Wage Growth Decomposition: Mobility, Learning, and Loss of Human Capital
Job Market Paper

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Abstract

I develop and calibrate a job search model of individual career dynamics with learning-by-doing at work and loss of skills in unemployment. The model assumes ex ante identical workers who stochastically move across several stages of their career. Cumulative wage growth is driven by human capital dynamics and job search. Workers' mobility decisions depend on changing offer arrival rates, human capital returns, and planning horizons. Firms post stage-specific offers taking workers' stage-specific search strategies into account. In the calibrated model, the range of equilibrium offers is broad initially, contracts in mid-career, and shifts downward in the last career stage. Mobility accounts for almost 60 percent of total wage growth among high school graduates during the first decade of a career and declines subsequently; among college graduates, mobility can even become a negative factor when they begin to search within a lower range of wage offers. The paper highlights the important role that the wage offer distributions play in wage growth analysis. It proposes a tractable stationary framework with rich life-cycle properties. The model features permanent wage scarring due to human capital loss, and implies that high school graduates lose up to almost 7 percent of their potential lifetime wage gain due to the depreciation of skills and foregone skill accumulation during unemployment.

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1 Introduction

In recent years, a growing literature has emerged that attempts to understand individual wage growth over the life-cycle.[1] One of the main objectives of this literature is to determine the factors that generate wage growth, which in particular include experience, mobility between employers, learning about match quality and perhaps negotiating a better contract. This paper focuses on job search and human capital accumulation and examines how they interact with loss of skills in unemployment to shape the life-cycle wage profile. It will be argued that these issues are closely linked through the direct impact of skills depreciation on human capital and its indirect impact on workers' job search strategies.

Worker heterogeneity over the life cycle plays a central role in the analysis. Thus, in the data, younger workers are found to be much more mobile than older ones (Menzio et al. (2015)). They are also found to have different human capital returns, and notably, the share of wage gainers declines with age while the share of wage losers rises (Rubinstein and Weiss (2007)). The model parsimoniously captures these life-cycle heterogeneity properties. It assumes that over the course of a career workers stochastically move through a finite number of stages, starting from being "young" and finishing with being "old." The stages are allowed to differ from one another in terms of workers' human capital dynamics and mobility rates. These stage-specific parameters are derived through calibration by targeting the observed aggregate life-cycle profiles of wages and transitions. While initially identical, workers become both between- and within-age heterogeneous over time.

For workers in different stages of their career, the endogenous distribution of wage offers is derived as an equilibrium in a wage-posting game, as in Burdett and Mortensen (1998). This distribution reflects the arrival rates of offers and the rates of human capital accumulation and loss specific to each stage, as well as the expected horizon of the workers. This distinguishes the approach from others in the literature on individual wage growth,[2] which assume a unified distribution of offers for all workers, with a preset functional form.

It is shown that the equilibrium offer distributions differ considerably across stages. The range of offers is relatively broad at the beginning of a career, and it is found that mobility accounts for 57 percent of total wage growth among high-school graduates in the first decade of their career, which is twice the proportion found in most previous studies.[3] Late in a career, it is found

[1] Adda et al. (2013), Altonji et al. (2013), Bagger et al. (2014), Bowlus and Liu (2013), Buchinsky et al. (2010), Carrillo-Tudela(2012), Schonberg (2010) and Yamaguchi (2010)

[2] Altonji et al. (2013), Bagger et al. (2013), Bowlus and Liu (2013), Yamaguchi (2010), Adda et al. (2013) and Menzio et al. (2015)

[3] However, it is lower than the estimate in Bowlus and Liu (2013).

that the entire equilibrium distribution of offers shifts downward. Thus, as an increasing proportion of workers reach their final career stage, late-life mobility turns out to be a *negative* factor in wage growth, on average.[4]

According to the results, workers lose from 3.1 to 6.6 percent of their potential lifetime wage gain, depending on their education level, due to their unemployment history. These losses are the result of both direct human capital depreciation and forgone accumulation of skills. A simulation demonstrates that although college graduates enjoy higher lifetime earnings than high school graduates, their earnings are more sensitive to unemployment history.

The results are novel in that they highlight the senstivity of wage growth decompostion to the assumptions made concerning offer distributions. Note that the decomposition of a cross-section of wages into the unobserved worker effect and the firm effect is, to some extent, arbitrary since once one distribution is assumed, the other can be derived as a residual. Allowing the distribution of offers (fixed firm effects in the model) to be set in equilibrium imposes greater discipline on the decomposition exercise. As the abovementioned results show, this method yields novel predictions about the role of mobility in life-cycle wage growth, both in qualitative and quantitative terms. Finally, by regarding a career as a stochastic movement of workers through a number of different stages, the model offers a novel theoretical framework that combines rich life-cycle features with stationarity and tractability.

The analysis implies that the role of job search in wage growth hinges to a large extent on the reservation cutoff of the unemployed. It is only when the lowest acceptable offer is sufficiently low that mobility becomes a significant channel of life-cycle wage increase (other things held equal). The issue of low reservation wage is closely related to another aspect of search models: their ability to generate sufficient residual wage dispersion (i.e. wage differences among observationally similar workers). Hornsten et al. (2011) were the first to note that search models generate enough frictional wage dispersion[5] only if the unemployed have a low reservation cutoff even when the value of continued search is high.[6] Combining this with the results obtained here leads to the conclusion that the importance of the search mechanism in life-cycle wage growth and its ability to generate sufficient wage dispersion among observationally similar workers appear to be interrelated,[7] with the reservation wage of the unemployed

[4] Notably, this novel effect arises only for college graduates since, according to the calibration, unemployment is especially damaging for them, and their offer arrival rate is particularly low. Therefore, they have especially low reservation wages, and, in equilibrium, firms take this into account.

[5] Measured as the ratio between the mean and the minimal wage in the market.

[6] In response to this claim, a small body of literature has emerged which searches for mechanisms that can do the job. See, for example, Burdett et al. (2011), Carrillo-Tudela (2012), Ortego-Marti (2012), and Tjaden and Wellschmied (2014).

[7] Jolivet et al. (2006) write that "job search models of the labor market hypothesize a very tight correspondence between the determinants of labor turnover and individual wage dynamics on one hand, and the determinants of wage dispersion on the other."

being a key factor in both.

The paper is organized as follows: Section 2 reviews the background literature. Section 3 presents a formal model of a career in which a worker's life cycle involves transitions through a number of stages. A wage-posting equilibrium is characterized for each stage, where each stage is regarded as a separate labor submarket. In Section 4, the number of stages is determined and stage-specific parameters are calibrated. Section 5 discusses stage-specific distributions of offers and compares offered wages generated by the model to actual data. In Section 6, the model is simulated by letting the workers live out their careers, move through the various stages, and sample offers from changing distributions. The resulting average wage path is analyzed, as well as its three additive components: the (positive) impact of actual experience; the (negative) impact of unemployment history; and the impact of mobility. Section 7 concludes.

2 Background Literature

The paper is related to two literatures. The first is the relatively new literature on the sources of cumulative wage growth. The research in this field attempts to decompose observed wage growth into the effect of unobserved human capital and that of job shopping.[8] The second is the literature on the impact of unemployment on careers, which focuses on the differences between the career outcomes of workers who experience unemployment and those who do not. These two literatures are described in a more detailed manner below. In addition, since one of the calibration outcomes is the rate of human capital accumulation by age, the literature on learning abilities and productivity over the life cycle will also be referred to in subsection 2.3.

2.1 Wage growth decomposition

Reduced-form studies that attempt to disentangle return on mobility from return on experience do so by estimating a set of regressions. The main regression equation usually estimates the Mincer wage function, which reflects decreasing returns on experience and sometimes tenure, while controlling for education and demographics. Schonberg (2007) estimates this equation and measures the impact of mobility using the differences in wage growth between job-stayers, job-switchers, and those who move to unemployment.[9] Buchinsky et al. (2010) highlight the fact that experience and seniority are themselves endogenous results of workers' decisions, and they explicitly include the equations that describe a decision to move to employment or switch jobs. Adda et al. (2013)

[8] Additional components are sometimes included, such as learning about match quality or effects of tenure.

[9] Using NLSY data.

estimate a wage equation including experience and tenure effects, allowing the transition rates to vary by state of the business cycle, experience, and skill level.[10] Altonji et al. (2013) estimate a rich model of earnings dynamics,[11] which includes the characterization of wage rates, work hours, employment, and job changes over the life cycle. Except for Schonberg (2007), who exploits the wage differential between stayers and movers, the impact of search in the papers mentioned above is measured by first estimating the parameters of the unobserved offers distribution, which is usually assumed to be log-normal, and then simulating the model that generates offers from it.

The reduced-form analysis described above usually concludes that human capital is by far the dominant driver of earnings growth, whether at the beginning of a career (except, perhaps, the very first year or two in the market) or over a longer horizon. A common result is that among high school graduates– the most frequently analyzed group of workers–human capital accumulation accounts for approximately three-quarters of wage growth during the first decade of a career.

There are few structural models that combine mobility and human capital accumulation. One of the pioneering studies that does combine these two mechanisms is Burdett, Coles, and Carrillo-Tudela (2011). In their model, workers accumulate experience at a given rate when employed and face an exogenous arrival rate of offers while on the job. The authors show that this rich wage process results in the convenient decomposition of wages into the two additive components of human capital and mobility. They focus on cross-sectional wage dispersion.[12] The current model involves a framework that is technically related to theirs and applies it to the analysis of individual wage profiles.

The existing papers that adopt a structural approach to individual wage dynamics differ in the way they model the two mechanisms of human capital accumulation and mobility. Yamagouchi (2010) and Bagger et al. (2013) combine human capital accumulation and on-the-job search in the framework of multilateral bargaining, as developed by Postel-Vinay and Robin (2002). Menzio et al. (2012) adopt a directed search framework with a finite lifetime.[13] Yamagouchi (2010) and Menzio et al. (2012) find that mobility plays only a minor role and its effect is concentrated at the very beginning of a career. Bagger et al. (2013) arrive at the opposite conclusion using data from Denmark rather than the U.S.

[10] Using German panel data.

[11] Using PSID data.

[12] Carrillo-Tudela (2012) extends the work of Burdett et al. (2011) by allowing for firm productivity heterogeneity and by calibrating the model to match the average wage profiles of young British workers. Nonetheless, her focus is on wage variance decomposition. The same is true for Tjaden and Wellschmied (2014), who add to it the depreciation of human capital in unemployment.

[13] Menzio et al.'s main focus is on explaining the decline in mobility over the life cycle. They claim that mobility falls endogenously with age because over time workers tend to search for a job in a submarket where pay is high, but they need to wait longer to get an offer.

Bowlus and Liu (2013) demonstrate that using the same US data as in previous studies and accounting for the endogenous interactions between search behavior and investment in human capital produces novel results in terms of wage growth decomposition. They show that the presence of human capital accumulation dramatically reduces reservation rates at the beginning of a career, when the returns on human capital are high. With initially low reservation rates, search is highly worthwhile and mobility explains a hefty 88 percent of wage growth among high school graduates in the first decade of their career.[14]

The current paper is conceptually related to Bowlus and Liu (2013), in the sense that it includes an element of the interaction between search and human capital processes by way of the effect of a worker's current human capital dynamics on the distribution of offers he faces. However, it differs in several respects: First, the entire distribution of offers, which changes over the course of a career, is endogenous and not just the reservation wage of the unemployed. Second, the process of productivity loss in unemployment is taken into account, which has an impact both on the reservation cutoff of the unemployed and on the entire distribution of offers at various stages of a career. Finally, the entire structure of a career is steady state and has a stochastic, rather than deterministic, life cycle.

2.2 Unemployment and subsequent career outcomes

In a parallel strand of the literature on individual wage outcomes, researchers have attempted to estimate the effects of layoffs on wage profiles in econometric reduced-form studies. Addison and Portugal (1989), using the U.S. Displaced Workers Survey, find that a 10 percent increase in unemployment duration lowers accepted wages by about 1 percentage point. Jacobson et al. (1993), using U.S. data, find losses amounting to 25 percent of the pre-displacement wage even five years after displacement. Gregory and Jukes (2001) estimate the effects of unemployment on the subsequent earnings of British men and find that the wage penalty after a six-month unemployment spell is 13 percent for the young and almost twice that for the old. Davis and von Wachter (2011) explore how earnings losses differ during the period following separation according to the timing of the separation within the business cycle. They estimate that workers in the U.S. can lose from 10 to 24 per cent of lifetime earnings as a result of displacement. Jarosch (2015) shows that in Germany the wage of a displaced worker is 10 percent lower than that of a stayer even 20 years after the displacement. Other recent studies that document substantial and persistent losses of earnings following displacement include Jung and Kuhn (2013) and Saporta (2013).

Despite ample evidence of the persistent negative impact of unemployment episodes on wages, both structural and reduced-form models of individual wage

[14] This estimate includes both the direst impact of mobility and its interaction with human capital accumulation. The direct impact itself accounts for 75 percent of the total wage growth

dynamics that focus on sources of wage growth have not included any mechanism that would generate such an effect.[15] In the current model, the negative impact of unemployment is reflected in the loss of human capital, and it is thus a process that tends to undo human capital accumulation on the job. The role of human capital depreciation is two-fold: First, it is a direct component of wage profile decomposition. The model implies that this component has only a moderately negative effect on average, but combined with forgone earnings, can amount to a non-negligible loss of potential cumulative wage gain. Second, substantial depreciation of human capital induces workers to lower their reservation rate. This is essentially the main force that reduces the reservation cutoffs of the unemployed in the last stage of their career, thus shifting downward the entire distribution of offers late in the life cycle.

2.3 Age, learning ability, and individual productivity

The calibration indicates that human capital accumulation rates decline with age, and for a significant share of workers in both education groups late-in-career productivity falls in employment (negative human capital accumulation). This outcome is related to the effects of aging on learning abilities and productivity. With regard to the former, the psychological literature presents ample evidence that some crucial learning-related abilities decline over the life cycle, such as the encoding of new memories of episodes or facts, working memory and processing speed (Hedden (2004) and Salthouse (2004)). In more recent research, Janascek et al. (2012) also find that the ability to unconsciously recognize regularities and patterns declines over age. Finally, Craik and Bialystok (2006), referring to the development of cognitive ability, note that "change can occur at any time [and] development depends on interactions among genetic, environmental and social factors." Indeed, the stochastic nature of ageing in the model conforms to this fact. With regard to the impact of age on individual productivity, it has generally been found that job performance often does decrease with age, but not in all tasks and less so in occupations where age-resistant abilities (such as verbal skills) are important (see Skirrbekk (2003, 2008) for a survey of this literature).

3 Theoretical Model

3.1 General setup

[15] The only exception is Altonji et al. (2013) who include the deterioration of general human capital in their wage equation. They find that the average impact of this wage component is small and that cumulative losses of human capital after 30 years of a career are negligible on average.

Fujita (2012) constructs a search model to explain the downward trend in the separation rate in the US during the past 30 years as a result of the increased probability of loss of skills in unemployment.

The modeling of a career is based on two facts. First, workers of different ages are heterogeneous in their human capital returns and in their chances to move across labor market states. Second, the return on human capital varies within-age and, in particular, the share of workers whose wages are growing declines with potential experience (Rubinstein and Weiss (2007)). To capture these two phenomena, it is assumed that over the life cycle workers move across several career stages and that their human capital dynamics and mobility opportunities change accordingly. These changes are assumed to arise from biological aging, life-cycle changes in family circumstances, etc. In other words, they are treated as random exogenous events that individuals have no control over. Two initially identical workers who simultaneously start their careers may be in different stages at a particular point in time, even though they have the same potential experience. At that point in time, the two workers accumulate or lose human capital at different rates and also receive job offers at different rates. Note that workers with higher potential experience are more likely to have already moved on to subsequent stages and therefore there is a *positive correlation* between potential experience and career stage. Though not directly observed, the gradual movement of workers across career stages can explain the average returns on experience over the life cycle, as well as the life cycle dynamics of transition rates in the labor market.

The workers in the model know their current stage-specific parameters and how those parameters will change in the future, though they do not know exactly when this will occur (since transitions across stages are random). The workers decide, at each stage, what the lowest acceptable offer is, given the distribution of available offers.

The firms operate according to a constant returns to scale technology. Each firm hires all types of workers and it is assumed that each worker can only perform a job that was advertised for workers of his type (i.e., workers in his career stage). Thus, there is no problem of incentive compatibility: from a worker's point of view each career stage means participating in a separate labor market. The firms know how mobile workers are in each career stage and what their human capital rates are and post stage-specific offers, as in Burdett and Mortensen (1998) and Burdett et al. (2011). In equilibrium, the lowest offer they post in each stage will equal the lowest offer that is acceptable in that stage; the highest offer will provide the firm with the same expected stage-specific profit. As a result, different distributions of offers arise endogenously, depending on the search and human capital parameters of each career stage and the expected horizon of the workers.

The approach used here has two advantages over the deterministic ageing approach. First, in that approach it would not be possible to solve for equilibrium distributions at different career stages since the model would have no stationary steady state. Second, under deterministic aging all workers with the same potential experience have the same rates of human capital accumulation and depreciation and search with the same efficiency. This implausible property is usually overcome in the existing literature by explicitly assuming permanent within-age heterogeneity. (In Bagger, Fontaine, Postel-Vinay, and Robin (2013), for example, there is a continuum of permanent worker fixed effects that define workers' initial human capital level and offer arrival rates. In Bowlus and Liu (2013), there are two types of workers who permanently differ in their offer arrival rates.) In the current model, workers are initially identical, but develop a dynamic within- and between-age heterogeneity ex post due to stochastic movement across stages.

While focusing on life-cycle changes in the equilibrium distributions of offers, the analysis takes as given the dynamics of offer arrival rates and human capital returns. An alternative would be to assume, for example, optimal search effort by workers as in Mortensen (2003) or a matching function as in Mortensen and Pissarides (1994) in order to underpin offer arrival rates and optimal investment in human capital as in Ben-Porath (1967) in order to explain the return on experience and unemployment. In contrast, in the current model, the exact nature of a worker's propensity to move and of the human capital processes is immaterial in a firm's wage-posting decision, as long as it is consistent with the observed arrival rates of offers and human capital returns. It is likely that introducing such mechanisms would not impact the main results obtained here while possibly rendering the model intractable. Therefore, the introduction of such mechanisms is left for future research.

3.2 Workers

Assume that time is continuous and that the economy is in a steady state. The life of a worker is divided into N stages $\{S_1, ..., S_N\}$. A unit measure of workers participates at each point in time in stage $S_k, k \in \{1, ..., N\}$, such that the total measure of workers in the economy is N. An individual worker starts his career in the first stage S_1. There is a measure ϕ of workers starting their career at each instant. A worker leaves S_1 according to the Poisson rate ϕ and moves on to S_2, where he remains until the transition shock ϕ occurs again. He then moves on to S_3, and so on. When a worker reaches the last stage, S_N, he stays there until the transition shock ϕ occurs for the last time in his career and he exits the economy for good. These assumptions guarantee that the measure of workers in each stage remains constant at every point in time.

A career starts in the first stage S_1, which the worker enters as an unemployed worker with unit productivity, $y_0 = 1$, which is the same for all workers. Once his career begins, a worker's productivity starts to evolve. Productivity is determined by general human capital, which grows when a worker is employed and accumulating experience, and declines when he is unemployed. Thus, it is assumed that when a worker is employed, his productivity grows automatically due to learning-by-doing. A worker who is not employed loses part of his skills, or, alternatively, finds it more difficult to keep up with technological advances, his productivity declines, and unlearning by not doing occurs.[16] This damage increases with the duration of non-employment. The rates of human capital accumulation and depreciation are ρ and η, respectively, such that x periods of employment increase productivity by a factor of $\exp(\rho \cdot x)$, while q periods of unemployment decrease productivity by a factor of $\exp(-\eta \cdot q)$. Human capital technology (the rates ρ and η) is exogenous and stage-specific. This is in contrast to the model of Ben-Porath (1967) in which workers decide in each period on the amount of time they wish to invest in human capital accumulation. In the current model, workers take it as given that as they move on to subsequent stages, their returns in employment and unemployment will change. Note that in a given stage the order in which spells of employment and unemployment occur does not affect productivity – it is only the cumulative durations of these two states that matters.

Human capital is preserved in job-to-job transitions and is carried across stages, such that a worker starts a new stage in a career with the same productivity with which he finished the previous one. A worker's productivity summarizes his entire labor market history, including periods of employment and non-employment, over all stages that he has lived through up to that point in time. For tractability, it is assumed that each transition across stages involves unemployment. When a transition shock occurs, an employed worker is separated from his current job and becomes unemployed in the next stage, searching for a job.

Search technology is stage-specific and in each stage it is defined by the Poisson offer arrival rates λ_0 and λ_1 for unemployed and employed workers, respectively. The offers are piece rates $\theta \in [0, 1]$ which stipulate a share of the flow productivity y that the worker receives at each instant, such that his wage will be $\theta \cdot y$. Piece rate offers originate from the cumulative distribution $F(\theta)$ – a stage-specific distribution found in a wage-posting equilibrium (as in Burdett and Mortensen (1998) and Burdett et al. (2011) and derived below). A worker who is in stage s can only sample offers from the distribution $F^s(\theta)$ and is unable to perform jobs that are advertised for workers in other stages. This rules out the problem of incentive compatibility among workers and implies that each stage is a separate labor submarket. Jobs are destroyed exogenously at a stage-specific Poisson rate δ and the worker's instantaneous discount rate

[16] The term "unlearning by not doing" was coined by Coles and Masters (2000). They show that when there is a loss of skills during unemployment, a firm with a vacancy that hires a worker does not take into account the externality that they create for other firms by improving the quality of their potential workers. In this setting, a number of Pareto-rankable equilibria arise, in which everyone would be better off if firms posted numerous vacancies and workers did not spend much time in unemployment, where they lose human capital. The worst case is an equilibrium with few vacancies and long unemployment spells which results in a poorly-skilled workforce.

is denoted by r. Income in unemployment is a fixed share b of a worker's human capital.

The value function of an unemployed worker in stage s with productivity yi is given by $W^{U,S}(y)$:

$$
\begin{aligned}
rW^{U,s}(y) \quad = \quad & b \cdot y + \frac{\partial W^{U,s}(y)}{\partial t} + \\
& + \lambda_0^s \int_{\underline{\theta}^s}^{\overline{\theta}^s} \max \left[W^{E,s}(y, \theta') - W^{U,s}(y), 0 \right] dF^s(\theta') + \\
& + \phi \cdot \left(W^{U,s+1}(y) - W^{U,s}(y) \right) \qquad (1)
\end{aligned}
$$

The first term in the flow value of unemployment is the flow income in unemployment $b \cdot y$. Note that the parameter b, which determines the income of the unemployed worker, is by assumption not stage-specific, i.e., unemployed workers in all stages receive the same share of their productivity as income in unemployment. The second term, $\frac{\partial W^{U,s}(y)}{\partial t}$, is the negative change in the value of unemployment over time, due to the depreciation of human capital, which reduces current productivity. Note that the value of unemployment $W^{U,s}(y)$ will be decreasing over time as long as the rate of human capital depreciation is not equal to zero. The third term is the search option of the unemployed in stage s. Unemployed workers receive an offer θ' sampled from the stage-specific distribution $F^s(\cdot)$ at rate λ_0^s which they either accept and become employed with the value $W^{E,s}(y, \theta')$ or decline and remain unemployed with their current value $W^{U,s}(y)$. Finally, an unemployed worker in stage s might experience a transition shock ϕ, following which he moves on to the next stage and his value is that of being unemployed in stage $s+1$ with the same human capital y.[17]

The value function of an employed worker in stage s depends both on the worker's productivity y and the piece rate θ paid by a firm:

$$
\begin{aligned}
rW^{E,s}(y, \theta) \quad = \quad & \theta \cdot y + \frac{\partial W^{E,s}(y, \theta)}{\partial t} + \\
& + \lambda_1^s \int_{\underline{\theta}^s}^{\overline{\theta}^s} \max \left[W^{E,s}(y, \theta') - W^{E,s}(y, \theta), 0 \right] dF^s(\theta') + \\
& + \delta^s \cdot \left(W^{U,s}(y) - W^{E,s}(y, \theta) \right) + \\
& + \phi \cdot \left(W^{U,s+1}(y) - W^{E,s}(y, \theta) \right) \qquad (2)
\end{aligned}
$$

[17] In the last stage, i.e., when $s = N$, the value in the event of a transition shock becomes zero, and the last element in the value function is simply $-\phi W^{U,s}(y)$.

The first term on the right-hand side is the flow wage of the worker, which is a share θ of his productivity. The second term, $\frac{\partial W^{E,s}(y,\theta)}{\partial t}$, is the positive increment to the value due to learning-by-doing. Since each instant of employment adds to the worker's productivity, the value of employment grows over time. The third term is the on-the-job search option. An employed worker receives an outside offer θ' that arrives from the stage-specific distribution $F^s(\theta')$ and compares the value of remaining with the current firm, $W^{E,s}(y,\theta)$, to the value of moving to the poaching firm, $W^{E,s}(y,\theta')$. Note that since there are no firm-specific skills, the entire stock of human capital y is moved to the new job in the case that a worker decides to accept the offer. An exogenous separation shock δ^s might destroy the match. Finally, an employed worker can experience a transition shock ϕ, in which case he loses his value and becomes an unemployed worker in the next stage $s+1$.

Note that both value functions are stage-specific (hence the superscript s) and account for the effect of a shortening horizon through the future component $W^{U,s+1}(y)$. This component is the expected value of being unemployed in the next submarket and, by the chain rule, also includes the values of all subsequent stages. It can be seen that the future component $W^{U,s+1}(y)$ declines from stage to stage: it is high in early stages since it subsumes many future stages, while in the last stage it is simply zero.

3.3 Optimal strategies

The optimal strategies available to the workers are straightforward. All unemployed workers in stage s will accept any offer above the reservation cutoff $\theta^{R,s}$. The cutoff arises because the value of unemployment is independent of a particular piece rate, whereas the value of employment is an increasing function of it. Notably, this cutoff is common to all workers in the same stage, since flow incomes and dynamic components in the value functions are proportionate to productivity, and individual human capital has no impact on the *relative* attractiveness of employment over unemployment.[18]

The cutoff in each stage does depend, however, on the key stage-specific mobility and human capital parameters $\lambda_0, \lambda_1, \delta, \rho, \eta$, as well as on the expected horizon of the workers. The reservation cutoff decreases with the relative value of unemployment. For example, a low arrival rate of offers λ_0 or a high loss of skills rate η, will make an unemployed workersmore willing to move to employment, thereby lowering $\theta^{R,s}$. If a stage is characterized by a high on-the-job offer arrival rate λ_1, a low match destruction rate δ or a high human capital accumulation rate ρ, then unemployed workers in this stage will also accept lower offers.

The effect of a shortening horizon is present in the model even though the length of a career is stochastic. This is because a permanent exit from the

[18] See Appendix A for the complete derivation.

market can only occur in the last stage of a career. Workers are forward-looking and take into account that the human capital accumulated in the first stage will serve them throughout their career and correspondingly productivity losses will be a permanent drag on value. This is especially relevant at the beginning of a career, when the expected horizon is the longest. This also enhances the relative attractiveness of employment and drives down the reservation cutoff, an effect which weakens with time.[19]

For employed workers, the optimal strategy is trivial and involves accepting any offer that is above their current piece rate. This is because human capital is perfectly transferable across firms, and all firms are identical.

The optimal strategies of workers are important in the model because they are taken as given by the firms, which decide which offers to post. In particular, and as will be shown below, the minimal posted offer is equal to the reservation cutoff of the unemployed, while a high frequency of job-to-job transitions will induce firms to set higher piece rates in order to better retain workers.

3.4 Firms

There is measure one of identical firms, operating according to the same constant returns to scale technology. Each firm posts N offers, one for each type of worker. Within each stage, the firm hires every worker for whom the respective offer is sufficiently attractive (see Burdett and Mortensen (1998)). The expected profit from posting an offer θ in a submarket s is:[20]

$$\begin{aligned}\pi^s(\theta) \quad = \quad & \overline{y}_0^s \cdot (1-\theta) \cdot \lambda_0^s U^s \cdot \left[\int_{x=0}^{\infty}\int_{q=0}^{\infty} e^{\rho^s \cdot x} e^{-\eta^s \cdot q} \frac{\partial^2 P^{U,s}(x,q)}{\partial x \partial q} \int_{\tau=0}^{\infty} e^{-r\tau} e^{-q^s(\theta)\cdot\tau} e^{\rho^s \tau} d\tau\right] + \\ & + \overline{y}_0^s \cdot (1-\theta) \cdot \lambda_1^s (1-U^s) \cdot \left[\int_{x=0}^{\infty}\int_{q=0}^{\infty}\int_{\underline{\theta}^s}^{\theta} e^{\rho^s \cdot x} e^{-\eta^s \cdot q} \frac{\partial^3 P^{E,s}(x,q,\theta)}{\partial x \partial q \partial \theta} \int_{\tau=0}^{\infty} e^{-r\tau} e^{-q^s(\theta)\tau} \cdot e^{\rho^s \tau} d\tau\right] \quad (3)\end{aligned}$$

The first term is the expected profit from hiring an unemployed worker. The measure of such workers in each stage, U^s, can easily be found from inflow-outflow steady-state conditions and equals $\frac{\phi+\delta^s}{\phi+\delta^s+\lambda_0^s}$. The term $e^{\rho^s \cdot x} e^{-\eta^s \cdot q}$ is the initial productivity of the worker at the time when he is hired by the firm. The unemployed workers accumulate stochastic labor market histories

[19] Mathematically, it can be seen from the value functions that the same additional future component, $\phi W^{U,s+1}$, which is present in both states, will increase the value of employment more than it increases the value of unemployment (due to the dynamic components of the value functions). Therefore, this difference will be larger in the early stages of a career.

[20] This expected profit does not take into account the stage-specific costs of posting an offer. In order to close the model, it is assumed that these stage-specific costs are such that the firms are indifferent between the stages when posting offers for any type of worker, and, WLOG, they post in all the stages.

in stage s, and their cumulative spells of employment (x) and unemployment (q) are distributed in steady state according to $P^{U,s}(x,q)$. Therefore, the integral $\int_{x=0}^{\infty}\int_{q=0}^{\infty} e^{\rho^s \cdot x} e^{-\eta^s \cdot q} \frac{\partial^2 P^{U,s}(x,q)}{\partial x \partial q}$ summarizes the average productivity of an unemployed worker hired in stage s. As long as the match survives, the productivity of the worker will grow at the rate ρ^s. The match will last for at least τ with probability $e^{-q^s(\theta)\cdot\tau}$, where $q^s(\theta) = \phi + \delta^s + \lambda_1^s \cdot (1 - F^s(\theta))$ is the total separation rate, which includes the transition shock ϕ, exogenous separation δ^s, and job-to-job transition $\lambda_1^s \cdot (1 - F^s(\theta))$. Therefore, the integral $\int_{\tau=0}^{\infty} e^{-r\tau} e^{-q^s(\theta)\cdot\tau} e^{\rho^s \tau} d\tau$ summarizes the expected productivity gain of a worker in this match.

The second term is the expected profit from poaching an employed worker. Here it is important that only those workers who are currently employed at piece rates below θ will be attracted by the offer θ. In this case, the joint distribution of employment and unemployment spells and piece rates, denoted by $P^{E,s}(x,q,\theta)$, must to be taken into account. The term $\int_{x=0}^{\infty}\int_{q=0}^{\infty}\int_{\underline{\theta}^s}^{\theta} e^{\rho^s \cdot x} e^{-\eta^s \cdot q} \frac{\partial^3 P^{E,s}(x,q,\theta)}{\partial x \partial q \partial \theta}$ thus summarizes the average productivity of a worker poached from a firm that paid him a piece rate below θ.

Finally, $\overline{y}_0^s$ is the average productivity with which workers start stage s. It has no impact on the distribution of offers in stage s (see Appendix C).

3.5 Steady-state equilibrium

Following Burdett and Mortensen (1998), a constant profit condition is imposed in order to derive the equilibrium distribution of offers. The idea is that a firm posting a very high offer will receive a low level of profit per worker, but will be very successful in attracting and retaining workers. Thus, the measure of its employees at each instant will be high. Correspondingly, a firm that posts a very low offer will receive a high level of profit per worker but will have a smaller measure of employees, since its workers will frequently accept outside offers and leave. A constant profit condition requires that both these extremes, as well as any offer between them, will yield the same expected profit. In this way, an entire non-degenerate distribution of offers arises across initially identical firms. Burdett et al. (2011) apply this type of equilibrium to a model with on-the-job search and human capital accumulation and study cross-sectional wage dispersion. Carrillo-Tudela (2012) addresses the same question but adds firm productivity differentials to their model. In order to study life-cycle wage profiles, this concept is applied here with on-the-job search, human capital accumulation, human capital depreciation, and segmented career stages.

For each stage $s, s \in \{1, ..., N\}$, a steady-state equilibrium is a tuple: $\left\{\theta^{R,s}, F^s(\cdot), U^s, P^{U,s}(x,q), P^{E,s}(x,q,\theta)\right\}$ such that

(i) $\theta^{R,s}$ is the optimal reservation piece rate of any unemployed worker in stage s.

(ii) $F^s(\cdot)$ satisfies the constant profit condition:

$$\begin{aligned} \pi^s(\theta) &= \overline{\pi}^s > 0 \text{ for all } \theta \text{ where } dF^s(\theta) > 0 \\ \pi^s(\theta) &\leq \overline{\pi}^s \qquad \text{for all } \theta \text{ where } dF^s(\theta) = 0 \end{aligned} \tag{4}$$

(iii) U^s, $P^{U,s}(x,q)$ and $P^{E,s}(x,q,\theta)$ are consistent with steady-state turnover.

The following useful result from Burdett et al. (2011) applies here as well:

Lemma 1. In the equilibrium defined above, for all $s, s \in \{1,...,N\}$: (i) $F^s(\cdot)$ contains no mass points, (ii) $F^s(\cdot)$ has a connected support, and (iii) $\underline{\theta}^s$=$\theta^{R,s}$. Condition (iii) implies that in each stage the lowest offer in the distribution equals the stage-specific reservation cutoff of the unemployed. The proof is presented in Appendix B.

The characterization of the equilibrium distribution of offers, including its upper and lower bounds, can be found in Burdett, Coles, and Carrillo-Tudela (2011). It is applicable here as well, with two major modifications: First, since both actual experience and unemployment history matter for productivity in the current model, the solution involves finding the steady-state joint distributions of histories and piece rates $P^{U,s}(x,q)$ and $P^{E,s}(x,q,\theta)$, for each stage. Second, since workers are forward-looking, their optimal reservation cutoff in each stage depends on the expected value of the future. Therefore, the cutoff and entire distribution of offers has to first be found for the last stage and then, working backwards, for all preceding stages. Appendix C presents a full description of the solution.

At this point, it is clear that the equilibrium distribution of offers in each stage of a career, the resulting life-cycle profile of the piece rate (the mobility component of a wage), and the life-cycle profile of productivity gains and losses (the human capital component of a wage) all depend on the stage-specific parameters.

4 Calibration

We use a quarterly model with three stages: "young", "middle" and "old". The transition parameter is set at $\phi = 1/36$ in order for the composition of the population to change symmetrically over a career, with "young" being most prevalent in the beginning and "old" being most prevalent as we approach 40 years of potential experience, the endpoint of the analysis. This transition rate implies that on average each career stage will last 9 years (with a standard deviation of 18 years). Figure 1 illustrates how the proportions of worker types change over a life cycle:

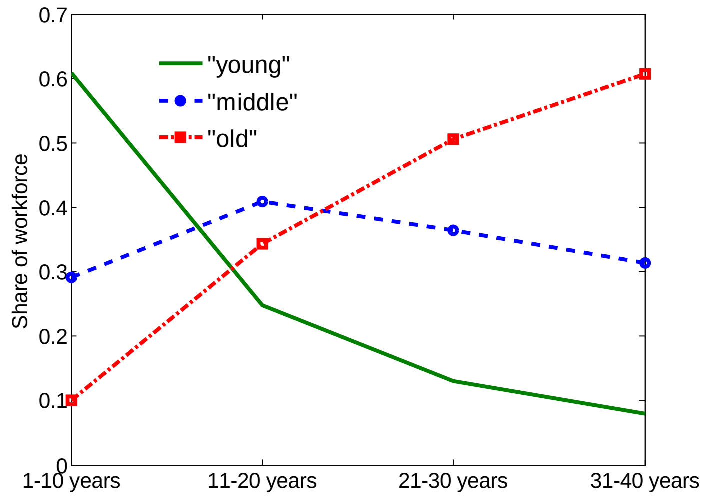

Figure 1: Labor force composition by type

Figure 1 illustrates the fact that career stages and potential experience are not deterministically linked, but rather are positively correlated through workforce composition, with workers who have already reached the last stage of their career becoming more and more prevalent as potential experience increases.

There are five stage-specific parameters to calibrate for each stage $s \in \{1, 2, 3\}$:$(\lambda_0^s, \lambda_1^s, \delta^s, \rho^s, \eta^s)$. The parameters can be divided into two categories: mobility rates $\lambda_0^s, \lambda_1^s, \delta^s$ and human capital accumulation and depreciation rates ρ^s, η^s. Data on mobility by age is used to calculate $\lambda_0^s, \lambda_1^s, \delta^s$ and the human capital parameters ρ^s, η^s are set in such a way that the average log-wage profile in the simulation matches the one from the data. The non-stage-specific parameters b and r are set at 0.4 and 0.0099 respectively, per quarter.

4.1 Mobility parameters $\lambda_0^s, \lambda_1^s, \delta^s$

According to the model, an age-specific transition rate observed in the data is a combination of the transition rates of three underlying types, i.e. "young", "middle" and "old", with weights set according to dynamic workforce composition, as in Figure 1 above. This correspondence is used to calibrate the stage-specific rates $\lambda_0^s, \lambda_1^s, \delta^s$ (see Appendix D, E). Note that in equilibrium δ^sevents correspond to the flow from employment to unemployment and λ_0^s events correspond to the flow from unemployment to employment. These two (weighted) rates can be compared directly to age-specific transition probabilities in the data. Job-to-job flows, however, arise in the model in the combined event of receiving an offer and accepting it (the probability of which depends on the current wage). Therefore, the actual job-to-job transition rate should be compared to a weighted aggregation of transition rates at all possible wage levels in the model. The detailed solution is presented in Appendix D and is along the lines of Nagypal (2008), Hornstein et al. (2011), and Ortego-Marti (2012).

The dashed lines in Figure 2 represent actual transition rates for college and high school graduates over a 40-year career (as calculated by Menzio et al. (2015) using the 1996 cohort of the SIPP panel[21]). The solid lines represent the transition rates implied by the calibration.

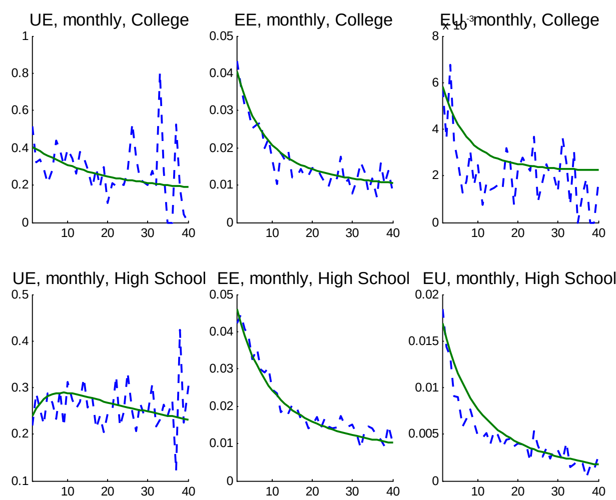

Figure 2: Mobility over the life-cycle: actual and fitted transition rates

Mobility generally declines with potential experience, as Figure 2 illustrates. This is driven by the changing type composition of the workforce, as more mobile "young" workers become extinct later in their career. Calibrated Poisson rates $\lambda_0^s, \lambda_1^s, \delta^s$ differ substantially across stages:

Table 1

	δ^s		λ_0^s		λ_1^s	
	HSG	**CG**	**HSG**	**CG**	**HSG**	**CG**
"*young*"	0.062	0.024	0.778	1.586	0.574	0.580
"*middle*"	0.008	0.003	1.656	1.181	0.084	0.039
"*old*"	0.0003	0.008	0.462	0.334	0.044	0.073

[21] I thank Ludo Visschers for providing this data.

Three points should be noted regarding the calibrated mobility parameters: First, once workers stop being "young", on-the-job search becomes much less efficient. Second, for both education levels the chance to leave unemployment drops sharply upon becoming an "old" worker. Finally, for college graduates the job-finding rate declines monotonically over the life cycle, whereas for high school graduates it is highest in the middle stage. The latter fact is driven by a slight hump in the empirical job-finding rate profile for high school graduates (see Figure 2).

4.2 Human capital parameters ρ^s, η^s

Given the transition rates, human capital parameters are now calibrated in order to match the life-cycle log-wage profiles of high school graduates and college graduates in the United States. In what follows, the construction of average log-wage profiles and the matching criterion are described.

Repeated cross-sections are taken from the CPS March Supplements for the years 1996–2006. The subsample is limited to white male workers, who are employed full-time and have positive potential experience (potential experience = age - years of education - 6). Observations with a missing value for real hourly wage are dropped, as are observations for which the hourly wage is below the federal minimum. Potential experience is limited to no more than 40 years. There are 59,162 observations for college graduates (completed 16 years of education) and 86,177 for high school graduates (completed 12 years of education). These samples combine individuals who entered the labor market as early as 1956 and as late as 2005. In order to take into account the cohort effects that can bias wage experience profiles due to an increase in the return on higher education that occurred in the second half of the twentieth century, the following simple regression was estimated for both education levels:

$$\ln w_{i,C,X,t} = \sum_{C=1956}^{2005} \beta_C \cdot D_{C,i} + \sum_{X=1}^{40} \beta_X \cdot D_{X,i,t} + \varepsilon_{i,t} \qquad (5)$$

The regression decomposes the log-wage of individual i, who belongs to cohort $D_{C,i}$ and has potential experience X at time t, into the effect of his cohort $D_{C,i}$, the effect of his experience at time t $D_{X,i,t}$, and the individual i.i.d. error term $\varepsilon_{i,t} \sim N(0, \sigma^2)$. The dummy variable $D_{C,i}$ equals 1 if individual i belongs to cohort C, defined according to the year of labor market entry. The dummy variable $D_{X,i,t}$ equals 1 if individual i has potential experience X in year t. The profile of coefficients β_X over potential experience levels is the life-cycle log-wage profile net of cohort effects. Notably, as shown in Figure 5, the removal of cohort effects has little impact on the workers, who have only a high school degree. By contrast, the profile of those with a college diploma is corrected upwards for high experience levels, reflecting the fact that most experienced workers are those who belong to the earliest cohorts, i.e., those who entered the labor market before the period of growth in the return on higher education. These workers are therefore "disadvantaged" relative to less experienced college graduates from more recent cohorts. The regression with cohort effects corrects for this bias.

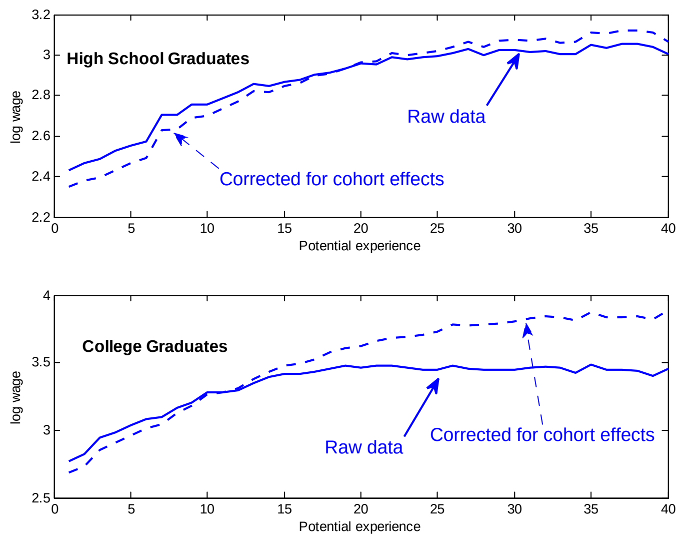

Figure 3: Wage profiles and cohort effects

For each education group, a search is carried out for combinations of ρ^s, η^s for all $s = 1, 2, 3$ that minimize the distance between the simulated average log-wage profile and the actual log-wage profiles net of cohort effects constructed above:

$$\min_{\rho^s . \eta^s} MSE = \frac{1}{40} \sum_{x=1}^{40} \left(\ln w_x^{data} - \ln w_x^{simulation} \right)^2, \tag{6}$$

where $\ln w_x^{data}$ is the β_X coefficient from the above regression (5), and $\ln w_x^{simulation}$ is the average log-wage profile obtained from simulating careers for an artificial sample of 10,000 workers over 40 years of potential experience. Figure 4 depicts the normalized $\ln w_x^{data}$ and $\ln w_x^{simulation}$ profiles[22] under the parameter combination that minimizes the distance between them, as in (6).

[22] The profiles are normalized to start from 0.

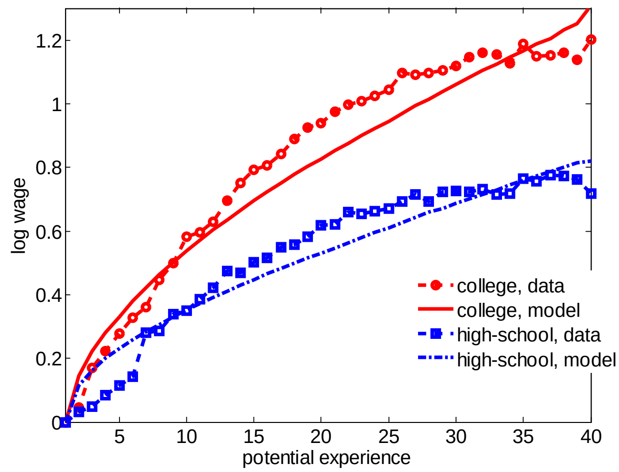

Figure 4: Cumulative wage growth: data and calibrated model

The mean square errors are $MSE(HSG) = 0.004$ (s.d. 0.0002) and $MSE(CG) = 0.006$ (s.d. 0.0007) for high school and college graduates, respectively.

Table 2 summarizes the resulting calibrated human capital parameters:

Table 2. Calibrated stage-specific human capital parameters

	ρ^s		η^s	
	HSG	**CG**	**HSG**	**CG**
"*young*"	0.008	0.017	0.000	0.000
"*middle*"	0.008	0.012	0.002	0.004
"*old*"	−0.006	−0.015	0.015	0.02

Several observations can be made based on Table 2: First, few benchmarks exist in the literature for the negative return from an additional period of unemployment η. Ortego-Marti (2012) uses PSID and runs a regression relating current log wage to accumulated unemployment history, and reports an estimate that is equivalent to $\eta = 0.036$ in the current model. Saporta-Eksten (2013) calibrates the loss-of-skills rate to be 0.0125 per quarter.[23] These estimates are not dependent on level experience and approximate the calibrated values for the last career stage.

Second, the calibration shows that returns to experience become negative in the last career stage. In other words, productivity declines even if a person is working. This effect is greater for college graduates than for high school

[23] 5% annually.

graduates. This may be because college graduates tend to be employed in technologically-oriented occupations and as technology advances older workers find it especially difficult to keep up. It is important to keep in mind that since each potential experience level combines workers of all three types, declining productivity among the "old" does not mean that all workers late in their career will be losing skills even in employment. What it does mean is that over the course of a career there is an increasing share of workers whose productivity declines even in employment, reaching 60 percent of the workforce at 31 to 40 years of potential experience. Finally, the model is framed in real terms, and a decline in real wage does not mean that these workers actually see a decline in their nominal wage.

Third, workers lose skills when unemployed only starting from the second stage and the losses increase from stage to stage. The rate of unlearning by not doing is never higher than the rate of learning. The increase in the negative return from unemployment with age is consistent with the evidence for the U.S. in Davis and Von Wachter (2011) and Johnson and Mommaerts (2011). Gregory and Jukes (2001) report a similar pattern using data from the U.K.

In summary, human capital technology, like mobility rates, varies substantially over the life-cycle. The negative impact of unemployment on wages is always driven more by foregone on-the-job human capital accumulation than by skills depreciation in unemployment, since ρ is always higher than η. This is particularly true for young workers, most of whom do not lose skills in unemployment at all and learn quickly on the job. The last stage is characterized by human capital losses in employment, and by even greater depreciation in unemployment. Finally, human capital processes are overall more intensive for college graduates than for high school graduates.

5 Stage-specific equilibrium distributions of offers, $F^s(\theta)$

The stage-specific parameters of human capital and mobility determine the endogenous equilibrium distributions of offers in each stage (see Appendix C for the detailed equilibrium solution). Figures 5a and 5b present the bounds of the equilibrium $F(\theta)$ for each of the three stages and for each of the two education levels.

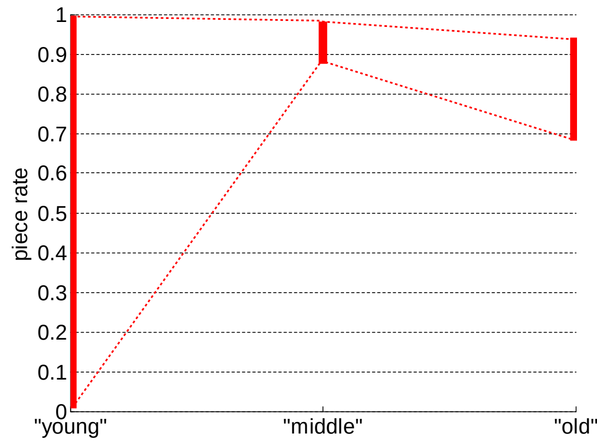

Figure 5a: College graduates: support of the equilibrium offers distribution

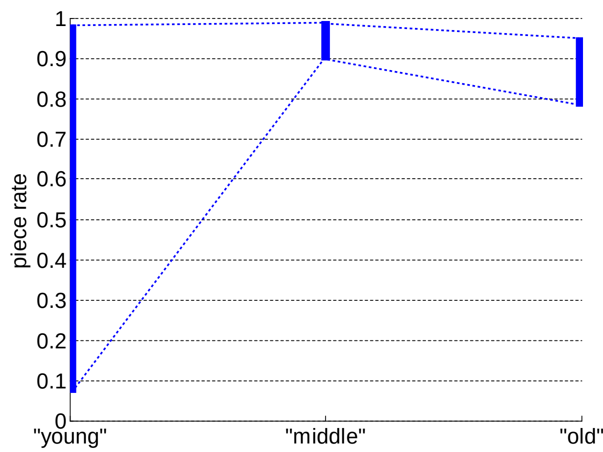

Figure 5b: High-School Graduates: support of the equilibirum offers distribution

In both education groups, there are marked differences between the distributions from which workers in different stages sample wage offers, as shown in Figure 5.

At the beginning of a career, the human capital accumulation rate ρ is especially high and the expected lifespan is long, which makes employment particularly attractive since human capital is general and each additional unit accumulated will serve a worker throughout his career, thus increasing earnings.

Offers arrive frequently on the job, as reflected in a high λ_1. Taken together, these factors drive down the reservation rate of unemployed workers in the first stage and correspondingly the lower bound of the equilibrium distribution $F^1(\theta)$. The fact that "young" workers are highly mobile between jobs increases the competition among firms, thus pushing up the highest equilibrium offer. As a result, there is a broad range of equilibrium offers for workers in the first stage of their career. In mid-career, the expected horizon shortens and human capital accumulation slows. While skill loss in unemployment is still moderate, on-the-job search becomes less efficient (lower λ_1), while the job-finding rate is not much lower than for the "young" (among high school graduates it is even higher). All this results in "middle-aged" unemployed workers valuing unemployment relatively more, thus raising their reservation cutoff. Consequently, the range of offers contracts. Finally, those who reach the "old" stage of their life cycle face far worse conditions in terms of the job-finding rate in unemployment λ_0, and especially in terms of human capital depreciation η. This again lowers their cutoffs , despite the fact that employment itself is associated with some loss of productivity, since conditions in unemployment are even worse. Therefore, relative to the "middle" stage of their career, workers in the last stage sample offers from a distribution that has shifted downward.

Comparing distributions of offers (piece rates) in the model to the data is not straightforward. First, piece rates cannot be observed directly. Second, only accepted offers show up in the surveys. Given these constraints, the empirical distribution of wages of low-tenured workers is used as a proxy for the distribution of wage offers. The data is taken from the CPS Outgoing Rotation Groups for 1999–2006 (see Appendix F for details). The sample is restricted to full-time white male workers who are employed now but were unemployed in the previous month. For some potential experience levels, there are only a few observations for such newly-hired workers, and therefore they are grouped into eight 5-year intervals in order to increase the size of each bin. These workers' real hourly wages are compared with those of newly-hired workers in the simulation (the product of the offered piece rate and the unemployed worker's human capital). Two statistics are used in the comparison: the 5th percentile of the distribution and the coefficient of variation. The former was chosen because previous analysis suggests that much of the difference between offer distributions across stages occurs in the lower part of the distribution. Comparing the coefficient of variation makes it possible to abstract from the scale of wages both in the data and in the model.

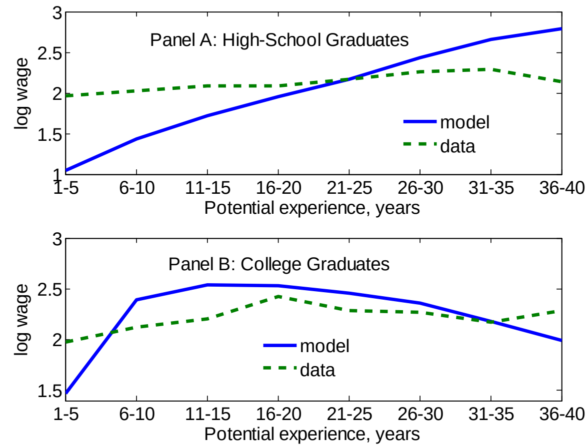

Figure 6: 5th percentile of log wage offers: model and data

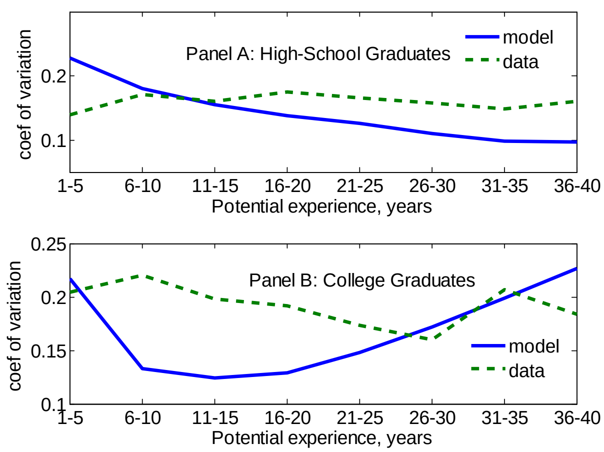

Figure 7: Coefficient of variation of log wage offers

Figures 6 and 7 indicate that both in the data and in the model wage offer distributions for college graduates vary over a career in a different manner than those of high school graduates. For college graduates, the profile of the 5th percentile is slightly hump-shaped, and there is a U-shape in the coefficient of variation. These non-monotonic patterns are captured by the model. For high school graduates, on the other hand, both the 5th percentile and the coefficient of variation remain almost constant over a career, while the model predicts that they change monotonically. However, the results of the comparison should be treated with caution in view of the noise in the data. Thus, even when bunching by 5-year intervals the number of just-hired workers in the CPS Outgoing Rotation Groups is very small: around 200 points on average in each interval for high school graduates and around 60 points for college graduates (for comparison, in the simulated sample it is over 37,000 for high school graduates and over 25,000 for college graduates).

6 Simulation and wage growth decomposition

A simulation is performed for a sample of 100,000 workers, who start from the first stage, S_1, with a corresponding distribution of offers and stochastically move on to later stages. Each worker's unemployment history is tracked, including, at each level of potential experience, current stage, employment status, actual productivity, productivity losses accumulated due to unemployment episodes up to the present, and current piece rate in the case of employment. Since the wage in the model is a combination of piece rate and actual productivity, which in turn is the product of total accumulated human capital and total human capital lost, it is straightforward to decompose the actual log wage of an employed worker in the simulation into the additive effects of mobility ($\ln \theta$), accumulated human capital, and human capital lost due to unemployment history.

6.1 Human capital accumulation

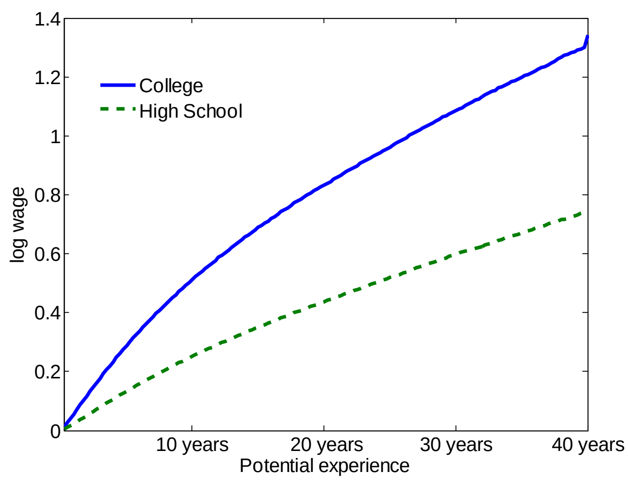

Figure 8a: human capital accumulation

Figure 8a presents the productivity profile resulting from actual experience accumulation. One can see how the gap between college graduates and high school graduates widens over time. The reason for this is twofold: First, the (positive) return on experience is always higher for college graduates than for high school graduates. Therefore, a given level of actual experience will benefit college graduates more in terms of human capital. Second, matches tend to last longer, and unemployment spells tend to be shorter for college graduates (according to calibrated λ_0 and δ) and therefore college graduates will have accumulated more actual experience than less-educated workers. For college graduates, productivity growth due to actual experience accumulation accounts for 62 percent of the wage increase during the first 10 years of a career (versus 44 percent for high school graduates), and for 85 percent over 40 years (versus 72 percent for high school graduates). Thus, human capital plays an important role in total wage growth. However, as will become clear, its estimated role is still much more modest than most previous studies have concluded.

6.2 Mobility

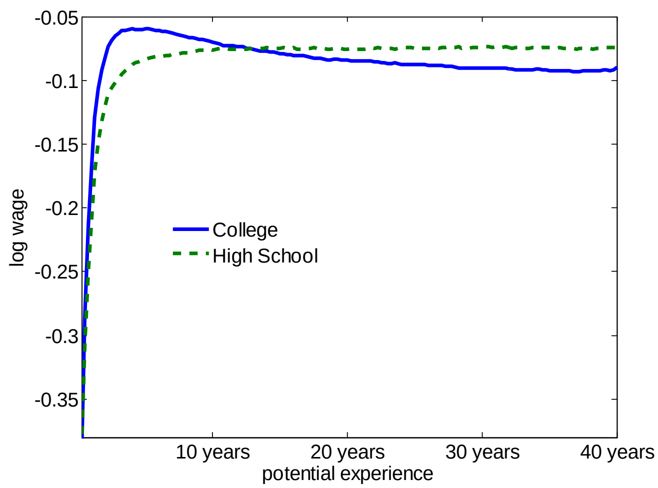

Figure 8b: Mobility component of wages

Figure 8b presents log piece rate profiles for college and high school graduates, i.e., the mobility component of log wages. Initially this component increases concavely, reflecting the diminishing returns of search as workers succeed in climbing up the wide ladder of offers in the first stage of their career. For college graduates, the average piece rate starts to *decrease* after rising initially and drops by 0.03 log points by the end of a career. Such a phenomenon could not occur in a framework where all workers sample job offers from the same distribution. Indeed, a common finding in the literature is that the average search component remains flat after an initial rise, because the returns from moving up the given wage ladder are quickly exhausted. In the current model, a similar dynamics applies within each stage, except that in the last stage, as compared to mid-career, the entire distribution of offers endogenously shifts downward, especially for college graduates who lose more in terms of job-finding probability and ability to retain skills in unemployment as they become "old". As the share of workers entering the last stage of their career builds up, there is a resulting decline in the average search component.

For high school graduates such an effect does not arise, because conditions in the last career stage do not deteriorate so drastically in their case. In fact, the endogenous distributions of offers for high school graduates do not differ much between the last two stages. Thus, the wage ladder remains approximately the same, and the average log piece rate stays flat.

For high school graduates, search accounts for 57 percent of cumulative wage growth (versus 40 percent for college graduates) during the first ten years of a career, and 30 percent (19 percent for college graduates) over the 40-year life cycle. Comparing these values to those found in other studies is again not straightforward. First, each study usually refers to a different sample and a different career horizon. Luckily, most studies include a subsample of high school graduates and regard the first 10 years of a career as an important milestone. Second, not all studies report estimates of the search input. It is nonetheless common that a study reports the estimated share of wage growth explained by the human capital component, which can be used to make inferences about the role of search. Thus, regardless of the assumptions made in each study regarding additional sources of wage growth, the share of human capital sets the upper bound for the combined role of all other factors of wage dynamics, including mobility. The higher the proportion of wage growth explained by human capital accumulation, the lower will be the proportion that is potentially explained by search.

The table below summarizes the findings of several studies regarding the role of human capital accumulation in wage growth for high school graduates in the U.S. during the first 10 years of their career, and the implied upper bound for the role of mobility (calculated as if it were the only additional factor of wage growth). The total wage growth (in log points) is included when reported by the authors in order to highlight the comparability of the different studies.

Table 3. The role of mobility in total wage growth: comparison to other studies

HSG 10 years of career	Sample	Mobility (implied)	total log wage growth
Present study	**SIPP 1996-2000, CPS 1996-2006**	**56%**	**0.53**
Altonji et al. (2013)	PSID 1975-1996	26%	0.51
Buchinsky et al. (2010)	PSID 1975-1992	22%	0.51
Menzio et al. (2015)	SIPP 1996-2000	24%	0.42[24]
Schonberg (2007)	NLSY 1979-1994	28%	0.55
Yamagouchi (2010)	NLSY 1979-2002	23%	0.53
Bowlus and Liu (2013)	SIPP 1996-2000, NLSY 1979-1994	88%	not reported

[24] Between 21 and 31 years

As can seen from the table, the implied contribution of mobility to wage growth ranges from 22 to 28 percent (ignoring Bowlus and Liu (2013)). The current model predicts a much higher upper bound of 56 percent, and the actual input is even higher at 57 percent, due to the negative returns from unemployment component.

An important factor accounting for the major role of mobility in search models is the low cutoff wage of unemployed workers at the beginning of their career. When young workers start their career with a low wage, and are easily able to move between jobs, then, other things being equal, they will benefit greatly from search. This is exactly the mechanism present in the current model, where two complementary factors drive down the reservation cutoff of the young. The first is within-stage: intensive human capital accumulation in the first stage, combined with efficient on-the-job search,[25] make employment more valuable. The second factor is the effect of the shortening horizon, which makes early employment even more attractive. The worker takes into account, through the value function, that general human capital accumulated on the job will benefit them over their entire career. This incentive is strongest in the first stage of a career, thus inducing young unemployed workers to accept very low offers and increasing the importance of mobility.

Though the estimate obtained of the role of mobility is twice that found in previous studies, it still falls short of the results of Bowlus and Liu (2013). This can be partly attributed to the endogeneity of the entire distribution of offers in the current model, as opposed to Bowlus and Liu (2013) where only the reservation cutoff is endogenous. Note that the range of the wage increase due to search depends not only on the lowest offer from which the ascent starts, but also on the upper bound of offers. In Bowlus and Liu (2013), the distribution of offers is exogenously set to be log-normal, with no upper bound, whereas in the current model the upper bound is fixed and derived endogenously, based on the equilibrium constant profit condition. This may limit the role of search relative to the setting in which, theoretically, the offers can be unboundedly high.

6.3 Human Capital Depreciation

[25] λ_0^1 is also high, thereby increasing the reservation rate.

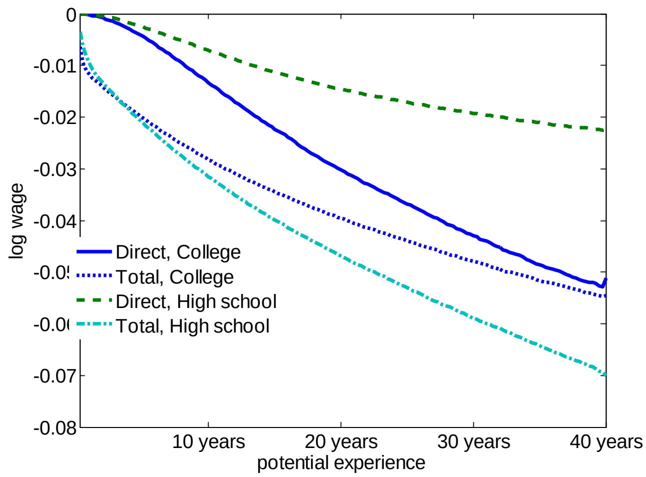

Figure 8c: losses from unemployment - loss of skills and foregone earnings

There are two sources for the negative impact of unemployment on wages in the model. First, there is a direct loss due to human capital depreciation, which quantitatively is very small (see Figure 8c for illustration). The direct losses amount to around 2 percent of cumulative wage growth over 40 years of a career on average for high school graduates and 3 percent for college graduates.[26] This direct component is more significant for college graduates, because they experience more intensive skills depreciation. Second, there is an indirect loss due to foregone on-the-job human capital accumulation. The total of both the direct and indirect components is non-negligible. For college graduates, direct and total losses converge towards the end of a career, because employment is not very different from unemployment for the "old" in terms of human capital, while high school graduates in the last stage are substantially better off when they have a job, and the indirect losses are substantial. Thus, over 6 percent of total wage growth is lost on average by high school graduates due to unemployment after 40 years in the labor market (versus about 3 percent for college graduates).

Unemployment spells in the U.S. are typically brief and infrequent and this is the case in the simulation as well. After 40 years of a simulated career, an average college graduate will have spent 15 months in unemployment (25 months for a high school graduate). These low averages conceal the heterogeneity of histories, such that some workers accumulate a great deal of unemployment, while others accumulate almost none. By comparing average wage profiles for these different types of careers, it is possible to asses the cost to those workers who are unlucky enough to experience long periods of unemployment.

[26] Notably, Altonji et al. (2013), which is the only study that estimates losses in general human capital due to cumulative unemployment spells, report losses that are -0.02 log points during the first 30 years of a career, which is very close to the result obtained here.

Figure 9 below presents average log-wage profiles for career scenarios that vary according to the amount of unemployment history accumulated during 40 years in the labor market.

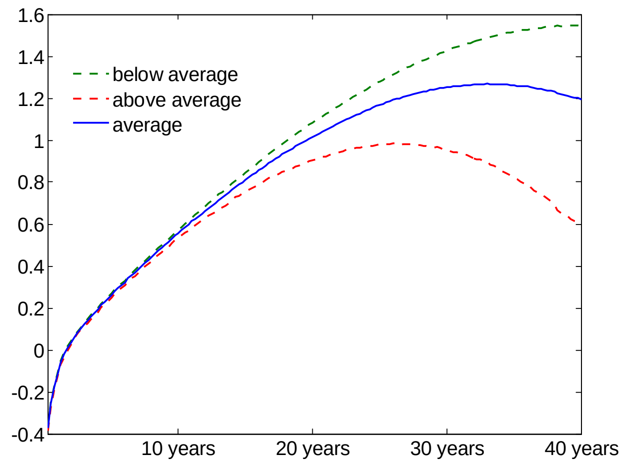

Figure 9a: College graduates: unemployment history and log-wage profiles

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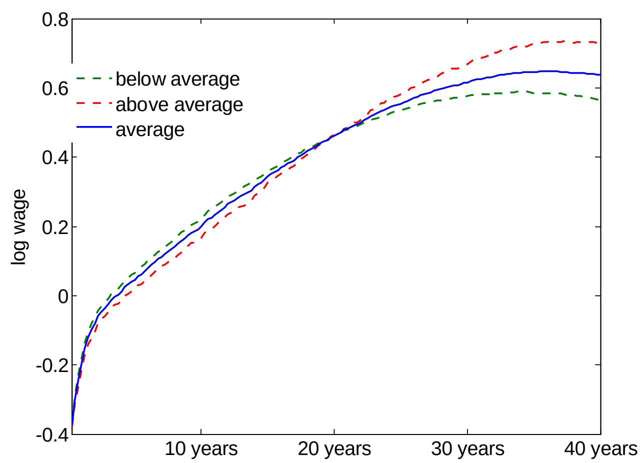

Figure 9b: High-school graduates: unemployment history and log-wage profiles

Figure 9a shows that the wage profiles for college graduates differ significantly according to the amount of unemployment accumulated. The wage profiles diverge in the second half of a career, when the wage profile of workers with long accumulated unemployment curves downward much more than the average. This is in contrast to high school graduates for whom different unemployment histories are not translated into marked differences in average wage profiles.

To numerically evaluate the size of the damage from long accumulated unemployment, the following simple calculation can be made: First, the simulated hourly wage profiles are scaled up so that they correspond to the actual ones not only in terms of wage growth, but also in terms of level. Technically, this involves multiplying all wages by a constant, which can be treated in the model as starting productivity upon entry into the labor market. Second, income earned over a quarter is the product of the wage and the proportion of the quarter actually spent in employment. Using that information, total wage income in a quarter can be calculated by multiplying the quarterly hourly wage by 40 (hours per week) $\times 13$ (weeks per quarter)$\times$proportion of the quarter in employment. Quarterly earnings are then discounted by a discount factor of $r = 0.0099$ to yield the present discounted value, for each worker, of a 40-year career. This is done for all workers who accumulated 40 years in the market. Table 4 summarizes the results of this exercise.

Table 4. Unemployment histories and lifetime earnings

Unemployment history	Share of all histories	Total, 2010 USD	% of average
College graduates			
average (1y3m)		1,659,000 (s.d. 515,130)	-
below average (8m)	61%	1,793,800 (s.d. 478,440)	+ 8.1%
above average (2y2m)	39%	1,443,200 (s.d. 498,060)	- 13.0%
high school graduates			
average (2y1m)		715,000 (s.d. 102,980)	-
below average (1y3m)	57%	727,860 (s.d. 118,150)	+1.8%
above average (3y3m)	43%	698,190 (s.d. 75,640)	-2.3%

An average college graduate will accumulate about 15 months of unemployment during a 40-year career, and his discounted income over 40 years will be around \$1.6 million, in 2010 prices. If he is lucky enough to be in the 61 percent of his cohort who accumulate less than 15 months of unemployment, his discounted lifetime income will be 8 percent higher. If, on the other hand, he is in the 39 percent who accumulate longer than average unemployment over 40 years, his lifetime income will be lower by 13 percent. An average high school graduate is expected to accumulate approximately \$0.7 million in 2010 prices, which is not significantly affected by total accumulated unemployment.

The life-cycle dynamics of job search and human capital depreciation rates are consistent with the results in Figure 9 and Table 4. For college graduates, the job-finding rate is low only in the last stage of a carreer, which implies that long unemployment spells mostly accumulate late in a worker's career, when unemployment is especially damaging to human capital. Therefore, their log-wage profiles bend downwards significantly more than the average late in a career. In contrast, high school graduates have about the same chance of experiencing long unemployment spells when they are "young" as when they are "old" , based on their job-finding rate λ_0. Therefore, they do not necessarily experience long unemployment in stages when it is most damaging to their skills, and the rate of depreciation is in any case relatively moderate for them. Their log-wage profiles will not deviate far from the average regardless of their unemployment history.

7 Conclusion

This study places unemployment into the context of life-cycle human capital evolution. Recent studies (for example, Jarosch (2015) and Saporta-Eksten (2013)) have shown that unemployment episodes also lead to increased employment risk in the future. It remains an open question whether this reduction in job stability and depreciation of (general) human capital are actually related. In a framework where less-productive matches are less resilient to match-specific shocks, longer unemployment spells lead to a higher risk of future endogenous separations. One could potentially test this prediction by testing whether longer unemployment spells following an exogenous layoff are associated with shorter employment durations in the future, while controlling for workers' pre-dislacement characteristics.

It would be interesting to incorporate precautionary savings and increased health risk in retirement within the model. As the evidence suggests, older workers have very low returns on experience and suffer from substantial skills depreciation in unemployment. If they spend down their assets on consumption smoothing during unemployment, they will have little chance to build them up again in order to shield themselves against high healthcare expenses in old age. They therefore might lower their reservation rates in order to get back to work. When these considerations are taken into account, the moral hazard problem among old unemployed workers becomes less severe, potentially making the optimal unemployment insurance profile U-shaped over the life cycle.[27] Furthermore, lower reservation rates are consistent with the substantial wage losses upon re-employment which is observed among older workers.[28] These questions are left to future research.

[27] Michelacci (2013) argues for a decreasing unemployment insurance based on the fact that young workers are liquidity-constrained and value the insurance more, in addition to having a greater incentive to get back to work in order to accumulate human capital.

[28] Johnson and Mommaerts (2011).

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