it. These profits are the result of an equilibrium in a z players game. One player is the pool and the other z-1 players are all the firms that quit the pool. Let $p^{o}_{z3}$ and $\pi^{o}_{z3}$ denote the license fees and profits of the quitting firms, and $P^{i}_{z3}$, $\Pi^{i}_{z3}$ denote the pool's license fee and profit, respectively.

Thus we get $P^{i}_{z3} = p^{o}_{z3} = \frac{1-\epsilon}{1+z}$, and $\Pi^{i}_{z3} = \pi^{o}_{z3} = \frac{(1-\epsilon)^2}{(1+z)^2}$.

*Proposition 7:*

*A pool is never part of a LSG equilibrium if the numbers of "parent" innovators satisfy s +sn≥3.*

The proof is in Appendix A.7

We now address the pool creation issue by suggesting a more elaborate pool formation process. It relies in part on the inverse pyramid structure, granting each firm the right to exclude some of its parent firms and ends in one of two possible outcomes. One outcome is the formation of a pool in the beginning of period 3 consisting of all period 1 and 2 firms, the other has all period 1 and 2 firms acting as "singletons" in the beginning of period 3 and interacting in a license setting game with (n+1)s firms.

We provide first an informal description of the formation process. Note that in order for a pool to form it must be the case that all s period 2 firms have successfully innovated. The process consists of three rounds of play. In the first round the period 2 firms play a simultaneous game where they indicate whether or not they wish to join a pool. This round is followed by a simultaneous offering game and finally a license setting game in the third round. In the offering game all the firms that agreed to join the pool in the first round play a simultaneous offering game with their parent firms. This offering game itself proceeds sequentially. The period 2 firm sequentially offers its parents a payment (possibly different for different parents) if they agree to join the pool. If all the parent firms of the period 2 firm agreed to join the pool, then the period 2 firm and its parents are members of the pool. The first time a refusal to join the pool is encountered, the offering game terminates. A period 2 firm which encountered a refusal can then decide whether it stays in the pool with all the "consenting" parents or whether it leaves the pool in which case this firm and all its parents are not in the pool[13]. Finally, in the third round, the pool generated in the two rounds and the singletons outside it engage in a LSG for the third period firm.

[13] The fact that the first rejection terminates the offering stage, increases the bargaining power of a period 2 firm. The increased bargaining power makes it profitable for a period 2 firm to form a pool since it reduces the payment necessary to induce period 1 firms to join the pool. It also leads to a tractable set of subgame perfect equilibrium outcomes.

More formally the formation process is given by the following extensive form game:

*Round 1*

Each period 2 firm j, where $j \varepsilon S=\{1,\ldots,s\}$, simultaneously announces $x_j \varepsilon \{Y,N\}$. The firms are then partitioned into two sets, $I=\{j\varepsilon S \mid x_j=Y\}$ and $O=\{j\varepsilon S \mid x_j=N\}$ and the process moves to round 2.

*Round 2*

The active firms in this round are the ones in I, the firms in O wait for the beginning of round 3 where they will play as singletons in the LSG. Each of the firms in I, simultaneously, plays in the following offering game, described below for a firm $i\varepsilon I$, with its parent firms:

*The Offering Game*

Firm i, following the natural order with $i_1$ first and $i_n$ last, offers $x_{is}$ to parent firm $i_s$. The parent firms sequentially accept or reject the offer. If all offers have been accepted, firm i pays the offers and the n+1 firms (firm i and its parent firms) move to round 3 where they play as a singleton player, setting a license fee for the bundle consisting of all n+1 innovations.

The offering game terminates when a rejection occurs. Let $i_j$* denote the first parent firm to reject an offer. Firm i then chooses whether to stay in I together with all its consenting parents or whether to exit the pool and move to round 3 where firm i and all its parents play as n+1 singletons.

All these offering games are carried out simultaneously and round 2 ends with a partition of all (n+1)s firms into one pool and several singletons based on the outcomes realized in the various offering games of round 2.

*Round 3*

Let K+1 denote the number of subsets of firms forming the partition. That is, one pool and K singletons. These K+1 subsets play the licensing game described as follows. Each subset sets a license fee paid by the period 3 firm for using patents belonging to the firms in it. The payoff of the pool player in the Nash Equilibrium of this game is equally shared among the period 2 firms that belong to it.

The payoffs of any of the (n+1)s firms from round 1-3 are the sum of payoffs from rounds 2 and 3.

We will show there exist a range of n and s values for which the subgame perfect equilibria of this game lead to one of two possible outcomes. The pool outcome, where all firms create a single patent pool or a singleton outcome where all firms stay as singletons. For values outside this range a single patent pool cannot be part of a SGPE equilibrium while the singleton outcome remains an equilibrium. We show by numerical examples that it is also possible for a partial pool to form in equilibrium. We recall that the profit of each player (singleton or the pool) in a LSG game with L players is given by $\frac{(1-\epsilon)^2}{(L+1)^2}$ . This leads to the following claim:

*Claim 3: For any* $(n, s) \in T$ *where T={(n,s)|* $\frac{(n+3)^2}{9n} < s < \frac{(n+3)^2}{4(n+1)}$*}, in any SGPE the number of singletons entering Round 3 is either 0 or (n+1)·s.*

The proof is in Appendix A.8

*Claim 4: The pool outcome, where all firms create a single patent pool, is a SGPE if* $(n, s) \in T$ *.*

The proof is in Appendix A.9

*Claim 5: The "singleton" outcome where all firms stay as singletons is a SGPE outcome.*

The proof is in Appendix A.10

*Remarks:*

1. When following a departure of a pool member, the pool moves first in the Stage 3 LSG and the departing firms move second we obtain a different and larger range of (n,s) pairs for which Claim 3 holds.

2. For (n,s) pairs outside the range described in Claim 3, while full cooperation does not occur in equilibrium, partial pools may still form in equilibrium as shown by a numerical example in Appendix A.8.
3. Note that the range of (n,s) pairs in Claim 3 allows for the formation of arbitrarily large pools, whereas in previous findings (Aoki and Nagaoka (2005, 2004)), the pool outcome occurred only for a small number of firms.

### 4.1 No Patents

Without the possibility for patent protection, as in Section 3.3, the probability of any period i innovation occurring is $(1-\varepsilon)^i$.

The effect of patent protection on the innovation probabilities is again not trivial to assess. However, in the case where pools are not allowed to form, the introduction of patent protection reduces the probability of the third period innovation. The license fees that the third period firm has to pay all the second period innovators greatly diminish its incentive to innovate. This result is formally stated in the following proposition.

*Proposition 8:*

*In the “no-pool” equilibrium, introduction of patent protection always decreases the probability of the third period innovation relative to the no-patent regime.*

The proof is in Appendix A.11

# 5. Conclusion

We studied sequential innovations and the endogenous formation of patent pools in two pyramidal structures. In the regular pyramid structure we provided a complete analysis of the patent, patent pools and no patens scenarios. We have shown that the introduction of patents and patent pools has ambiguous effects on the innovation probabilities and social welfare but is more justified the larger is the pyramid. In the patent pool scenario we constructed a simple protocol leading in SGPE to the formation of a stable pool of all the firms that have successfully innovated.

In the inverse pyramid structure we have shown that a patent pool is not stable and cannot be part of a SGPE following the simple protocol suggested for the regular pyramid case. However, we provided a more elaborate protocol taking into account the pyramid specific structure. In this protocol, each firm negotiates with its parent firms. If the offers it makes are rejected, it may exclude some or all of its parents form the pool. The SGPE induced by this protocol lead to two possible outcomes, either a pool joined by all the firms that have innovated or a singleton outcome where no pool is formed. We also show that in the absence of pools, patents are not justified in the inverse pyramid structure, since they reduce the probability to innovate the final innovation.

The inverse pyramid structure used in this paper can be applied to a standard production setting where a final marketable product is based on several essential components, which are produced by independent monopolies. In this case, a "pool" is formed by having these monopolies merge and decide jointly on the pricing of all the components. Our conclusions regarding the formation of a pool and its welfare implication carry through to such environments.

In this paper we assumed innovations are symmetric. That is, each innovation relies on the same number of "parent" firms and paves the way for the same number of follow-up innovations. Relaxing this assumption leads to more complex calculations but does not affect our main conclusions regarding pool formation.

Our findings augment those reported in several previous studies along the dimensions of the justification for patents, desirability of pools and pool stability. Boldrin and Levine (2005) and later Bessen and Maskin (2009) showed that patents slow down the innovative activity. Our regular pyramid model produces the same conclusion when the pyramid is narrow enough (Bessen and Maskin (2009) considered a "linear" pyramid with one innovation at each period). The same can be said about our inverse pyramid. We do find patents are justified when allowing for wider pyramids.

Shapiro (2003), Gilbert (2004), Lerner and Tirole (2004) and Llanes and Trento (2012) concluded pools are desirable when patents are close to being perfect complements. We reach the same conclusion in our richer structure which explicitly models the formation of pools, taking into account the incentives to join or leave a pool. Furthermore we take into account the influence of the pool on innovative activity in the periods preceding it.

Finally, we contribute to the study of pool formation. Llanes and Trento (2012) and Quint (2014) noted that the incentives to exit a pool increase, rendering it unstable when large enough. Aoki and Nagaoka (2004) provide a stable formation process when there are 4 firms. We showed pools are always stable in a regular pyramid structure whereas in an inverse pyramid structure stability was obtained only when using an elaborate formation protocol. This non-stability might explain why a patent pool is not formed, and the need (as in the AIDS example) for the intervention of government or non-profit organizations to facilitate their creation. Our formation protocol shows that granting later innovators a superior bargaining position might be a possible solution to the pool creation issue.

More work needs to be done on the effect of time dependent rewards to innovations and on extending the model to more periods. Such an extension would also enable us to study the optimal length of a patent as in Llanes and Trento (2012). The stability of pools is also closely related to the large literature on coalition formation and it remains to be seen whether several formation protocols raised in that context can be applied in innovative environments.

## APPENDIX – Proofs

### A.1 Proof of Proposition 1

a. The problem faced by periods 1 and 2 firms that have undertaken R&D is what license fees $p_{13}$ and $p_{23}$ should be set for the period 3 firm. A period 3 firm will undertake R&D and thus pay the license fee only when its current revenue exceeds R&D cost and license revenues. Thus the probability that a period 3 firm will undertake R&D as a function of the licenses set is: $q_3 = 1 - \epsilon - p_{13} - p_{23}$ and we obtain the following payoff functions for the period 1 and 2 firms:

$$\pi_{13} = \sum_{i=0}^{m*n} \binom{n \cdot m}{i} \cdot {q_3}^{i} \cdot (1 - q_3)^{m \cdot n - i} \cdot i \cdot p_{13}$$

$$\pi_{23} = \sum_{i=0}^{n} \binom{n}{i} \cdot {q_3}^i \cdot (1-q_3)^{n-i} \cdot i \cdot p_{23}$$

These reduce, substituting for $q_3$, to:

$$\pi_{13} = mnp_{13}(1 - \epsilon - p_{13} - p_{23})$$

$$\pi_{23} = np_{23}(1 - \epsilon - p_{13} - p_{23})$$

So, the Nash Equilibrium of this m+1 firms' LSG is given by: $p_{13} = p_{23} = \frac{1-\epsilon}{3}$.

Substituting we obtain: $q_3 = \frac{1-\epsilon}{3}, \pi_{13} = \frac{1}{9}mn(1-\epsilon)^2$ ,$\pi_{23} = \frac{1}{9}n(1-\epsilon)^2$

b. The probability that a period 2 firm will undertake R&D as a function of the license set is: $q_2 = 1 - \epsilon - p_{12} + \beta\pi_{23}$. The expected revenue of the period 1 firm from the licenses paid by period 2 firms is derived by taking the expectation over m≤s (m being the number of firms who decide to innovate in period 2). Thus, we obtain the following payoff function for the period 1 firm from period 2 onwards:

$$\pi_{12} = \sum_{m=0}^{s} \left( \binom{s}{m} {q_2}^m (1-q_2)^{s-m} (mp_{12} + \beta\pi_{13}) \right)$$

When $n < \frac{9(1+\epsilon)}{2\beta(1-\epsilon)^2}$ the solution is interior, the period 1 firm sets $p_{12} = \frac{1-\epsilon}{2}$ to maximize its profits. Substituting we obtain: $\pi_{12} = \frac{1}{324}s(9 + 2\beta n(1-\epsilon))^2(1-\epsilon)^2$ and $q_2 = \frac{1}{18}(9 + 2n\beta(1-\epsilon))(1-\epsilon)$,

When $n \geq \frac{9(1+\epsilon)}{2\beta(1-\epsilon)^2}$ we are at a corner solution, where $p_{12} = \frac{n\beta(1-\epsilon)^2}{9} - \epsilon$ is the largest license fee for which $q_2=1$[14], and the period 1 firm profit is given by: $\frac{1}{9}s(2n\beta(1-\epsilon)^2 - 9\epsilon)$. Note that the number of potential firms in period 3 does not

[14] $p_{12} \geq \frac{1-\epsilon}{2}$ at a corner solution, since reducing the license fee does not increase the probability of innovation in the second period as it is already at its maximum value of 1.

affect $q_2$ since the license fee charged by the period 1 firm extracts the surplus a period 2 firm expects to get from its followers.

c. Finally, at period 1, the probability of undertaking R&D is: $q_1 = 1 - \epsilon + \beta\ \pi_{12}$ . Substituting the revenues from b, we obtain the probability to innovate in period 1 for both interior and corner solutions.

### A.2 Proof of Claim 1

Note that period 3 firms whose parent firm does not belong to the pool, have to purchase both, a license for its parent firm and a license for the pool (since the period 1 firm belongs to the pool). Thus, the probabilities that a period 3 firm not in the pool and a period 3 firm in the pool will undertake R&D as a function of the license fees set are given by: $q_{z3} = 1 - \epsilon - p_{z3} - P_{z3}$ and $Q_{z3} = 1 - \epsilon - P_{z3}$. The payoff function of a firm not belonging to the pool and the pool's payoff function are respectively given by:

$$\pi_{z3} = \sum_{k=0}^{n} \binom{n}{k} q_{z3}{}^{k}(1 - q_{z3})^{n-k} k \cdot p_{z3} = np_{z3}(1 - P_{z3} - p_{z3} - \epsilon)$$

and

$$\Pi_{z3} = \left( \sum_{k=0}^{n(z-1)} \binom{n(z-1)}{k} Q_{z3}{}^{k}(1 - Q_{z3})^{n(z-1)-k} \cdot k + \sum_{k=0}^{n(m-z+1)} \binom{n(m-z+1)}{k} q_{z3}{}^{k}(1 - q_{z3})^{n(m-z+1)-k} \cdot k \right) P_{z3} = nP_{z3}((z-1) \cdot p_{z3} + m(1 - \epsilon - P_{z3} - p_{z3})$$

The first term represents the average number of firms which decide to innovate in period 3, those which rely on patents in the pool as well as those that do not (recall that they have to pay for the use of the period 1 patent).

The Nash equilibrium outcome of this LSG with 1+(m-(z-1))=m-z+2 players is given by:

$$P_{z3} = \frac{(m + z - 1)(1 - \epsilon)}{-1 + 3m + z}$$

$$p_{z3} = \frac{m(1-\epsilon)}{-1+3m+z}$$

Substituting these values into the payoff functions of the pool containing z firms and a firm outside the pool we obtain:

$$\Pi_{z3} = \frac{mn(1-m-z)^2(1-\epsilon)^2}{(1-3m-z)^2}$$

$$\pi_{z3} = \frac{m^2n(1-\epsilon)^2}{(1-3m-z)^2}$$

**A.3 Proof of Claim 2**

The payoff function of the period 1 firm is given by:

$$\widehat{\Pi_{z3}} = \sum_{k=0}^{n(z-1)} \binom{n(z-1)}{k} \widehat{Q_{z3}}^k \cdot (1-\widehat{Q_{z3}})^{n(z-1)-k} \cdot k\widehat{P_{z3}} + \sum_{k=0}^{n(m-z+1)} \binom{n(m-z+1)}{k} \widehat{q_{z3}}^k \cdot (1-\widehat{q_{z3}})^{n(m-z+1)-k} \cdot k\widehat{p_{z3}^1}$$

Whereas, the payoff function of each of the m-z+1 period 2 firms not in the pool is given by:

$$\widehat{\pi_{z3}} = \sum_{k=0}^{n} \binom{n}{k} \widehat{q_{z3}}^k \cdot (1-\widehat{q_{z3}})^{n-k} \cdot k\widehat{p_{z3}^2}$$

Where, $\widehat{Q_{z3}} = 1-\epsilon-\widehat{P_{z3}}$ and $\widehat{q_{z3}} = 1-\epsilon-\widehat{p_{z3}^1}-\widehat{p_{z3}^2}$

The unique Nash Equilibrium of this LSG is given by: $\widehat{P_{z3}} = \frac{1-\epsilon}{2}$ and $\widehat{p_{z3}^1} = \widehat{p_{z3}^2} = \frac{1-\epsilon}{3}$. Substituting these values into the payoff functions we obtain that the profits of the pool containing z firms, when the period 1 firm offers independent licenses, and the profit of a firm outside the pool are respectively given by:

$$\widehat{\Pi_{z3}} = \frac{1}{36} n(4m+5z-5)(1-\epsilon)^2$$

$$\widehat{\pi_{z3}} = \frac{1}{9}n(1-\epsilon)^2 = \pi_{23}$$

**A.4 Proof of Proposition 4**

As before the probability that a period 2 firm will innovate is given by:

$Q_2 = 1 - \epsilon - P_{12} + \beta\pi_{23}$ The expected revenue of the period 1 firm is again derived by taking the expectation over m≤s (m being the number of firms who decide to innovate in period 2). We let $\Pi_{12}$ be the profit of the firm in the first period and $P_{12}$ the license fees that firms from period 2 pay to the period 1 firm. Therefore, $\Pi_{12} = \sum_{m=0}^{s}\binom{s}{m}{Q_2}^{m}(1-Q_2)^{s-m}(mP_{12} + \beta(\Pi_3 - m\pi_{23}))$

When $n \leq \frac{4(1+\epsilon)}{\beta(1-\epsilon)^2}$ the solution is interior, the period 1 firm sets $P_{12} = \frac{1}{72}\left(36 - n\beta(1-\epsilon)\right)(1-\epsilon)$Substituting we obtain $\Pi_{12} = \frac{1}{64}s(4+n\beta(1-\epsilon))^2(1-\epsilon)^2$ and $Q_2 = \frac{1}{8}(4+n\beta(1-\epsilon))(1-\epsilon)$.

When $n > \frac{4(1+\epsilon)}{\beta(1-\epsilon)^2}$ we are at a corner solution, where $P_{12} = \frac{1}{9}(n\beta(1-\epsilon)^2 - 9\epsilon)$, is the largest license fee for which $q_2$=1, and the period 1 firm's profit is given by: $\frac{1}{4}s(n\beta(-1+\epsilon)^2 - 4\epsilon)$

Finally at period 1, the probability of undertaking R&D is:

$Q_1 = 1 - \epsilon + \beta\ \Pi_{12}$ and therefore $Q_1 = \min\{1 + \frac{1}{64}s\beta(-4+n\beta(-1+\epsilon))^2(-1+\epsilon)^2 - \epsilon, 1\}$ in the interior solution, and $Q_1 = \min\{1 + \frac{1}{4}s\beta(n\beta(-1+\epsilon)^2 - 4\epsilon) - \epsilon, 1\}$ in the corner solution.

Comparing these values to those derived in the no-pool case we see that as in previous studies the creation of a pool weakly increases the probability of innovation and the innovator's profits in each period[15].

[15] For the case where $Q_2$=$q_2$ =1 we need to add the condition that β<0.8.

### A.5 Proof of Proposition 5

Note that the probability to innovate in period 3 with patent protection exceeds the probability to innovate without patent protection if and only if $q_1 \cdot q_2 \cdot q_3 > (1-\epsilon)^3$ if there is no pool, and if and only if $Q_1 \cdot Q_2 \cdot Q_3 > (1-\epsilon)^3$ when a pool forms. First we can see that when n=s=1 these inequalities are not satisfied. However, when n and s increase, the probabilities to innovate in period 1 and 2 approach 1 whether or not there is a pool. Since $Q_3=(1-\epsilon)/2$ and $q_3=(1-\epsilon)/3$, for $\epsilon$ large enough patent protection increases the probability to innovate in period 3. The opposite is true for small enough n,s, $\epsilon$ .

### A.6 Proof of Proposition 6

It can be easily shown, by straight forward substitution, that

$$W(1-Q_1, 1-Q_2, 1-Q_3) - W(1-q_1, 1-q_2, 1-q_3) =$$

$$ns\beta^2\left(248832+\beta\left(69984s+n\left(64368+s\beta\left(53784+n\beta\left(13722+1163n\beta(1-\epsilon)\right)(1-\epsilon)\right)(1-\epsilon)\right)\right)(1-\epsilon)\right)(1-\epsilon)^2 > 0$$

when all innovation probabilities are less than 1.

We now proceed to analyze all the "corner" cases where some of the probabilities are 1. Note that since $Q_i \geq q_i$ there are 8 cases that need to be checked

Case 1: All innovation probabilities in periods 1 and 2 with or without a pool equal 1, the claim is verified by substitution. Since $W(0,0,Q_3) - W(0,0,q_3) = \frac{1}{72} ns\beta^2(1-\epsilon)(17\epsilon-5) > 0$

We now note that W is decreasing in its third argument, the innovation threshold in the third period, when it is above , the central planner's threshold, since $\frac{\partial W}{\partial V_3} = ns(1-v_1)(1-v_2)\beta^2(\epsilon - v_3)$. Hence the welfare in the pool case is larger than the welfare in a hypothetical scenario where the pool's innovation threshold in the third period is replaced by the no-pool threshold of the third period keeping everything else constant. The proof in all the following cases consists of showing that the welfare realized in the hypothetical scenario exceeds the welfare in the no-pool case.

Case 2: $Q_2, q_2<1, Q_1=q_1=1$:

$$w(0,1-Q_2,1-Q_3)-w(0,1-q_2,1-q_3)>w(0,1-Q_2,1-q_3)-w(0,1-q_2,1-q_3)=\frac{ns\beta^2(72+23n\beta(1-\epsilon))(1-\epsilon)^3}{10368}>0$$

Case 3: $Q_1=Q_2=q_1=1, q_2<1$

$$w(0,0,1-Q_3)-w(0,1-q_2,1-q_3)>w(0,0,1-q_3)-w(0,1-q_2,1-q_3)$$
$$=-\frac{1}{648}s\beta(16n^2\beta^2(-1+\epsilon)^4+81(1+\epsilon)(-1+3\epsilon)-18n\beta(-1+\epsilon)^2(3+7\epsilon))$$

Tedious calculations show this expression is non-negative for all values of n,s for which $Q_2=1$ and $q_2<1$.

Case 4: $Q_2=q_2=1$, $Q_1,q_1<1$.

In this case the following inequalities must hold:

$$n\beta(-1+\epsilon)^2>4(1+\epsilon)$$

$$ns\beta^2(-1+\epsilon)^2<4(\epsilon+s\beta\epsilon)$$

$$2n\beta(-1+\epsilon)^2>9(1+\epsilon)$$

$$ns\beta(1+\beta)(-1+\epsilon)^2<9(\epsilon+s\beta\epsilon)$$

Again, through several manipulations it can be shown these inequalities cannot be satisfied by any admissible set of parameters.

Case 5: $Q_2=Q_1=q_2=1$, $q_1<1$

In this case the following inequalities must hold:

$$2n\beta(-1+\epsilon)^2 > 9(1+\epsilon)$$

$$ns\beta^2(-1+\epsilon)^2 > 4(\epsilon + s\beta\epsilon)$$

$$ns\beta(1+\beta)(-1+\epsilon)^2 < 9(\epsilon + s\beta\epsilon)$$

$$2n\beta(-1+\epsilon)^2 > 9(1+\epsilon)$$

Similar to before it can be shown these inequalities cannot be satisfied by any admissible set of parameters.

Case 6 : $Q_2 = Q_1 = 1$, $q_2$, $q_1 < 1$

$$2n\beta(-1+\epsilon)^2 < 9(1+\epsilon)$$

$$ns\beta^2(-1+\epsilon)^2 > 4(\epsilon + s\beta\epsilon)$$

$$s\beta\left(9 - 2n\beta(-1+\epsilon)\right)^2(-1+\epsilon)^2 < 324\epsilon$$

$$n\beta(-1+\epsilon)^2 > 4(1+\epsilon)$$

These inequalities are inconsistent for $\beta$ close to 1 and s>1.

Case 7: $Q_2=1$, $q_2$, $q_1$, $Q_1<1$.

$ns\beta^2(-1+\epsilon)^2 < 4(\epsilon + s\beta\epsilon), 2n\beta(-1+\epsilon)^2 < 9(1+\epsilon), s\beta(9 - 2n\beta(-1+\epsilon))^2(-1+\epsilon)^2 < 324\epsilon, n\beta(-1+\epsilon)^2 > 4(1+\epsilon)$

These inequalities are inconsistent for $\beta$ close to 1.

Case 8: $Q_1=1$, $q_2$, $q_1$, $Q_2<1$.

$$w(0,1-Q_2,1-Q_3) - w(1-q_1,1-q_2,1-q_3) > w(0,1-Q_2,1-q_3) - w(1-q_1,1-q_2,1-q_3) = \frac{1}{2} + \frac{s\beta(-4+n\beta(-1+\epsilon))(-108+31n\beta(-1+\epsilon))(-1+\epsilon)^2}{1152} - \frac{(17496-54s\beta(-9+2n\beta(-1+\epsilon))(-27+8n\beta(-1+\epsilon))(-1+\epsilon)+s^2\beta^2(-3+n\beta(-1+\epsilon))(-9+2n\beta(-1+\epsilon))^3(-1+\epsilon)^2)(-1+\epsilon)^2}{34992} - \epsilon$$

Which is positive when $\beta$ is close to 1.

### A.7 Proof of Proposition 7

Assume there exist a pool containing z firms in an equilibrium of the LSG. Let h be the number of players in this LSG (the pool and all the other firms acting as singletons). Hence, the gain from unilaterally leaving the pool is given by $\frac{(1-\epsilon)^2}{(2+h)^2}$. Hence the equilibrium payoff for each pool's member must be at least $\frac{(1-\epsilon)^2}{(2+h)^2}$. However, the pool's profit is $\frac{(1-\epsilon)^2}{(1+h)^2}$ and for $\geq 2$, $\frac{(1-\epsilon)^2}{z(1+h)^2} < \frac{(1-\epsilon)^2}{(2+h)^2}$. Hence the pool does not generate enough profits to keep its members from deserting it and no pool will form in equilibrium.

### A.8 Proof of Claim 3

Assume by way of contradiction there is a SGPE where the number of singletons is $\theta = \theta_1 + \theta_2$, where $\theta_i$ is the number of period i firms that are not in the pool with $n\theta_2 \leq \theta_1 \leq ns$, $0 \leq \theta_2 < s$ and $0 < \theta < (n+1)s-1$. Then there must be at least one period 2 firm which we denote by α that enters stage 3 in a pool together with $\gamma \geq 0$ parent firms. To determine the offer Firm α made to each of its parents in this candidate for equilibrium we note that if Firm α's threat to leave and then exclude all its parents is credible the offers must be: $\frac{(1-\epsilon)^2}{(\theta+1+(\gamma+1)+1)^2}$ if $\theta < (n+1)s-\gamma-1$ and $\frac{(1-\epsilon)^2}{(\theta+(\gamma+1)+1)^2}$ if $\theta = (n+1)s-\gamma-1$[16] (these are the payoffs in the LSG once the threat is carried out). To show that the threat is credible note that the payoff to firm α from staying in the pool is: $\frac{(1-\epsilon)^2}{(s-\theta_2)(\theta+1+1)^2} - \frac{\gamma(1-\epsilon)^2}{(\theta+1+(\gamma+1)+1)^2}$ if $\theta < (n+1)s-\gamma-1$ and $\frac{(1-\epsilon)^2}{(\theta+1)^2} - \frac{\gamma(1-\epsilon)^2}{(\theta+(\gamma+1)+1)^2}$ if $\theta = (n+1)s-\gamma-1$. If Firm α were to leave the pool it would obtain $\frac{(1-\epsilon)^2}{(\theta+1+(\gamma+1)+1)^2}$ if $\theta < (n+1)s-\gamma-1$ and $\frac{(1-\epsilon)^2}{(\theta+(\gamma+1)+1)^2}$ if $\theta = (n+1)s-\gamma-1$.

Since for $(n,s) \in T$ (i) $\frac{(1-\epsilon)^2}{(\theta+1+(\gamma+1)+1)^2} > \frac{(1-\epsilon)^2}{(s-\theta_2)(\theta+1+1)^2} - \frac{\gamma(1-\epsilon)^2}{(\theta+1+(\gamma+1)+1)^2}$ and

(ii) $\frac{(1-\epsilon)^2}{(\theta+(\gamma+1)+1)^2} > \frac{(1-\epsilon)^2}{(\theta+1)^2} - \frac{\gamma(1-\epsilon)^2}{(\theta+(\gamma+1)+1)^2}$, we obtain that Firm α 's threat to leave and exclude its parents is credible. Since inequalities (i) and (ii) hold for any $\gamma \leq n$ firm

[16] Note that in this case $\gamma \geq 1$.

α's payoff outside the pool is higher, implying a partial pool cannot be part of an equilibrium[17]. The firm can increase its payoff by making unacceptable offers to its parents and leaving the pool, once rejected.

### A.9 Proof of Claim 4

The SGPE which leads to the creation of the pool with (n+1)s members can be described as follows: Each period 2 firm joins the pool in round 1. Each period 1 firm rejects any offer below $\frac{(1-\epsilon)^2}{(n+3)^2}$ and accepts any other offer. Each period 2 firm offers each parent firm $\frac{(1-\epsilon)^2}{(n+3)^2}$ in round 2 and leaves the pool prior to round 3 if it encounters a rejection. Finally each firm as well as the pool (if formed) play the unique Nash Equilibrium in the LSG in period 3.

This is a SGPE since no period 1 can gain by increasing its acceptance threshold. If it were to raise it, the period 2 firm offering it will (as we have shown in claim 3) leave the pool and the period 1 firm will remain with same payoff it was offered, in the new outcome. Lowering its threshold will have no effect on the equilibrium. A period 2 firm's decision to leave the pool when encountering a rejection is a best response as shown in claim 1. Furthermore, a period 2 firm cannot gain by making larger offers. It remains to show that a period 2 firm cannot gain by lowering one of its offers. To show that note that its payoff in the "candidate" SGPE is: $\frac{(1-\epsilon)^2}{4S} - \frac{n(1-\epsilon)^2}{(n+3)^2}$. The payoff of such a firm, if it chooses to leave the pool (either directly in round 1 or by making unacceptable offers in round 2) is: $\frac{(1-\epsilon)^2}{(n+3)^2}$ . Since $(n, s) \in T$ we obtain $\frac{(1-\epsilon)^2}{(n+3)^2} < \frac{(1-\epsilon)^2}{4S} - \frac{n(1-\epsilon)^2}{(n+3)^2}$, which shows a period 2 firm cannot gain by leaving the pool and hence cannot increase its payoff by reducing its offer.

In the following paragraph, we present an example regarding the second remark showing that for $(n, s) \notin T$ a partial pool may form in equilibrium. Let n=20 and s=2. For

[17] Inequality (i) , when $\theta_2 = 0$, holds only when $\gamma \leq n-1$. However in the case where Firm α as well as all its parents are in the pool, there must be another period 2 firm with one or more parents outside the pool. This firm can now be used to show we are not in an equilibrium.

simplicity, we denote by 1,2 the period 2 firms and by 1i,2i (i=1,…,20) the period 1 parent firms of both respectively. We now describe three possible strategies for period 1 firms in the offering game. Strategy A is to accept any offer greater than or equal to $\frac{(1-\epsilon)^2}{(1+1+21)^2}$ and reject any smaller offer. Strategy B is to be accept any offer greater than or equal to $\frac{(1-\epsilon)^2}{(1+1+21+1)^2}$ and reject any smaller offer. Strategy C is to accept any offer greater than or equal to $\frac{(1-\epsilon)^2}{(1+1+1)^2}$ and reject any smaller offer. Assume firms 1i (i=1,…,20) follow strategy B, firms 2i (i=1,…,19) follow strategy A and firm $2_{20}$ follows strategy C. Furthermore, firms 1 and 2 offer each of their "parent" firms $\frac{(1-\epsilon)^2}{(1+1+21+1)^2}$ and $\frac{(1-\epsilon)^2}{(1+1+21)^2}$ respectively if they agree to join the pool. It can be easily verified these strategies form a SGPE equilibrium where a partial pool consisting of 41 firms is formed.

**A.10 Proof of Claim 5**

A SGPE yielding it is given by all period 2 firms refusing to join a pool in round 1. To see this is part of a SGPE profile strategies note that if a period 2 firms decides to join a pool in round 1, it will, by claim 3, make unacceptable offers to its parent firms and reach round 3 as a singleton (together with all its "singleton" parents) and thus will not increase its payoff. Hence, opting out of a pool in round 1 is part of a SGPE equilibrium.

**A.11 Proof of Proposition 8**

The probabilities to innovate in each period with patent protection are given by:

$$q_1 = 1 - \epsilon + \beta \cdot q_1^{n-1} \cdot q_2 \cdot p_{12} + \beta^2 \cdot q_1^{sn-1} \cdot q_2^{s} \cdot q_3 \cdot p_{13}$$

$$q_2 = 1 - \epsilon - n \cdot p_{12} + \beta \cdot q_1^{n(s-1)} \cdot q_2^{s-1} \cdot q_3 \cdot p_{23}$$

$$q_3 = 1 - \epsilon - sp_{23} - nsp_{13}$$

From previous results we get: $p_{23} = p_{13} = q_3 = \frac{1-\epsilon}{s(n+1)+1}$ . Therefore, we can obtain two upper bounds for the innovation probabilities in period 1 and 2 given by $\overline{q_1}, \overline{q_2}$ where:

$\overline{q_1} = 1 - \epsilon + p_{12} + \beta q_3 \cdot p_{13}$, $\overline{q_2} = 1 - \epsilon - np_{12} + \beta q_3 \cdot p_{13}$. Now note that

$ns\overline{q_1} + s\overline{q_2} = s(n+1)\overline{q_{12}}$ where $\overline{q_{12}} = 1 - \epsilon + \beta q_3 \cdot p_{13}$. Hence, $\overline{q_{12}}^{s(n+1)} \geq \overline{q_1}^{sn} \cdot \overline{q_2}^{s}$. The difference between the probabilities for realization of the innovation in period 3 without and with patent is:

$$(1-\epsilon)^{s(n+1)+1} - q_1^{sn} q_2^{s} q_3 > (1-\epsilon)^{s(n+1)+1} - \overline{q_1}^{sn} \cdot \overline{q_2}^{s} \cdot q_3 >$$

$$(1-\epsilon)^{s(n+1)+1} - \overline{q_{12}}^{s(n+1)} \cdot q_3 = \left( (1-\epsilon)^{s(n+1)} - \frac{\left(1-\epsilon+\frac{\beta(1-\epsilon)^2}{(1+s(n+1))^2}\right)^{s(n+1)}}{1+s(n+1)} \right)(1-\epsilon)$$

The R.H.S can by written as $\left( 1 - \frac{\left(1+\frac{\beta(1-\epsilon)}{(1+s(n+1))^2}\right)^{s(n+1)}}{1+s(n+1)} \right)(1-\epsilon)^{s(n+1)}(1-\epsilon)$ which is positive whenever $1 - \frac{\left(1+\frac{1}{(1+s(n+1))^2}\right)^{s(n+1)}}{1+s(n+1)}$ is positive. Letting s(n+1) be x, we can see that the previous term is indeed positive for any x>3.

## REFERENCES


Aoki, R. and S. Nagaoka (2004) “The Consortium Standard and Patent Pools”, *The Economic Review*, 55, 345-356.

Aoki, R. and S. Nagaoka (2005), “Coalition Formation for a Consortium Standard through a Standard Body and a Patent Pool: Theory and Evidence from MPEG2, DVD and 3G”, Hitotsubashi University Institute of Innovation Research, IIR, Working Paper 05-01.

Baron, J. and T. Pohlmann (2013), "Who Cooperates in Standards Consortia: Rivals or Complementors?", *Journal of Competition Law and Economics*, 9, 905-929.

Bessen, J. and E. Maskin (2009), "Sequential Innovation, Patents and Imitation", *RAND Journal of Economics*, 40, 611–635.

Boldrin, M. and D.K. Levine (2005), “The Economics of Ideas and Intellectual Property”, *Proceedings of the National Academy of Sciences*, 102, 1252–1256.

Boutin, A. (2016), "Screening for Good Patent Pools through Price Caps on Individual Licenses", *American Economic Journal: Microeconomics,* 8, 64-94.

Brenner, S. (2009), “Optimal Formation Rules for Patent Pools”, *Economic Theory*, 40, 373-388.

Galasso A., and M. Schankerman (2013), “Patents and Cumulative Innovation: Causal Evidence from the Courts”, *The Quarterly Journal of Economics*, 130, 317-369.

Gilbert, R.J. (2004), “Antitrust for Patent Pools: A Century of Policy Evolution”, *Stanford Technology Law Review* 3.

Green, J.R., and S. Scotchmer (1995), "On the Division of Profit in Sequential Innovation", *RAND Journal of Economics,* 26, 20–33.

Heller, M.A. (1998), "The Tragedy of the Anticommons: Property in the Transition from Marx to Markets", *Harvard Law Review* 111, 621–688.

Heller, M.A. and R. S. Eisenberg (1998), “Can Patents Deter Innovation? The Anticommons in Biomedical Research”, *Science*, 280, 698-701.

Hopenhayn, H., Llobet, G. and M. Mitchell (2006), “Rewarding Sequential Innovators: Prizes, Patents, and Buyouts”, *Journal of Political Economy*, 114, 1041–1068.

Lerner, J., and J. Tirole (2004), “Efficient Patent Pools”, American Economic Review, 94, 691-711.

Llanes, G., and S. Trento (2012), "Patent Policy, Patent Pools, and the Accumulation of Claims in Sequential Innovation”, *Economic Theory* 50, 703 –725.

Quint, D., (2014), “Pooling with Essential and Nonessential Patents”, *American Economic Journal: Microeconomics,* 6, 23–57.

Remembering the Laser Diode, [Editoria], (2012, November). Nature Photonics 6, 795, Retrieved from http://www.nature.com/nphoton/journal/v6/n12/full/nphoton.2012.310.html

Scotchmer, S. (1991), "Standing on the Shoulders of Giants: Cumulative Research and the Patent Law", *Journal of Economic Perspectives*, 5, 29–41.

Scotchmer, S. (1996), "Protecting Early Innovators: Should Second-Generation Products be Patentable", *RAND Journal of Economics,* 27, 322–331.

Shapiro, C. (2003), “Antitrust Limits to Patent Settlements”, *RAND Journal of Economics*, 34, 391-411.

Shapiro, C. (2001), "Navigating the Patent Thicket: Cross Licenses, Patent Pools, and Standard Setting," in Innovation Policy and the Economy Vol. 1 Chap. 4, NBER and MIT press, Cambridge.

Tirole, J. (2015), "Market Failures and Public Policy." *American Economic Review*, 105, 1665-1682.