Optimal Execution of Open-Market Stock Repurchase Programs*

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Abstract

We provide a theoretical investigation of the execution of open-market stock repurchase programs. Our model suggests that the execution depends on availability of free cash and information asymmetry. The results highlight important features of open-market stock repurchase programs: they leave the firm the option to avoid payout when cash is needed for operations, yet they also disburse free cash as long as the stock is not severely overpriced. Because they are preformed at management discretion, however, repurchase programs also re-distribute wealth among shareholders. The model generates predictions about the completion rate of the programs and about the bid-ask spread during the repurchase period that might explain inconsistencies among earlier empirical studies.

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1 Introduction

Over the last two decades, announcements of open-market stock repurchase programs (henceforth, "open-market programs") have become common practice.[1] Yet, empirical evidence points to great variability associated with their execution. First, there is great variability documented about actual completion rates. In the US, Stephens and Weisbach (1998) and Jagannathan, Stephens, and Weisbach (2000) document that average actual repurchase rates are only between 70–80%. Frequently only a small fraction of the quantity of stock announced is actually repurchased, and many announcing firms do not repurchase at all. Actual repurchase rates are even lower outside the US. Ikenberry, Lakonishok, and Vermaelen (2000) find average actual repurchase rates to be as low as 28% of the quantity announced in Canada, and Rau and Vermaelen (2002) find average actual repurchase rates of only 37% in the UK. In addition, there is great variability in the timing of the repurchase trade. Some firms repurchase the full quantity announced immediately after they announce. Others repurchase gradually, or wait for a long period and then repurchase all or part of the quantity announced (Stephens and Weisbach (1998), Cook Krigman and Leach (2004)).

There is also disagreement among empirical studies about the affect of the open-market program announcements on liquidity, measured by the bid–ask spread. In the US, Barclay and Smith (1988) find widening of the spread. However, Miller and McConnell (1995) find no widening of the bid–ask spread, whereas Wiggins (1994), Franz, Rau, and Tripathy (1995), and Cook, Krigman, and Leach (2004) actually find narrowing of the bid–ask spread during periods of actual repurchases. Outside the US, Brockman and Chung (2001) and Ginglinger and Hamon (2003) find widening of the bid–ask spread during actual repurchase periods in Hong Kong and France, respectively.[2]

Why is there variability in actual repurchase and in the bid-ask spread during the repurchase period across firms and across countries? What is the optimal way to execute an open-market repurchase program? Are there any implications for regulatory bodies? The purpose of this paper is to develop a theoretical framework with which to answer these questions. Unlike most earlier theoretical investigations of open-market programs, we build on the motivation to distribute free cash in order to avoid its waste. Increasing empirical evidence suggests that the availability of free cash, and the need to avoid its waste, play an important role in decisions to

[1] See, for example, Stephens and Weisbach (1998), Jagannathan, Stephens, and Weisbach (2000), and Grullon and Michaely (2002).

[2] Most of the above studies also have findings on market depth consistent with their findings about bid–ask spread. That is, studies that find narrowing of the bid–ask spread also find an increase in market depth measured by price impact on order imbalances, and studies that find widening of the spread also find a decrease in market depth.

announce and execute open-market programs. For example, Grullon and Michaely (2004) find that the program announcement return is higher for firms that are more likely to overinvest, and Stephens and Weisbach (1998), and Oswald and Young (2004) find that actual repurchases depend on the availability of free cash.[3]

For most of the analysis, we take the program announcement as given in order to focus on the execution. Assuming uncertainty and asymmetric information about the firm value, we show that the execution is the solution to an optimization problem over waste-prevention benefits from paying out free cash and gains (or losses) from the informed trade of the firm. Specifically, if the firm learns that it does not have free cash it refrains from executing the program so as not to hurt investment. If, instead, the firm learns that it does have free cash, it will always execute the repurchase when the stock is undervalued, because in this case it benefits from preventing the waste of free cash and it also accrues trading gains from the (informed) repurchase trade. However, when the stock is overvalued, the firm faces a tradeoff between waste prevention gains and trading losses. Hence, the firm is less likely to execute the repurchase when the stock is overvalued; the higher the overvaluation, the less likely is the execution. Thus, open-market programs enhance value to shareholders by distributing free cash but also result in wealth transfers among shareholders because of the informed/strategic trade of the firm.

The model provides predictions about actual repurchase rates and the bid–ask spread that might explain the discrepancy among the empirical studies cited above: When uncertainty about the firm value is relatively high, actual repurchases are driven by the motivation to take advantage of information through strategic trading, and, hence, open-market programs are characterized with low completion rates and high bid–ask spreads. In this case, expected wealth transfers among shareholders (expropriations) because of the firm's informed trade are more significant than expected value enhancement through the disbursement of free cash. In contrast, when uncertainty about the firm value is relatively low, actual repurchases are driven by the motivation to distribute free cash in order to avoid its waste, and open-market programs are thus characterized with higher completion rates and lower bid–ask spreads. In this case, expected value enhancement through the disbursement of free cash is more significant than expected wealth expropriations. These results naturally generate testable predictions about how the execution will depend on firm characteristics, such as value vs. growth, large vs. small, etc.

Because the model suggests that open-market programs enhance expected firm value but at the same time result in wealth expropriations, the model also has important regulatory impli-

[3]On the agency costs of free cash flow see, for example, Jensen (1986) Stultz (1990) and Bates (2005).

cations: Whenever wealth expropriations are more significant than the enhancement of value, open-market programs and their execution should be regulated or even forbidden. In contrast, when the situation is reversed, regulatory bodies should encourage open-market programs and avoid regulating the execution, as regulation could discourage executions and thereby exacerbate the waste of free cash and wealth expropriations among shareholders groups. We show that this implication is broadly consistent with the cross-country evidence (see Section 4).

The model highlights two important features of open-market programs that have been largely ignored in the theoretical literature and that might explain their increasing popularity. First, the model suggests that open-market programs provide the firm with financial flexibility, i.e., the firm retains the option not to eventually repurchase, should the availability of free cash change.[4] Most firms have considerable amounts of cash on their balance sheet at the time they announce a repurchase program. Our thrust is that, at the time they make the announcement, whether this cash is free or not is yet to be determined. Second, in most of the existing literature, the trading gains associated with the (informed) repurchase trade are generally viewed as a negative property of open-market programs (e.g., Barclay and Smith (1988)) whereas the model here suggests that these trading gains do not represent a zero sum game. Specifically, if free cash disbursement is value enhancing, then when the firm executes the repurchase under uncertainty and asymmetric information, it is privately informed not only about the true value of the stock, but also about the value enhancement through the repurchase trade. This increases the motivation to execute the repurchase even when the stock is overvalued.

Most earlier theoretical investigations of open-market repurchase programs, focus on signaling undervaluation motivation. Signaling motivation, however, seems inconsistent with the noncommitting nature of open-market programs confirmed with low actual repurchase rates and does not explain the mixed results on the bid–ask spread. Further, even among the signaling papers, very few consider the optionality of the programs (i.e., distinguish between announcement and actual repurchase). The later group includes Ikenberry and Vermaelen (1996), Bhattacharya and Dittmar (2003), and Oded (2005). All three papers build signaling stories based on the optionality of open-market programs, but abstract from the disbursement of free cash. Brennan and Thakor (1990) also consider the optionality of open-market programs, but their focus is on wealth transfers associated with the execution rather than on signaling undervaluation. Interestingly, the agency costs of free cash are largely ignored in theoretical work about repurchases in general and for open-market repurchase programs in particular. To our knowledge, the only theoretical papers that consider free cash distribution as a motivation in

[4] Supporting evidence on the flexibility of open-market programs is provided in Jagannathan, Stephens, and Weisbach (2000), Guay and Harford (2000), and Brav et al (2005).

repurchase policy are Chowdhry and Nanda (1994), and Lucas and McDonald (1998). However, these studies do not distinguish between announcement and actual repurchase, and thus apply more to tender offers than to open-market programs.

The rest of this paper is organized as follows. The assumptions are set up and discussed in Section 2. Section 2 also demonstrates the main idea using a numerical example. A general formulation and solution is given in Section 3. Implications that were briefly discussed above and possible extensions are discussed in Section 4. Section 5 concludes.

2 Assumptions and Example

There are three dates indexed by $t = 0, 1, 2$. All agents are risk neutral, the interest rate is zero, and there are no taxes or transaction costs. Consider an equity-financed firm. At $t = 0$, the firm owns a project and some cash, where it is unclear what portion of the cash will be needed to finance the project and what portion of the cash is free cash. At $t = 1$ the firm generates assets in place with value of $\tilde{A} \in \{A,\ A + X\}$ with equal probability, where $0 < X$, and realizes *free* cash $\tilde{C} \in \{0, C\}$ with equal probability, where $0 < C$, and where $\tilde{A}$ and $\tilde{C}$ are independent.[5] Thus, there are four equally likely outcomes for the firm value V at $t = 1$: $V_1 \in \{A, A + C, A + X, A + X + C\}$. We will generally omit the time index for $t = 1$, as most of the action happens on this date. At $t = 2$ the firm is sold/dismantled, and investors get the value of their shares. The firm is run by a manager who maximizes the terminal value per share.[6] Information is symmetric at $t = 0$. However, at $t = 1$ only the manager observes the realization of $\tilde{A}$ and $\tilde{C}$, whereas all other agents know only the distribution of these variables. The practical interpretation here is that the manager observes the realized value of the firm's projects and in addition observes what portion of the firm's cash is actually needed for the projects, whereas the rest becomes free cash. The shareholders and the market do not observe this information yet. At $t = 2$, all information is publicly known.

There are N shares outstanding at $t = 0$, and we normalize the values of A, X, C, V to be values per share using lowercase letters a, x, c, v, respectively. At $t = 0$ the firm can announce a repurchase program that it may execute at $t = 1$.[7] The firm can buy back shares at $t = 1$

[5]See also Section 4 on the assumption that $\tilde{A}$ and $\tilde{C}$ are independent

[6]It will be shown in Section 3.3.2 that, in our set up, this objective function is equivalent to maximizing the expected wealth of the original shareholders. Whose value the firm is maximizing is still an open question in corporate finance. See, for example, Myers and Majluf (1984).

[7]Since our focus is optimal execution of the program, we will later take it as given that the firm has a program it can execute at $t = 1$. In the model, for the firm, announcing always dominates not announcing, and, since at $t = 0$ all information is symmetric, the announcement has no signaling content. We will consider the case

only with free cash (otherwise the value of assets in place is severely damaged). If the firm does have free cash but does not distribute it with a repurchase at $t = 1$, a portion $(1 - \delta)$ of the free cash is lost, where $\delta \in (0, 1]$.[8] Without loss of generality, we assume that the firm will repurchase whenever indifferent. Like most payout policy models, we assume that borrowing is not allowed.

At $t = 1$ there is a market for the stock. Liquidity traders place quantity bids $Q_A < \frac{N}{3}$ and $Q_B < \frac{N}{3}$ they want to buy and sell respectively. The market maker sets prices p_A, p_B in the buy and sell markets, respectively, before investors place their quantity bids (anticipating the possibility of informed trade from the firm side) to earn zero expected profit.[9]

The following example demonstrates how uncertainty in the value of assets in place and uncertainty of free cash interact to determine the program execution.

Example 1: Consider a case with high uncertainty of the value of assets in place and relatively low uncertainty of free cash (shown in Figure 1a). At $t = 0$ firm F has $N = 10$ shares. At $t = 1$, the value of assets in place is realized to be either 7 or 12 with equal probability, and free cash is realized to be 2 or 0 with equal probability. Thus, $a = 0.7$, $x = 0.5$, $c = 0.2$, and there are four possible states as described in Figure 1a. Assume that at $t = 1$ liquidity buyers place quantity orders $Q_A = Q_B = 3$, and suppose further that if the free cash is not distributed at $t = 1$ then $\delta = 0.8$, i.e. the waste rate is $1 - \delta = 0.2$. Suppose that the firm does not announce a repurchase program. Then the expected firm value at $t = 2$ is

$$0.5(7 + 12 + 0.8 \times 2) = 10.3.$$

Since there is no informed trade at $t = 1$, this is the price the market maker will sell and buy for at $t = 1$. That is, without a repurchase program $p_A = p_B = 1.03$. If, instead, the firm does announce a program at $t = 0$, at $t = 1$ it will buy shares only in the upper state in Figure 1a.

without a program only for comparison. We also take it as given that firms must make their programs publicly known (announce) beforehand. The only country in which firms are not required to announce their programs beforehand is the US, and even there, announcing is the norm.

[8] We take the agency problem as given, as we want to focus on the execution itself. Thus, we refrain from modeling the reasons for the waste and do not model any benefits for the manager. Models that assume the manager does not benefit from the waste of free cash include Chowdhry and Nanda (1994). See also Section 4 on this assumption. We exclude $\delta = 0$ to simplify the analysis. Our results would hold also for $\delta = 0$.

[9] The market mechanism we use is standard and is empolyed, for example, in Noe (2002). We focus on $t = 1$ because this is where the repurchase takes place, but it could be assumed that the market opens also at $t = 0$ and $t = 2$. The restriction on liquidity trade is without loss of generality in order to limit the discussion to the feasible range of the results.

To earn zero expected profit, the market maker must set p_A such that

$$3[(p_A - 0.7) + (p_A - (0.7 + 0.8 \times 0.2)) + (p_A - (1.2))] + (p_A - (\frac{7+5}{10 - \frac{2}{p_A}}))(3 + \frac{2}{p_A}) = 0$$

which upon solution implies $p_A = 1.1093$. The implied average terminal stock value at $t = 2$ is

$$0.25[0.7 + 0.86 + 1.2 + (\frac{7+5}{10 - \frac{2}{1.1093}})] = 1.056.$$

This is also the price at which the market maker buys for at $t = 1$ (no adverse selection on sell market), that is, $p_B = E[p_2] = 1.056$.[10] In comparison to the case where the firm does not announce a program, liquidity buyers lose 0.16, original shareholders gain a total 0.26 (i.e. 0.026 per share regardless of when they sell). Social wealth increases because of the repurchase by

$$0.26 - 0.16 = 0.1.$$

Now, consider instead, a case with low uncertainty of the value of assets in place relative to the uncertainty of the free cash (shown in Figure 1b). Specifically, consider firm G, for which everything is the same as for firm F, except that at $t = 1$ the value of the assets in place is realized to be either 9 or 10 with equal probability (the free cash is still either 2 or 0 with equal probability). Thus, for firm G, $a = 0.9$, $x = 0.1$, $c = 0.2$. There are four possible states as described in Figure 1b. If the firm does not announce a repurchase program, the expected firm value at $t = 2$ is

$$0.5(9 + 10 + 0.8 \times 2) = 10.3$$

and $p_A = p_B = 1.03$ (same as for firm F). However, if firm G announces a program at $t = 0$, at $t = 1$ it will buy shares not only in the upper state in Figure 1b, but rather in both states in which it has free cash. To earn zero expected profit, the market maker must set p_A such that

$$3[(p_A - 0.9) + (p_A - 1)] + (p_A - (\frac{9}{10 - \frac{2}{p_A}}))(3 + \frac{2}{p_A}) + (p_A - (\frac{9+1}{10 - \frac{2}{p_A}}))(3 + \frac{2}{p_A}) = 0$$

which upon solution implies $p_A = 1.0829$. The implied average terminal stock value at $t = 2$ is

$$0.25[0.9 + 1.0 + \frac{9}{10 - \frac{2}{1.0829}} + \frac{9+1}{10 - \frac{2}{1.0829}}] = 1.0576.$$

[10] One can verify that the firm will not buy in the state with low asset value with free cash: if it does not repurchase, terminal value per share would be $0.7 + 0.8 * 0.2 = 0.86$. If it does repurchase, terminal value per share would be $7/(10 - (2/1.1093)) = 0.8540$, and hence the firm is better off not repurchasing.

This is also the price at which the market maker buys for at $t = 1$ (no adverse selection on sell market), that is $p_B = E[p_2] = 1.0576$.[11] In comparison to the case where the firm does not announce a program, liquidity buyers lose 0.0760, original shareholders gain a total 0.2760 (i.e. 0.0276 per share regardless of when they sell). Social wealth increases because of the repurchase by

$$0.2760 - 0.0760 = 0.2.$$

Table 1 highlights the differences between Firm F and firm G in the above example.

Example 1 demonstrates that, when uncertainty in the value of assets in place relative to the uncertainty of free cash is low (Firm F in Table 1), an open-market program will result in higher completion rate and lower bid–ask spread in comparison to the case in which uncertainty in the value of assets in place relative to uncertainty in the free cash is high (Firm G in Table 1). Furthermore, when uncertainty about the value of assets in place is low, the increase in social wealth is higher and there is also less wealth transfer from liquidity/outside investors to insiders. For both firms, the inherited flexibility of open-market programs leads to informed trade from the firm side. Managers repurchase to enhance the value of terminal shares. This value enhancement comes partly at new shareholders expense and partly because the repurchase prevents the waste of free cash.

3 The Formal Model

Because informed trade is possible only in the buy market, we focus on this market and denote $Q_A \equiv Q$, $p_A \equiv p$. Given the assumptions in Section 2, the market maker's zero-expected-profit condition is

$$\sum_j [\Pr\{j\}(p - v_{2|v_j,r_j})(Q + r_j|_p)] = 0 \tag{1}$$

where j indicates the four possible outcomes (states) of the firm value V at $t = 1$, where r_j is the number of shares the firm repurchases at $t = 1$ in state j, and $v_{2|v_j,r_j}$ is the value of each share at $t = 2$ depending on v_j, the value per share in state j realized at $t = 1$, and on r_j.

Definition 1 *Equilibrium is a set* $(\{r_j\}, p)$ *consisting of a repurchase strategy* $\{r_j\} \in (0, \frac{C}{p})$ *set by the manager given* $\{v_j\}, p$ *to maximize the terminal value per share* v_2*, and a price* p *set*

[11]One can verify that the firm will indeed buy in state with low asset value with free cash: If it does not repurchase, terminal value per share would be $0.9 + 0.8 * 0.2 = 1.06$. If it does repurchase, terminal value per share would be $9/(10 - (2/1.0829)) = 1.1039$, and hence the firm is better off repurchasing.

by the market maker, such that condition (1) is satisfied.

It is immediate to show that if the firm does not announce a repurchase program $p = E[v_2] = a + \frac{x}{2} + \frac{\delta c}{2}$. Henceforth we take it as given that the firm announces a repurchase program at $t = 0$, and we focus on the optimal execution (see also footnote 7).

Lemma 1 *In any equilibrium, the firm never repurchases in the states $v = a, v = a + x$ and it always repurchases with all available cash in the state $v = a + x + c$.*

Proofs of all Lemmas and Propositions appear in the Appendix.

Accordingly, we can write the market maker's zero-expected-profit condition (1) as:

$$(p - a)Q + (p - v_{2|v=a+c})(Q + r|_{p,v=a+c}) + (p - (a + x))Q + (p - (\frac{A + X}{N - \frac{C}{p}}))(Q + \frac{C}{p}) = 0. \quad (2)$$

This condition essentially requires that the average of the differences between the price that the market maker is willing to sell for and the terminal value of a share, weighted by the quantity he sells in each state, is equal to zero. The first and the third terms correspond to the states with low and high asset value, respectively, where the firm has no cash and therefore does not repurchase. The last term corresponds to the state with high asset value and cash ($v = a + x + c$). By Lemma 1, in this state the firm will always repurchase. In this state the terminal value per share is $\frac{A+X}{N-\frac{C}{p}}$ and the market maker sells $Q + \frac{C}{p}$ shares. The second term corresponds to the interesting state with low asset value and with cash ($v = a + c$) and in which the decision to repurchase depends on the model parameters. In this term, the value of r (repurchase) is either $\frac{C}{p}$ or 0, depending on whether or not the firm repurchases in this state, and $v_{2|v=a+c}$ is either $\frac{A}{N-\frac{C}{p}}$ or $a + \delta c$ depending on whether or not the firm repurchases in this state, respectively. An important feature of repurchases under asymmetric information that is reflected in (2) is the nonlinearity in value introduced through the firm's trade. Specifically, when the firm does repurchase to take advantage of its private information, the per-share value increases not only because trading gains are added to the value of the terminal shares, but also because these trading gains are shared by a reduced number of shares.

Definition 2 *A Full Repurchase Equilibrium is an equilibrium in which the firm repurchases at $t = 1$ whenever it has free cash, i.e. in both states $v = a + c$ and $v = a + x + c$. A Partial Repurchase Equilibrium is an equilibrium in which the firm repurchases at $t = 1$ only when it has free cash and the asset value is high, i.e. only in state $v = a + x + c$.*

In any full repurchase equilibrium, condition (2) becomes

$$(p-a)Q+(p-\frac{A}{N-\frac{C}{p}})(Q+\frac{C}{p})+(p-(a+x))Q+(p-(\frac{A+X}{N-\frac{C}{p}}))(Q+\frac{C}{p})=0, \tag{3}$$

whereas in any partial repurchase equilibrium, condition (2) becomes

$$(p-a)Q+(p-(a+\delta c))Q+(p-(a+x))Q+(p-(\frac{A+X}{N-\frac{C}{p}}))(Q+\frac{C}{p})=0. \tag{4}$$

The following Lemma presents the solution for the price p of (3) and (4) in a full repurchase equilibrium and in a partial repurchase equilibrium, respectively.

Lemma 2 *In any full repurchase equilibrium, the price p at which the market maker sells at $t=1$ is*

$$p=\frac{(a+\frac{x}{2}+c)-\frac{Nc}{2Q}+\sqrt{\left((a+\frac{x}{2}+c)-\frac{Nc}{2Q}\right)^2+c\left((2a+x+2c)\frac{N}{Q}-(2a+x)\right)}}{2}. \tag{5}$$

In any partial repurchase equilibrium, the price p at which the market maker sells at $t=1$ is

$$p=\frac{a+\frac{x}{2}+(1+\frac{\delta}{4})c-\frac{Nc}{4Q}+\sqrt{\left(a+\frac{x}{2}+(1+\frac{\delta}{4})c-\frac{Nc}{4Q}\right)^2+c\left((a+x+c)\frac{N}{Q}-(3a+x+\delta c)\right)}}{2}. \tag{6}$$

The firm's decision about whether or not to repurchase in the state $v=a+c$ depends on the one hand on how deep the undervaluation is and on the other hand on how severe the waste is. Specifically, since by assumption the manager maximizes the value of the terminal shares, she will not buy if the terminal stock value without repurchase is higher than the terminal stock value with the repurchase, that is, if

$$a+\delta c>\frac{Na}{N-\frac{C}{p}},$$

which after rearrangement is equivalent to

$$p>\frac{a}{\delta}+c. \tag{7}$$

Otherwise, that is, if $p \leq \frac{a}{\delta} + c$, the firm will always repurchase (recall that without loss of generality we have assumed that the firm will repurchase whenever indifferent). The important and nonintuitive insight, which is reflected in (7), is that when coming to decide whether to repurchase or not, the manager does not compare the value to the price, but rather compares the projected terminal values under each alternative.[12]

The following Lemma combines condition (5) with the requirement that (7) does *not* hold to give a necessary and sufficient condition for a full repurchase equilibrium; it also combines conditions (6) and (7) to give a necessary and sufficient condition for a partial repurchase equilibrium.

Lemma 3 *A necessary and sufficient condition for a full repurchase equilibrium is*

$$\frac{a}{\delta}\left(4 + 2\frac{x}{a} - \frac{4}{\delta}\right) \leq c\left(\frac{1}{\delta}(\frac{2N}{Q} + 4) - \left(2 + \frac{x}{a}\right)\left(\frac{N}{Q} + 1\right)\right). \tag{8}$$

A necessary and sufficient condition for a partial repurchase equilibrium is

$$\frac{a}{\delta}\left(4 + 2\frac{x}{a} - \frac{4}{\delta}\right) > c\left(\frac{1}{\delta}\left(\frac{N}{Q} + 4\right) - 1 - (1 + \frac{x}{a})(\frac{N}{Q} + 1)\right). \tag{9}$$

Conditions (8) and (9) are the basis for our results. We first demonstrate the results for special cases.

3.1 Special Cases

3.1.1 Special Case 1: $\delta = 1$

Consider first the case in which $\delta = 1$, that is, the case where there is no cash waste regardless of whether the firm repurchases or not. In this case the sole purpose of repurchasing is trading gains based on asymmetric information. The repurchase also does not increase social wealth.

Proposition 1 *Suppose $\delta = 1$. If both*

$$\frac{x}{a} < \frac{2}{\frac{N}{Q} + 1} \tag{10}$$

[12]Note that $p > a + \delta c$ is not enough to assure no repurchase. That is, to assure no repurchase it is not enough that the price is higher than the terminal stock value without a repurchase. That assurance requires the stronger restriction, reflected in (7), that the terminal stock value without a repurchase be higher than the terminal stock value with a repurchase.

and

$$\frac{2x}{2-\frac{x}{a}\left(\frac{N}{Q}+1\right)} \leqslant c \tag{11}$$

hold, the outcome is a full repurchase equilibrium. Otherwise, the outcome is a partial repurchase equilibrium.

The intuition for Proposition 1 is as follows. When $\delta = 1$, cash waste does not affect repurchase policy. A full repurchase equilibrium requires that the firm will buy in the state with low value of asset in place and free cash ($v = a + c$). Otherwise, a partial repurchase equilibrium prevails (the firm always buys in the other state with free cash $v = a + x + c$). For the firm to repurchase in the state with low value of assets in place and free cash, the price must not be higher than the stock value in this state. This is because when $\delta = 1$ condition (7) becomes $p \leq a + c$. The equilibrium price, in turn, must provide the market maker with zero expected profit and hence reflects the expected value, pushed somewhat higher to reflect the level of adverse selection associated with the repurchase. Adverse selection, however, is positively correlated with both variability in the value of assets in place, $\frac{x}{a}$, and with the level of free cash when the firm does have cash, c (i.e. the variability of free cash). When $\frac{x}{a}$ is significant, the price that the market maker sets to earn zero expected profit will always be too high for the firm to repurchase in the state with low asset value and with cash ($v = a + c$), no matter what the value of c is. However, when $\frac{x}{a}$ is sufficiently low, its effect on the price becomes less significant so that with enough variability in free cash (when c is high enough), the stock value with free cash will be higher than the price that gives the market maker zero expected profit even if the value of assets in place is realized to be low and a partial repurchase equilibrium cannot hold and a full repurchase equilibrium will prevail.[13]

Figure 2 demonstrates the results in Proposition 1 by means of a graph. The figure illustrates how the decision of whether or not to repurchase in the state $v = a + c$ depends on the variability in the value of assets in place, $\frac{x}{a}$, and the variability in free cash, c. The vertical dashed line indicates where condition (10) holds with equality. To the right of this line the variability in the firm value, introduced through the variability in assets in place, is too high so that it pushes the stock price too high for a full repurchase equilibrium to exist, and therefore a partial repurchase equilibrium prevails. To the left of the dashed line, equilibrium type depends on the level of free cash. Specifically, the solid curved line indicates where condition (11) holds with equality. Below this curved line, variability in the firm value due to variability in cash

[13]For simplicity we have assumed a cash distribution of $\{0, C\}$. If, instead, we chose two positive values of cash (as is for assets in place), the qualitative results should not change. However, this will significantly complicate the analysis.

is too low (c is too low), so that in the state with low asset value and cash the firm will not repurchase and hence partial repurchase equilibrium prevails. Above this line (c is sufficiently high), a full repurchase equilibrium prevails. Note that a deeper market (larger $\frac{Q}{N}$) means that the dashed line gets pushed to the right and the solid curved line gets pushed down, so that the region in which full equilibrium prevails widens and the region in which partial equilibrium prevails narrows. Also, in the region where $\frac{x}{a} < \frac{2}{\frac{N}{Q}+1}$, the smaller $\frac{x}{a}$ is, the lower the required level of c for a full repurchase equilibrium to exist.

3.1.2 Special Case 2: $x = 0$

Suppose there is no variability in the value of assets in place i.e., $x = 0$.

Proposition 2 *When $x = 0$ a full repurchase equilibrium always exists and a partial repurchase equilibrium never exits.*

Intuitively, when there is no variability in the value of assets in place, only variability in free cash determines the variability in the firm value. Because the market maker sets a price to earn zero expected profit, the firm will be undervalued whenever it has free cash. Thus, regardless of the waste rate, the firm will always repurchase when it has free cash. When $\delta < 1$, however, with $x = 0$ the repurchase completely prevents the waste of free cash.

3.2 The general case

In the general case, a full repurchase equilibrium and a partial repurchase a equilibrium are not mutually exclusive. Thus, we have to analyze them separately. We first investigate existence of full repurchase equilibrium based on condition (8) and then investigate existence of partial repurchase equilibrium based on condition (9).

3.2.1 Full repurchase equilibrium

Proposition 3: *(Existence of full repurchase equilibrium)*

A full repurchase equilibrium always exists if

$$\delta \leq \frac{2}{2+\frac{x}{a}} \tag{12}$$

and never exists if

$$\frac{2}{\left(2+\frac{x}{a}\right)} \frac{\left(\frac{N}{Q}+2\right)}{\left(\frac{N}{Q}+1\right)} \leq \delta. \tag{13}$$

In the range

$$\frac{2}{2+\frac{x}{a}} < \delta < \frac{2}{\left(2+\frac{x}{a}\right)} \frac{\left(\frac{N}{Q}+2\right)}{\left(\frac{N}{Q}+1\right)} \tag{14}$$

a full repurchase equilibrium exits if $c \geq c^F$ *where*

$$c^F\left(a, x, \delta, \frac{N}{Q}\right) \equiv \frac{\frac{a}{\delta}\left(4+2\frac{x}{a}-\frac{4}{\delta}\right)}{\frac{1}{\delta}\left(\frac{2N}{Q}+4\right)-(2+\frac{x}{a})(\frac{N}{Q}+1)}. \tag{15}$$

and does not exist otherwise.

Proposition 3 suggests that existence of full repurchase equilibrium depends primarily on the relation between δ and $\frac{x}{a}$. Only when δ and $\frac{x}{a}$ meet particular joint conditions does the relation between these variables and c also matter. Specifically, when both δ and $\frac{x}{a}$ are low, full repurchase equilibrium always holds, and when both δ and $\frac{x}{a}$ are high, full repurchase equilibrium never holds. Otherwise, if both are neither too small nor too high, existence will depend on the variability of free cash c, where there is some level of c, c^F, above which a full repurchase equilibrium exists and below which it does not exist. Thus, in this region, for sufficiently high c a full repurchase equilibrium will exist.

Figure 3 demonstrates the results of Proposition 3 by means of a graph. The figure illustrates how existence of full repurchase equilibrium depends on the variability in the value of assets in place, $\frac{x}{a}$, and the waste rate captured by δ. The solid (curved) line indicates where condition (12) holds with equality. The dotted (curved) line indicates where condition (13) holds with equality. In the area above the dotted line a full repurchase equilibrium never exists. In the area below the solid line a full repurchase equilibrium always exists. In the area captured between the lines (where (14) holds), full repurchase equilibrium exits if $c > c^F$, where c^F is defined in (15).

The intuition for Proposition 3 is as follows. In the area above the dotted line, δ is high (free cash waste is not significant) and hence the intuition in Proposition 1 for the case with high $\frac{x}{a}$ still goes though. That is, if δ and $\frac{x}{a}$ are high (up and to the right of the dotted line in the figure), the waste of free cash is not material; whereas the adverse selection introduced though the variability in value of assets in place is strong, so that the ask price that the market maker sets is very high. Consequently, in the state in which the value of assets in place is realized to be low the firm is better off not repurchasing. Although in this case some free cash is lost, thereby reducing shareholders' value, the alternative of paying too much for the shares would hurt share value even more. Between the curved lines, the effect of free cash waste becomes significant, so that the intuition of Proposition 1 no longer holds. Specifically, in this region, variability in the value of assets in place still motivates no repurchase in the state with low asset value (with free cash) but the potential benefit from preventing free cash loss now significantly motivates a repurchase. Consequently, in this region, existence of a full repurchase equilibrium depends on the level of free cash c when the firm does have cash (variability in free cash). Higher variability in free cash magnifies the benefit from waste prevention more than it magnifies the loss from paying a higher price set by the market maker to compensate for higher adverse selection. This is not only because benefits from waste prevention are higher, but also because these benefits are shared by a reduced number of shares. Thus, in this region, there is some level of c given in (15), above which a full repurchase equilibrium exists and below which it does not exist. Below the solid line, a full repurchase equilibrium always exists. Intuitively, in this region the waste rate is so high that the firm is willing to buy back shares even if the price is very high. This is because the alternative is losing most of the cash and severely damaging firm value.

Last, note that higher $\frac{N}{Q}$ does not affect the solid line but does push the dotted line down, decreasing the prevalence of full repurchase equilibrium. This is because lower liquidity increases the effect of adverse selection on price (pushes it up) so that, other things equal, when existence of full repurchase equilibrium does depend on adverse selection (i.e., when the waste does not dominate), a full repurchase equilibrium is less likely to prevail.

3.2.2 Partial repurchase equilibrium

Proposition 4: *(Existence of partial repurchase equilibrium)*
4A. *(Case 1) Suppose*

$$\frac{x}{a} < \frac{4}{\frac{N}{Q} - 2}$$

then

$$\frac{2}{2 + \frac{x}{a}} < \frac{2(\frac{N}{Q} + 2) - \frac{N}{Q}}{(2 + \frac{x}{a})\left(\frac{N}{Q} + 1\right) - \frac{N}{Q}}.$$

In this case a partial repurchase equilibrium never exits if

$$\delta \leq \frac{2}{2 + \frac{x}{a}} \tag{16}$$

and always exists if

$$\frac{2\left(\frac{N}{Q} + 2\right) - \frac{N}{Q}}{(2 + \frac{x}{a})\left(\frac{N}{Q} + 1\right) - \frac{N}{Q}} \leq \delta. \tag{17}$$

In the range

$$\frac{2}{2+\frac{x}{a}} < \delta < \frac{2(\frac{N}{Q}+2)-\frac{N}{Q}}{\left(2+\frac{x}{a}\right)\left(\frac{N}{Q}+1\right)-\frac{N}{Q}} \tag{18}$$

a partial repurchase equilibrium exits if $c < c^P$ *where*

$$c^P\left(a,x,\delta,\frac{N}{Q}\right) \equiv \frac{\frac{a}{\delta}\left(4+2\frac{x}{a}-\frac{4}{\delta}\right)}{\frac{1}{\delta}\left(\frac{N}{Q}+4\right)-1-\left(1+\frac{x}{a}\right)\left(\frac{N}{Q}+1\right)} \tag{19}$$

and does not exist otherwise.

4.B *(Case 2) Suppose*

$$\frac{x}{a} > \frac{4}{\frac{N}{Q}-2}$$

then

$$\frac{2(\frac{N}{Q}+2)-\frac{N}{Q}}{\left(2+\frac{x}{a}\right)\left(\frac{N}{Q}+1\right)-\frac{N}{Q}} < \frac{2}{2+\frac{x}{a}}.$$

In this case a partial repurchase equilibrium never exits if

$$\delta \leq \frac{2(\frac{N}{Q}+2)-\frac{N}{Q}}{\left(2+\frac{x}{a}\right)\left(\frac{N}{Q}+1\right)-\frac{N}{Q}} \tag{20}$$

and always exists if

$$\frac{2}{2+\frac{x}{a}} \leq \delta. \tag{21}$$

In the range

$$\frac{2(\frac{N}{Q}+2)-\frac{N}{Q}}{\left(2+\frac{x}{a}\right)\left(\frac{N}{Q}+1\right)-\frac{N}{Q}} < \delta < \frac{2}{2+\frac{x}{a}} \tag{22}$$

a partial repurchase equilibrium exits if $c > c^P$ *where* c^P *is defined in (19) and does not exist otherwise.*

4.C *(Case 3) Suppose*

$$\frac{x}{a} = \frac{4}{\frac{N}{Q}-2}$$

then

$$\frac{2(\frac{N}{Q}+2)-\frac{N}{Q}}{\left(2+\frac{x}{a}\right)\left(\frac{N}{Q}+1\right)-\frac{N}{Q}}=\frac{2}{2+\frac{x}{a}}.$$

In this case a partial repurchase equilibrium never exits if

$$\delta \leq \frac{2}{2+\frac{x}{a}}$$

and always exists otherwise.

Proposition 4 is rather long and seems complex because the relation between the restrictions on δ changes with $\frac{x}{a}$, and three separate cases must thus be considered. However, the same results are obtained in all three cases: existence of partial repurchase equilibrium depends primarily on the relation between δ and $\frac{x}{a}$. Only in a certain region does the relation between these variables and c also matter. Specifically, when both δ and $\frac{x}{a}$ are low partial repurchase equilibrium never holds, and when both δ and $\frac{x}{a}$ are high partial repurchase equilibrium always holds. Otherwise, existence of partial repurchase equilibrium depends on the variability in free cash c as follows. If δ is high but $\frac{x}{a}$ is low, there is some level of c below which a partial repurchase equilibrium exists and above which it does not exist, and if $\frac{x}{a}$ is high but δ is low a partial repurchase equilibrium exists above that same level of c and does not exist below that level.

Figure 4 demonstrates the results of Proposition 4 by means of a graph. The figure illustrates how the existence of partial repurchase equilibrium depends on the variability in the value of assets in place, $\frac{x}{a}$, and the waste rate captured by δ (high waste rate is low δ). As in Figure 3, the solid (curved) line indicates where condition (16) holds with equality. The dashed (curved) line indicates where condition (17) holds with equality. In the area above both lines a partial repurchase equilibrium always exists. In the area below both lines a partial repurchase equilibrium never exists. In the area captured between the lines, to the left of their crossing point, a partial repurchase equilibrium exists if $c < c^P$, where c^P is defined in (19), whereas in the area captured between these lines to the right of their crossing point a partial repurchase equilibrium exits if $c > c^P$. (A partial repurchase equilibrium does not exist at the crossing point.)

The intuition for Proposition 4 is as follows. In the area above the solid line, δ is relatively high (the waste is not significant), so that the intuition of Proposition 1 still goes through. That is, if $\frac{x}{a}$ and δ are high (to the right of the solid line in the figure), the effect of the variability in value of assets in place on the price is strong enough to deter the firm from repurchasing if the value of assets in place is realized to be low, regardless of the realized level of free cash. Above the solid line and to the left of the dashed line (i.e., between the curved lines to the left of their crossing point), the variability of assets in place is low enough to render the variability in the level of cash important. If variability in free cash is low (when c is sufficiently low), partial equilibrium will exist. Otherwise, if variability in free cash is high (when c is sufficiently high), a partial repurchase equilibrium cannot exist, because in the states in which the firm does have free cash, given the price set by the market maker to earn zero expected profit, repurchasing results in higher terminal stock value even if the value of assets is place is realized to be low.

The situation is different than in that of Proposition 1 when δ becomes significantly low, i.e., below the solid line, because in this region the waste of free cash becomes significant so that the firm will repurchase regardless of the price and therefore a partial repurchase equilibrium never holds. This is in turn because the unappealing alternative is to "watch the free cash disappear" without contributing to the firms' value. However, a partial repurchase equilibrium may still be viable even if the waste rate is high (δ low) if variability in the value of assets in place is sufficiently high (between the curved lines and to the right of their crossing point). In this region, because variability in the value of assets in place is high, higher variability in free cash magnifies wealth expropriations through adverse selection more than it magnifies benefits from waste prevention. As a result, the price that assures zero expected profit to the market maker assuming that the firm buys only in the high state increases very quickly in c. Consequently, in this region, there is some level of c, given in (19), below which a partial repurchase equilibrium cannot exist but above which it can.

Last, note that higher $\frac{N}{Q}$ (lower liquidity) does not affect the solid line but does push the dashed line down and thereby increases the prevalence of partial repurchase equilibrium. This is because lower liquidity increases the effect of adverse selection on price (pushes it up), so that other things equal a partial repurchase equilibrium is more likely to prevail.

3.2.3 Coexistence of full and partial repurchase equilibrium

Figure 5 combines Figures 3 and 4 to demonstrate the ranges of existence for both partial and full repurchase equilibria, depending on $\frac{x}{a}$ and δ. In the area below both the solid and the dashed lines (i.e., to the left of their crossing point and below the solid line) both $\frac{x}{a}$ and δ are low, hence only full repurchase equilibrium exists. However, in the area below the solid line above the dashed line (i.e., to the right of their crossing point between the lines) δ is low but $\frac{x}{a}$ is now relatively high, so a partial repurchase equilibrium can also exist if $c < c^P$. In the area above the dotted line both δ and $\frac{x}{a}$ are high, hence only a partial repurchase equilibrium exists. However, in the area below the dotted line but above both the dashed and solid lines $\frac{x}{a}$ is high but δ is relatively not as high so that a full repurchase equilibrium can also exist if $c > c^F$.

In the area above the solid line but below the dashed line (to the left of their crossing point) existence of both equilibria depends on the level of c. If c is sufficiently low to render $c < c^P$, only a partial repurchase equilibrium exists, and if c is sufficiently high to render $c > c^F$, only a full repurchase equilibrium exists. If $c^F < c < c^P$, i.e., if

$$\frac{\frac{a}{\delta}\left(4+2\frac{x}{a}-\frac{4}{\delta}\right)}{\frac{1}{\delta}\left(\frac{2N}{Q}+4\right)-(2+\frac{x}{a})\left(\frac{N}{Q}-1\right)} < c < \frac{\frac{a}{\delta}\left(4+2\frac{x}{a}-\frac{4}{\delta}\right)}{\frac{1}{\delta}\left(\frac{N}{Q}+4\right)-1-(1+\frac{x}{a})(\frac{N}{Q}+1)} \tag{23}$$

both partial and full repurchase equilibria can exist. Consider condition (23) on c. If $\delta = 1$ both limits on c are identical, and the range is thus empty. In this case, as we have seen earlier (special case $\delta = 1$), a full repurchase equilibrium and a partial repurchase equilibrium are mutually exclusive. For any $\delta < 1$ we get a range of values for c in which both equilibria exist. Within the discussed area (the triangle shaped area in Figure 5), this range on c indicated in (23) widens with the decrease in δ.

Proposition 5: *When existence of full repurchase equilibrium depends on c, the value of c^F increases in $\frac{x}{a}$ and δ. When existence of partial repurchase equilibrium depends on c, the value of c^P increases in $\frac{x}{a}$ and δ for*

$$\frac{x}{a} < \frac{4}{\frac{N}{Q}-2}$$

and decreases with $\frac{x}{a}$ and δ otherwise.

The following example demonstrates coexistence of partial and full repurchase equilibria.

Example 2: Suppose $A = 7, X = 1, C = 5, N = 10, Q_A = 1$, and $\delta = 0.55$. Then there exists a partial repurchase equilibrium for $p = 1.808$. There also exists a full repurchase equilibrium for $p = 1.627$. In Figure 5, these equilibria are between the dashed and the solid lines to the right of their mutual crossing point. To understand the coexistence result, suppose the market maker sets the price to $p = 1.808$. Condition (4) becomes

$$(1.808-0.7)+(1.808-(0.7+0.55\times0.5))+(1.808-(0.7+0.1))+(1.808-\frac{7+1}{10-\frac{5}{1.808}})(1+\frac{5}{1.808}) = 0.$$

The market maker gains in the states, $\{a, a+x\}$ and loses in the states $\{a+c,\, a+x+c\}$. He makes zero expected profit. The firm does not repurchase in the state $a+c$ and will not deviate; if it did, the terminal value per share would be $\frac{7}{10-\frac{5}{1.808}} = 0.968$, which is lower than 0.975 (the terminal value without a repurchase in that state). Thus, a partial repurchase equilibrium exists. Now suppose the market maker sets a price $p = 1.627$. Condition (3) becomes

$$(1.627-0.7)+(1.627-\frac{7}{10-\frac{5}{1.627}})(1+\frac{5}{1.627})+(1.627-(0.7+0.1))+(1.627-\frac{7+1}{10-\frac{5}{1.627}})(1+\frac{5}{1.627})=0.$$

The market maker gains in the states, $\{a, a+c\}$ and loses in the state $\{a+x,\ a+x+c\}$. He makes zero expected profit. The firm repurchases whenever it has free cash. The firm does repurchase in the state $a+c$ and will not deviate; if it did, the terminal value per share would be 0.975, which is lower than $\frac{7}{10-\frac{5}{1.627}} = 1.014$ (the terminal value with a repurchase in that state). The intuition for the coexistence here is as follows. Going from the partial repurchase equilibrium to the full repurchase equilibrium, the market maker reduces the price and therefore his gain in the states $\{a, a+x\}$ is reduced and his loss in the state $v = a+x+c$ is increased. However, with that lower price, it now pays the firm to repurchase in the state $v = a+c$. The market maker now makes money in the state $v = a+c$, not only at the liquidity buyers expense, but also at the firm's expense. This additional gain in the state $v = a+c$ compensates the market maker for lower gains in the other states, and again he ends up with zero expected profit. The firm pays 1.627 per share to realize only 1.0147 on the terminal date. However, it is happy to do so; if it does not, 45% of the cash will be lost, which is more than its trading loss $1-\frac{1.014}{1.627} = 37.6\%$. Example 2 uses extreme waste rate ($\delta = 0.55$) in order to demonstrate the intuition. In the other coexistence ranges, examples with lower losses to the firm (or higher gains) can be given.

3.3 The good equilibrium and the bad equilibrium

Once we have established the areas of existence, the question is whether we can state that the full repurchase equilibrium is better than the partial repurchase equilibrium as Examples 1 and 2 suggest. This is our goal in this subsection. More specifically, we will demonstrate that, in a full repurchase equilibrium, completion rate and social wealth are higher and bid–ask spread and wealth expropriations are lower. We first consider completion rate and social wealth and then turn to consider bid–ask spread and social wealth. When performing the analysis, we must make sure we do not compare apples to oranges. For example, it will not be correct to compare the social wealth improvement in a full repurchase equilibrium with a high δ to the social wealth improvement in a partial repurchase equilibrium with a low δ (relative to no repurchase).

3.3.1 Completion rate and social wealth

By definition, completion rate in a full repurchase equilibrium is higher than completion rate in a partial repurchase equilibrium. Similarly, implications about social wealth are immediate from the analysis of full versus partial repurchase equilibrium and the value of δ. Specifically, completion rate is 50% in a full repurchase equilibrium and only 25% in a partial repurchase equilibrium.[14] In a full repurchase equilibrium all the loss incurred in the case of no repurchase is saved. In a partial repurchase equilibrium, only half of this loss is saved. Expected social wealth increases by $0.5(1-\delta)C$ with a full repurchase equilibrium, but only by $0.25(1-\delta)C$ with a partial repurchase equilibrium (relative to no repurchase).

3.3.2 Prices, bid–ask spread, and wealth expropriation

In this subsection we consider prices, bid–ask spread, and wealth expropriations. We will show that, consistent with Examples 1 and 2, the bid–ask spread and wealth expropriations (transfers from the liquidity buyers to the original shareholders) are lower in a full repurchase equilibrium than in a partial repurchase equilibrium.[15] Because here we need to consider the bid price, in this subsection we revert back to specifically indicating bid and ask prices p_A, p_B and quantities Q_A, Q_B at $t=1$. (At $t=0,2$ or without a repurchase there is no information asymmetry/adverse selection rendering this notation irrelevant).

Lemma 4 *In any equilibrium, full or partial,* $p_0 = p_B = E[p_2]$.

The following properties are helpful for the analysis. First, because at $t=2$ all information is public, then $p_2 = v_2$, and accordingly maximizing v_2 is equivalent to maximizing p_2. Second, since maximizing p_2 results in maximizing $E[p_2]$, and since by Lemma 4 $p_B = E[p_2]$, maximizing value of terminal shares is equivalent to maximizing value of original shareholders' wealth. In comparison to the situation without repurchase, original shareholders who sell at $t=1$ gain

[14] Of course, this is because we have assumed four states with equal probabilities and that the firm has cash available for repurchase only in two states. Assuming a different distribution of the states would result in different completion rates, but would not alter the qualitative result.

[15] The bid–ask spread is one measure of liquidity. There are others, such as depth which is the price impact of order imbalances (see O'Hara (1995)). Cook, Krigman, and Leach (2004) consider also the number of trading days. In our model, we could measure average trade volume. Consequently, expected increase in liquidity with a repurchase, in comparison to no repurchase, is $\frac{C}{4p}$ in partial repurchase equilibrium and $\frac{C}{2p}$ in full repurchase equilibrium. Thus, trade volume is higher in a full repurchase equilibrium. We could also measure depth with bid–ask spread/volume change. However because, in the model, bid–ask prices are set before investors place their bids, predictions about depth would be consistent with the prediction about the bid–ask spread. Thus we will only use the bid–ask spread to measure liquidity.

the difference between expected terminal price without repurchase and expected terminal price with it, where $E[p_2] = p_B$. Original shareholders who sell at $t = 2$ gain the same. Third, since $E[p_2] = p_B$, and since the loss per share of liquidity buyers is $p_A - E[p_2]$, the bid–ask spread is equal to the loss per share of liquidity buyers. Accordingly, aggregate loss of liquidity buyers is

$$Q_A \left(p_A - E[p_2]\right).$$

The focus of the analysis in this subsection is thus on the difference $p_A - E[p_2]$.

Note also that, since the market maker gets zero expected profit, the expected gain of original shareholders can be calculated as the sum of the increase in social wealth and the loss of the liquidity buyers. As noted above, we do not care when original shareholders sell, because there is no adverse selection in the sell market and hence they get the same price which reflects $E[p_2]$ regardless of when they sell. There is thus no need to distinguish between original shareholders groups (short term who sell at $t = 1$ and long term who sell at $t = 2$). Propositions 6 and 7 generalize the results in Examples 2 and 1, respectively.

Proposition 6: *Whenever a full repurchase equilibrium and a partial repurchase equilibrium coexist, the full repurchase equilibrium leads to higher completion rate, higher social wealth, lower bid–ask spread, and lower wealth expropriations.*

Proposition 7: *Fix all parameters except a, x and fix $(a + \frac{x}{2})$. Then, in a full repurchase equilibrium, the bid–ask spread and wealth transfers are independent of $\frac{x}{a}$, whereas in a partial repurchase equilibrium they increase with $\frac{x}{a}$.*

Propositions 6 and 7 together establish that a full repurchase equilibrium is better than a partial repurchase equilibrium in all four dimensions defined above (completion rate, social wealth, bid–ask spread, and wealth expropriations), not only when they coexist, but also when we move along horizontal lines in Figure 5, as long as the expected value of assets in place $(a + \frac{x}{2})$ is fixed. We suggest that this is as far as a comparison can go without comparing apples to oranges.

4 Implications robustness and further research

The model generates testable predictions about actual repurchase characteristics. Specifically, a full repurchase equilibrium means higher completion rate, more social wealth, lower bid–ask spread, and less wealth expropriation. A full repurchase equilibrium is more likely to prevail when $\frac{x}{a}$ is low (low variability of assets in place), which could be interpreted as low uncertainty of assets value, i.e., in mature industries as opposed to growth industries, in large firms as opposed to small firms, etc. Low $\frac{x}{a}$ could also be interpreted as low information asymmetry, which is associated with high efficiency/transparency of the financial markets. A full repurchase equilibrium is also more likely to prevail when δ is low (high potential for waste of free cash), which could be interpreted as low management efficiency, low governance quality, or as luck of positive NPV projects. In general, the model also predicts that, when the values of $\frac{x}{a}$ and δ are not extreme, high uncertainty of free cash c will increase the likelihood of a full repurchase equilibrium.[16] Thus, the model predicts different levels of completion rates and different levels of widening of the spread according to the above firm/industry characteristics. However, it cannot predict narrowing of the spread (see empirical evidence in the introduction). This is because our benchmark at $t = 0$ is a firm with no asymmetric information and no program, in which case the bid–ask spread is zero. It might be possible to generate narrowing of the spread in a setup in which information is already asymmetric at $t = 0$ and in which the announcement reveals some sort of information. The tension between information revealed and adverse selection created will determine whether, following the announcement, the outcome would be narrowing, widening, or no change of the spread.

For the social planner, the model suggests that if financial markets function well so that uncertainty about the firm value is relatively low, the benefits in social wealth from allowing/not regulating repurchase programs are significant in comparison to the wealth transfers they engender. In this case the social planer should encourage repurchase programs and not regulate them (as is the situation in the US). However, when uncertainty, and hence information asymmetry about firms' value, is high, repurchase programs should be prohibited or strictly regulated with reporting requirements to reduce information asymmetry and wealth transfers (as is the situation in most counties with financial markets that are less efficient). Table 2 reviews the regulation level in five countries in which open-market programs are allowed: US, Canada, UK, France, and Hong Kong. The table suggests that indeed regulation of the programs is significantly higher outside the US. This implication might also clarify the mixed evidence about completion rates and bid–ask spread discussed in the Introduction: In the US, where markets are most efficient, uncertainty about the firm value, and hence information asymmetry, is relatively low. Accordingly, in the US, where open-market programs are the least regulated, they are associated with high actual repurchase rates and low bid–ask spreads. In other countries, markets are less efficient and naturally information asymmetry about firms is high; hence repurchase programs are more regulated, completion rates are relatively low, and following the announcement, bid–ask spread increases. Although Table 2 includes only

[16] When $\frac{x}{a}$ is high and δ is low, there is a region in which a partial repurchase equilibrium could exist if c is sufficiently high. However, in this region, a full repurchase equilibrium always exists regardless of c (See section 3.2.3). See also further in this section on implications of coexistence.

five countries, these are the countries with relatively more efficient financial markets in comparison to the rest of the world. In most other countries financial markets are less efficient and, consistent with this argument, repurchase programs are completely forbidden. The mixed evidence about bid–ask spread within the US described in the Introduction could be explained by noting that the Barclay and Smith (1988) sample is from an earlier time period, before the legislation of Rule 10b-18 in 1982 that deregulated repurchase programs in the US had its effect on repurchase practice, whereas the rest of the studies were based on samples taken after the 1984 deregulation of repurchases.[17]

The situation in which the waste rate and uncertainty in the value of assets in place have comparable affect, and thus both full repurchase equilibrium and partial repurchase equilibrium coexist, is most interesting for regulatory bodies, because this is where easing regulation of actual repurchases could increase social wealth and reduce wealth expropriations. With high regulation (not easy for firms to repurchase) we might end with partial repurchase equilibrium, whereas low regulation would increase the likelihood of full repurchase equilibrium. To clarify this argument, we refer back to Example 2. In the partial repurchase equilibrium in this example, the market maker set a relatively high price, and the firm repurchased only in the state with high valuation of assets in place and free cash. In the full repurchase equilibrium, the market maker set a lower price, and the firm repurchased whenever it had free cash, even if asset value was low. We suggest that, other things equal, if actual repurchases are highly regulated it is more likely that the market maker will anticipate low completion rate and thus set a high price so that a partial repurchase equilibrium will prevail, whereas if the firm can freely trade the market maker anticipates more repurchase trade and hence is willing to set a lower price so that a full repurchase equilibrium will prevail.[18]

The assumption that the variables $\tilde{A}$ and $\tilde{C}$ are independent may seem too strong. For example, it could be argued that when the firm has no good investments, so that assets-in-place value is low, is also when free cash is abundant (negative correlation between free cash and assets-in-place value). Although this is probably true for many firms/projects, the situation

[17] Rule 10b-18 sets conditions under which the SEC will not file charges against repurchasing firms. These conditions are guidelines, not requirements. In setting these conditions (about actual repurchase trade) the SEC essentially deregulated repurchase programs and stimulated a dramatic growth in repurchase activity in the US. See also Grullon and Michaely (2002)).

[18] It could be argued instead that when both equilibria coexist, only the full repurchase equilibrium would stand refinements (e.g., the intuitive criterion). However, robustness of Nash equilibrium refinements is still an open question (see for example Kreps (1990, page 418). We prefer to suggest that deregulation of the programs might increase the likelihood that the full equilibrium will prevail, in the same manner that in bankruptcies chapter 11 can help increase the likelihood of reorganization (the good equilibrium) over inefficient liquidation (the bad equilibrium).

where a project turns out to be only marginally profitable but still consumes all the cash is also very common (positive correlation). Thus, although correlations probably exist, they may be in either direction, suggesting that the assumption that $\tilde{A}$ and $\tilde{C}$ are independent should not be viewed as biasing the results.

The model takes the free-cash waste as given and assumes that insiders do not benefit from the waste. This is because our focus is on how the waste affects the execution and not on the agency problem itself. If the agency problem is endogenized, i.e., if insiders benefit from the (noncontractible, nonverifiable) waste of free cash, our results about program execution would be weakened because these private benefits motivate cash retention. However, as long as the benefit from retaining free cash is not too high, the results should still hold. Furthermore, once the agency problem is endogenized, all aspects of it should be considered—those that discourage actual repurchase and those that motivate actual repurchase. For example, increasing evidence suggests that granting of stock options to insiders motivates actual repurchases to hide the dilution associated with these options.[19]

Last, in our three-date set up, we are not able to explain the diversity of actual repurchase timing patterns documented, for example, in Cook, Krigman, and Leach (2004). However, a dynamic setup in which the variables $\tilde{A}$ and $\tilde{C}$ are revealed over time could generate diverse actual repurchasing patterns. In such an alternative setup the firm should consider how its private information is revealed through its trade. Similarly, in our model, firms either use all available cash to repurchase or do not repurchase at all. Although this result is, in general, consistent with the empirical evidence (see, for example, Stephens and Weisbach (1998)), a dynamic setup may support repurchase with only part of the free cash.

5 Conclusion

In this paper we perform a theoretical investigation of the execution of open-market stock repurchase programs. Our model suggests that the execution depends on availability of free cash and information asymmetry. The results highlight important features of open-market stock repurchase programs: they leave the firm the option to avoid payout when cash is needed for operations, yet they also disburse free cash as long as the stock is not severely overpriced. Because they are preformed at management discretion, however, repurchase programs also redistribute wealth among shareholders. The model generates predictions about the completion rate of the programs and about the bid-ask spread during the repurchase period that might explain inconsistencies among earlier empirical studies.

[19] See, for example, Fenn and Liang (2001), Kahle (2002).

6 Appendix - Proofs of Lemmas and Propositions

Proof of Lemma 1: In states $v = a, v = a+x$ the firm does not have cash. Thus, by assumption, it cannot buy. In the state $v = a+x+c$ the firm will repurchase whenever the value of a terminal share with repurchase is higher than the value without it, i.e., whenever

$$a + x + \delta c < \frac{A + X}{N - \frac{C}{p}} = \frac{a + x}{1 - \frac{c}{p}},$$

which can be rearranged to

$$p < \frac{a + x}{\delta} + c,$$

which always holds, otherwise, the firm will never repurchase, implying that the market maker is making a positive profit.■

Proof of Lemma 2: In any full repurchase equilibrium, condition (3) must hold, and, in addition, condition (7) must *not* hold. Upon substitution of $A = Na, C = Nc, X = Nx$ and rearrangement we can write (3) as

$$p^2 - p\left((a + \frac{x}{2} + c) - \frac{Nc}{2Q}\right) + \frac{c(2a + x)}{4} - \frac{(2a + x + 2c)\, Nc}{4Q} = 0$$

or,

$$p = \frac{(a + \frac{x}{2} + c) - \frac{Nc}{2Q} \pm \sqrt{\left((a + \frac{x}{2} + c) - \frac{Nc}{2Q}\right)^2 + c\left((2a + x + 2c)\, \frac{N}{Q} - (2a + x)\right)}}{2}.$$

It is immediate to show that only the '+' solution is feasible to get (5).

Next, in any partial repurchase equilibrium, condition (4) must hold, and, in addition, condition (7) must hold. Upon substitution of $A = Na, C = Nc, X = Nx$ and rearrangement, we can write (4) as

$$p^2 - ((a + \frac{x}{2} + (1 + \frac{\delta}{4})c) - \frac{Nc}{4Q})p + \frac{c(3a + x + \delta c)}{4} - \frac{Nc(a + x + c)}{4Q} = 0,$$

or,

$$p = \frac{(a + \frac{x}{2} + (1 + \frac{\delta}{4})c) - \frac{Nc}{4Q} \pm \sqrt{\left(a + \frac{x}{2} + (1 + \frac{\delta}{4})c - \frac{Nc}{4Q}\right)^2 + c\left((a + x + c)\frac{N}{Q} - (3a + x + \delta c)\right)}}{2}.$$

We can further reduce to the '+' solution since only the positive solution is feasible to get (6).■

Proof of Lemma 3: Upon substitution of (5) into the complement of (7) (i.e., into $p \leq \frac{a}{\delta}+c$), a necessary and sufficient condition for a full repurchase equilibrium is

$$\frac{(a+\frac{x}{2}+c)-\frac{Nc}{2Q}+\sqrt{\left((a+\frac{x}{2}+c)-\frac{Nc}{2Q}\right)^2+c\left(\frac{N}{Q}\left(2a+x+2c\right)-\left(2a+x\right)\right)}}{2} \leq \frac{a}{\delta}+c \tag{24}$$

which can be rearranged to

$$c\left(\frac{N}{Q}\left(2a+x+2c\right)-\left(2a+x\right)\right) \leq 4\left(\frac{a}{\delta}+c\right)^2-4\left(\frac{a}{\delta}+c\right)\left((a+\frac{x}{2}+c)-\frac{Nc}{2Q}\right)$$

and further rearranged to get (8).

Upon substitution of (6) into (7), a necessary and sufficient condition for a partial repurchase equilibrium is

$$\frac{a+\frac{x}{2}+(1+\frac{\delta}{4})c-\frac{Nc}{4Q}+\sqrt{\left(a+\frac{x}{2}+(1+\frac{\delta}{4})c-\frac{Nc}{4Q}\right)^2+c\left((a+x+c)\frac{N}{Q}-(3a+x+\delta c)\right)}}{2} > \frac{a}{\delta}+c. \tag{25}$$

which can be rearranged to

$$c(\frac{N}{Q}(a+x+c)-(3a+x+\delta c)) > 4\left(\frac{a}{\delta}+c\right)^2-4\left(\frac{a}{\delta}+c\right)\left(a+\frac{x}{2}+(1+\frac{\delta}{4})c-\frac{Nc}{4Q}\right),$$

and further rearranged to get (9).■

Proof of Proposition 1: When $\delta = 1$, conditions (8) and (9) become

$$2x > c\left(2-\frac{x}{a}\left(\frac{N}{Q}+1\right)\right)$$

and

$$2x \leqslant c\left(2-\frac{x}{a}\left(\frac{N}{Q}+1\right)\right) \tag{26}$$

respectively, and are mutually exclusive. Thus, full repurchase is the outcome whenever (26) holds. Otherwise, the outcome is partial repurchase. Now if

$$2 \leq \frac{x}{a}\left(\frac{N}{Q}+1\right),$$

which is

$$\frac{2}{\frac{N}{Q}+1} \leq \frac{x}{a},$$

condition (26) never holds (because x, c are positive) so that the firm will never fully repurchase (always partially repurchase.) Otherwise, if

$$x < \frac{2a}{\frac{N}{Q}+1},$$

condition (26) can be written as (11). Thus, in this case, the firm will fully repurchase if (11) holds and partially repurchase otherwise.■

Proof of Proposition 2: If $\delta = 1$, it is immediate from Proposition 1 that the firm will always fully repurchase. So consider the case with $0 < \delta < 1$. Conditions (8) and (9) become

$$\frac{4a}{\delta}\left(1-\frac{1}{\delta}\right) \leq c\left(\frac{2}{\delta}\left(\frac{N}{Q}+2\right)-2\left(\frac{N}{Q}+1\right)\right) \tag{27}$$

and

$$\frac{4a}{\delta}\left(1-\frac{1}{\delta}\right) > c\left(\frac{1}{\delta}\left(\frac{N}{Q}+4\right)-\left(\frac{N}{Q}+2\right)\right) \tag{28}$$

respectively. Condition (27) always holds because the L.H.S. is negative and the multiplier of c in the R.H.S. is always positive. Condition (28) never holds because the L.H.S. is negative and the multiplier of c in the R.H.S. is always positive.■

Proof of Proposition 3: Consider the condition for full repurchase equilibrium (8). The L.H.S. of (8) is negative whenever

$$4 + 2\frac{x}{a} < \frac{4}{\delta}$$

that is, whenever

$$\delta < \frac{2}{2+\frac{x}{a}}. \tag{29}$$

The R.H.S. of (8) is negative whenever

$$\frac{2}{\delta}(\frac{N}{Q}+2) < \left(2+\frac{x}{a}\right)\left(\frac{N}{Q}+1\right)$$

that is, whenever

$$\frac{2}{\left(2+\frac{x}{a}\right)}\frac{(\frac{N}{Q}+2)}{\left(\frac{N}{Q}+1\right)} < \delta. \tag{30}$$

Note that

$$\frac{2}{2+\frac{x}{a}} < \frac{2}{\left(2+\frac{x}{a}\right)}\frac{(\frac{N}{Q}+2)}{\left(\frac{N}{Q}+1\right)},$$

and hence, whenever (29) holds, (30) never holds, and whenever (30) holds, (29) never holds. Now suppose that condition (29) holds. Then condition (30) does *not* hold, and we can write (8) as $c \geq c^F$ where c^F is defined in (15). In this case, in c^F the numerator is negative, whereas the denominator is positive and, because c is positive, in this situation the condition $c \geq c^F$ always holds, i.e., (8) always holds.
Conclusion: If condition (29) holds (i.e., if the inequality in (12) holds strictly), full repurchase equilibrium always exists.

Next, if condition (30) holds, we can write (8) as $c \leq c^F$, where c^F is defined in (15). Since condition (30) holds, the denominator in c^F is negative. But if condition (30) holds, condition (29) never holds, so that c^F is negative. Since c is positive, in this situation, the condition $c \leq c^F$ never holds, i.e., (8) never holds.
Conclusion: Whenever (30) holds (i.e., if the inequality in (13) holds strictly), full repurchase equilibrium never exists.

In the intermediate region for δ indicated in (14), both conditions (29) and (30) do not hold. Because (30) does not hold, (8) can be written as $c \geq c^F$, where c^F is defined in (15). Because condition (29) also does not hold, c^F is positive (because in this region both the numerator and the denominator of the c^F are positive). Thus, in this region, existence of full repurchase equilibrium depends on the value of c, i.e., a full repurchase equilibrium exists if c is sufficiently high such that $c \geq c^F$.

Last, because it is always the case that

$$\frac{2}{2+\frac{x}{a}} < \frac{2}{\left(2+\frac{x}{a}\right)}\frac{(\frac{N}{Q}+2)}{\left(\frac{N}{Q}+1\right)},$$

then, when (12) holds with equality, condition (8) always holds (i.e. a full repurchase equilibrium always exists), and when (13) holds with equality, condition (8) never holds (i.e., a full repurchase equilibrium never exists).■

Proof of Proposition 4: Consider the condition for partial repurchase equilibrium (9). The L.H.S. of (9) is negative whenever

$$4 + 2\frac{x}{a} < \frac{4}{\delta}$$

which can also be written as

$$\delta < \frac{2}{2 + \frac{x}{a}}. \tag{31}$$

The R.H.S. of (9) is negative whenever

$$\frac{1}{\delta}\left(\frac{N}{Q} + 4\right) < 1 + \left(1 + \frac{x}{a}\right)\left(\frac{N}{Q} + 1\right)$$

which can also be written as

$$\frac{2\left(\frac{N}{Q} + 2\right) - \frac{N}{Q}}{\left(2 + \frac{x}{a}\right)\left(\frac{N}{Q} + 1\right) - \frac{N}{Q}} < \delta. \tag{32}$$

The relation between conditions (31) and (32) depends on whether or not

$$\frac{2}{2 + \frac{x}{a}} \leq \frac{2\left(\frac{N}{Q} + 2\right) - \frac{N}{Q}}{\left(2 + \frac{x}{a}\right)\left(\frac{N}{Q} + 1\right) - \frac{N}{Q}}$$

which after rearrangement is

$$\frac{x}{a} \leq \frac{4}{\frac{N}{Q} - 2}. \tag{33}$$

We need to consider three cases: the case where (33) holds with strict inequality (case 1), the case when it does not hold (case 2), and the case when it holds with equality (case 3).

Case 1: Suppose first that in condition (33) the inequality holds strictly. Then

$$\frac{2}{2 + \frac{x}{a}} < \frac{2\left(\frac{N}{Q} + 2\right) - \frac{N}{Q}}{\left(2 + \frac{x}{a}\right)\left(\frac{N}{Q} + 1\right) - \frac{N}{Q}}$$

and, hence, whenever (31) holds, (32) never holds, and whenever (32) holds, (31) never holds. Now suppose that (31) holds. We can then write (9) as $c < c^P$, where c^P is defined in (19). Because in case 1, (31) hold implies that (32) does not hold, then in c^P the numerator is negative whereas the denominator is positive so that c^P is negative. Because c is always positive then in this situation of case 1 the condition $c < c^P$can never hold, i.e., condition (9) can never hold.
Conclusion: In case 1, if condition (31) holds (i.e., if the inequality in (16) holds strictly), partial repurchase equilibrium never exists.

Next, if condition (32) holds, we can write condition (9) as $c > c^P$. Since condition (32) holds, the denominator is negative. But we have seen that if condition (32) holds then condition (31) never holds, so that c^P is negative. Since c is positive, in this situation of case 1, condition $c > c^P$ always holds, i.e., (9) always holds.
Conclusion: In case 1, if condition (32) holds (i.e., if the inequality in (17) holds strictly), a partial repurchase equilibrium always exists.

In the intermediate region for δ

$$\frac{2}{2+\frac{x}{a}} < \delta < \frac{2\left(\frac{N}{Q}+2\right)-\frac{N}{Q}}{\left(2+\frac{x}{a}\right)\left(\frac{N}{Q}+1\right)-\frac{N}{Q}} \tag{34}$$

both conditions (31) and (32) do not hold. Because condition (32) does not hold, (9) can be written as $c < c^P$. Because condition (31) also does not hold, in this region, c^P is positive (because in this region both the numerator and denominator of c^P are positive). Thus, in this region, existence of partial repurchase equilibrium depends on the value of c, i.e., a partial repurchase equilibrium exists if c is sufficiently low such that $c < c^P$.

Last, because in case 1 it is always the case that

$$\frac{2}{2+\frac{x}{a}} < \frac{2(\frac{N}{Q}+2)-\frac{N}{Q}}{\left(2+\frac{x}{a}\right)\left(\frac{N}{Q}+1\right)-\frac{N}{Q}},$$

then when (16) holds with equality, condition (9) never holds (i.e., a partial repurchase equilibrium never exists), and when (17) holds with equality, condition (9) always holds (i.e., a partial repurchase equilibrium always exists).

Case 2: Suppose now, instead, that condition (33) does not hold. Then

$$\frac{2\left(\frac{N}{Q}+2\right)-\frac{N}{Q}}{\left(2+\frac{x}{a}\right)\left(\frac{N}{Q}+1\right)-\frac{N}{Q}}<\frac{2}{2+\frac{x}{a}}.$$

Now if

$$\delta<\frac{2\left(\frac{N}{Q}+2\right)-\frac{N}{Q}}{\left(2+\frac{x}{a}\right)\left(\frac{N}{Q}+1\right)-\frac{N}{Q}}, \tag{35}$$

we can write (9) as $c<c^P$. But in case 2, if (35) holds, condition (31) always holds, and hence, in c^P the numerator is negative and the denominator is positive so that c^P is always negative. Because c is always positive, in this situation of case 2 the condition $c<c^P$ never holds, i.e., condition (9) never holds.

Conclusion: In case 2, if condition (35) holds (i.e., if the inequality in (20) holds strictly), partial repurchase equilibrium never exists.

Next, if

$$\frac{2}{2+\frac{x}{a}}<\delta, \tag{36}$$

condition (35) never holds so that we can write (9) as $c>c^P$. But in case 2, if (36) holds, condition (32) always holds and (31) never holds, and hence, in c^P the denominator is negative and the numerator is positive so that c^P is negative. Because c is positive, the condition $c>c^P$ always holds, i.e., condition (9) always holds.

Conclusion: In case 2, if condition (35) holds (i.e., if the inequality in (21) holds strictly), equilibrium with partial repurchase always exists.

In the intermediate region for δ

$$\frac{2\left(\frac{N}{Q}+2\right)-\frac{N}{Q}}{\left(2+\frac{x}{a}\right)\left(\frac{N}{Q}+1\right)-\frac{N}{Q}}<\delta<\frac{2}{2+\frac{x}{a}}$$

both conditions (31) and (32) hold. Because condition (32) holds, condition (9) can be written as $c>c^P$. Also, condition (31) holds. Thus, in this region of δ, c^P is positive (because in this region, both the numerator and the denominator in c^P are negative). Thus, in this region, existence of partial repurchase equilibrium depends on the value of c, i.e., a partial repurchase equilibrium exists if c is sufficiently high such that $c>c^P$.

Last, because in case 2 it is always the case that

$$\frac{2(\frac{N}{Q}+2)-\frac{N}{Q}}{(2+\frac{x}{a})\left(\frac{N}{Q}+1\right)-\frac{N}{Q}}<\frac{2}{2+\frac{x}{a}},$$

then when (20) holds with equality, condition (9) never holds (i.e., a partial repurchase equilibrium never exists), and when (21) holds with equality, condition (9) always holds (i.e., a partial repurchase equilibrium always exists).

Case 3: Suppose now that condition (33) holds with equality. Then conditions (31) and condition (32) are mutually exclusive and there is no intermediate area to consider. There are only two regions to consider, and the analysis and the results in these regions are as in cases 1 and 2. That is, if

$$\delta\leq\frac{2}{2+\frac{x}{a}}=\frac{2\left(\frac{N}{Q}+2\right)-\frac{N}{Q}}{(2+\frac{x}{a})\left(\frac{N}{Q}+1\right)-\frac{N}{Q}},$$

a partial repurchase equilibrium never exists, and it always exists otherwise.■

Proof of Proposition 5: Existence of full repurchase equilibrium depends on c only in the region (14) of δ. Consider the definition of c^F in (15). The numerator in c^F is increasing in $\frac{x}{a}$ and the denominator is increasing in $\frac{x}{a}$, hence c^F is decreasing in $\frac{x}{a}$. In the region (14), the numerator in c^F is increasing in δ because, by derivation, it is increasing in δ if $\delta<\frac{4}{2+\frac{x}{a}}$, which contains the region (14) (since by assumption $N/Q>3$). The denominator of c^F is decreasing in δ and hence c^F is increasing in δ, establishing the first part of proposition 5.

For existence of partial repurchase equilibrium, first consider the case where $\frac{x}{a}<\frac{4}{\frac{N}{Q}-2}$. In this case, existence of partial repurchase equilibrium depends on c only in the region (18) of δ. In the region (18) both the numerator and denominator of c^P (defined in (19)) are positive. The numerator is increasing in δ because, by derivation, it is increasing in δ if $\delta<\frac{4}{2+\frac{x}{a}}$, which contains the region (18) (since by assumption $N/Q>3$). The denominator of c^P is decreasing in δ and hence c^P is increasing in δ. Also, the numerator in c^P is increasing in $\frac{x}{a}$, whereas the denominator is decreasing in $\frac{x}{a}$, so that c^P is increasing in $\frac{x}{a}$. Next, consider the case where $\frac{x}{a}>\frac{4}{\frac{N}{Q}-2}$. In this case, existence of partial repurchase equilibrium depends on c only in the region (22) of δ. In the region (22) both the numerator and the denominator of c^P are negative. The numerator is increasing in δ (becomes less negative) because, by derivation, it is increasing in δ if $\delta<\frac{4}{2+\frac{x}{a}}$, which contains the region (22). The denominator is decreasing in δ (becomes more negative), so that c^P is decreasing in δ. The numerator in c^P is increasing in $\frac{x}{a}$ (becomes less negative) and the denominator is increasing in $\frac{x}{a}$ (becomes more negative) and hence in this region $c^P$is decreasing in $\frac{x}{a}$, establishing the second part of Proposition 5.■

Proof of Lemma 4: There is no adverse selection in the sell market. In order to make zero expected profit in the sell market, the market maker must set $P_B = E[p_2]$. At time zero all information is symmetric, and, since we take it as given that a program has been announced, the competitive price for which shares would be traded at $t = 0$ (bid or ask) must be equal to the expected sell price (at $t = 1$ or $t = 2$).■

Proof of Proposition 6: The result for completion rate and social wealth is established in subsection 3.3.1. Wealth expropriation is an increasing function of the bid–ask spread, so it is sufficient to show that the bid–ask spread is higher. Consider the market maker conditions (3) and (4), and let p^F and p^P denote the prices for which they hold, respectively. Coexistence implies that $\frac{A}{N-\frac{C}{p^F}} \geq a + \delta c$ and $\frac{A}{N-\frac{C}{p^P}} < a + \delta c$, hence it must be the case that $p^P > p^F$. Next, since $p^P > p^F$, then $\frac{A+X}{N-\frac{C}{p^F}} > \frac{A+X}{N-\frac{C}{p^P}}$, and $\frac{A}{N-\frac{C}{p^F}} > \frac{A}{N-\frac{C}{p^P}}$, hence $E[v_2^F] > E[v_2^P]$; and since $v_2 = p_2$ (no adverse selection at $t = 2$), then $E[p_2^F] > E[p_2^P]$.■

Proof of Proposition 7: First, wealth transfers are an increasing function of the bid–ask spread, hence they increase whenever the bid–ask spread increases and decrease whenever the bid–ask spread decreases. Next, in a full repurchase equilibrium when $a+\frac{x}{2}$ is fixed, the market maker condition is independent of $\frac{x}{a}$, hence p and $E[p_2]$ are independent of x. In a partial repurchase equilibrium, when $a + \frac{x}{2}$ is fixed, an increase in $\frac{x}{a}$ implies that x is increasing. Consider the market maker condition in a partial repurchase equilibrium. Given that $a + \frac{x}{2}$ is fixed, $E[v_2]$ increases in x. This is because the changes in v_2 in the states a and $a + x$ offset each other while the decrease in v_2 in the state $a + c$ is smaller than the increase in the state $a+x+c$ because of the nonlinearity introduced though the firm's trade in the later state. But, if $E[v_2]$ increases in x, p must increase in x to provide the market maker with zero expected profit. Furthermore, in the state $a + x + c$ the market maker is losing not only on a per share basis but also because the quantity traded is higher than in all other states. Thus, for the market maker to break even, it must be that the increase in p is larger than the increase in $E[v_2]$, establishing that $p - E[v_2]$ is increasing in $\frac{x}{a}$. Now since $E[v_2] = E[p_2] = p_B$, then the bid–ask spread is increasing in $\frac{x}{a}$ (when $a + \frac{x}{2}$ are fixed).■

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Figure 1: Graphs depicting Firms *F* and *G* in Example 1.

Figure 1a: Firm *F* – a firm for which uncertainty about assets in place is high relative to uncertainty of free cash. A=7, X=5, C=2, N=10, Q=3, δ=0.8.

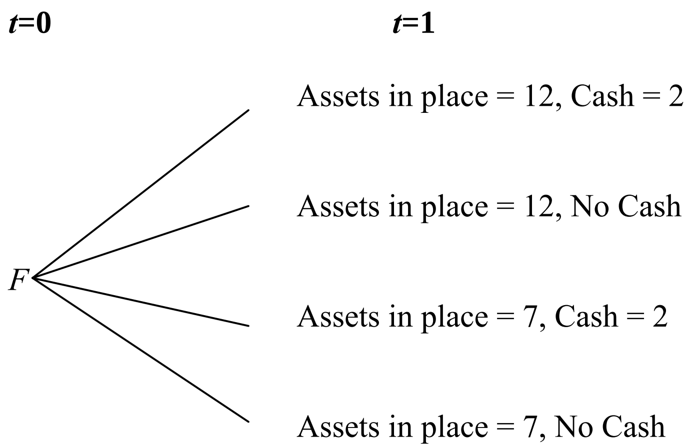

Figure 1b: Firm *G* – a firm for which uncertainty about assets in place is low relative to uncertainty of free cash. A=9, X=1, C=2, N=10, Q=3, δ=0.8.

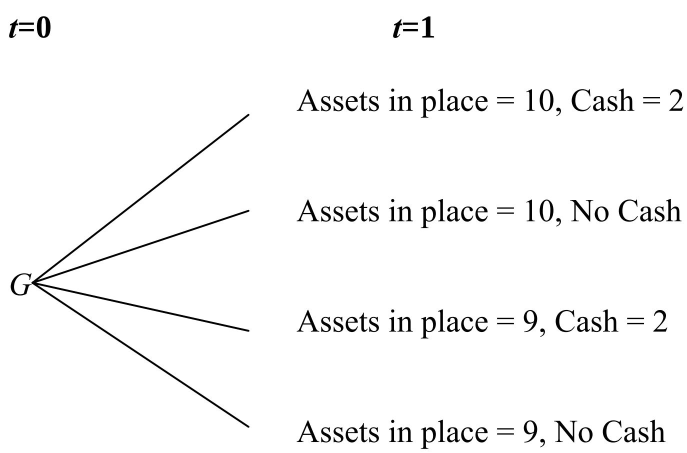

Figure 2: Existence of partial repurchase equilibrium and full repurchase equilibrium when δ=1.

This figure demonstrates the results in Proposition 1 by means of a graph. The figure illustrates how the decision of whether or not to repurchase in the state $v = a+c$ depends on the variability in the value of assets in place, x/a, and the variability in free cash, c. The vertical dashed line indicates where condition (10) holds with equality. To the right of this line the variability in the firm value, introduced through the variability in assets in place, is too high so that it pushes the stock price too high for a full repurchase equilibrium to exist, and therefore a partial repurchase equilibrium prevails. To the left of the dashed line, equilibrium type depends on the level of free cash. Specifically, the solid curved line indicates where condition (11) holds with equality. Below this curved line, variability in the firm value due to variability in cash is too low (c is too low), so that in the state with low asset value and cash the firm will not repurchase and hence partial repurchase equilibrium prevails. Above this line (c is sufficiently high), a full repurchase equilibrium prevails.

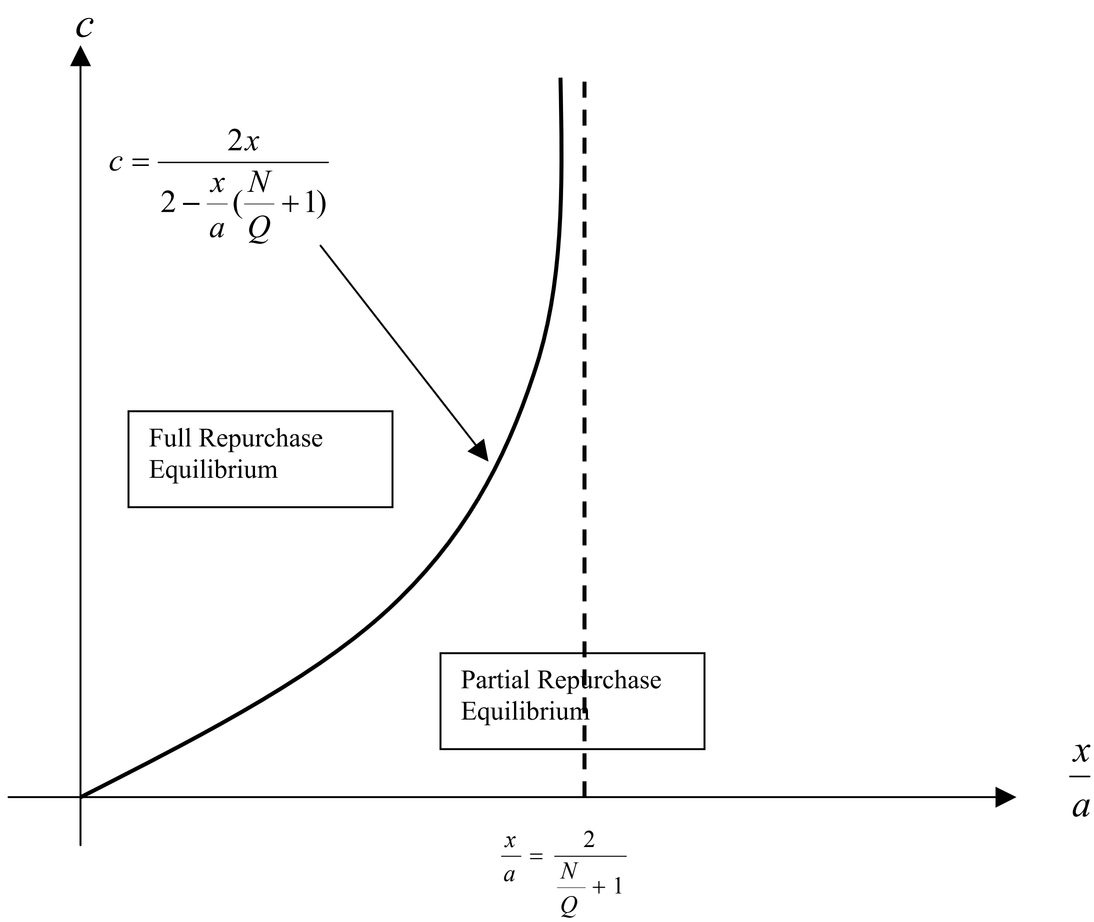

Figure 3: Existence of full repurchase equilibrium.

This figure demonstrates the results of Proposition 3 by means of a graph. The figure illustrates how existence of full repurchase equilibrium depends on the variability in the value of assets in place, x/a, and the waste rate captured by δ. The solid (curved) line indicates where condition (12) holds with equality. The dotted (curved) line indicates where condition (13) holds with equality. In the area above the dotted line a full repurchase equilibrium never exists. In the area below the solid line a full repurchase equilibrium always exists. In the area captured between the lines where (14) holds, full repurchase equilibrium exits if $c>c^F$, where c^F is defined in (15).

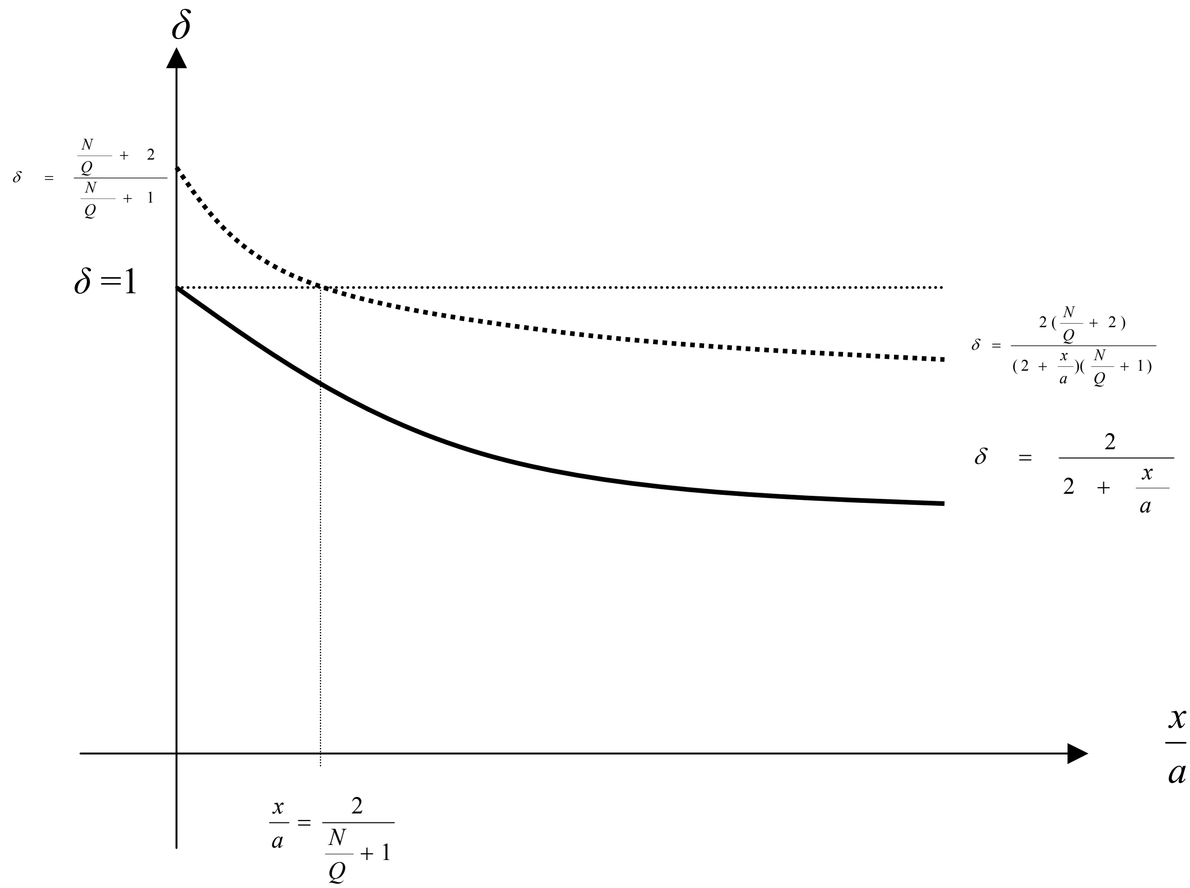

Figure 4: Existence of partial repurchase equilibrium.

This figure demonstrates the results of Proposition 4 by means of a graph. The figure illustrates how the existence of partial repurchase equilibrium depends on the variability in the value of assets in place, x/a, and the waste rate captured by δ (high waste rate is low δ). As in Figure 3, the solid (curved) line indicates where condition (16) holds with equality. The dashed (curved) line indicates where condition (17) holds with equality. In the area above both lines a partial repurchase equilibrium always exists. In the area below both lines a partial repurchase equilibrium never exists. In the area captured between the lines, to the left of their crossing point, a partial repurchase equilibrium exists if $c<c^P$, where c^P is defined in (19), whereas in the area captured between these lines to the right of their crossing point a partial repurchase equilibrium exits if $c>c^P$. (A partial repurchase equilibrium does not exist at the crossing point.)

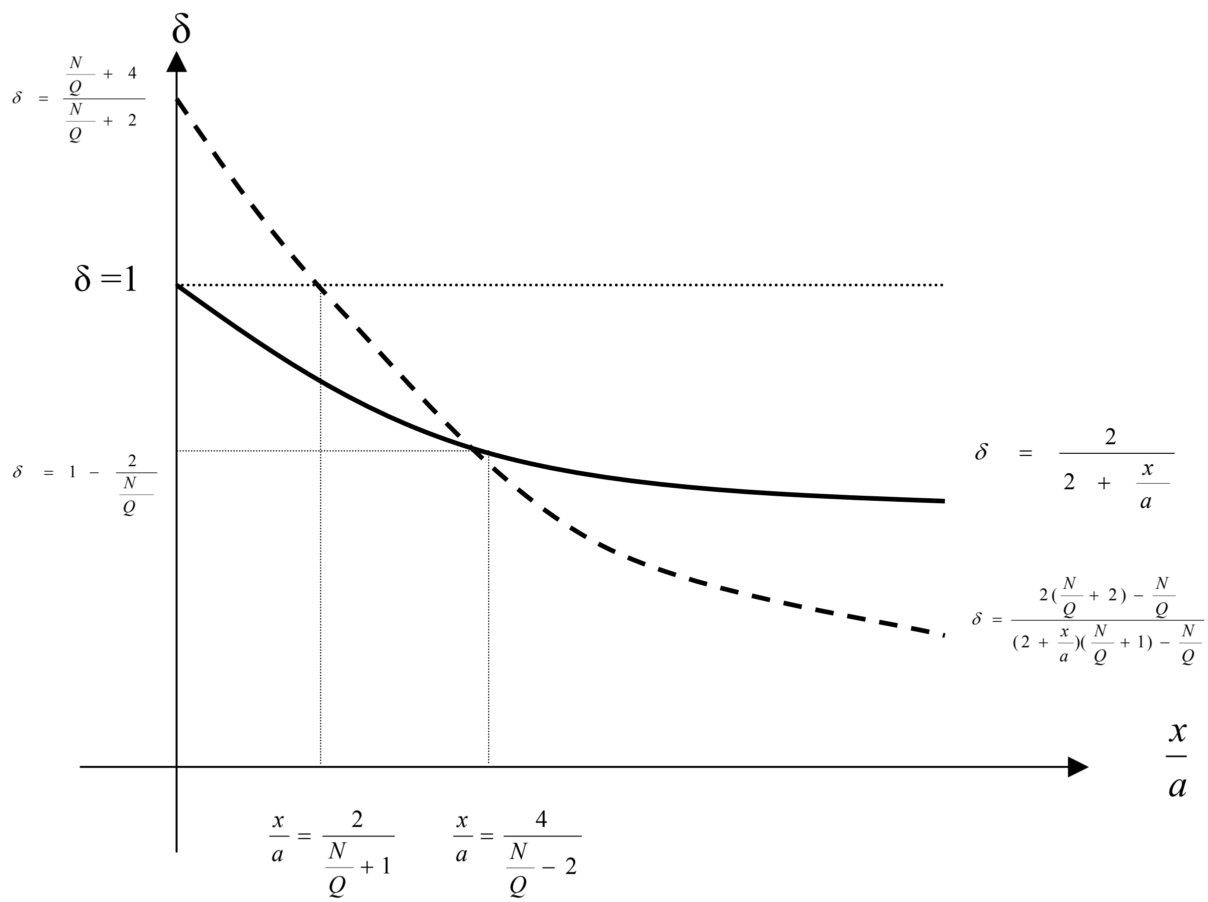

Figure 5: Existence of partial repurchase equilibrium and full repurchase equilibrium.

Figure 5 combines Figures 3 and 4 to demonstrate the ranges of existence for both partial and full repurchase equilibria depending on *x*/*a* and δ. In the area below both the solid and the dashed lines (i.e. to the left of their crossing point and below the solid line) both *x*/*a* and δ are low, hence only full repurchase equilibrium exists. However, in the area below the solid line above the dashed line (i.e., to the right of their crossing point between the lines) δ is low but *x*/*a* is now relatively high, so a partial repurchase equilibrium can also exist if $c<c^P$. In the area above the dotted line both δ and *x*/*a* are high, hence only a partial repurchase equilibrium exists. However, in the area below the doted line but above both the dashed and solid lines *x*/*a* is high but δ is relatively not as high so that a full repurchase equilibrium can also exist if $c>c^F$.

In the area above the solid line but below the dashed line (to the left of the crossing point of these lines) existence of both equilibria depends on the level of *c*. If c is sufficiently low to render $c<c^P$ only a partial repurchase equilibrium exists, and if *c* is sufficiently high to render $c>c^F$, only a full repurchase equilibrium exists. If $c^F<c<c^P$, both partial and full repurchase equilibria can exist.

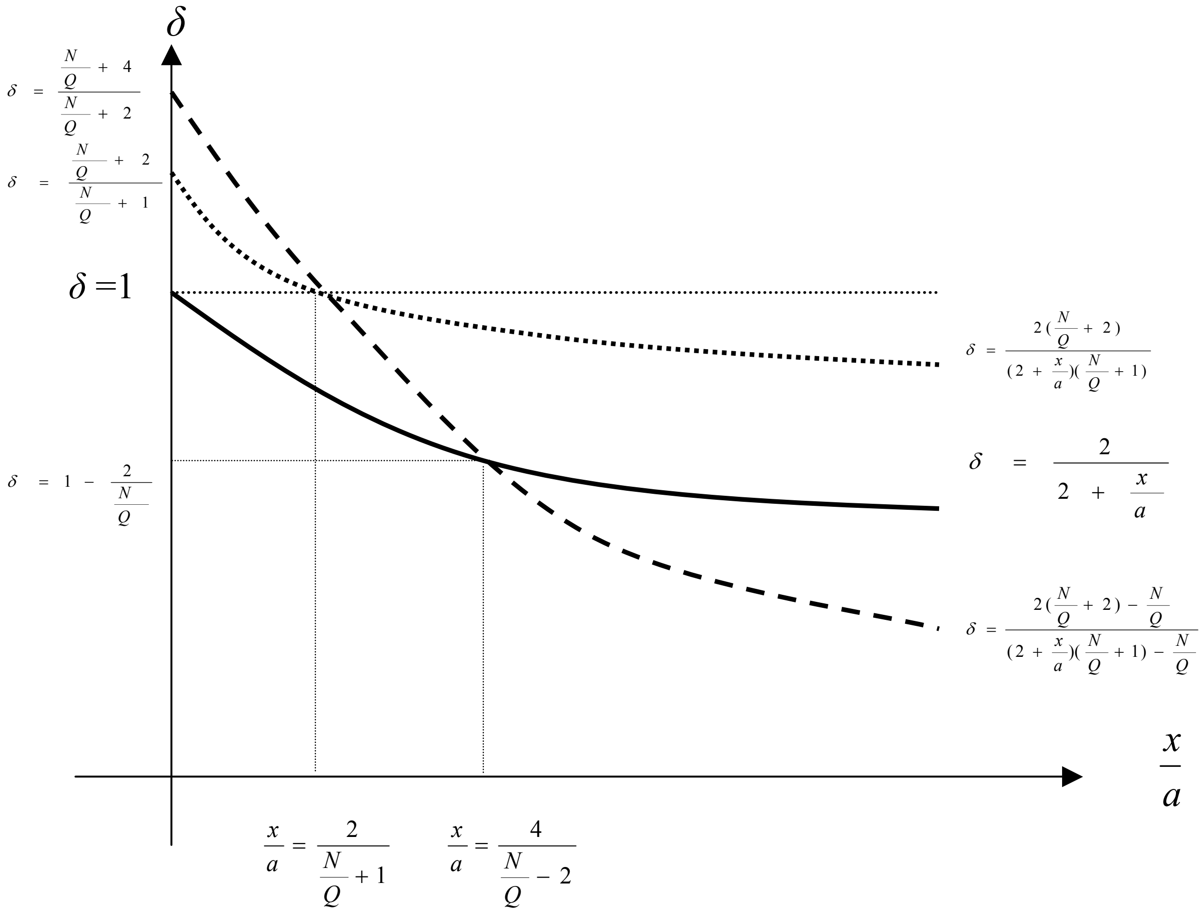

Table 1: Comparison of Firm *F* and Firm *G* in Example 1.

	Firm *F*	Firm *G*
A	7	9
X	5	1
C	2	2
N	10	10
Q_A	3	3
δ	0.8	0.8
Stock price without a repurchase program ($p_A = p_B = E[p_2]$)	1.03	1.03
Completion rate = probability that repurchase is executed	25%	50%
Ask price with repurchase (p_A)	1.1093	1.0829
Bid price with repurchase p_B = average terminal value	1.0560	1.0760
Bid–ask spread = $p_A - p_B$	0.0533	0.0253
Original shareholders' wealth - increase relative to without repurchase	0.260	0.2760
Liquidity buyers loss - relative to without repurchase	0.160	0.0760
Social wealth increase - relative to without repurchase	0.1	0.2

Table 2: Level of regulation of open-market repurchase programs in five countries in which the programs are allowed.

Regulation/Country	US*	Canada	UK	France	Hong Kong
Approval	Board	Board	Shareholders	Shareholders	Shareholders
Period	No	12M	No	18M	12M
Size	No	5%	15%**	10%	10%
Disclosure	No	Monthly	Daily	Monthly	Daily
Timing	No	Moderate	Moderate	High	High
Volume	No	High	Low	High	Low
Price	No	High	Low	High	No
Insider Trade	No	Yes	Yes	Yes	Yes

Legend:

- Approval: Who approves the program (Board or Shareholders).
- Period: Restriction on the time period of the program (in months (M), or , No restriction).
- Size: Restriction on fraction of total outstanding shares that the firm can buy back (in percentage, or No restriction).
- Disclosure: Frequency of reporting requirement of actual repurchases (Daily, Monthly, No requirement).
- Timing: Restrictions on the timing of actual repurchases (High, Moderate, No restriction). A timing restriction is, for example: no repurchase in the last hour of the trading day.
- Volume: Restrictions on the volume of actual repurchase trade when repurchasing (High, Low, No restriction).
- Price: Restrictions on the actual repurchase price (High, Low). A price restriction is, for example: the firm is not allowed to bid the price up.
- Insider Trade: Are insiders prohibited from trading for their own portfolio during the repurchase period (Yes, No).

* In the US firms that want to be protected under the Safe Harbor Act (Rule 10b-18) need to follow guidelines on actual repurchases (timing, volume and price). However, unlike in the other countries above, in the US these guidelines are not requirements.

** In the UK the program size limit is per year.