# A Model of Community Standards.

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#### Comments welcome.

#### Abstract

I introduce a new model of community standards relevant to the judicial determination of obscenity. In the model, standards are defined as subjective judgments restricted only by a simple reasonableness condition. A set of individual standards is then methodically aggregated to form the community standard. I define several axioms which reflect legal concerns expressed by the judiciary. The axioms require that the community standard (a) preserve unanimous agreements about the entire standard, (b) become more permissive when all individuals become more permissive, and not discriminate, ex ante, (c) between individuals and (d) between works. I then show that the only method which satisfies these properties is *unanimity rule*, in which a work is considered obscene if and only if all members of the community consider it to be obscene. I also consider several variants of the model and provide characterizations in these related models.

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# 1 Introduction

In 1957, the United States Supreme Court ruled that obscenity is "utterly without redeeming social importance" and is not protected by the U.S. Constitution. The court held that "contemporary community standards" are to be used in determining whether particular works are obscene.<sup>1</sup> The Supreme Court has never explained what "community standards" are or how, if at all, they are related to the standards of the individuals who comprise the community. Lower courts have provided only limited guidance describing the community standard as an "aggregation or average".

I introduce a new model in which community standards are formed by aggregating a set of individual standards. In the model, standards are defined as judgments categorizations of possible works as either "obscene" or "not obscene." Every possible judgment is allowed provided it satisfies the following restriction: neither individuals nor the community may consider one-hundred percent of the works to be obscene. I define several basic normative properties of aggregation methods which reflect legal concerns expressed by the judiciary. I then show that the only method which satisfies these properties is *unanimity rule*, in which a work is considered obscene if and only if all members of the community consider it to be obscene.

<sup>&</sup>lt;sup>1</sup>Roth v. United States, 354 U.S. 476 (1957). The Supreme Court retained the community standards test when it refined the definition of obscenity sixteen years later in *Miller v. California* 413 U.S. 15 (1973).

#### **1.1** The Problem of Community Standards

In communities that are perfectly homogeneous, where each individual's belief is identical, it should be simple to determine the community standard. However, as the Supreme Court has recognized, few communities are perfectly homogeneous. For this reason the Court has required the jury to consider the views of a diverse set of individuals, including the young and the old, the religious and the irreligious, the sensitive and the insensitive.<sup>2</sup> But when the community is heterogeneous, it is not obvious how the conflicting views of the citizenry should be combined.

Some commentators, including Sadurski (1987), have argued that the community standard is an average or median in a mathematical sense. But as another commentator has pointed out, "the notion of an average standard ... implies the existence of a spectrum of tolerance that can be ranked along a single dimension, from least tolerant to most tolerant. The problem with this approach is that a single dimension of tolerance does not exist." (Boyce, 2008).<sup>3</sup> No court nor commentator has yet identified an objective method to order judgments or levels of tolerance along a single dimension.

A different approach was taken by Lord Patrick Devlin in his classic work, *The Enforcement of Morals* (Devlin, 1965). Lord Devlin argued that it was proper for governments to prohibit behavior felt to be immoral by the community. He suggested

<sup>&</sup>lt;sup>2</sup>See Roth v. United States, 354 U.S. 476 (1957) and Pinkus v. United States, 436 U.S. 293 (1978). <sup>3</sup>Boyce (2008), however, assents to the principle that community standards "must in some sense

be an aggregate of the standards of the individuals who comprise the community."

that, in some sense, unanimous agreement within a society is necessary to justify regulation of immorality: "the moral judgment of society must be something about which any twelve men or women drawn at random might after discussion be expected to be unanimous." To ascertain the moral standards of the community, Lord Devlin's understanding of the Law of England can be described in the following way. First, the community consists of all "right-minded" or "reasonable" persons within the society.<sup>4</sup> Next, an act is deemed immoral if and only if every reasonable person believes the act to be immoral. "Immorality then, for the purpose of the law, is what every right-minded person is presumed to consider to be immoral." (Devlin, 1965).<sup>5</sup>

While the U.S. Supreme Court adopted the principle that certain acts (the distribution and sale of obscene material) can be criminalized on the grounds of offense to community morals, American courts have never adopted a specific rule to ascertain the moral standards of the community. Individual jurors are instructed to ascertain these standards on the basis of their experience and familiarity with the community, and are not instructed as to the method through which differing beliefs should be combined.<sup>6</sup> For over fifty years the Supreme Court has simply ignored this question,

<sup>&</sup>lt;sup>4</sup>Whether an individual is "right-minded" or "reasonable" does not seem to be directly connected to the specific content of that individual's beliefs; otherwise Devlin's rule would be circular and illdefined.

<sup>&</sup>lt;sup>5</sup>Whether Devlin's rule is certainly practicable is a debatable proposition. He certainly felt that the rule would lead to convictions in 1958, but whether that should remain the case in the more tolerant environment of the twenty-first century is unclear. However, the mere possibility that some communities would find little to prohibit does not invalidate Devlin's rule. He argued that a community should be able to prohibit that which it found immoral, and not that every community must find some works to be immoral.

<sup>&</sup>lt;sup>6</sup>The views of the individual jurors themselves are combined through the unanimous jury rule which closely corresponds to Devlin's rule: an individual is convicted of an immoral act only when every juror considers the act to be immoral.

allowing the incarceration of defendants convicted under a vague and murky legal doctrine.

## 1.2 The Model

The basic model can be described as follows. First, there is a community, which can be any group of individuals. The Supreme Court has required that the community be defined in geographic terms and contain all adults in that community, including the young, the old, the religious, the irreligious, the sensitive, and the insensitive.<sup>7</sup> Lord Devlin (1965) seems to have argued that the community consists only of reasonable persons. Others might propose to restrict the definition to clerics, to parents, or to some other community of interest. The model is general enough to include all of these as special cases.

Next, there is an infinite set of all possible works. We might loosely understand this as the set of possible artworks but it might also include literary works, scientific publications, and other forms of human expression. The space of works is modeled as a non-atomic measure space. The decision to use a non-atomic measure space rather than a discrete space is made to simplify the exposition. Parallel conclusions would be reached if the space of works were modeled as discrete and appropriate modifications were made to the axioms.

Individuals from the community have standards as to which works in the set are <sup>7</sup>See Roth v. United States, 354 U.S. 476 (1957) and Pinkus v. United States, 436 U.S. 293 (1978). obscene. An individual standard is simply a division of the set into two groups: the obscene and the non-obscene (or permissible). Individual standards are assumed to be well-informed and made after deliberation and reflection. There is a single restriction on allowable standards: the set of works judged to be obscene must be of less than full measure. Reasonable individuals should all believe that some works, even those lacking serious literary, artistic, political, and scientific value, are non-obscene.<sup>8</sup> I do not require individuals to believe that some works must be obscene — there is no reason why individuals *must* be offended by anything.

These individual standards are then aggregated to form a community standard. The community standard is subject to the same restriction as the individual standards: the set of works judged to be obscene must be of less than full measure. I place no other restrictions on the class of allowable standards. Individual standards and community standards are assumed to be subjective.

An aggregation rule is a systematic method of deriving the community standard from the individuals judgments. Aggregation rules are studied through the axiomatic approach: several normative properties are formalized as axioms and the unique rule satisfying these axioms is characterized.

I suggest two distinct approaches to understanding aggregation rules. First, the aggregation rule may be understood as an actual procedure used to determine whether a work is obscene. It specifies how the standards of the members of the community

<sup>&</sup>lt;sup>8</sup>Individuals who do not satisfy this restriction would be found to be unreasonable as a matter of law.

(or of a jury) are to be combined.

Second, an aggregation rule may be understood as a jury instruction. As mentioned above, the community standards are to be determined by the trier of fact as part of a mental exercise. The aggregation rule instructs the trier of fact on how to aggregate these many envisioned individual standards into a single community standard. Legislators attempting to codify community standards into law might undertake a similar thought exercise.

#### 1.3 The Main Result

I introduce four axioms. Each is, in some way, a desirable property for any objective aggregation rule.

The first axiom, *homogeneity*, requires that if there is a single standard shared by every member of the community, then that standard is also the community standard. In some sense, if this axiom is not satisfied, then the community standard must be derived from something other than the individual judgments.

The second axiom, *responsiveness*, requires the community standard to "respond" in the same direction (more permissive or less) as the community. If every individual standard becomes more permissive, then the community standard should become more permissive as well. Responsiveness prevents the perverse result in which a defendant is convicted *because* the individuals in the community became more tolerant.

The third axiom, *anonymity*, requires that the aggregation rule not discriminate

between individuals. In general, the law requires equal treatment of individuals. More specific to this case, the Supreme Court has explicitly held that the views of all adult members of the community must be taken into account in determining the community standard.

The fourth axiom, *neutrality*, requires that the aggregation rule not discriminate, ex ante, between works. This axiom assumes that all judgments are subjective and is relevant when there is no method by which works can be objectively compared. No court nor commentator has yet identified a plausible method of comparison. The lack of an objective method is largely what makes even personal views on obscenity difficult to define through a rule. Supreme Court Justice Potter Stewart believed that obscenity could only be prohibited if hard-core pornography but could not define even that term. He only knew it when he saw it.<sup>9</sup> A natural method to compare works would be to judge them by their parts; however, this is method was expressly disallowed by the Supreme Court.<sup>10</sup>

Together, these four axioms characterize the unanimity rule, under which a work is deemed obscene when every individual considers it to be obscene.

## 1.4 Multiple Standards

The U.S. Supreme Court has held that contemporary community standards are to be used in evaluating two elements of obscenity: (a) whether the work appeals to

<sup>&</sup>lt;sup>9</sup>Concurring opinion in Jacobellis v. Ohio, 378 U.S. 184 (1964).

<sup>&</sup>lt;sup>10</sup>Roth v. United States, 354 U.S. 476 (1957).

the prurient interest, and (b) whether the work is patently offensive.<sup>11</sup> This implies that there are, at least, three types of judgments individuals can make: (1) which works appeal to the prurient interest, (2) which works are patently offensive, and (3) which works are obscene; that is, which both appeal to the prurient interest and are patently offensive.

The first two types of judgments are not logically related. As a matter of law, a work may appeal to the prurient interest but not be patently offensive; alternatively, a work may be patently offensive but not appeal to the prurient interest. Were one judgment to imply the other, there would be no need for both elements to appear in the test. Each of the first two types of judgments, however, is clearly related to the third. If a work both appeals to the prurient interest and is patently offensive, then it also appeals to the prurient interest.

If there is a single community standard for obscenity, as has been assumed in this paper, then the judgments being aggregated are of the third type. We might label the resulting standard the prurient interest and patently offensive community standard. However, one could infer from the Supreme Court opinions that there are two community standards, (a) the prurient interest community standard and (b) the patently offensive community standard.

<sup>&</sup>lt;sup>11</sup>*Miller v. California*, 413 U.S. 15 (1973). The full test provided in Miller is: (a) whether the average person, applying contemporary community standards would find that the work, taken as a whole, appeals to the prurient interest; (b) whether the work depicts or describes, in a patently offensive way, sexual conduct specifically defined by the applicable state law; and (c) whether the work, taken as a whole, lacks serious literary, artistic, political, or scientific value. The third element is an "objective" standard and does not vary from community to community. The test provided in Miller remains the current law.

A model of two community standards would take the following form. Individuals would make two separate judgments about which works (1) appeal to the prurient interest and (2) are patently offensive. The judgments would then be aggregated to form (a) the prurient interest community standard and (b) the patently offensive community standard. These two community standards need not be aggregated independently — it is conceivable, for example, that the individual judgments about which works are patently offensive are somehow relevant in determining the prurient interest community standard.

The main result of this paper does not change in the case of two (or more) standards. Even if we allow for interdependent aggregation, unanimity rule is the unique aggregation rule that satisfies the four axioms.

#### 1.5 Other Standards

The model introduced in this paper is general and can be applied to problems other than the question of which works are legally obscene. I will describe three different types of legal standards to which the model can be applied.

First, standards of offensiveness are used to determine whether speech, or other forms of expression, may be prohibited on the grounds that it is offensive. Obscenity doctrine provides the clearest example of a prohibition on offensive expression; other examples include the prohibitions on the broadcast of indecent and profane speech regulated by the Federal Communications Commission. Second, standards of proof are used to determine whether defendants are guilty (or liable) in criminal (and civil) cases. Commonly used standards of proof include (a) the proof beyond a reasonable doubt standard, (b) the clear and convincing standard, and (c) the preponderance of the evidence standard. Here, instead of a set of works, we have a set of cases as in Kornhauser (1992a,b) and Lax (2007), and individuals choose the subset of cases that lead to conviction. The results of the paper support the use of unanimity rule in determining which works are obscene.

Third, standards of behavior are used to evaluate behavior in civil and criminal trials. Examples of standards of behavior include the reasonable person standard studied by Rubinstein (1983), the business judgment rule, and fiduciary duties. To model this standard, we replace the set of works with a set of actions, and individuals have multiple standards, one for each set of circumstances, describing which actions are unreasonable in that circumstance.

# 2 The Model

#### 2.1 Notation and the Model

The **community** is a set  $N \equiv \{1, ..., n\}$  of individuals. The space of **works** is denoted by  $(W, \Sigma, \mu)$ , where W is the set of works,  $\Sigma$  is the  $\sigma$ -algebra of subsets of works, and  $\mu$  is a measure on  $(W, \Sigma)$ . The space  $(W, \Sigma)$  is assumed to be isomorphic to  $([0, 1], \mathscr{B})$ , where  $\mathscr{B}$  is the set of Borel subsets of [0, 1]. I assume that  $\mu$  is countably additive, non-atomic, non-negative, and finite.<sup>12</sup> Let  $\Phi$  be the set of all automorphisms of  $(W, \Sigma)$  that preserve the measure  $\mu$ .

Let  $\mathcal{J} \equiv \{J \in \Sigma : \mu(J) < \mu(W)\}$  be the set of **judgments**. The requirement that judgments must be of less than full measure is a reasonableness condition that reflects the idea that not all works can be obscene, or should be prohibited. Let  $M \equiv \{1, ..., m\}$  denote the set of **issues**. For example, if there is only a single standard of obscenity then m = 1, while if there is both a standard of "appeal to the prurient interest" and "patently offensive" then m = 2. The set M can be finite or countably infinite. A **standard** is an M-vector of judgments, one for each issue. The set of standards is denoted  $\mathcal{S} \equiv \mathcal{J}^M$ . A **profile** is an N-vector of standards,  $S = (S_1, ..., S_n) \in \mathcal{S}^N$ , where  $S_i$  represent individual i's standard. I write  $S_{ij}$  to denote individual i's judgment about issue j. A rule  $f : \mathcal{S}^N \to \mathcal{S}$  is a function mapping each profile into a **community standard**, denoted  $f(S) = (f_1(S), ..., f_m(S))$ .

For any two sets S and T of the form  $\mathcal{J}^K$ , I define  $\square$  as the coordinatewise intersection, so that  $(S \sqcap T)_k \equiv S_k \cap T_k$ , and I define  $\sqcup$  as the coordinatewise union, so that  $(S \sqcup T)_k \equiv S_k \cup T_k$ . Note that there exist  $S, T \in \mathcal{J}^K$  such that  $S \sqcup T \notin \mathcal{J}^K$ . I define  $S \sqsubseteq T$  to mean that  $S_k \subseteq T_k$  for every  $k \in K$ . When  $S \sqsubseteq T$  I write that S is as **permissive** as T, because every work that a particular person permits in profile T is permitted by that person in profile S. I define  $(\phi S)_k \equiv \phi(S_k)$ .

<sup>&</sup>lt;sup>12</sup>The space of actions is taken from the model of non-atomic games studied in Aumann and Shapley (1974) and Dubey and Neyman (1984).

## 2.2 Axioms

The first axiom, *homogeneity*, requires that if the community is perfectly homogeneous, so that every individual in the community has identical views about the entire standard, then this commonly held belief is the community standard. In some sense, if this axiom is not satisfied, then the community standard must be derived from something other than the individual judgments. This axiom excludes *degenerate* rules, under which the community standard is predetermined and does not change as a result of the opinions.<sup>13</sup>

**Homogeneity:** If  $S_i = S_j$  for all  $i, j \in N$ , then  $f(S) = S_1 = ... = S_n$ .

Suppose that the individual standards change and that every individual's new standard is as permissive as was that individual's old standard (so that  $S_i \subseteq S_i^*$ for all  $i \in N$ ). The second axiom, responsiveness, requires the resulting community standard to be as permissive as the prior community standard (so that  $f(S) \subseteq f(S^*)$ ). In other words, the community standard must "respond" in the same direction (more permissive or less) as the individuals in the community. Responsiveness prevents the perverse result in which a defendant is convicted because the individuals in the community became more permissive. This axiom excludes variable threshold rules, under which the degree of consent required to deem a work obscene varies.

**Responsiveness:** If  $S \sqsubseteq S^*$ , then  $f(S) \sqsubseteq f(S^*)$ .

<sup>&</sup>lt;sup>13</sup>The examples provided in this section are not meant as an exhaustive list of all rules excluded by these axioms.

The principle of anonymity requires that each individual's view must be treated equally. Individuals' names are switched through a permutation  $\pi$  of N. For a given permutation,  $\pi(i)$  is the new name of the individual formerly known as i. For a given profile S,  $\pi S \equiv (S_{\pi(1)}, ..., S_{\pi(n)})$  is the profile that results once names are switched. The third axiom, *anonymity*, requires that permutations of the individuals' names do not affect the community standard. This axiom excludes *dictatorships*, under which a pre-selected individual decides which works are obscene.

**Anonymity:** For every permutation  $\pi$  of N,  $f(S) = f(\pi S)$ 

The principle of *neutrality* is similar. It requires that a rule not discriminate, ex ante, between works on the basis of their names. Differences between works in the community standard should come from the beliefs and not from the rule. Works' names switched through an automorphism  $\phi \in \Phi$ . For a given profile S,  $f(\phi S)$  is the community standard derived from the profile that results when the names are switched; while  $\phi f(S)$  is the community standard that results when the names are switched only after the aggregation. The neutrality axiom requires that these two community standards be the same. This axiom excludes rules that deem a particular work obscene regardless of the opinions.

**Neutrality:** For every automorphism  $\phi \in \Phi$ ,  $\phi(f(S)) = f(\phi S)$ .

## 2.3 The Unanimity Rule

Under the "unanimity rule", a work is considered obscene if it is considered obscene by every individual. If there are multiple issues, then for each issue a work is prohibitable only when it is considered prohibitable by every individual.

**Unanimity Rule:** For every  $S \in \mathcal{S}^N$ ,  $f(S) = \sqcap_{i \in N} S_i$ .

The main result of this paper is that a rule satisfies all four axioms if and only if it is unanimity rule.

**Theorem 2.1.** The unanimity rule is the only rule that satisfies homogeneity, responsiveness, anonymity, and neutrality. Moreover, all four axioms are independent.

*Proof.* Step 1: I show that any work considered obscene by every individual must be considered obscene by the community, or that  $\Box_{i \in N} S_i \sqsubseteq f(S)$  for all  $S \in \mathcal{S}^N$ .

Let  $S \in \mathcal{S}^N$ . Define  $S^*$  as the profile such that  $S_j^* \equiv \prod_{i \in N} S_i$  for all  $j \in N$ . By homogeneity,  $f(S^*) = \prod_{i \in N} S_i$ . Because  $S^* \sqsubseteq S$ , responsiveness implies that  $f(S^*) \sqsubseteq f(S)$ . Thus  $\prod_{i \in N} S_i \sqsubseteq f(S)$ .

Step 2: I show that if there is a profile T such that (a)  $T_{ik} \cup T_{jl} = W$  unless i = jand k = l, and (b)  $\mu(T_{ik}) = \mu(T_{jk})$  for all  $i, j \in N$  and  $k \in M$ , then  $f(T) \sqsubseteq \sqcap_{i \in N} T_i$ .

Let  $T \in \mathcal{S}^N$  such that conditions (a) and (b) are met. Without loss of generality, let  $w \notin T_{11}$ . To prove that  $f(T) \sqsubseteq \sqcap_{i \in N} T_i$ , it is sufficient to show that  $w \notin f_1(T)$ .

Suppose, contrariwise, that  $w \in f_1(T)$ . Then, by neutrality,  $W \setminus T_{11} \sqsubseteq f_1(T)$ . By anonymity and neutrality,  $W \setminus T_{i1} \sqsubseteq f_1(T)$  for all  $i \in N$ . Thus  $\sqcup_{i \in N} (W \setminus T_{i1}) \sqsubseteq$   $f_1(T)$ . By step 1,  $\sqcap_{i \in N} T_{i1} \sqsubseteq f_1(T)$ , which implies that  $f_1(T) = W$ . But this is a contradiction, which proves that  $w \notin f_1(T)$ , and therefore that  $f(T) \sqsubseteq \sqcap_{i \in N} T_i$ .

Step 3: I show that any work not considered obscene by every individual must not be considered obscene by the community, or that  $f(S) \sqsubseteq \sqcap_{i \in N} S_i$  for all  $S \in \mathcal{S}^N$ .

Let  $S \in S^N$ . Without loss of generality, let  $w \notin S_{11}$ . To prove that  $f(S) \sqsubseteq \sqcap_{i \in N} S_i$ , it is sufficient to show that  $w \notin f_1(S)$ . Let T be a profile such that: (1)  $T_{ik} \cup T_{jl} = W$ unless i = j and k = l, (2)  $\mu(T_{ik}) = \mu(T_{jk})$  for all  $i, j \in N$  and  $k \in M$ , (3)  $w \notin T_{11}$ , and (4)  $S \sqsubseteq T$ . By step 2,  $f(T) \sqsubseteq \sqcap_{i \in N} T_i$ . Because  $S \sqsubseteq T$ , responsiveness implies that  $f(S) \sqsubseteq f(T) \sqsubseteq \sqcap_{i \in N} T_i$ . Because  $w \notin T_{11}$  it follows that  $w \notin f_1(S)$ . This proves that  $f(S) \sqsubseteq \sqcap_{i \in N} S_i$ .

Step 4: Steps 1 and 3 directly imply that  $f(S) = \bigcap_{i \in N} S_i$ . The independence of the axioms is proved in the appendix.

#### 2.4 Independence

Unanimity rule is clearly *independent* in the sense that the community standard's judgment about a particular work given a particular issue depends only on the individual judgments about that work given that issue. This *independence* property can be broken into two strong axioms, *work-independence* and *issue-independence*. A rule is *work-independent* if the determination as to whether a particular work is obscene depends only on the opinions about that particular work.

Work-Independence: If there exists  $w \in W$  and  $S, S' \in \mathcal{S}^N$  such that  $w \in S_{ij}$  if

and only if  $w \in S'_{ij}$  for all  $i \in N$  and  $j \in M$ , then  $w \in f_j(S)$  if and only if  $w \in f_j(S')$ .

A rule is *issue-independent* if the collective judgment for each issue depends only on the opinions about that issue.

**Issue-Independence:** If there exists  $j \in M$  and  $S, S' \in \mathcal{S}^N$  such that  $S_{ij} = S'_{ij}$  for all  $i \in N$ , then  $f_j(S) = f_j(S')$ .

It has long been known that when there is only a single issue (m = 1) and the set of works is finite, the unanimity rule is the unique rule satisfying homogeneity, responsiveness, anonymity, neutrality, and work-independence. (Monjardet, 1990; Nehring and Puppe, 2006). If there are multiple issues (m > 1), then it is clear that unanimity rule would be the unique rule satisfying these five axioms and issue-independence. In the infinite setting described in subsection 2.1, neither of these strong independence axioms must be assumed, but both are implied by the combination of homogeneity, responsiveness, anonymity, and neutrality.

Given this prior result, a natural question is whether either independence axiom is somehow implied by the model or some (non-full) subset of the axioms. The answer to this question is no — while all four axioms together are sufficient to imply work-independence and issue-independence, all four are also necessary to rule out non-independent rules. **Theorem 2.2.** The combination of the homogeneity, responsiveness, anonymity, and neutrality axioms is sufficient to imply work-independence and necessary to exclude rules that violate work-independence.

**Theorem 2.3.** Let  $m \ge 2$ . The combination of the homogeneity, responsiveness, anonymity, and neutrality axioms is sufficient to imply issue-independence and necessary to exclude rules that violate issue-independence.

# **3** Other results

#### 3.1 Finite Set of Works

In the previous section I assumed that the set of works is continuous and that each judgment must be of less than full measure. In this subsection I examine the implications of this assumption by allowing W to be finite and requiring only that there be at least one non-obscene work.

Consider the model specified in Section 2.1, with the following changes. Let  $\mathcal{W}$  describe an infinite set of works, and let  $W \subseteq \mathcal{W}$  be a collection of works. For each  $W \subseteq \mathcal{W}$ , let  $\mathcal{J}_W \equiv 2^W \setminus W$  be the set of non-full subsets of W, and let  $\mathcal{S}_W \equiv \mathcal{J}_W^N$  be the set of standards over W. For each  $W \subseteq \mathcal{W}$ , let  $f^W : \mathcal{S}_W^N \to \mathcal{S}_W$  be a function mapping from an N-vector of standards into a single standard. Let  $\Phi_W$  denote the set of permutations of W.

The axioms all have natural analogues in this setting, where f is replaced by  $f^W$ ,

S is replaced by  $S_W$ , and  $\Phi$  is replaced by  $\Phi_W$ . The following characterization of the unanimity rule follows directly from Monjardet (1990) and Nehring and Puppe (2006).<sup>14</sup>

**Theorem 3.1.** The unanimity rule is the only rule that satisfies homogeneity, anonymity, neutrality, work-independence, and issue-independence. Moreover, all five axioms are independent.

Proof. Issue-independence and work-independence imply that, for each issue  $j \in M$ and each work  $w \in W$ , there exists a group of coalitions  $G_{jw} \subseteq 2^N$  such that  $w \in f_j^W(S)$  if and only if  $\{i \in N : w \in S_{ij}\} \in G_{jw}$ . Neutrality implies that there exists a single such group of coalitions  $G_j$  for each issue j such that  $G_j = G_{jw}$  for all  $w \in W$ . Anonymity implies that there is a collection of quotas,  $Q_j \subseteq \{0, ..., n\}$ , such that  $w \in f_j^W(S)$  if and only if  $|\{i \in N : w \in S_{ij}\}| \in Q_j$ . Homogeneity implies that  $Q_j \neq \{0, ..., n\}$ .

Let  $j \in M$ , let  $x \in \{0, ..., n-1\}$ , and let  $S \in \mathcal{S}_W^N$  such that, for all  $w \in W$ ,  $|\{i \in N : w \in S_{ij}\}| = x$ . Then  $f_j^W(S) = \emptyset$  if  $x \in Q_j$  and  $f_j^W(S) = W$ , otherwise. Clearly  $f_j^W(S) \neq W$  and therefore  $\{0, ..., n-1\} \subseteq Q_j$ . Because  $Q_j \neq \{0, ..., n\}$  it follows that  $f^W(S) = \prod_{i \in N} S_i$ .

The independence of the axioms is proved in the appendix.

Without the independence axioms, the four axioms of homogeneity, responsive-

<sup>&</sup>lt;sup>14</sup>Both Monjardet (1990) and Nehring and Puppe (2006) used stronger axioms which additionally included responsiveness. However, as I show in the proof, responsiveness is implied by the other five axioms.

ness, anonymity, and neutrality are not by themselves sufficient to characterize the unanimity rule. The other rules that satisfy these axioms have a special property their outcomes differ from the unanimity rule outcome only when individuals consider a very small number of works to be non-obscene.

To formalize this concept, let  $S_{W^{mn}} = \{S \in S_W : |W \setminus S_j| \ge m * n \text{ for all } j \in M\}$ be the set of standards in which each individual considers at least m \* n works to be acceptable for each issue. A rule has the *MN-Property* if, whenever each individual considers at least m \* n works to be acceptable for each issue, the outcome coincides with the unanimity rule outcome.

**MN-Property:** For each  $S \in \mathcal{S}_{W^{mn}}^N$ ,  $f^W(S) = \sqcap_{i \in N} S_i$ .

The four axioms are sufficient to imply the MN-Property.

**Lemma 3.2.** If an aggregation rule satisfies homogeneity, responsiveness, anonymity, and neutrality, then it satisfies the MN-Property.

Proof. Let  $S \in \mathcal{S}_{W^{mn}}^N$ . Let  $S' \equiv (\prod_{i \in N} S_i)^N$ , the N-vector for which each element is  $\prod_{i \in N} S_i$ . Clearly,  $S' \sqsubseteq S$ . By homogeneity,  $f^W(S') = \prod_{i \in N} S_i$ . Responsiveness implies that  $\prod_{i \in N} S_i \sqsubseteq f^W(S)$ .

Let  $w \notin S_{11}$ . To show that  $f^W(S) \sqsubseteq \sqcap_{i \in N} S_i$ . it is sufficient to show that  $w \notin f_1^W(S)$ . Let  $S^*$  be a profile such that (a)  $w \notin S_{11}^*$ , (b)  $S_{ik}^* \cup S_{jl}^* = W$  unless i = jand k = l, (c)  $|W \setminus S_{ij}^*| = 1$  for all  $i \in N$  and  $j \in M$ , and (d)  $S \sqsubseteq S^*$ . Note that such a profile  $S^*$  is guaranteed to exist for all  $S \in S_{Wmn}^N$ . Anonymity and neutrality imply that either  $(W \setminus \square_{i \in N} S_i^* 1) \sqsubseteq f_1^W(S^*)$  or (2)  $(W \setminus \square_{i \in N} S_i^* 1) \sqcap f_1^W(S^*) = \emptyset$ . Because  $\square_{i \in N} S_i^* \sqsubseteq f^W(S^*)$ , (1) would imply that  $f_1^W(S^*) = W \notin J_W$ , therefore (2) must be true, implying that  $w \notin f_1^W(S^*)$ . Because  $S \sqsubseteq S^*$ , responsivness implies that  $f_1^W(S) \sqsubseteq f_1^W(S^*)$ , and therefore  $w \notin f_1^W(S)$ .

Lemma 3.2 explains why the axioms imply one result in the continuous model and another in the finite model. Any rule that satisfies the four axioms will coincide with unanimity rule when the set of non-obscene works is "large" relative to the number of individuals and issues — and not relative to the size of the entire set of works. In the continuous case, a set F with measure  $\mu(F) = \frac{\mu(W)}{100}$  is, in some sense, the same relative size as a finite single-element set G out of a hundred-element set W. Both F and G are one percent of the whole. However, while G has one element, F has uncountably many elements, and thus only F is large relative to any integers n and m. Similarly, if W were countably infinite and the set of non-obscene works was also required to be countably infinite, the four axioms would imply unanimity rule.

For every work, it is reasonable to assume that there are similar works about which every individual would feel exactly the same way. Take a painting and add a small spot of blue paint; there is probably a place on the painting (or picture frame) where the spot would not affect any individual's judgment about the painting.

Formally, we can describe the set of similar issues in the following way. For each  $W \subseteq \mathcal{W}$ , let  $W' \subseteq \mathcal{W}$  be a "similar" set of works, so that |W| = |W'| and  $W \cap W' = \emptyset$ . For each  $w \in W$  let  $w' \in W'$  denote its counterpart. Let  $\psi : \mathcal{S}_W \to \mathcal{S}_{W \cup W'}$  be the replication function such that  $w \in S_{ij}$  if and only if  $w, w' \in \psi(S)_{ij}$ . For each  $S \in \mathcal{S}_W^N$ , let  $\psi(S) = (\psi(S_1), ..., \psi(S_n))$ . For a set  $W \in \mathcal{W}$ , let  $\psi(W) = W \cup W'$ .

A natural requirement is that the community standard preserve replications. For a given profile S,  $f^{\psi(W)}(\psi(S))$  is the community standard derived from the replicated profile, and  $\psi(f^W(S))$  is the community standard derived from the profile and then replicated. The next axiom, *replication invariance*, requires that these two community standards be the same.

**Replication Invariance:** For each  $W \subseteq W$  and  $S \in \mathcal{S}_W^N$ ,  $\psi(f^W(S)) = f^{\psi(W)}(\psi(S))$ .

Replication invariance, when combined with the other four axioms, is sufficient to characterize the unanimity rule without a direct assumption of independence.

**Theorem 3.3.** An aggregation rule satisfies homogeneity, responsiveness, anonymity, neutrality, and replication invariance if and only if it is unanimity rule. Furthermore, the five axioms are independent.

Proof. Let  $S \in \mathcal{S}_W^N$  and let  $z \equiv \min \{x \in \mathbb{N} : x \ge \log_2(m * n)\}$ . For all x > 1, let  $\psi^x(S) = \psi(\psi^{x-1}(S))$ . Repeated application of the replication invariance axiom implies that  $\psi^z(f^W(S)) = f^{\psi^z(W)}(\psi^z(S))$ . Because  $\psi^z(S) \in \mathcal{S}_{\psi^z(W)^{mn}}^N$ , it follows from Lemma 3.2 that  $f^{\psi^z(W)}(\psi^z(S)) = \prod_{i \in N} \psi^z(S_i)$ . Therefore,  $f^W(S) = \prod_{i \in N} S_i$ .

The independence of the axioms is proved in the appendix.  $\Box$ 

## 3.2 Ordered Works

The neutrality axiom implicitly assumes that there is no objective ordering on the set of works. No objective method to compare works (with respect to obscenity) has ever been developed by courts or by commentators. However, there are circumstances in which this assumption might appear to be too strong. In this subsection I consider the case where there is only a single issue, and the set of works is simply the real line. The non-obscene sets are taken to be open convex intervals of the real line, with the interpretation that if x and y are non-obscene, then  $z \in [x, y]$  should also be non-obscene.

Consider the model specified in Section 2.1, with the following changes. Let the set of works  $W = \mathbb{R}$  be the real line, and let  $\mathcal{J}$  denote the set of convex open intervals in  $\mathbb{R}$ . Here elements of  $\mathcal{J}$  correspond to judgments about which works are non-obscene or permissible. To simplify the model, let m = 1. Let  $\Phi$  denote the set of strictly monotonic mappings  $\phi : \mathbb{R} \to \mathbb{R}$ . When  $S \sqsubseteq T$  I write that T is as permissive as S. A rule  $f^*$  is the *least permissive* if, for every rule f and all profiles  $S \in \mathcal{S}^N$ ,  $f^*(S) \sqsubseteq f(S)$ .

The *median-rule* is the rule in which the highest and lowest endpoints of the set of works considered non-obscene by the community standard are the median highest and median lowest in the community. (If n is even, then the median-rule uses the  $\frac{n}{2}$ <sup>th</sup> highest and lowest endpoints.

Median-rule: For all  $S \in \mathcal{S}^N$ 

$$f^{med}(S) = \left\{ x \in \mathbb{R} : |\{i \in N : S_i \cap [x, \infty) \neq \emptyset\}|, |\{i \in N : S_i \cap (-\infty, x] \neq \emptyset\}| \ge \frac{n}{2} \right\}.$$

The median-rule is one of many rules that satisfies the four axioms in this setting. However, every other rule is more permissive than the median-rule.

**Proposition 3.4.** The median-rule is the least permissive rule that satisfies homogeneity, responsiveness, anonymity, and neutrality.

Proof. I first show that the median-rule satisfies the four axioms. To show that the median-rule satisfies homogeneity, let S' be a standard, let  $S \equiv (S')^N$ . If  $w \notin S_1$ , convexity implies that either  $S_i \cap [w, \infty) = \emptyset$  for all  $i \in N$  or that  $S_i \cap (-\infty, x] = \emptyset$  for all  $i \in N$  which implies that  $w \notin f^{med}(S)$ . If  $w \in S_1$ , convexity implies that either  $S_i \cap [w, \infty) \neq \emptyset$  for all  $i \in N$  or that  $S_i \cap (-\infty, x] \neq \emptyset$  for all  $i \in N$  which implies that  $x \notin f^{med}(S)$ . If  $w \in S_1$ , convexity implies that either  $S_i \cap [w, \infty) \neq \emptyset$  for all  $i \in N$  or that  $S_i \cap (-\infty, x] \neq \emptyset$  for all  $i \in N$  which implies that  $w \notin f^{med}(S)$ .

To show that the median-rule satisfies responsiveness, let  $S \sqsubseteq T$  and let  $w \in f^{med}(S)$ . I will show that  $w \in f^{med}(T)$ . That  $w \in f^{med}(S)$  implies that both  $|\{i \in N : S_i \cap [w, \infty) \neq \emptyset\}| \ge \frac{n}{2}$  and  $|\{i \in N : S_i \cap (-\infty, w] \neq \emptyset\}| \ge \frac{n}{2}$ . Because  $S_i \sqsubseteq T_i$  for all  $i \in N$ ,  $S_i \cap [w, \infty) \neq \emptyset$  implies that  $T_i \cap [w, \infty) \neq \emptyset$  and  $S_i \cap (-\infty, w] \neq \emptyset$  implies that  $T_i \cap (-\infty, w] \neq \emptyset$ . It follows that  $w \in f^{med}(T)$ .

To show that the median-rule satisfies anonymity is trivial. To show that the median-rule satisfies neutrality, let  $S \in \mathcal{S}^N$  and  $\phi \in \Phi$ . It is sufficient to show that either condition (a)  $S_i \sqcap [x, \infty) \neq \emptyset$  if and only if  $\phi(S_i) \sqcap [\phi(x), \infty) \neq \emptyset$  and  $S_{ij} \sqcap (-\infty, x] \neq \emptyset$  if and only if  $\phi(S_i) \sqcap (-\infty, \phi(x)] \neq \emptyset$ , or condition (b)

 $S_i \sqcap [x, \infty) \neq \emptyset$  if and only if  $\phi(S_i) \sqcap (-\infty, \phi(x)] \neq \emptyset$  and  $S_i \sqcap (-\infty, x] \neq \emptyset$  if and only if  $\phi(S_i) \sqcap [\phi(x), \infty) \neq \emptyset$  must be true for all  $x \in \mathbb{R}$  and  $i \in N$ .

Let  $i \in N$  and  $x \in \mathbb{R}$ . If  $x \in S_i$  then trivially  $\phi(x) \in \phi(S_i)$  and the conditions hold. If  $x \notin S_i$ , it must be true that (1)  $S_i \sqcap [x, \infty) \neq \emptyset$  or (2)  $S_i \sqcap (-\infty, x] \neq \emptyset$  but not both.

First, assume that x > y implies that  $\phi(x) > \phi(y)$ . If (1), let  $y \in S_i \sqcap [x, \infty)$ . Because  $\phi(y) > \phi(x)$  it follows that  $\phi(y) \in S_i \sqcap [\phi(x), \infty)$  and  $\phi(y) \notin S_i \sqcap (-\infty, \phi(y)]$ and (a) holds. If (2), let  $z \in S_i \sqcap (-\infty, x]$ . Because  $\phi(z) < \phi(x)$  it follows that  $\phi(z) \in S_i \sqcap [\phi(z), \infty)$  and  $\phi(z) \notin S_i \sqcap (-\infty, \phi(z)]$  and (a) holds.

Alternately, assume that x > y implies that  $\phi(x) < \phi(y)$ . If (1), let  $y \in S_i \sqcap [x, \infty)$ . Because  $\phi(y) < \phi(x)$  it follows that  $\phi(y) \notin S_i \sqcap [\phi(x), \infty)$  and  $\phi(y) \in S_i \sqcap (-\infty, \phi(y)]$ and (b) holds. If (2), let  $z \in S_i \sqcap (-\infty, x]$ . Because  $\phi(z) > \phi(x)$  it follows that  $\phi(z) \notin S_i \sqcap [\phi(z), \infty)$  and  $\phi(z) \in S_i \sqcap (-\infty, \phi(z)]$  and (b) holds. This proves that the median-rule satisfies neutrality.

This proves that  $f^{med}$  satisfies homogeneity, responsiveness, anonymity, and neutrality. To complete the proof I must show that if a rule f satisfies homogeneity, responsiveness, anonymity, and neutrality, then  $f^{med}(S) \sqsubseteq f(S)$  for all profiles  $S \in \mathcal{S}^N$ .

Let  $S \in \mathcal{S}^N$ . Let  $t^- \equiv \inf(f^{med}(S))$  and let  $t^+ \equiv \sup(f^{med}(S))$ . For each individual  $i \in N$ , let  $a_i \equiv \inf S_i$  and  $b_i \equiv \sup S_i$ . Note that  $S_i = (a_i, b_i)$ . If  $b_i \neq b_j$  for all  $i, j \in N$ , let  $S^+ \in \mathcal{S}^N$  such that  $S_i^+ = (b_i - \varepsilon, b_i)$ , with  $\varepsilon$  chosen suitably small such that  $S_i^+ \sqcap S_j^+ = \emptyset$  for all  $i, j \in N$  and  $S^+ \sqsubseteq S$ . If there exists  $i, j \in N$  such that  $b_i = b_j$  then construct the profile  $S^+$  so that  $S_i^+ = (b_i - \varepsilon, b_i), S_j^+ = (b_i - 3\varepsilon, b_i - 2\varepsilon)$ , etc., again with  $\varepsilon$  chosen suitably small such that  $S_i^+ \sqcap S_j^+ = \emptyset$  for all  $i \neq j$ ,  $\sup(S_{i+1}^+) \neq \inf(S_i^+)$ , and  $S^+ \sqsubseteq S$ .

Similarly, if  $a_i \neq a_j$  for all  $i, j \in N$ , let  $S^- \in S^N$  such that  $S_i^- = (a_i, a_i + \varepsilon)$ , with  $\varepsilon$  chosen suitably small such that  $S_i^- \sqcap S_j^- = \emptyset$  for all  $i, j \in N$  and  $S^- \sqsubseteq S$ . If there exists  $i, j \in N$  such that  $a_i = a_j$  then construct the profile  $S^-$  so that  $S_i^- = (a_i, a_i + \varepsilon)$ ,  $S_j^- = (a_i + 2\varepsilon, a_i + 3\varepsilon)$ , etc., again with  $\varepsilon$  chosen suitably small such that  $S_i^- \sqcap S_j^- = \emptyset$ for all  $i \neq j$ ,  $\inf(S_{i+1}^-) \neq \sup(S_i^-)$ , and  $S^- \sqsubseteq S$ .

Let  $S_{(i)}^+$  denote the *i*-th 'highest' element of  $S^+$ , such that i > j implies that x > y for all  $x \in S_{(i)}^+$  and all  $y \in S_{(j)}^+$ . Let  $S_{(i)}^-$  denote the *i*-th 'lowest' element of  $S^-$ , such that i > j implies that x < y for all  $x \in S_{(i)}^-$  and all  $y \in S_{(j)}^-$ . Let  $z = \min\{x \in \mathbb{N} : x \ge \frac{n}{2}\}$ . Note that  $t^+ = \sup(S_{(z)}^+)$ , and that  $t^- = \inf(S_{(z)}^-)$ .

For  $X \subseteq \mathbb{R}$ , let  $conv(X) = \{y \in \mathbb{R} : \text{ there exists } x, z \in X \text{ such that } x \ge y \ge z\}.$ Let  $S^{++} \equiv (conv(\cup_{i \in N} S_i^+), ..., conv(\cup_{i \in N} S_i^+))$ . By homogeneity,  $f(S^{++}) = conv(\cup_{i \in N} S_i^+)$ . Because  $S^+ \subseteq S^{++}$ , responsiveness implies that  $f(S^+) \sqsubseteq conv(\cup_{i \in N} S_i^+)$ .

For all  $i \in N$ , either  $S_{(i)}^+ \sqsubseteq f(S^+)$  or  $S_{(i)}^+ \sqcap f(S^+) = \emptyset$ . To see why, let  $i \in N$ , let  $x, y \in S_{(i)}^+$ , and let  $\phi \in \Phi$  such that, for all  $i \in N$ ,  $\phi\left(\inf\left(S_{(i)}^+\right)\right) = \inf\left(S_{(i)}^+\right)$ ,  $\phi\left(\sup\left(S_{(i)}^+\right)\right) = \sup\left(S_{(i)}^+\right)$ , and where  $y = \phi(x)$ . Then  $\phi(S^+) = S^+$ . By neutrality,  $x \in f(S^+)$  if and only if  $y = \phi(x) \in f(\phi(S^+)) = f(S^+)$ .

Next, for all  $i \in N$ ,  $S_{(i)}^+ \sqsubseteq f(S^+)$  if and only if  $S_{(n-1-i)}^+ \sqsubseteq f(S^+)$ . To see why, let  $i \in N$ , let  $x \in S_{(i)}^+$ , let  $y \in S_{(n-1-i)}^+$ , and let  $\phi \in \Phi$  such that, for all  $i \in N, \ \phi\left(\inf\left(S_{(i)}^{+}\right)\right) = \sup\left(S_{(n-1-i)}^{+}\right), \ \phi\left(\sup\left(S_{(i)}^{+}\right)\right) = \inf\left(S_{(n-1-i)}^{+}\right), \ \text{and where}$  $y = \phi(x).$  Let  $\pi$  be the permutation such that  $\pi(i) = n-1-i$ . Then  $\pi\phi(S^{+}) = S^{+}$ . By anonymity and neutrality,  $x \in f(S^{+})$  if and only if  $y = \phi(x) \in f(\pi\phi(S^{+})) = f(S^{+})$ .

Suppose, contrarivise, that  $S_i^+ \sqcap f(S^+) = \emptyset$  for all  $i \in N$ . This implies that  $f(S^+) \sqsubseteq conv (\sqcup_{i \in N} S_i^+) \setminus (\sqcup_{i \in N} S_i^+)$ . Let  $v \in f(S^+)$ , and without loss of generality, assume that  $v \in \left[ \sup \left( S_{(i+1)}^+ \right), \inf \left( S_{(i)}^+ \right) \right] \right]$  for some i < n. If  $v = \inf \left( S_{(i)}^+ \right)$ , construct a profile  $S^v$  such that  $S_{(j)}^+ = S_{(j)}^v$  for all  $j \neq i$ , let  $S_{(i)}^v \equiv \left( v - \delta, \sup \left( S_{(i)}^+ \right) \right) \right)$  for some small  $\delta$ , and let  $x \in S_{(i)}^+$ . If  $v \neq \inf \left( S_{(i)}^+ \right)$ , construct a profile  $S^v$  such that  $S_{(j)}^+ = S_{(j)}^v$  for all  $j \neq i + 1$ , let  $S_{(i+1)}^v \equiv \left( \inf \left( S_{(i+1)}^+ \right), v + \delta \right)$ , and let  $x \in S_{(i+1)}^+$ . Let  $\phi \in \Phi$  such that, for all  $i \in N$ ,  $\phi \left( \inf \left( S_i^+ \right) \right) = \inf \left( S_i^v \right)$ ,  $\phi \left( \sup \left( S_i^+ \right) \right) = \sup \left( S_i^v \right)$ , and  $v = \phi(x)$ . Then  $\phi(S^+) = S^v$ . Because  $S^+ \sqsubseteq S^v$ , responsiveness implies that  $f(S^+) \sqsubseteq f(S^v)$  and therefore  $v \in f(S^v)$ . Because  $x \notin f(S^+)$ , neutrality implies that  $v = \phi(x) \notin f(\phi(S^+)) = f(S^v)$ . This contradiction proves that there exists  $i \in N$  such that  $S_{(i)}^+ \sqsubseteq f(S^+)$ .

By convexity, if  $i \ge j \ge k$  and  $S_{(i)}^+, S_{(k)}^+ \sqsubseteq f(S^+)$  then  $S_{(j)}^+ \sqsubseteq f(S^+)$ . For all  $i \in N$ either  $i \ge z \ge n-1-i$  or  $n-1-i \ge z \ge i$  which implies that  $S_{(z)}^+ \sqsubseteq f(S^+)$ . Because  $S^+ \sqsubseteq S$  it follows that  $f(S^+) \sqsubseteq f(S)$  and therefore  $S_{(z)}^+ \sqsubseteq f(S)$ . A similar argument can be used to show that  $S_{(z)}^- \sqsubseteq f(S)$ . By convexity,  $conv \left\{ S_{(z)}^-, S_{(z)}^+ \right\} = (t^-, t^+) = f^{med}(S) \sqsubseteq f(S)$ .

#### 3.3 Related issues

I have assumed that the issues in M are not logically related and do not imply one another. That would not be a reasonable assumption if, for example, we were to include three issues, "appeal to prurient interest," "patent offensiveness," and "obscenity." The last issue is the intersection of the previous two.

To describe this formally, consider the model specified in Section 2.1, with the following changes. Let  $M \equiv \{a, b, a \land b\}$ , with the interpretation a= "appeals to the prurient interest", b= "patently offensive", and  $a \land b =$  "obscene". Let  $S \subseteq \mathcal{J}^M$  be the set of standards such that, for all  $S_i \in S$  and  $S_{ia} \sqcap S_{ib} = S_{i(a \land b)}$ .

If we add an additional assumption of issue-independence, this formal setup allows us to remove two unecessary axioms: responsiveness and neutrality. The combination of the issue-independence, homogeneity, and anonymity axioms is sufficient to characterize the unanimity rule. This theorem is related to the doctrinal paradox of Kornhauser and Sager (1986) which was first formalized by List and Pettit (2002).

**Theorem 3.5.** An aggregation rule satisfies homogeneity, anonymity, and issueindependence if and only if it is unanimity rule. Furthermore, the three axioms are independent.

Proof. Issue-independence implies that there are functions  $g_a, g_b, g_{a\wedge b} : \mathcal{J}^N \to \mathcal{J}$ such that, for all  $S \in \mathcal{S}^N$ ,  $f(S) = \left(g_a\left((S_{ia})_{i\in N}\right), g_b\left((S_{ib})_{i\in N}\right), g_{a\wedge b}\left(\left(S_{i(a\wedge b)}\right)_{i\in N}\right)\right)$ such that, for all  $x, y \in \mathcal{J}^N$ ,  $g_a(x) \sqcap g_b(y) = g_{a\wedge b}(x \sqcap y)$ . Furthermore,  $g_{a\wedge b}(x)$  must be responsive. To see why, assume that  $x \sqsubseteq z$ . Clearly,  $g_a(x) \sqcap g_b(z) = g_{a\wedge b}(x) =$   $g_a(z) \sqcap g_b(x)$ . This implies that  $g_{a \wedge b}(x) \sqsubseteq g_a(z) \sqcap g_b(z)$  and therefore  $g_{a \wedge b}(x) \sqsubseteq g_{a \wedge b}(z)$ .

Homogeneity implies that, for all  $x \in \mathcal{J}^N$ ,  $g_a(x) = g_b(x) = g_{a\wedge b}(x)$ . To see why, suppose, contrariwise, that there is an  $x \in \mathcal{J}^N$  such that  $g_a(x) \neq g_b(x)$ . We know that  $g_a(x) \sqcap g_b(x) = g_{a\wedge b}(x)$ , this implies that either  $g_a(x) > g_{a\wedge b}(x)$  or  $g_b(x) > g_{a\wedge b}(x)$ or both. Without loss of generality, assume that  $g_a(x) > g_{a\wedge b}(x)$ . For all  $z \in \mathcal{J}^N$ ,  $g_a(x) \sqcap g_b(z) = g_{a\wedge b}(x \sqcap z)$ . Let  $z \equiv (g_a(x))^N$ , the N-vector for which every element is equal to  $g_a(x)$ . By homogeneity,  $g_b(z) = g_a(x)$  which implies that  $g_a(x) \sqcap g_a(x) =$  $g_a(x) = g_{a\wedge b}(x \sqcap z)$ . But because  $g_{a\wedge b}(x) \ge g_{a\wedge b}(x \sqcap z)$ , this violates the assumption that  $g_a(x) > g_{a\wedge b}(x)$  and proves that, for all  $x \in \mathcal{J}^N$ ,  $g_a(x) = g_b(x)$ . Therefore,  $g_a(x) = g_{a\wedge b}(x)$ . Let  $g(x) \equiv g_a(x)$ .

Let  $x \in \mathcal{J}^N$ , and let  $\pi$  be the permutation such that  $\pi(n) = 1$  and, for all i < n,  $\pi(i) = i + 1$ . By anonymity,  $g(x) = g(\pi x)$ . It follows that  $g(x) = g(x) \sqcap g(\pi x) = g(x \sqcap \pi x)$ . By induction, this implies that  $g(x) = g(x \sqcap \pi x \sqcap \pi \pi x \sqcap \dots) = g(\sqcap_{i \in N} x_i, \dots, \sqcap_{i \in N} x_i)$ . From homogeneity it follows that  $g(x) = \sqcap_{i \in N} x_i$  which implies that for all  $S \in \mathcal{S}^N$ ,  $f(S) = (\sqcap_{i \in N} S_{ia}, \sqcap_{i \in N} S_{ib}, \sqcap_{i \in N} S_{i(a \land b)}) = \sqcap_{i \in N} S_i$ .

The independence of the axioms is proved in the appendix.

# 4 Conclusion

I have introduced a new model of community standards used in determining whether potentially obscene material is protected by the free speech and press guarantees of the United States Constitution. In the model, both individual and community standards are taken to be judgments — categorizations of possible works as either "obscene" or "not obscene." Every possible judgment is allowed provided it satisfies the following restriction: neither individuals nor the community may consider all works to be obscene. Community standards are derived systematically from the individual standards. Every possible method of deriving the community standards is considered. The methods are they evaluated according to normative axioms.

The axioms require that the community standard (a) preserve unanimous agreements about the entire standard, (b) become more permissive when all individuals become more permissive, and not discriminate, ex ante, (c) between individuals and (d) between works. Together, these four axioms characterize the unanimity rule, under which a work is deemed obscene when every individual considers it to be obscene. Every other conceivable method of deriving a community standard from individual standards must violate one or more of these axioms. Whether this result is positive or negative depends on the specific interpretation of the model.

If the jury is taken to be a perfectly representative sample of the society, then unanimity rule coincides with the unanimous jury rule, the dominant rule in criminal trials in the United States.<sup>15</sup> Similarly, if we assume that the community consists of all reasonable persons who live in a society, then the result support Lord Devlin's

<sup>&</sup>lt;sup>15</sup>In civil cases, the unanimous jury rule is used in Federal courts, in the District of Columbia, and in twenty-seven states out of fifty. In criminal cases, the unanimous jury rule is used everywhere but Puerto Rico. The correspondence is not perfect, however. The rule generally requires that a jury must unanimously agree to find for either the plaintiff or the defendant. When the jury is not unanimous the result is a mistrial, which is a victory for the defense except that the case can be retried.

argument that community standards are connected to unanimity rule.

However, there are strong reasons for believing that unanimity rule is not always used in the United States. The primary reason is that there are still convictions for obscenity. American society has become much more diverse in the past half-century, even in places generally thought to be conservative bastions. Empirical research supports the claim that many of these convictions are for material considered nonobscene by a many individuals in the relevant communities. (Linz et al., 1991, 1995).

There is an additional problem which occurs if the accused is a member of the community. In most criminal prosecutions the defendant's incentives are generally not aligned with those of the tribunal. Lord Devlin dealt with this problem by allowing the court to infer what the defendant's honest belief would be if the defendant was reasonable and had thought about the act in question. If the defendant's actual views are relevant, then unanimity rule may be unworkable in the United States. The self-incrimination clause of the Fifth Amendment to the United States Constitution prevents the court from asking the defendant to reveal facts (including beliefs) that would lead to conviction.

If, despite this, we decide to press forward with the unanimity rule, and if the relevant community consists of all reasonable individuals within the relevant geographical region, then the unanimity rule could be implemented through a jury instruction. Jurors would be instructed to find a work obscene only if every reasonable person in the community would consider it obscene. However, for this rule to be meaningful, whether a person is deemed 'reasonable must not depend on that persons judgment

If unanimity rule is not used, however, then the law can take one of two paths. First, the law could rely upon a rule that violates one of the four axioms. The rule would not respect unanimous judgments of the society, or convict individuals because society becomes more permissive, or discriminate between individuals or works.

Second, the law could cut the connection between the judgments of individuals in the community and the applicable legal standard. There is nothing, per se, wrong with such an approach. It would, however, represent a total sea change in the approach of the Supreme Court.

# Appendices

# A Proofs

## A.1 Proof of Theorem 2.1: Independence of the Axioms

**Claim.** The homogeneity, responsiveness, anonymity, and neutrality axioms are independent.

*Proof.* I present four rules. Each violates one axiom while satisfying the remaining three. This is sufficient to prove the claim.

**Rule 1:** Consider the degenerate rule in which  $f_j(S) \equiv \emptyset$  for all  $j \in M$  and all

 $S \in \mathcal{S}^N$ . This trivially satisfies the responsiveness, anonymity, and neutrality axioms but violates homogeneity.

**Rule 2:** Consider the rule in which  $f(S) \equiv \bigsqcup_{i \in N} S_i$ , if  $\bigsqcup_{i \in N} S_i \in S$ , and  $\sqcap_{i \in N} S_i$ otherwise. This trivially satisfies the homogeneity, anonymity, and neutrality axioms. To see why it violates responsiveness, let S be a profile such that (a)  $\bigsqcup_{i \in N} S_i = W^M$ for all  $i \in N$ , (b)  $\sqcap_{i \in N} S_i = \varnothing^M$  for all  $i \in N$ , and (c)  $S_{1j} \neq \varnothing$  for all  $j \in M$ . Let  $S^*$ be a profile where  $S_i^* = S_i \sqcap S_1$  for all  $i \in N$ . Clearly  $S^* \sqsubseteq S$ . Because  $\bigsqcup_{i \in N} S_i \notin S$ , it follows that  $f(S) = \varnothing^M$ , while  $f(S^*) = S_1$ . Because  $S_1 \not\sqsubseteq \varnothing^M$  the example shows that this rule violates responsiveness.

**Rule 3:** Consider the rule in which  $f(S) \equiv S_1$  for all  $S \in S^N$ . This trivially satisfies the homogeneity, responsiveness, and neutrality axioms but violates anonymity.

**Rule 4:** Let  $w^* \in W$ . Consider the rule in which, for all issues  $j \in M$ ,  $f_j(S) \equiv (\bigcap_{i \in N} S_{ij}) \cup \{w \in W : w \in \bigcup_{i \in N} S_{ij} \text{ and } w = w^*\}$ . This trivially satisfies the homogeneity, responsiveness, and anonymity axioms but violates neutrality.

#### A.2 Proof of Theorem 2.2

*Proof.* Any rule that satisfies the four axioms is necessarily unanimity rule, which satisfies work-independence. To show that all four axioms are necessary to exclude rules which violate work-independence, I provide four rules. Each violates one of the four axioms in addition to work-independence.

**Rule 1:** Consider the degenerate rule in which, for all  $j \in M$ ,  $f_j(S) \equiv \prod_{i \in N} S_{ij}$  if

 $\mu(\bigcap_{i \in N} S_{ij}) > 0$ , else  $f_j(S) \equiv \emptyset$ . This trivially satisfies the responsiveness, anonymity, and neutrality axioms but violates homogeneity and work-independence.

**Rule 2:** Consider the rule in which  $f(S) \equiv \bigsqcup_{i \in N} S_i$ , if  $\bigsqcup_{i \in N} S_i \in S$ , and  $\sqcap_{i \in N} S_i$ otherwise. This satisfies homogeneity, anonymity, and neutrality but violates responsiveness and work-independence..

**Rule 3:** Consider the rule in which  $f_j(S) \equiv \bigcap_{\{k \in N: \mu(S_{1j} \cup S_{kj}) < \mu(W)\}} S_k$  for all  $S \in S^N$ . This satisfies homogeneity, responsiveness, and neutrality but violates anonymity and work-independence.

**Rule 4:** Let  $w', w^* \in W$ . Consider the rule in which, for all issues  $j \in M$ ,  $f_j(S) \equiv (\bigcap_{i \in N} S_{ij}) \cup \{w \in W : \{w, w'\} \sqsubseteq \bigcup_{i \in N} S_{ij} \text{ and } w = w^*\}$ . This trivially satisfies the homogeneity, responsiveness, and anonymity axioms but violates neutrality and work-independence.

#### A.3 Proof of Theorem 2.3

*Proof.* Any rule that satisfies the four axioms is necessarily unanimity rule, which satisfies issue-independence. To show that all four axioms are necessary to exclude rules which violate work-independence, I provide four rules. Each violates one of the four axioms in addition to issue-independence.

**Rule 1:** Consider the degenerate rule in which, for all  $j \in M$ ,  $f_j(S) \equiv \prod_{i \in N} S_{i1}$ . This trivially satisfies the responsiveness, anonymity, and neutrality axioms but violates homogeneity and issue-independence. **Rule 2:** Consider the rule in which, for all  $j \in M$ ,  $f_j(S) \equiv \sqcap_{i \in N} S_{ij} \sqcap$ 

 $\{w \in W : \text{ for all } j \in M, \{i \in N : w \in S_{ij}\} \in \{\emptyset, N\}\}$ . This trivially satisfies the homogeneity, anonymity, and neutrality axioms but violates responsiveness and issueindependence.

**Rule 3:** Consider the rule in which  $f_1(S) \equiv \bigcap_{i \in N} S_{i1}$  and, for j > 1,  $w \in f_j(S)$  if and only if  $w \in S_{1j}$  and  $w \in S_{kj}$  for all  $k \in \{i \in N : w \in S_{i1} \text{ if and only if } w \in S_{11}\}$ . This trivially satisfies the homogeneity, responsiveness, and neutrality axioms but violates anonymity and issue-independence.

**Rule 4:** Consider the rule in which  $f_1(S) \equiv (\bigcap_{i \in N} S_{ij}) \cup \{w \in W : w \in \bigcup_{i \in N} S_{ij} \text{ and } w = w^*\}$ and, for j > 1,  $f_j(S) \equiv \bigcap_{i \in N} S_{ij}$ . This trivially satisfies the homogeneity, responsiveness, and anonymity axioms but violates neutrality and work-independence.  $\Box$ 

#### A.4 Proof of Theorem 3.1: Independence of the Axioms

**Claim.** The homogeneity, anonymity, neutrality, work-independence, and issue-independence axioms are independent.

*Proof.* I present five rules. Each violates one axiom while satisfying the remaining four. This is sufficient to prove the claim.

**Rule 1:** Consider the degenerate rule in which  $f_j^W(S) \equiv \emptyset$  for all  $j \in M$  and all  $S \in \mathcal{S}^N$ . This trivially satisfies the anonymity, neutrality, work-independence, and issue-independence axioms but violates homogeneity.

**Rule 2:** Consider the rule in which  $f^W(S) \equiv S_1$  for all  $S \in \mathcal{S}^N$ . This trivially sat-

isfies the homogeneity, neutrality, work-independence, and issue-independence axioms but violates anonymity.

**Rule 3:** Let  $w^* \in W$ . Consider the rule in which, for all issues  $j \in M$ ,  $f_j^W(S) \equiv (\bigcap_{i \in N} S_{ij}) \cup \{w \in W : w \in \bigcup_{i \in N} S_{ij} \text{ and } w = w^*\}$ . This trivially satisfies the homogeneity, anonymity, work-independence, and issue-independence axioms but violates neutrality.

**Rule 4:** Consider the rule in which  $f^W(S) \equiv \bigsqcup_{i \in N} S_i$ , if  $\bigsqcup_{i \in N} S_i \in S$ , and  $\sqcap_{i \in N} S_i$ otherwise. This trivially satisfies the homogeneity, anonymity, neutrality, and issueindependence axioms, but violates work-independence.

**Rule 5:** Let  $U \equiv \{w \in W : w \in S_{ij} \text{ whenever } w \in S_{kj} \text{ for all } i, k \in N \text{ and } j \in M\}$ . Consider the rule in which  $f_j^W(S) = U \sqcap_{i \in N} S_{ij}$ . This rule clearly satisfies the homogeneity, anonymity, neutrality, and work-independence axioms but violates issueindependence.

#### A.5 Proof of Theorem 3.3: Independence of the Axioms

**Claim.** The homogeneity, responsiveness, anonymity, neutrality, and replication invariance axioms are independent.

*Proof.* I present five rules. Each violates one axiom while satisfying the remaining four. This is sufficient to prove the claim.

**Rule 1:** Consider the degenerate rule in which  $f_j^W(S) \equiv \emptyset$  for all  $j \in M$  and all  $S \in S^N$ . This trivially satisfies the responsiveness, anonymity, neutrality, and replication invariance axioms but violates homogeneity.

**Rule 2:** Consider the rule in which  $f^W(S) \equiv \bigsqcup_{i \in N} S_i$ , if  $\bigsqcup_{i \in N} S_i \in S$ , and  $\sqcap_{i \in N} S_i$ otherwise. This trivially satisfies the homogeneity, anonymity, neutrality, and replication invariance axioms but violates responsiveness.

**Rule 3:** Consider the rule in which  $f^W(S) \equiv S_1$  for all  $S \in \mathcal{S}^N$ . This trivially satisfies the homogeneity, responsiveness, neutrality, and replication invariance axioms but violates anonymity.

**Rule 4:** Let  $w^* \in W$ , and let  $g : \mathcal{W} \to \mathbb{R}$  be a function mapping each element of  $\mathcal{W}$  to a unique element of the real line, such that (a)  $g(w) \geq g(w^*)$  for all  $w \in \bigcup_{i=1}^{\infty} \psi^k(w^*)$  and (b)  $g(w^*) \geq g(w)$  for all  $w \in \bigcup_{w \in W \setminus \{w^*\}} \bigcup_{i=1}^{\infty} \psi^k(w)$ . Without loss of generality, assume that  $\arg \max_{w \in W} g(W \setminus \bigcap_{i \in N} S_{ij}) \in S_{(1)j}$ . Let  $X_j \equiv$  $\{x \in W : g(x) > \max_{w \in W} (W \setminus \bigcap_{i \neq (1)} S_{ij})\}$ , and let  $V_j \equiv \{x \in W : g(x) \geq g(w^*)\}$ . Consider the rule in which, for all issues  $j \in M$ ,  $f_j^W(S) \equiv \bigcap_{i \in N} S_{ij} \sqcup (X_j \sqcap V_j)$ . This satisfies the homogeneity, responsiveness, anonymity, and replication invariance axioms but violates neutrality.

**Rule 5:** Let  $P_j \equiv \{w \in W : |\{i \in N : w \in S_{ij}\}| \ge |\{i \in N : v \in S_{ij}\}|$  for all  $v \in W\}$ . Consider the rule where  $f_j^W(S) \equiv W \setminus P_j$  when  $|W \setminus S_{ij}| = 1$  for all  $i \in N$ , and where  $f_j^W(S) \equiv \prod_{i \in N}$  otherwise. This rule satisfies the homogeneity, responsiveness, anonymity, and neutrality axioms but fails replication invariance.

## A.6 Proof of Theorem 3.5: Independence of the Axioms

**Claim.** The homogeneity, anonymity, and issue-independence axioms are independent.

*Proof.* I present three rules. Each violates one axiom while satisfying the remaining two. This is sufficient to prove the claim.

**Rule 1:** Consider the degenerate rule in which  $f_j(S) \equiv \emptyset$  for all  $j \in M$  and all  $S \in \mathcal{S}^N$ . This satisfies anonymity and issue-independence but violates homogeneity.

**Rule 2:** Consider the rule in which  $f(S) \equiv S_1$  for all  $S \in S^N$ . This satisfies homogeneity and issue-independence but violates anonymity.

**Rule 3:** Let  $w^* \in W$ . Consider the rule in which, for issues  $j \in \{a, b\}$ ,  $f_j(S) \equiv (\bigcap_{i \in N} S_{ij}) \cup \{w \in W : w \in \bigcup_{i \in N} S_{ij} \text{ and } w = w^*\}$ , and where  $f_{a \wedge b}(S) \equiv f_a(S) \sqcap f_b(S)$ . This satisfies homogeneity and anonymity but not issue-independence.

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