Abstract

This study deals with the differences in quality of decision-making between groups with a different number of voters and voters with different skills. In particular, we focus our attention on the differences between a jury and judges in court, represented by eleven and one/three voters, respectively. In each group, individual's skills express his or her probability of making the correct decision. Given individuals' skills, the collective probability of each group for making a correct decision can be calculated. We are interested in examining the difference between a jury's collective probability of making a correct decision (that is a large group, where decision-makers are not experts) and a group of judges (that is a small group, where decision-makers are experts) or even a single expert judge. In light of this comparison, we draw some general conclusions regarding the quality of a decision of a small group of experts versus a large number of non-experts.

The seminal Condorcet jury theorem is presented in the context of a group of individuals that has to choose one out of two alternatives, one of which is preferred, however its identity is unknown. Condorcet jury theorem consists of two parts. According to the first one, the probability that a group of individuals that applies the simple majority rule to choose the correct alternative is higher than the probability that a single member would choose this alternative. According to the second part of the theorem, when the number of group's members goes to infinity, the probability to choose the correct alternative goes to one. We review some studies of group decision-making that generalized the theorem, expended it and examined its validity.

The model used in this study is a decision making model that can be applied to a single or a group of individuals, where each individual chooses one out of two options

(e.g., innocent or guilty) under two possible states of nature (the defendant is either innocent or guilty). We assume that individuals' skills are homogeneous, each individual has a probability p to decide correctly and this probability is greater than a half. In addition, we assume that the probability that a judge would make the correct decision is higher than the one of a member of the jury. Consequently, we examine the quality versus the quantity of voters. On the one hand, one or three judges with higher skills relative to a jury (quality), and on the other hand eleven members of the jury (quantity).

We use the model presented by Ben-Yashar (2013), in which she examined the effect of adding two voters to a group on the probability of choosing correctly. We present the difference between the collective probability of n individuals to make a correct decision and the one of $n-2$ individuals to make a correct decision by:

$$D^{n-(n-2)} = \binom{2k-1}{k-1} p^{k+1}(1-p)^{k} - (\binom{2k-1}{k} p^{k}(1-p)^{k+1}$$

To compare the collective probabilities of making a correct decision between the groups, we calculate the difference between the collective probability to make the correct decision by the larger group (eleven individuals) and the one of the smaller group (one or three individuals). We prove that when individuals' skill are homogeneous, this difference is positive, since the larger the group is, the higher is the collective probability of making a correct decision. The first derivative with respect to p reveals an inflection point and the second derivative reveals the maximal point (a result we have obtained for both the comparison between eleven individuals to a single one and for the comparison between eleven to three individuals). By simulations, we show a graph in which the difference function increases and then decreases and that the difference function has a single maximum. We find that when

the difference is maximal, the group size has the highest influence on the collective probability of making a correct decision and on the graph's edges, where the skills are close to a half or to one, the group size has a minor influence. We show how to calculate the skills that are needed from the small group so that their collective probability of making a correct decision will be equal to the collective probability of making a correct decision by the large group. We find that the addition that is needed to the skills of the small group is highest, when the difference between the collective probability of making a correct decision by the large group to the collective probability of making a correct decision by the small group is maximal and is very low when the homogenous skills are on the graph's edges.

We then present the general case, where we examine a general difference function, describing the difference between the collective probability of a group of T and a group of n voters to make a correct decision ($T > n$). As in the simple case, we derived the function with respect to p, find an inflection point and by the second derivative find that the point we obtained is a maximum.

By numerical simulations, we examine the effect of increasing one/two of the groups and explain the results. We find that increasing the large group keeping the size of the small one unchanged, results in an increase of the difference between the collective probability of making a correct decision by a group of T versus a group of n individuals, a decrease of the marginal difference and a decrease of the skills that are needed to obtain the maximal difference. We find that the larger the large group is, the skills that are needed from the small group to obtain the same collective probability of making a correct decision as the large group increase, yet the marginal addition to the skills decreases.

We also find that when we increase the small group and keep the large group unchanged, the difference between the collective probability of group of T and a group of n individuals decreases, the marginal difference decreases and the skills that are needed to reach the marginal difference decrease too. We show that the larger the small group is, the skills that are needed from the small group to obtain the same collective probability of making a correct decision as the large group decreases and the marginal addition to the skills decreases too. Finally, we present a simulation in which we keep the difference between the number of voters in the large group to the number of voters in the small group unchanged, and we find that increasing the two groups by the same size results in a decrease of the difference between the collective probability of making a correct decision of the large group versus the small group and a decrease of the marginal difference and the skills that are needed to obtain the maximal difference. We find that the larger the groups are, the skills that are needed from the small group to obtain the same collective probability of making a correct decision as the large group decreases and the marginal addition to the skills decreases too. In conclusion, through these simulations, we study the properties of the difference function, the maximal difference, the marginal difference, the skills that are needed to reach the maximal difference and the tradeoff between the quantity versus quality of the voters in the group.

Finally, we explain for which skills the quantity advantage of the voters is higher and for which skills the quality advantage of the voters is higher. We explain why one or three judges are preferred for making decisions in court.

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