Convergence and Divergence in Growth Regressions*

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December 2014

Abstract

This paper presents three new results on growth regressions. First, it uses data on total factor productivity in addition to output, now available from PWT, to estimate convergence of output to productivity. This test improves the estimation of the rate of convergence and shows that it is close to the famous 2 percent measured by Barro. Furthermore, this estimation does not require use of any control variables. Second, by using a regression of productivity on the global frontier we show that productivity, and output per capita, can diverge significantly from the global frontier for many countries. Third, a regression of the estimated coefficients of divergence on various explanatory variables shows that our method can separate the long-run effect of such variables from their overall effect.

Keywords: Economic Growth, Growth Regressions, Global Frontier, Divergence, Convergence.

AEA Classification: O40, O47, O57.

^{*} The views in this paper are those of the authors and do not necessarily represent their affiliated institutions.

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1. Introduction

A major tool in the empirical research on economic growth is growth regressions, which follow the seminal contribution of Barro (1991). Over the years growth regressions have been criticized on various grounds. First, growth regressions use a large set of control variables to estimate convergence, and the choice of such variables seems to be arbitrary. Second, growth regressions find 'conditional convergence,' namely output per capita of each country converges to its own steady state. That implies that the distribution of output per capita across countries should converge to some long-run distribution. But studies that examine directly this distribution find that it diverges over time. Third, the measured rates of convergence in standard growth regressions differ too much across studies. Fourth, the control variables used in growth regressions are also viewed as explanatory variables for economic growth, but their estimated effects do not differentiate between short and long-run effects. This paper tries to overcome these critiques by extending growth regressions in two ways. First, we use data on total factor productivity, in addition to data on output. Second, test an extended model of growth regressions, with an additional assumption on how productivity adjusts to the global frontier. These two extensions enable us to estimate the rate of convergence without using control variables, to reconcile the conflicting findings on convergence and divergence, to explain why standard growth regressions come up with different rates of convergence, and to differentiate between short and long run effects of explanatory variables.

Our contribution is presented as an extension of the canonical growth regression model from the authoritative survey of this literature, Durlauf, Johnson and Temple (2005) (hereafter DJT). According to this model the ratio between output per worker and labor augmented productivity should converge to some long-run value. The speed of convergence is denoted b, which is usually assumed to be equal across countries. Usually growth regressions estimate this coefficient by using only data on output per worker, using additional variables as controls. In this paper we use the new PWT 8 data set, which includes data on labor, capital and labor share in addition to output. This enables

us to calculate productivity within the same data set, and with it we can estimate the coefficient of convergence b without using any control variables.

Next we add to the basic growth regression model a new assumption on how productivity itself changes over time. While previous models assume that productivity grows at a constant rate, we assume that each country's productivity follows the global productivity frontier, or the global technology frontier, in the following way. In the long-run each country follows the global frontier, but might follow it partially. More specifically, in the long-run a country might adopt in each period only d of the new technologies, where d is a country specific parameter that can be equal to or lower than 1. If d is equal to 1 the country's long-run productivity path follows the global frontier fully, but if d is less than 1, this long-run path diverges away from the frontier. In the short-run, productivity converges at a rate c to this long-run productivity path.

We then embed this assumption in the convergence model and get a dynamic process with two rates of convergence and one rate of divergence. The coefficient b measures convergence of output per worker to productivity, the coefficient c measures convergence of productivity to its long-run path, and the coefficient d measures by how much long-run productivity follows the global frontier. Using this model and using TFP of the US as the global frontier, we estimate for each country these three parameters, b, c and d. Again, these estimations do not use any control variables that are added in previous growth regression. The reason we don't need these control variables is that we use alternative variables, like productivity and the global frontier.

The estimation of these parameters leads to very interesting results. First, the rate of convergence of output to productivity b is between 1 to 2 percent, very close to the number found by Barro (1991). Second, the rate of convergence of productivity to its long-run path c is higher, around 8 percent. Third, the estimated coefficients of long-run following of the global frontier d indeed differ significantly across countries and for many countries are significantly lower than 1. Hence, economic growth in all countries is driven by the global frontier, but some countries follow it fully, while others only partially. These countries are diverging from the frontier in productivity and in output as well. This is the reason why we call d the 'coefficient of divergence.' Hence, our

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¹ A similar assumption is made by Philips and Sul (2007, 2009), but is used differently, as discussed below.

extended growth regression model can account for divergence between countries with d = 1, which follow the frontier fully, and countries with d < 1, which follow it only partially. Thus, this model can reconcile convergence in growth regressions with the findings of global divergence, as reported above.

Furthermore, our differentiation between the two rates of convergence, b for output and c for productivity, can explain why many growth regression studies differ in their estimated rate of convergence. We show that they actually measure a weighted average of b and c, which differ significantly, as described above. Since the weights in this average differ with the averaging of growth rates over studies, by 5 years, 10 years or even 25 years, the average rate of convergence should differ as well.

As explained above, we estimate the growth regression model with data on output, productivity and the global frontier, without using additional control variables. Examples for such control variables, which are used in many growth regressions, are variables that describe geography, education, institutions, ethnic diversity, fiscal policy and similar potential explanations to economic growth. Usually, adding these variables to standard growth regressions as controls is also considered to test whether they affect economic growth. But it cannot tell whether they have a short or a long-run effect. Instead, our extended model enables us to overcome this problem as well. We can test how various explanatory variables affect the country coefficient of divergence *d* and thus, how these variables affect economic growth in the long-run. Indeed, our empirical tests show that some variables, which affect growth in a standard growth regression, have no effect on the long-run rate of growth and vice versa.

This paper is mainly related to the literature of growth regressions, which began with Barro (1991), Mankiw, Romer and Weil (1992), Barro and Sala-i-Martin (1992) and developed into a huge literature.² An excellent summary of growth regressions and their critiques appears in DJT. One of the main critiques of growth regressions has been its arbitrary choice of control variables in the estimation of convergence.³ This issue is dealt with in this paper, where we show that the dynamic parameters can be estimated without any control variables. Another main critique on growth regressions is that 'conditional

² Earlier papers that influenced growth regressions are Baumol (1986) and Kormendi and Meguire (1985).

³ Note the title of Sala-i-Martin (1997), "I Just Ran 2 Million Regressions." See also Durlauf (2009).

convergence' implies that the distribution of output per capita across countries should converge to a limit distribution. In contrast, Bernard and Durlauf (1995, 1996), Quah (1996) and others, found that this distribution diverges over time.⁴ Our paper shows that the two results, conditional convergence and divergence, can be reconciled.

Our extended growth regression model fits a world in which each country does not invent most of its technologies, but adopts them from outside. A country might also adopt only part of the growing set of global technologies. There is a growing literature that tries to explain such partial adoption of technologies, beginning with Krugman (1979).⁵ Recently this view also gained support from data on technology adoption, collected and analyzed by Comin and others.⁶ This paper shows that countries might follow the global frontier only partially. A similar assumption is also made by Phillips and Sul (2007, 2009), but they use it mainly as a critique of growth regressions, while this paper instead embeds this assumption within the growth regression model in order to improve it. Another closely related research is Dowrick and Rogers (2002), who also show that rates of technical change differ significantly across countries, but their paper differs from ours both in methods and in data.

It is important to stress that our extended growth regression model could become relevant only recently, due to data availability. First, initial growth regressions had only 25 years of data, while we use 60 years of data and for some countries even 140 years of data. This enables us to estimate *d*, as the global frontier changes significantly over such a long period, and it could not be done for a much shorter period of time.⁷ Also the data on productivity became available only recently in PPP-adjusted data sets.

The paper is organized as follows. Section 2 presents the extended growth regression model. Section 3 presents dynamic and empirical implications of the extended model. Section 4 describes the estimation and data. Section 5 presents the panel

⁴ See also Pesaran (2007a), Philips and Sul (2007, 2009), Henderson and Russell (2005) and Di Vaio and Enflo (2011). Also related are the 'varying parameters models' by Liu and Stengos (1999), Durlauf *et al.* (2001) and Lee *et al* (1997, 1998).

⁵ See also Parente and Prescott (1994), Zeira (1998), Eaton and Kortum (1999), and Acemoglu, Aghion and Zilibotti (2006).

⁶ Examples are Comin and Hobijn (2010) and Comin and Mestieri (2013).

⁷ The longer data sets indeed enabled extensions of growth regressions from cross-sections to panels. These were also criticized, as in DJT, for the high variability of output relative to the low variability of most explanatory variables. We avoid this criticism by estimating without control variables.

cointegration estimation of convergence and divergence in 1950-2008. Section 6 estimates the panel cointegration model for a group of countries since 1870. Section 7 presents the results of estimation by differences. Section 8 estimates the effects of some explanatory variables on the long-run rate of growth. Section 9 contains various robustness checks. Section 10 summarizes, while the Appendix presents a theoretical model of convergence in an open economy with adjustment costs to investment.

2. The Extended Growth Regression Model

To explain our contributions, we use the canonical representation of the growth regression model, as described in DJT. Assume first that production in country j in period t is described by:

(1)
$$Y(j,t) = F[K(j,t), A(j,t)L(j,t)],$$

where Y(j,t) is output, L(j,t) is labor, K(j,t) is the amount of capital and A(j,t) is total factor productivity (TFP), which is assumed to be, as in DJT, labor augmenting. The function F is a standard CRS production function. Define output per worker in country j at time t as y(j,t) = Y(j,t)/L(j,t) and define efficiency output per worker to be $y^{E}(i,t) = y(j,t)/A(j,t)$, namely the ratio between output per worker and productivity.

Note that in the long-run marginal productivity of capital should be constant, either because it is equal to the subjective discount rate plus the depreciation rate in a closed economy model, or because it equals the global interest rate plus the rate of depreciation in an open economy model. The marginal productivity is constant only if the ratio between the capital-labor ratio and productivity K(j,t)/[L(j,t)A(j,t)] is constant. Note that the efficiency output per worker is:

$$y^{E}(j,t) = F\left[\frac{K(j,t)}{L(j,t)A(j,t)},1\right].$$

Hence, in the long-run the efficiency output per worker should be constant as well. As in DJT we denote this long-run efficiency output per worker by $y^{E}(j,\infty)$.

The basic assumption in the growth regression literature is that the efficiency output per worker converges to its long-run value, $y^{E}(j,\infty)$, through capital adjustment,

⁸ DJT assume a specific production function, Cobb-Douglas. We use a more general version.

and that it converges gradually. There are two possible mechanisms that can explain gradual adjustment of capital and output. One is derived from the Solow model, where capital accumulation is bounded by savings, since the economy is closed. 9 An alternative explanation is that capital is adjusted gradually due to adjustment costs, and it fits open economies as well. The gradual convergence of efficiency output per worker is described by the following log-linearized dynamic equation: 10

(2)
$$\ln y^{E}(j,t) = \{1 - [1 - b(j)]^{t}\} \ln y^{E}(j,\infty) + [1 - b(j)]^{t} \ln y^{E}(j,0).$$

The parameter b(i) measures the rate of convergence of efficiency output to its long-run value. Most growth regressions assume that it is equal across countries. 11 In Appendix 2 we describe an open economy model of economic growth, which yields equation (2). This model also shows that $v^{E}(j,\infty)$ should be considered to be equal across countries and also that b(i) should be around 2%.

In order to estimate the empirical implications of (2) the standard growth regression model adds two more assumptions. First, labor grows at a constant rate n(j):

(3)
$$L(j,t) = L(j,0) \exp[n(j)t],$$

and productivity grows at a constant rate g(j):

(4)
$$A(j,t) = A(j,0) \exp[g(j)t].$$

The rates of growth g(j) and n(j) can differ across countries, but g is usually assumed to be the same across countries. 12

From equations (2), (3) and (4) we derive the following presentation of the average growth rate of country *j* over *T* periods:

(5)
$$\frac{\ln y(j,T) - \ln y(j,0)}{T} = g(j) + \frac{1 - [1 - b(j)]^{T}}{T} \ln A(j,0) + \frac{1 - [1 - b(j)]^{T}}{T} \ln y^{E}(j,\infty) - \frac{1 - [1 - b(j)]^{T}}{T} \ln y(j,0).$$

This is the classical cross-section growth regression. 13 Estimation of this average growth rate over the initial output per worker $\ln y(i,0)$ should yield the rate of convergence b(i).

⁹ The Solow model was used by Mankiw, Romer and Weil (1992) and later by many others, as described in DJT. Barro and Sala-i-Martin (1992) used the Ramsey-Cass model, also of a closed economy.

¹⁰ Equation (2) is exactly the same as equation (1) in DJT, except for approximating $1 - \exp(-b)$ by b.

¹¹ A non-parametric study that differs with this assumption is Henderson (2010).

¹² See DJT.

¹³ This is equation (8) in DJT.

Since g(j), A(j,0) and $y^E(j,\infty)$ are all unobservable, the regressions controls for them by adding other variables like educational attainment, political stability, rate of saving, geographical characteristics, quality of institutions, ethnic diversity, religion, and many more. These additional variables are sometimes called 'explanatory variables,' since they can be viewed as explaining differences in growth rates across countries. Actually, there has been quite a proliferation of such explanatory variables in the literature and their total number has already passed 150. The arbitrary choice of these control variables is one of the main critiques of this literature. Another problem is that the estimated effects of such explanatory variable are on the sum $g(j) + [1 - (1 - b)^T]T^{-1}A(j,0)$ without differentiating between the long run effect on g(i) and the short run effect on A(i,0).

We avoid the problem of choosing control variable by not using equation (5) for estimation. Instead we estimate directly equation (2) as explained below. This is possible since we have data on productivity in addition to output, so we actually have data on efficiency output per worker y^E . This enables us to estimate the rate of convergence b(j) without any control variables. Another point of departure from the standard growth regression model is replacing assumption (4) by a more realistic model of productivity dynamics. Assume that productivity of each country converges to some long-run productivity path gradually. Denote this long-run productivity of country j at time t by: LRA(j,t). Then the convergence to this long-run path is described, similar to (2), by:

(6)
$$\ln A(j,t) - \ln LRA(j,t) = [1 - c(j)]^t [\ln A(j,0) - \ln LRA(j,0)].$$

The coefficient of convergence to the long-run path of productivity is c(j), which is assumed a-priori to differ across countries and also to differ from the coefficient of output convergence, b(j). We therefore now have two separate convergence coefficients, one for physical output and one for productivity, and in this paper we try to estimate both.

We next turn to specify the long-run productivity path. Assume that each country tries to follow the global productivity frontier, which is denoted by F. It can be also viewed as a global technology frontier and it is assumed to grow steadily over time:

(7)
$$\ln F(t) = \ln F(t-1) + g + v(t),$$

where g is the average rate of growth of the frontier and v(t) is a white noise. Assume that in the long-run a country can follow this frontier either fully or partially. It means

that a country follows only d(j) of the frontier, where this coefficient is assumed to be country specific and might be smaller than 1. We do not supply here a full explanation to this assumption, but it will be justified empirically below, where the parameter d(j) is estimated for each country and it is shown, that for many countries it is strictly below 1. We therefore assume that the long-run productivity path of country j satisfies:

(8)
$$\ln LRA(j,t) = a(j) + d(j) \ln F(t).$$

From the dynamic conditions (6) and (8) we derive the overall dynamics of the productivity of country *j*:

(9)
$$\ln A(j,t) - d(j) \ln F(t) =$$

$$= \{1 - [1 - c(j)]^t\} a(j) + [1 - c(j)]^t [\ln A(j,0) - d(j) \ln F(0)]$$

Namely, TFP of country j converges at a rate c(j) to a path which follows the frontier fully or partially. The gradual adjustment of productivity is similar to (2), but it has different economic justifications. One explanation could be that due to costs to adoption of technologies they are adopted gradually, as in Parente and Prescott (1994).

3. Empirical Implications of the Extended Model

3.1 Convergence of Output to TFP

The first empirical implication is the dynamic condition that describes how output adjusts to TFP and it is derived from the basic growth regression assumption, equation (2), by assuming that time 0 is actually t-1:

(10)
$$\ln y(j,t) - \ln A(j,t) - \ln y^{E}(j,\infty) =$$

$$= [1 - b(j)] \left[\ln y(j,t-1) - \ln A(j,t-1) - \ln y^{E}(j,\infty) \right]$$

Equation (10) means that output per worker, in logarithm, converges to a long-run growth path, which is described by: $\ln A(j,t) + \ln y^E(j,\infty)$. Empirically, equation (10) states that the logarithm of output per worker in each country is cointegrated with $\ln A(j,t)$, where the coefficient of cointegration is 1. The error correction coefficient is b(j) and the long-run distance between output per worker and cointegrated productivity is $\ln y^E(j,\infty)$. We therefore examine equation (10) by a cointegration test of $\ln y(j,t)$ on $\ln A(j,t)$. The test is required also because output per worker and productivity are non-stationary. Our hypothesis is that the cointegration coefficient should be similar to all countries and equal

to 1. The error correction coefficient should be the measured rate of convergence of output, which is the rate of convergence all growth regressions try to estimate. In this section we present cointegration as the method of estimation.

3.2. Convergence and Divergence of Productivity

The dynamics of TFP are derived from equation (9) by using t-1 as period 0:

(11)
$$\ln A(j,t) - d(j) \ln F(t) - a(j) =$$

$$= [1 - c(j)] [\ln A(j,t-1) - d(j) \ln F(t-1) - a(j)]$$

Equation (11) implies that productivity converges to a long-run path, which is described by: $d(j) \ln F(t) + a$. This implies that productivity should be cointegrated with the global frontier, where the coefficient of cointegration is d(j). The error correction coefficient is c(j) and the long-run distance between TFP and the cointegrated frontier is a(j). Hence, a cointegration test of productivity $\ln A(j,t)$ on the global frontier $\ln F(t)$ should yield the country parameter d(j) as a coefficient of cointegration and the rate of productivity convergence c(j) as the error correction coefficient.¹⁴

The important result of equation (11) is that despite the convergence of productivity to a path that follows the global frontier, this convergence does not exclude divergence of productivity if the coefficient d is lower than 1. In that case TFP diverges from the global frontier, because the country does not follow the global technology fully, but only partially. This is how convergence and divergence can be reconciled in the growth regression model once we introduce a more dynamic growth path of productivity. This point is further amplified in the next sub-section.

3.3. Convergence and Divergence of Output per Worker

From combining the dynamic conditions (2) and (9) together we get:

(12)
$$\ln y(j,t) - d(j) \ln F(t) = \left\{ 1 - [1 - b(j)]^t \right\} \ln y^E(j,\infty) + \left\{ 1 - [1 - c(j)]^t \right\} a(j) + \\ + [1 - b(j)]^t \left[\ln y(j,0) - \ln A(j,0) \right] + [1 - c(j)]^t \left[\ln A(j,0) - d(j) \ln F(0) \right]$$

Equation (12) shows that the difference between $\ln y(j,t)$ and $d(j) \ln F(t)$ converges in the long run to $\ln y^E(j,\infty) + a(j)$. This implies that output per worker $\ln y(j,t)$ and the global frontier $\ln F(t)$ should be cointegrated and the coefficient of cointegration is d(j), namely the same coefficient from equation (11). Hence, a cointegration test of

The cointegration test also measures a(j), but we do not use it in this paper.

equation (12) should also yield the parameters d(j) for each country. This can therefore be an additional test of the findings of (11). Note also that estimation of (12) does not identify the rates of convergence, b and c, since the error correction coefficient is a combination of these two rates, which changes over time.

Equation (12) further demonstrates how despite the convergence of each country to its long-run growth path, this path itself can diverge from the frontier for country j, if d(j) < 1. As a result such a country diverges also from the countries that follow the frontier fully. Thus, our tests reconcile the convergence found in growth regressions with the results of alternative empirical studies that examined the dynamics of the distribution of output per worker or per capita across countries over time and found divergence.

3.4. Varying Rates of Convergence in Growth Regressions

We next return to the standard growth regression model, as presented by equation (5), but instead of assuming that productivity follows (4), we use our extended model, as summarized by (9). Within the extended model the average growth rate over T periods is equal to:

$$\frac{\ln y(j,T) - \ln y(j,0)}{T} = \frac{1 - [1 - b(j)]^{T}}{T} \ln y^{E}(j,\infty) + \frac{1 - [1 - c(j)]^{T}}{T} [a(j) + d(j) \ln F(0)] + d(j)g + d(j) \frac{\sum_{1}^{T} v(\tau)}{T} - \frac{1 - [1 - b(j)]^{T}}{T} \ln y(j,0) + \frac{[1 - c(j)]^{T} - [1 - b(j)]^{T}}{T} \ln A(j,0).$$

If our extended model is the right one, then equation (13) implies that the regression coefficient of initial output $\ln y(j,0)$ reflects not only b, but also c in the coefficient of productivity A, since productivity is strongly correlated with output per worker across countries. If the regression coefficient of $\ln A(j,0)$ on $\ln y(j,0)$ in a cross-country regression in period 0 is R < 1, then the estimated coefficient of $\ln y(j,0)$ in (13) is actually equal to:

$$COEFF = \frac{-1 + (1 - R)(1 - b)^{T} + R(1 - c)^{T}}{T}.$$

Hence, this regression yields an average of b and c and it does not estimate exactly b. To clarify it calculate the measured rate of convergence, \widetilde{b} :

(14)
$$1 - \widetilde{b} = \left[1 + TCOEFF\right]^{\frac{1}{T}} = (1 - b) \left[1 - R + R\left(\frac{1 - c}{1 - b}\right)^{T}\right]^{\frac{1}{T}}.$$

Assume that convergence of productivity is more rapid than convergence of output, namely c > b. Then, if the regression dependent variable is the growth rate averaged over a short period, like T = 1, we get that $\widetilde{b} = (1 - R)b + Rc$, namely the estimated rate of convergence is much higher than the actual b. If on the contrary growth rates are averaged over a long period and T is high, then the estimated rate of convergence \widetilde{b} is close to the actual b.

This analysis shows that if our model is the right one, the rate of convergence, which is estimated in standard growth regressions, should be sensitive to the length of period of averaging growth rates. This holds either in a cross-country regression or in a panel regression. It is well known that the length of period changes significantly across studies of growth regressions from one year to more than 20 years. It is also known, as shown in DJT, that studies of growth regressions have found different rates of convergence. We raise here the possibility that these differences might have been the result of not differentiating between the convergence of output and the convergence of productivity, which is manifested when using different averaging of growth rates. Unlike standard growth regressions, our equations (10) and (11) enable us to estimate directly and separately b(j) and c(j) for each country and to overcome this problem.

4. Estimation of the Extended Model

In the empirical part of the paper we estimate for a large set of countries the dynamic conditions (10), (11) and (12) and derive for each country the coefficients b, c and d. These country coefficients are estimated by the method of panel cointegration, but we also use estimation of differences in some regressions for robustness. Finally, we test how the coefficients d of the various countries depend on standard explanatory variables, which are used in many growth regressions. This test is performed to show that our model can isolate the long-run growth effect of these variables from the overall effect. In this section we focus mainly on the estimation of the dynamic parameters, which requires data on output and on total factor productivity.

We use the new merged data of the Penn World Table and the Groningen Growth and Development Centre, PWT 8, as described in Feenstra, Inklaar and Timmer (2013). The PWT 8 includes data on output, employment, capital and the share of labor for a large panel of countries. The data set also includes calculated productivity but we prefer to calculate TFP ourselves, for two reasons. First, the productivity levels computed in WPT 8 use development accounting, namely they calculate productivity without human capital. This does not fit our model, since we want to examine the effect of total factor productivity on output, and that should include human capital as well. Second, we follow DJT in equation (1) by assuming that productivity is labor augmenting. This requires a slightly different method of calculating TFP growth rates from the standard Solow Growth Accounting. This method is described in Appendix 1 and as a result the rate of growth of total factor productivity when it is labor augmenting is described by the following equation:

(15)
$$\frac{A(j,t) - A(j,t-1)}{A(j,t-1)} = \frac{1}{s_L(j,t-1)} \left[\frac{Y(j,t) - Y(j,t-1)}{Y(j,t-1)} - \frac{K(j,t) - K(j,t-1)}{K(j,t-1)} \right] + \frac{K(j,t) - K(j,t-1)}{K(j,t-1)} - \frac{L(j,t) - L(j,t-1)}{L(j,t-1)}.$$

Since PWT 8 contains data on output, capital stock, labor and the labor share s_L for a set of 70 countries over the years 1970-2008 we can use this data to calculate TFP paths for these countries. More specifically, the data we use are real variables in 2005 prices based on national accounts, namely 'rgdpna' for GDP and 'rkna' for the stock of capital. Note that we do not use data after 2008 to avoid the effects of the recent financial crisis.

Using this data set we estimate equations (10) and (11). Since equation (12) does not require data on productivity, but only output per worker or output per capita, we estimate this equation using a wider data set that includes many more countries over a longer time period. In this estimation we use the Groningen-Maddison data set of real GDP per capita, in PPP adjusted Geary-Khamis 1990 US\$, for 139 countries over the

¹⁶ Since we estimate the dynamics of output and productivity and do not compare levels across countries, as in standard growth regressions, we do not PPP adjusted real variables.

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¹⁵ The data span over longer periods for some countries, but we limit our estimations to a balanced panel.

years 1950-2008. For a smaller set of 30 countries the data are much longer and span over 140 years, from 1870 to 2010.

An additional variable needed for our estimation is the global productivity frontier. For this variable we use the US total factor productivity, in estimating equation (11), or the US output per capita, in estimating equation (12). The United States is considered to be a leader of the global economy for a long period of time and its output per capita has grown quite steadily over more than a hundred and forty years and is also the highest among the developed large countries. Figure 1 presents a comparison of US GDP per worker in the blue curve to US TFP (labor augmented) in the red curve, both in natural logarithms. The two curves fit one another quite well, as expected from the discussion above, and they also have a fairly stable slope, namely a stable rate of growth. Hence this variable fits well our assumption (7) on the global frontier.

[Insert Figure 1 here]

To further examine the use of the US as representing the global frontier, we test whether US productivity satisfies equation (6). We run a regression of its growth rate on a constant for the period 1970-2008 and find that the coefficient is equal exactly to the mean growth rate in this period, which is 1.68 percent. We also run a similar test on US GDP per capita, which is used in the test of equation (12), for the periods 1870-2010 and 1950-2010, and find that the constant is equal exactly to the mean growth rate, 1.8% for the entire period 1870-2010 and 1.95% for the sub-period 1950-2010. We also run unit root tests and find that the first differences are stationary, for each sub-period examined.

In the panel cointegration estimation we use 5 years moving averages of output per worker and of TFP to reduce cyclical high-frequency autocorrelations of output and this is done also for the US TFP the global frontier. We therefore calculate for each year the following geometric average:

$$\ln y_5(i,t) = \frac{1}{5} \left[\ln y(i,t) + \ln y(i,t-1) + \ln y(i,t-2) + \ln y(i,t-3) + \ln y(i,t-4) \right]$$

We calculate similarly the averages of TFP of all countries. Equations (10) and (11) remain intact after averaging and hence the estimated coefficients of b, c, and d should

not be affected by averaging.¹⁷ As for equation (12) the averaging affects its error correction coefficient, but in the estimation of this equation we focus only on the coefficient of cointegration d, which is not affect by averaging.

5. Measuring Convergence in 1970-2008

We begin by estimating the dynamic equation (10) by running a panel cointegration test of output per worker on TFP. The panel is balanced and covers 69 countries over the period 1970-2008. The data is taken from WPT 8, as explained above. The results of the estimation are presented in Table 1. The first 6 columns present the results with averaging over 5 years, while the seventh column presents the results from averaging over 10 years. The first column presents the regression results for the whole sample, while the following columns presents average results for different regions. Column 2 presents the results for the OECD countries, column 3 for the Sub-Saharan African countries (SSA), column 4 for Central and South America (CSA), column 5 for East Asia and column 6 for the countries of the Middle East and North Africa (MENA). The panel cointegration excludes two countries, which are outliers, Turkey and Indonesia. In regression 7 with 10 years averages we exclude two outliers, Indonesia and Morocco.

[Insert Table 1 here]

The results of Table 1 fit our model quite well. The average coefficient of cointegration is 1.07, which is very close to 1, as expected by the model. It is quite close to 1 in most regions, except for East Asia, but in all regressions 1 is within the 95% confidence interval. The estimated rate of convergence is 2.3 percent, very close to the original number found by Barro (1991) and Barro and Sala-i-Martin (1992). Note that these original growth regressions used averages of growth rates over a long period of time and thus their estimates should have been close to the true rate of convergence, as explained in sub-section 3.4. An interesting and surprising finding in Table 1 is that the rate of convergence is very similar in all regions, which also shows that this result is

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¹⁷ Actually, due to cyclical autocorrelation it is better to average over a long period of time, but that reduces the number of observations. We find that 5 years averages are a good balance between the two concerns, but we present below, in the section on robustness checks, also results for averaging over 10 years.

robust. It is also interesting to note that the measured rate of convergence is even lower when we use 10 years average of output and productivity and is around 1.6%. Our guess is that this figure should be closer to the true rate of convergence of output, as it removes additional cyclical autocorrelation. Note that our measuring of the rate of convergence does not require any additional control variables.

6. Divergence of Productivity in 1970-2008

We next turn to estimate the dynamic equation (11) using a panel cointegration test of each country's TFP over the global frontier for the same 70 countries over the period 1970-2008. The data are the same from WPT 8. We run this test in order to estimate the divergence coefficient d of each country and the rate of convergence of productivity c. Table 2 presents the results, where the first column contains the results for the full sample, columns 2-6 present the results for the global regions defined above, and column 7 presents the results with averaging over 10 instead of 5 years. In this estimation we removed two countries, Cyprus and Zimbabwe, which were outliers.

[Insert Table 2 here]

The average value of d across the 70 countries is 0.7 and it is significantly lower than 1. This means that our initial hypothesis, that countries might follow the global frontier partially and not fully, is indeed supported strongly by the data. This means that despite the usual convergence found in Section 4, as in all growth regressions, there is also divergence of productivities of countries from the frontier and as a result from one another as well. The divergence of productivities leads, according to the results of Section 4, to divergence of output per worker as well. In other words, the growth regressions find convergence of output to productivity, and of productivity to a long-run productivity path, but this long-run path might diverge from the frontier and thus lead to divergence of output across countries. Note from column 7 that the average value of d does not change much when we move to averaging TFP over 10 years instead of 5 years. Furthermore, in an unbalanced cointegration test, which enables us to include many more countries, actually 121 after excluding Belo-Russia, Tajikistan, Kuwait, Rwanda and

Georgia, we find that the average coefficient of cointegration d is 0.86, which is also lower than 1.¹⁸

A further examination of the results of Table 2 indicates that d follows a regional pattern to some extent. It seems that most of the world lags significantly behind the frontier. Not surprisingly the most miserable region is Sub Saharan African countries with d around 0.1. Namely, these countries catch up only one tenth of the growth of the global frontier year by year. But South and Central American countries are lagging behind as well with a coefficient of divergence smaller than a third. Interestingly, the value of d for East Asia is above 1. This is caused by the famous Asian Tigers: Hong Kong, Korea, Singapore, Taiwan and recently China. These countries went through a rapid 'catch up' through much of the period. Since this process has not ended yet it might be captured in the regression as having d higher than 1. We therefore treat the high values of d in this region with some caution in some of the tests below. Note that in the estimation we do not constrain the coefficient d to be between 0 and 1 as the theory implies. The main reason is the possibility of misspecification in the estimation of (11), especially if the coefficient a is changing during the period of the estimation, which might appear as a different d. We therefore follow Eberhardt and Teal (2013), who claim that unconstrained heterogeneous estimation is preferred, since it reduces bias of average estimates, where the noise created by misspecification at the country-level is filtered out.

Another interesting result of Table 2 is that the value of c, which measures the rate of convergence of productivity to its long-run path, is around 9-10%. This result is robust across regions. It is much higher than the rate of convergence of output to productivity b. This finding reinforces the point made in sub-section 3.4. The estimation of the rate of convergence in standard growth regressions is a weighted average of 2 and 9 percent and as a result it can be anywhere between these two values, depending among other things on how growth rates are averaged. Indeed the estimated rates of convergence in many growth regression studies fluctuate significantly, as shown in a meta-analysis by Abreu, De Groot and Florax (2005). They examine a sample of 610 regressions and find that most of them are between 2% and 10%.

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¹⁸ This test is not reported in Table 2 and is available upon request.

7. Divergence of Output: 1950-2008 and 1870-2010

In this section we test the dynamic condition (12) that describes the convergence of output per worker to long-run productivity, which might diverge from the global frontier. Since this equation does not include productivity, we will be able to estimate it over a much wider set of countries. Note, that the test will enable us to estimate the coefficient d(j) for each country j, although it will not enable us to identify the coefficients b and c as is clear from (12). This is still beneficial, since the above estimations of b and c show that they differ little across countries and regions, while d differs much more. Hence it is worthwhile to extend its estimation over more countries and longer periods of time. In the following estimations we use output per capita and not output per worker in order to keep the number of countries and periods of time as large as possible as employment data are fewer significantly. All the data used from here on are from the Groningen data set, as described in Section 4.

7.1 Output and the Frontier: 1950-2008

In this sub-section we present a panel cointegration of equation (12) for 139 countries over the period 1950-2008 using the Groningen data set, as described above in Section 4. The panel cointegration uses 5 year averages. The results for the whole sample are presented in Table 3. The main result of this table is that most of countries are diverging from the global frontier, since the coefficient d is equal on average to 0.69, it is significantly lower than 1 and it is significantly heterogeneous across countries. Hence, these results, as the results of Section 6, show that convergence and divergence are not contradictory.

[Insert Table 3 here]

We have tested the ADF of cointegration for the various countries and the results came out very supportive. Only for 5 countries the probability of not being cointegrated was higher than 10% and only for 9 countries the probability of not being cointegrated was higher than 7%. Most of these countries suffered from intense conflicts and severe

interruptions of economic activity. We therefore treat these countries as outliers from here on. An additional group of countries that deserves attention are the oil-producing countries, which experienced very high levels of output in the 1970s and declining output since then. Such countries might bias d downward. Table 3 also presents the results of the panel cointegration without the outliers and without the oil-producing countries. The second column in Table 3 shows that removing these countries indeed increases d, but not by much and it is still significantly lower than 1.

[Insert Table 4 here]

Table 4 presents the results by regions and shows that *d* follows a regional pattern to some extent. The regions are the same as before but OECD, SSA, SCA, EA are now joined by the category Other Countries, which stands for MENA and the East European countries. Table 4 paints a clear picture of divergence from the frontier. While the OECD countries follow the frontier fully with *d* very close to 1, and while in South East Asia *d* is higher than 1, around 1.6, which is discussed below, the rest of the world lags behind the frontier. Not surprisingly the most miserable region is South Saharan Africa, but Latin American countries are lagging quite behind as well and so are MENA and East Europe. This supports our main assumption, that *d* is significantly lower than 1 for many countries and it is quite variable across countries. It also fits the results on divergence of productivity in Section 6.

At this point we should discuss the problem caused by the famous Asian Tigers: Hong Kong, Korea, Singapore and Taiwan. These countries went through a rapid 'catch up' through much of the period. In terms of equation (9) this can be viewed as a change in d(i), but it can also be a result of a change in a(i) over these years. In that case it biases the estimates of the parameter d for these countries, which is on average equal to 2.5. Without the Asian Tigers the estimated average of d in the whole sample goes down

¹⁹ The countries not cointegrated with probability above 10% are Bangladesh, Indonesia, Kenya, Laos and Vietnam. The countries with probability between 10% and 7% are Ghana, Cambodia, Nepal and Senegal.

²⁰ Mankiw, Romer and Weil (1992) have eliminated these countries from their analysis.

²¹ We define countries as oil producers if their oil rents exceed 30% of GDP in 1975-2000. The countries are Bahrain, Republic of Congo, Equatorial Guinea, Gabon, Ghana, Kuwait, Libya, Nigeria, Oman, Qatar, Saudi Arabia, and the United Arab Emirates.

to .58 and it is lower for East Asia as well. We therefore treat the high values of d of in this region with some caution in some of the tests below.

7.2 Output and the Frontier: 1870-2010

In this sub-section we extend the analysis further to the past. Our data allow us to examine patterns of economic growth over a longer period of time, 1870-2010, although for a smaller set of 30 countries, in addition to the US.²² Most of these countries are now developed, namely OECD countries. Note that this period has experienced not only significant economic growth, but also two World Wars and the Great Depression. The results of the panel cointegration regression are presented in Table 5.

[Insert Table 5 here]

Table 5 shows that the developed countries indeed followed the global frontier and did not diverge from it. The average d, without Sri Lanka, an outlier in this estimation, is 0.99 and is not significantly different than 1. The only countries in this long-run sample that diverge from the frontier are: Argentina with d equal to 0.61, India with d equal to 0.02, New Zealand with d equal to 0.73, Uruguay with d equal to 0.59, and South Africa with d equal to 0.7. Interestingly, the other Latin American countries in this sample, Brazil, Chile, Columbia, Peru and Venezuela, follow fully the global frontier. We know that Latin American countries have grown better until WWII and much worse later. This might be one reason for the differences between the estimation in 1950-2008 and the estimation over the longer period 1870-2010 in Latin America.

8. Effects of Explanatory Variables on Global Divergence

In this paper we estimate the convergence and divergence of output across countries without using any control variable. But these country specific explanatory variables might have an effect on the parameters b(j), c(j) and d(j). Actually, we have seen that both b(j) and c(j) are quite uniform across countries, but d(j) differs significantly across

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²² The sample for 1870-2010 includes Argentina, Australia, Austria, Belgium, Brazil, Canada, Chile, Colombia, Denmark, Finland, France, Germany, Greece, India, Indonesia, Italy, Japan, Netherlands, New Zealand, Norway, Peru, Portugal, South Africa, Spain, Sri Lanka, Sweden, Switzerland, United Kingdom, Uruguay and Venezuela.

countries. Hence, we suspect that d should depend on some of the explanatory variables, and indeed in this section we run cross-country regressions of d over a set of common explanatory variables and find that it is affected by some of these. Note that the goal of this estimation is not to find the ultimate explanation for divergence across countries. We use this test in comparison with a standard growth regression over the same sample and with the same explanatory variables in order to show that the two sets of regressions yield different results. Namely, the regression of d over the explanatory variables identifies the long-run effect of these variables, which differs from the overall effect on output, which is revealed by standard growth regressions. The goal of this section is only to highlight the ability of our method to isolate the long-run effect of explanatory variables.

In order to achieve this goal we pick a standard set of explanatory variables, which are used in many growth regressions:

- 1. TROPIC is the share of land in a country that is tropical (Gallup *et al.*, 2010).
- 2. COAST is the share of land in a country that is within 100 km from a coast or from a navigable river (Gallup *et al.*, 2010).
- 3. Y_50 is the natural logarithm of the GDP per capita in the country at 1950.
- 4. ETHNIC is a measure for ethnic fractionalization in a country.
- 5. EDU is average years of schooling of people above age 15 over the period 1950-2010 (Barro and Lee, 2013).
- 6. OPEN is a measure of openness of a country. It is a measure of trade policy over the years 1965-1990, which has been introduced by Sachs and Warner (1995).²³
- 7. ICRG is average measure of quality of institutions during the period 1982-1997 according to the International Country Risk Guide (Knack and Keefer, 1995).
- 8. G/Y is the share of public expenditures in GDP, averaged in the years 1950-1960, taken from Feenstra, Inklaar, and Timmer (2013).

Note that variables 1-2 reflect the geographical explanation to growth. Variables 3-4 reflect the history of the country, namely its initial conditions, both economic and

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²³ This is a variable that classifies an economy as closed according to the following five criteria: (i) if its average tariff rate exceeded 40%; (ii) if its non-tariff barriers covered more than 40% of imports; (iii) if it had a socialist economic system; (iv) if it had a state monopoly of major exports; or (v) if its black-market premium exceeded 20% during either the decade of the 1970s or the decade of the 1980s.

social. Variable 5 is human capital and variables 6-8 reflect institutional explanations to economic growth. As mentioned above, these variables were chosen not only because they are used in many growth regressions, but also because they are potentially related to following the global technology frontier, which lies at the heart of this paper. As explained by Sachs (2001), geography is a barrier to technology transfer, since technology might be region-specific, especially in agriculture or health. This is also implied by Parente and Prescott (1994) and by Zeira (1998). Human capital also affects the ability to adopt new technologies, as pointed by Galor and Moav (2000) and Zeira (2009). Institutions are crucial to adoption of technology, as claimed by Acemoglu, Johnson and Robinson (2005) and others, especially institutions that affect international trade, as stressed by Grossman and Helpman (1991).

[Insert Table 6 here]

Before we turn to the direct estimation, we present the matrix of correlations between these variables in Table 6. This table can already give us some preliminary insights into the relationship between these variables and economic performance. For example, being in the tropics is strongly negatively correlated with most other variables, like education and institutions. It is also clear that the quality of institutions is strongly correlated with openness and with initial output. This is probably the reason that some of these variables come out insignificant in the regressions. As a result, we omit in the following analysis the variable ICRG.

The regressions are presented in Tables 7 and 8. The first one, Table 7, presents the results of the estimation where the dependent variable is the average rate of growth over the years 1950-2008, which we denote by AVG. Since Initial output in 1950 is one of the explanatory variables, this is a standard growth regression, and we include it as a point of reference, that enables us to compare it to the results of the other regression. Table 8 presents the regressions with d as the dependent variable. These regressions therefore show how the explanatory variables affect the rate of divergence from the frontier, namely how they affect the long-run rate of growth of a country. We also report in the text on the regressions with e as the dependent variable.

All the regressions in the two tables include constants and are OLS in a crosssection of countries. In each table we present three separate regressions. One is with all the countries for which the data is available without outliers. Data availability reduces the number of countries in the regression to 90. In the next regression we omit the South East Asian countries, and in the third we omit both the SEA countries and the OECD countries. The reasons for these omissions are as following. First, there is a bias in the estimation of d among the SEA countries and it is too high above 1. This is mainly because most of the rapid growth in these countries happened toward the end of the period explored, and thus the cointegration procedure tends to confuse the convergence in these countries with a new trend. Another reason for considering omission of these countries is that they are clearly countries that change their pattern of growth during the period covered by the data. Since they change their d and probably also change their coefficient a, it is preferred not to include these countries when testing for a statistical regularity between explanatory variables and these coefficients. For very different reasons we also find the inclusion of the OECD countries as problematic in the estimation of the effects on d. The main reason is that in these countries d is around 1, which is a corner solution, since in the long-run countries cannot adopt technologies at a higher rate than the frontier. Being at such a corner, therefore, might make these countries insensitive to the explanatory variables. The OECD countries may have more or less education, larger or smaller government, better or worse institutions, but they all have d around 1, since it is a corner solution. Thus, including the OECD countries in the estimation reduces its ability to identify relationships between d and the explanatory variables. Hence, the third regressions in Tables 7 and 8 omit not only the SEA countries, but the OECD countries as well.

[Insert Table 7 here]

Table 7 presents the results of the standard growth regression on this set of explanatory variables for the three sets of countries, full, without SEA and without SEA and OECD. There are 6 variables that are significant throughout, in addition to the constant. These variables are TROPIC, which reduces growth, proximity to coast, which

increases growth, initial output Y_50, which has a negative effect on economic growth as expected, education, which has a positive effect on growth, openness, also with a positive effect on growth and the share of government in GDP, which has a negative effect on economic growth. These results resemble results of many other studies. Ethnic fractionalization appears to be insignificant.

[Insert Table 8 here]

Table 8 presents the effects of the same explanatory variables on the long-run coefficient d and we can compare these results to those in Table 7. One variable that still has a negative significant effect is the tropics. Note that its effect is increasing as we narrow the set of countries in the test. In the most relevant group, without South East Asia and OECD, the effect of TROPIC is around half. Namely being in the Tropics can reduce d by almost 0.5 relative to the developed countries. Hence, this variable can account for much of the divergence of Africa and Latin America. Another variable that affects d positively and significantly is openness. Hence, it has a significant positive effect on growth both in the short and in the long-run. But for the other variables the results in Table 9 differ significantly from the results in Table 8. Initial output becomes less and less significant as we narrow the sample. Education and the share of government in output become insignificant once we begin to narrow the set of countries. The result on education is quite surprising.²⁴ One possible interpretation is that education affects only the level of output but not its long-run rate of growth. Another interesting result is that although ethnic fractionalization does not affect growth in the standard growth regression, it has a negative significant effect on d, namely on long-run growth.

Thus, Tables 7 and 8 demonstrate that the dynamic estimation suggested in this paper enables us to differentiate between short-run and long-run effects of various explanatory variables on economic growth. Of the variables used in this section, which is to a large extent an arbitrary set of variables, we found that initial output, education and fiscal policy do have an effect on economic growth in a standard growth regression, but do not have a significant effect on the long-run rate of growth, while ethnic

²⁴ For similar results and a more through analysis of the effect of education on growth see Delgado, Henderson, and Parmeter (2014).

fractionalization does not have a significant effect in a growth regression, but tends to have a significant effect on long-run growth.

9. Conclusions

Durlauf (2009) claims that one of the problems of early growth regressions was that they were used as empirical tests to judge between two conflicting theories, neoclassical growth and endogenous growth. He is right of course, because endogenous growth theory focuses mainly on global technical change, while growth regressions test economic growth across countries. Thus they are not really comparable. But this paper claims that the two phenomena, global technical change and individual countries' growth performances, are strongly related, because each country adopts global technologies. The big question is how much.

In a world where the global technology expands continuously and countries can choose whether to adopt a new technology or not, the growth path of each country reflects, among other things, how much it follows the global technology frontier. The main claim in this paper, hence, is that growth regressions should include the global technology frontier. Note, that we do not criticize previous studies for not including this variable, since previous data spanned over much shorter time periods, during which changes in the global frontier were relatively small. Nowadays, that the data cover much longer time periods, the global frontier cannot be left outside any more.

This paper adds the global technology frontier in growth regressions by specifying explicitly how a country might adopt global technologies either fully or partially. We then estimate this specification and find that most countries run more slowly than the frontier. We show that this finding reconciles the results of growth regressions with the results on the dynamics of the distribution of output across countries. Another benefit of our approach is that we can estimate convergence and divergence without controlling for explanatory variables, which avoids some of the critiques on growth regressions. We also show how our approach can help in separating the effects of explanatory variables on growth into long-run and short-run effects. Our regressions demonstrate that this difference is significant.

A second innovation in this paper for growth regressions is the explicit use of data on productivity. Since the main prediction of the growth regression model, as presented for example in DJT, is that output converges to productivity, once we use data on productivity we can measure the rate of convergence much more accurately. Here again we benefit from the new availability of data. The recent PWT 8 contains data on capital and labor share, which enables us to use measures of productivity and of output per worker that are uniform, as they are produced in the same data set.

This paper is of course quite preliminary and might lead in the future to many potential research lines. One possible direction can be estimation of the dynamic coefficients by use of alternative methods to panel cointegration like differences, non-parametric estimation, rolling regressions, or other methods. Another possible direction of research is to extend the second stage regressions to more explanatory variables and to take better care of endogeneity problems. All such potential extensions are waiting for future research.

Appendix:

1. Growth Accounting of Labor Augmented Productivity

Assume that productivity is labor augmenting, as in the growth regression model (1) in the paper and in DJT.

$$Y(t) = F[K(t), A(t)L(t)].$$

The differential of the change in output between period t-1 and t is described by the following equation, where the derivatives are taken in period t-1:

$$Y(t) - Y(t-1) = F_K(t-1)[K(t) - K(t-1)] +$$

$$+ F_L(t-1)A(t-1)[L(t) - L(t-1)] + F_LL(t-1)[A(t) - A(t-1)].$$

Divide by output at time t-1 and get:

$$\begin{split} \frac{Y(t)-Y(t-1)}{Y(t-1)} &= \frac{F_K(t-1)K(t-1)}{Y(t-1)} \frac{K(t)-K(t-1)}{K(t-1)} + \\ &+ \frac{F_L(t-1)A(t-1)L(t)}{Y(t-1)} \frac{L(t)-L(t-1)}{L(t-1)} + \frac{F_L(t-1)A(t-1)L(t-1)}{Y(t-1)} \frac{A(t)-A(t-1)}{A(t-1)}. \end{split}$$

Since $F_K(t-1) = MPK(t-1)$ and $F_L(t-1)A(t-1) = MPL(t-1)$ we can rewrite this equation with the shares of capital and labor in output, s_K and s_L respectively, and get:

$$\begin{split} &\frac{Y(t)-Y(t-1)}{Y(t-1)} = \left[1-s_L(t-1)\right] \frac{K(t)-K(t-1)}{K(t)} + \\ &+ s_L(t-1) \frac{L(t)-L(t-1)}{L(t)} + s_L(t-1) \frac{A(t)-A(t-1)}{A(t)}. \end{split}$$

We can derive the rate of growth of productivity from this equation:

(A.1)
$$\frac{A(t) - A(t-1)}{A(t-1)} = \frac{1}{s_L(t-1)} \left[\frac{Y(t) - Y(t-1)}{Y(t-1)} - \frac{K(t) - K(t-1)}{K(t-1)} \right] + \frac{K(t) - K(t-1)}{K(t-1)} - \frac{L(t) - L(t-1)}{L(t-1)}.$$

The rate of growth of this labor augmenting productivity is very similar to the rate of growth of productivity which is multiplicative in the production function as in 'Solow's Growth Accounting.' It can be shown that it is equal to (A.1) multiplied by $s_L(t-1)$. Namely, the rate of growth of productivity that is labor augmenting should be around 1.5 higher than the rate of growth of the standard TFP.

2. Convergence in a Small Open Economy

Consider a small open economy with full capital mobility facing a constant global interest rate r. Output in the economy in period t is described by the following Cobb-Douglas production function:

(A.2)
$$Y(t) = K(t)^{\alpha} [A(t)L(t)]^{1-\alpha},$$

where Y(t) is output, L(t) is labor and K(t) is the amount of capital invested prior to t. Capital depreciates at a rate δ . Productivity A and population N increase at constant rates:

(A.3)
$$A(t) = A(0)e^{gt}$$
, and $N(t) = N(0)e^{nt}$,

where g and n are positive numbers.²⁵ Each person supplies 1 unit of labor per period. Investment has adjustment costs, which are assumed to be quadratic and of CRS:

(A.4)
$$a(t) = \frac{1}{2z} \frac{[K(t+1) - K(t)]^2}{K(t)}.$$

The parameter z is an inverse measure of the intensity of these costs.

Due to the constant returns to scale of the production and the adjustment cost functions, the value of each firm is proportional to its capital and marginal q is equal to average q, as shown in Hayashi (1982). Hence, the market value of capital V(t) satisfies:

$$(A.5) V(t) = q(t)K(t+1),$$

where q(t) is the economy wide value of one unit of capital. Denote the wage rate in period t by w(t). Profit maximization by firms leads to the following two first order conditions. Equilibrium wage is:

(A.6)
$$w(t) = (1 - \alpha)K(t)^{\alpha} A(t)^{1-\alpha} L(t)^{-\alpha}$$
.

The rate of capital accumulation is:

(A.7)
$$\frac{K(t+1) - K(t)}{K(t)} = z[q(t) - 1].$$

We next introduce the equilibrium conditions. Labor market equilibrium requires:

$$(A.8) L(t) = N(t).$$

Due to capital mobility and lack of risk, the returns on capital and on lending are equal, so that:

²⁵ Note that this open economy model fits the canonical growth regression model of DJT but it can be applied also to the extended model.

(A.9)
$$q(t)(1+r) = MPK(t+1) + q(t+1) - d + \frac{z}{2}[q(t+1) - 1]^{2},$$

In order to describe the dynamics of the economy we transform the dynamic variables to better fit the empirical model. Instead of the price of capital we use: Q(t) = q(t) - 1, and instead of marginal productivity of capital we use its natural logarithm: $x(t) = \ln[MPK(t)]$. From (A.9) we get:

(A.10)
$$Q(t)(1+r) = \exp[x(t+1)] + Q(t+1) - (r+\delta) + \frac{z}{2}Q(t+1)^{2}.$$

The dynamics of x are derived from (A.3) and (A.7):

(A.11)
$$x(t+1) = x(t) + (1-\alpha)\{g + n - \ln[1 + zQ(t)]\}.$$

The equilibrium solution to this dynamic system, (A.10) and (A.11), is a saddle path, which is described by a function: Q(t) = Q[x(t)], where Q is monotonic increasing. Using a linear approximation we get that the steady state of the system is described by:

$$(A.12) Q^* = \frac{g+n}{z},$$

And:

(A.13)
$$x^* = \ln(r+\delta) + \ln\left[1 + \frac{g+n}{z} \frac{r - (g+n)/2}{r+\delta}\right].$$

We next turn to connect the model more to the growth regression model. Note that efficiency output per worker, $v^{E}(t)$, satisfies:

(A.14)
$$\ln y^{E}(t) = -\frac{\alpha}{1-\alpha} [x(t) - \ln \alpha].$$

Hence, efficiency output per worker converges to a steady state $\ln y^E(\infty)$ along the saddle path, which can be calculated from (A.12) and (A.13) and is equal to:

(A.15)
$$\ln y^{E}(\infty) = \frac{\alpha}{1-\alpha} \left\{ \ln \alpha - \ln(r+\delta) - \ln \left[1 + \frac{g+n}{z} \frac{r - (g+n)/2}{r+\delta} \right] \right\} \cong \frac{\alpha}{1-\alpha} \left[\ln \alpha - \ln(r+\delta) \right].$$

Note that since r is the same for all countries, and α and δ are technological parameters that should also be the same for all countries, $\ln y^E(\infty)$ should also be equal across countries if they are small open economies.

From (A.11) and (A.14) we derive the dynamics of efficiency output per worker:

(A.16)
$$\ln y^{E}(t+1) = \ln y^{E}(t) + \alpha z Q \left[\ln \alpha - \frac{1-\alpha}{\alpha} \ln y^{E}(t) \right] - \alpha (g+n).$$

Hence, the coefficient of convergence of y^E in the neighborhood of the steady state is equal to:

(A.17)
$$b = (1 - \alpha)zQ'(x^*).$$

One way to find b is to calculate the slope of the saddle path at the steady state, $Q'(x^*)$. This slope is the positive solution of the following quadratic equation:

(A.18)
$$(1-\alpha)z(1+g+n)[Q'(x^*)]^2 + \left[r-g-n+(1-\alpha)ze^{x^*}\right]Q'(x^*) - e^{x^*} = 0.$$

Another way to estimate b is to examine the dynamics of capital accumulation using a first order approximation around the steady state. We get:

(A.19)
$$\ln K(t+1) - \ln K(t) = n + g + zQ'(x^*) \frac{MPK(t) - MPK^*}{MPK^*}.$$

Hence:

$$(A.20) \ b = (1-\alpha)MPK * \frac{\partial [\ln K(t+1) - \ln K(t)]}{\partial MPK(t)} \cong (1-\alpha)(r+\delta) \frac{\partial [\ln K(t+1) - \ln K(t)]}{\partial MPK(t)}.$$

This equation enables us to roughly estimate the expected size of b. We can assume, for example by comparing China today with the US, that the effect of MPK on the rate of growth of capital should be somewhere between 0.3 and 0.5. According to standard assumptions $r + \delta$ is around 0.1 and $1 - \alpha = 0.65$. Hence, the rate of self convergence b should be somewhere between 1.7% and 3.2%. Therefore, the open economy model yields a rate of convergence that fits the data well, unlike the closed economy model used by Barro and Sala-i-Martin (1992).

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Figures and Tables

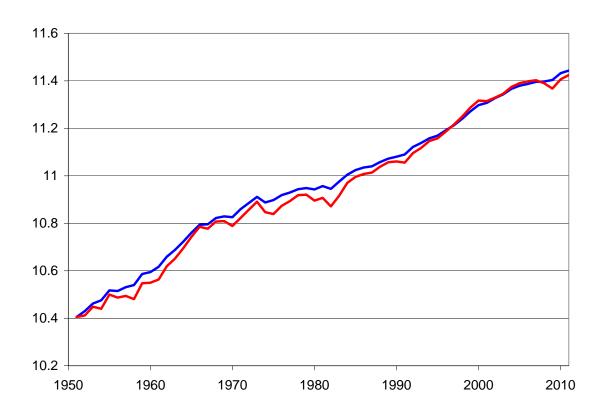


Figure 1: Natural Logarithm of US GDP per worker and US TFP in 1950-2010

Coefficient	Full	OECD	SSA	CSA	EA	MENA	10 Years
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Cointegration	0.998***	1.123***	0.894*	0.792***	1.096***	1.853*	0.743***
Coefficient	(0.14)	(0.12)	(0.63)	(0.26)	(0.24)	(0.13)	(0.17)
b	0.023***	0.026***	0.013***	0.031***	0.010	0.021***	0.013***
	(0.003)	(0.005)	(0.004)	(0.006)	(0.016)	(0.004)	(0.003)
No. of	67	24	12	17	9	6	67
Countries							

^{1.} Standard errors in parenthesis.

2. Significance levels are denoted: * of 10%, ** of 5%, *** of 1%.

Table 1: Cointegration Estimation of Rate of Convergence *b* in 70 Countries 1970-2008

Coefficient	Full	OECD	SSA	CSA	EA	MENA	10 Years
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
d	0.571***	0.657***	0.156	0.231**	1.393***	0.732*	0.532***
	(0.13)	(0.14)	(0.43)	(0.12)	(0.48)	(0.52)	(0.22)
С	0.090***	0.096***	0.046***	0.10***	0.093***	0.098***	0.052***
	(0.007)	(0.01)	(0.02)	(0.01)	(0.02)	(0.02)	(0.005)
No. of	68	24	12	17	10	6	68
Countries							

Standard errors in parenthesis.
 Significance levels are denoted: * of 10%, ** of 5%, *** of 1%.
 Table 2: Cointegration Test of TFP to Global Frontier in 1970-2008

Coefficient	Whole Sample	Without Oil & Outliers
d	0.688***	0.708**
	(0.093)	(0.072)
Error Correction	0.0389***	0.0405***
	(0.002)	(0.002)
Test for	$\chi^2 = 23.95$	$\chi^2 = 16.63$
d=1	P=0.00	P=0.00
Hausman Test for	$\chi^2 = 2.80$	$\chi^2 = 9.23$
Heterogeneity	P=0.094	P=0.002
Countries	139	124

^{1.} Standard errors in parenthesis.

Table 3: Panel Cointegration of b and d 1950-2008

Coefficient	OECD	SSA	LAC	E_SEA	Other Countries
d	1.060***	0.201*	0.634**	1.617***	0.623***
	(0.078)	(0.115)	(0.098)	(0.322)	(0.177)
Error Correction	0.0344***	0.0424***	0.0468***	0.0348***	0.0438***
	(0.006)	(0.005)	(0.004)	(0.006)	(0.007)
Test for	$\chi^2 = 0.59$	$\chi^2 = 48.54$	$\chi^2 = 14.05$	$\chi^2 = 3.67$	$\chi^2 = 4.31$
d=1	P=0.4425	P=0.000	P=0.000	P=0.056	P=0.038
Hausman Test for	$\chi^2 = 0.54$	$\chi^2 = 1.18$	$\chi^2 = 7.33$	$\chi^2 = -43.00$	$\chi^2 = 9.71$
Heterogeneity	P=0.464	P=0.277	P=0.007	P=0.0000	P=0.002
Countries	21	42	23	13	25

Table 4: Panel Cointegration of b and d by Regions, 1950-2008

Parameter	Coefficient	Z	P>z	Test $d = 1$	Hausmann Test for Heterogeneity
d	0.993 (0.060)	16.49	0.000	χ^2 (1)=0.01 P> χ^2 (1): 0.908	χ^2 (2)=2.86 P> χ^2 (2): 0.091
Error Correction	0.0232 (0.004)	6.51	0.000		
1. Standard errors	are in parenthes	is.			

Table 5: Panel Cointegration Estimation of b and d, 1870-2010

^{2.} Hausman null hypothesis: difference in coefficient not systematic.

	TROPICS	COAST	ETHNIC	Y_50	EDU	OPEN	G/Y
TROPICS	1.0000						
COAST	-0.1794	1.0000					
ETHNIC	0.5729	-0.5279	1.0000				
Y_50	-0.4754	0.3517	-0.3811	1.0000			
EDU	-0.5709	0.4554	-0.5503	0.7405	1.0000		
OPEN	-0.3205	0.3301	-0.3812	0.4930	0.5353	1.0000	
G/Y	-0.0466	-0.2029	0.1393	-0.2273	-0.0785	-0.2162	1.0000
ICRG	-0.5740	0.4079	-0.5705	0.6879	0.7884	0.7054	-0.1777

Table 6: Correlations between the Explanatory Variables

Dependent Variable: AVG						
Explanatory Variable	(1)	(2)	(3)			
-	Whole	Without	Without SEA			
	sample	SEA	and OECD			
TROPIC	-0.704***	-0.938***	-0.906***			
	(0.235) 0.008***	(0.242) 0.007***	(0.281)			
COAST		0.007***	0.007**			
	(0.003) -0.857***	(0.003)	(0.004)			
Y_50	-0.857***	-0.648***	-0.529***			
	(0.178)	(0.194)	(0.222)			
ETHNIC	-0.766*	-0.569	-0.530			
	(0.452)	(0.422)	(0.574)			
EDU	0.149***	0.123**	0.156**			
	(0.059) 1.109***	(0.060) 0.754***	(0.076) 1.190***			
OPEN						
	(0.231)	(0.234)	(0.471)			
G/Y	-2.558***	-1.707**	-1.883**			
	(0.898)	(0.859) 6.527***	(0.986)			
CONST.	8.012***		5.509***			
	(1.267)	(1.343)	(1.566)			
\mathbb{R}^2	0.61	0.60	0.52			
F PROB.	0.0000	0.0000	0.0000			
OBS.	90	77	57			
1. Robust standard errors in parer	ntheses.					

Robust standard errors in parentheses.
 Significance levels of 99%, 95% and 90% are denoted by ***, **, and * respectively.
 Table 7: Effect of Explanatory Variables on growth rate 1950-2010

Dependent Variable: d						
Explanatory Variable	(1)	(2)	(3)			
	Whole	Without	Without SEA			
	sample	SEA	and OECD			
TROPIC	-0.287**	-0.389***	-0.459***			
	(0.152)	(0.133)	(0.144)			
COAST	0.004**	0.002	0.003			
	(0.002) -0.468***	(0.002) -0.240**	(0.002)			
Y_50	-0.468***	-0.240**	-0.225*			
	(0.127)	(0.122)	(0.137)			
		4.4	***			
ETHNIC	-0.432*	-0.417**	-0.574**			
	(0.282) 0.088**	(0.218)	(0.304)			
EDU		0.053	0.048			
	(0.039) 0.591***	(0.038)	(0.043) 0.875***			
OPEN		0.319**	0.875***			
	(.150) -1.290**	(0.150)	(0.291)			
G/Y		-0.483	-0.927			
	(0.578) 4.051***	(0.513) 2.517***	(.680)			
CONST.	4.051***	2.517***	2.580***			
	(0.946)	(0.835)	(0.966)			
\mathbb{R}^2	0.45	0.44	0.48			
F PROB.	0.0000	0.0000	0.0000			
OBS.	90	77	57			

Robust standard errors in parentheses.
 Significance levels of 99%, 95% and 90% are denoted by ***, **, and * respectively.
 Table 8: Effect of Explanatory Variables on d