

Segregation, Informativeness and Lorenz Dominance*

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Abstract

It is possible to partially order cities according to the informativeness of neighborhoods about their ethnic groups. It is also possible to partially order cities with two ethnic groups according to the Lorenz criterion. We show that a segregation order satisfies four well-established segregation principles if and only if it is consistent with the informativeness criterion. We then use this result to show that for the two-group case, the Lorenz and the informativeness criteria are equivalent.

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1 Introduction

Sociologists and economists have long been interested in how to adequately measure segregation. While early studies restricted attention to segregation between two groups, i.e., blacks and whites, or men and women, later ones developed measures for multigroup cases.¹ One of the difficulties of measuring segregation is that it is not clear what segregation actually means. Massey and Denton [13] identified five dimensions, namely evenness, exposure, concentration, centralization and clustering, each of which captures some particular aspect of the concept. The one which is of interest to us is that of evenness. Evenness refers to the similarity among distributions of members of different groups across locations. The more similar these distributions are, the less is the degree of segregation. Although the number of segregation indices is very large, it is safe to say that most of the literature, both theoretical and empirical, focuses on the evenness dimension.²

For the two-group case, the literature on segregation borrowed the device of the Lorenz curve and built what is known as segregation curves. Recall that for each fraction p , the Lorenz curve depicts the proportion of total income that is owned by the poorest proportion p of the population. A segregation curve is essentially a Lorenz curve where one group, say blacks, is treated as population, and the other group, say whites, is treated as income. Segregation curves appear in the literature as early as in Duncan and Duncan [5].

Segregation curves are used to partially order cities, and we refer to the resulting ranking as the Lorenz partial order. Specifically, given two cities, their corresponding segregation curves may or may not cross. If they do not cross then the city whose

¹See Reardon and Firebaugh [14] for an enumeration and analysis of various multigroup segregation measures. For the two-group case, Massey and Denton [13] provide a comprehensive survey.

²For papers that model segregation differently, see Echenique and Fryer [6] and Ballester and Vorsatz[2].

segregation curve lies below that of the other one is deemed, according to the Lorenz criterion, the more segregated one. It was not until the mid eighties that the Lorenz partial order began to be justified on first principles as a valid criterion for segregation measurement. The first contribution in that direction was James and Taeuber [10] who developed a set of principles against which segregation indices could be compared. These principles, which they called *organizational equivalence*, *transfers*, and *composition invariance* became widely accepted and well-established axioms for segregation measures. Indeed, together with *anonymity*, they are considered to be properties that any measure that aims at capturing the evenness dimension of segregation must satisfy. Later, Hutchens [9] showed that, restricted to cities where the members of one group is evenly distributed across locations, any segregation measure that complies with the above principles conforms with the Lorenz criterion.

This paper addresses the question of whether an appropriate extension of the Lorenz criterion can be found for the multigroup case. By appropriate extension we mean one that epitomizes unanimous agreement among all the segregation measures that satisfy an appropriate generalization of the above four principles. We will show that we can answer this question in the affirmative. To do so, we draw on the literature on the value of information and order the cities according to the informativeness of their neighborhoods about the ethnicity of a randomly selected resident. Specifically, given a city, the location of a randomly selected individual is a signal that provides information about the ethnic group he belongs to. In that sense, the collection of distributions of the various ethnic groups across locations can be seen as an experiment in the sense of Blackwell [3, 4], one in which locations play the role of signals and ethnic groups play the role of states of nature. We can then borrow Blackwell's partial order on experiments and apply it to partially order cities. More concretely, a city whose locations are more informative than another city's locations will be considered more segregated than the latter.

It turns out that this partial order is the one we are looking for. That is, it satisfies the appropriate extension of the principles of anonymity, organizational equivalence,

transfers and composition invariance to the multigroup case, and furthermore, any segregation measure or even partial order that satisfies these principles must agree with it. As a corollary we obtain that, when restricted to cities with only two groups, the partial order derived from the segregation curves coincides with the partial order derived from the informativeness of the city's neighborhoods. In that sense, not only is the latter partial order applicable to the multigroup case, but it is also a generalization of the standard order based on segregation curves.

The fact that any partial order that satisfies the above mentioned axioms must be consistent with the partial order derived from the segregation curves was stated without proof by James and Taeuber [10]. As mentioned earlier, a proof of this result for the case where all locations contain the same number of members of one group (e.g., all occupations contain the same number of women), was proved by Hutchens [9]. Frankel and Volij [7] noted that any order that satisfies three of the four axioms and weak form of the fourth one must be consistent with the partial order associated with the informativeness of the cities' experiments restricted to the class of cities with the same ethnic distribution.³ We prove this result for the case of all cities, independently of their ethnic distribution.

The paper is organized as follows. After introducing the basic notation in Section 2, Section 3 presents four minimal properties that measures of the evenness dimension of segregation should satisfy. Section 4 defines Blackwell's partial order and shows that any segregation order that satisfies the above four properties must be consistent with it. Section 5 shows that, restricted to the two-group case, the Blackwell and the Lorenz orders coincide. Section 6 concludes.

³See also Grant, Kajii and Polak [8], and Andreoli and Zoli [1] for related results.

2 Notation

The basic model of segregation measurement consists of a list of locations containing different numbers of members of various groups. Papers that focus on residential racial segregation refer to the locations as neighborhoods, and to the groups as ethnic groups. Papers dealing with occupational gender segregation usually use occupations as locations and classify the groups by gender. For our purposes we will use the language of racial residential segregation, and refer to the list of neighborhoods as cities.

Let G be a finite set of ethnic groups. This set will remain fixed for the whole analysis and in Section 5 it will be further restricted to contain only two groups. A *neighborhood* n is characterized by its racial composition. For each ethnic group g , T_n^g denotes the number of residents of n that belong to g . The racial composition of n is then the vector $(T_n^g)_{g \in G}$. A *city* is a finite collection of nonempty neighborhoods such that for each ethnic group g , at least one neighborhood has a positive number of residents of that group. Formally, a city is a system $\langle N, ((T_n^g)_{g \in G})_{n \in N} \rangle$ such that N is the set of neighborhoods, for each ethnic group $g \in G$, $\sum_{n \in N} T_n^g > 0$, and for each $n \in N$, $\sum_{g \in G} T_n^g > 0$.

Given a city $X = \langle N, ((T_n^g)_{g \in G})_{n \in N} \rangle$, we denote by $T^g(X)$ the total number of residents of group g : $T^g(X) = \sum_{n \in N} T_n^g$. When it is clear to which city we are referring, we will simply write T^g . We will denote by t_n^g the proportion of individuals of ethnic group g that reside in neighborhood n . Formally, $t_n^g = T_n^g / T^g$. Similarly, $p_n^g = T_n^g / \sum_{g \in G} T_n^g$ is the proportion of residents of n that belong to ethnic group g . The *ethnic distribution of a neighborhood* n is given by $(p_n^g)_{g \in G} = (T_n^g)_{g \in G} / \sum_{g \in G} T_n^g$, and the *ethnic distribution of a city* X is given by $(T^g)_{g \in G} / \sum_{g \in G} T^g$.

For any positive integer k , I_k denotes the $k \times k$ identity matrix. We will sometimes apply certain operations on matrices by postmultiplying them by special Markov matrices.⁴ A *splitting matrix* is one that is obtained from an identity matrix by splitting

⁴Recall that a Markov matrix is a non-negative matrix with each row summing to one. Recall also that a permutation matrix is one that is obtained by permuting the columns of an identity matrix.

some of its columns into several columns. Permuting the columns of a splitting matrix also results in a splitting matrix. A *merging matrix* is one that is obtained from a permutation matrix by replacing some of its columns by their sum. Note that a product of merging matrices is also a merging matrix. Also note that when a matrix is postmultiplied by a splitting matrix, some of its columns are split into several proportional columns, and that when a matrix is postmultiplied by a merging matrix, some of its columns are replaced by their sum.

3 Properties of segregation orders

Let \mathcal{C} be the set of all cities. A *segregation order* is a partial order on \mathcal{C} . For any X and $Y \in \mathcal{C}$, $X \succcurlyeq Y$ means that X is at least as segregated as Y according to \succcurlyeq .⁵ Clearly, not all partial orders on \mathcal{C} are reasonable segregation orders. In this section we introduce several properties that are generally agreed to be required from good segregation orders.

The first one, anonymity, requires that segregation does not depend on the labeling of the ethnic groups. In order to formalize this property, we need a preliminary definition.

We say that two cities, $X = \langle N_X, ((T_n^g)_{g \in G})_{n \in N_X} \rangle$ and $Y = \langle N_Y, ((T_{n'}^g)_{g \in G})_{n' \in N_Y} \rangle$, are *equivalent* if there is a one-to-one mapping $\varphi : N_X \rightarrow N_Y$ such that for all $n \in N_X$, $(T_n^g)_{g \in G} = (T_{\varphi(n)}^g)_{g \in G}$.

Equivalent cities differ only in the names of their neighborhoods. The anonymity axiom requires that equivalent cities are equally segregated:

Anonymity (ANON) A segregation order \succcurlyeq satisfies *anonymity* if for any two equivalent cities X and Y we have $X \sim Y$.

The next property is what James and Taeuber [10] call the principle of organizational equivalence. It holds that the segregation of a city is unaffected if one of its

⁵Given \succcurlyeq , the associated relations \succ and \sim are defined as usual: $X \succ Y \Leftrightarrow X \succcurlyeq Y$ and not $Y \succcurlyeq X$, and $X \sim Y \Leftrightarrow X \succcurlyeq Y$ and $Y \succcurlyeq X$.

neighborhoods is divided into two neighborhoods with the same ethnic distribution, or equivalently if two such neighborhoods are combined into one. Table 1 illustrates such a division. Organizational equivalence requires that before and after the change segregation be the same.

	X		Y		
	A	B	A_1	A_2	B
Blacks	30	50	20	10	50
Whites	120	50	80	40	50

Table 1: Splitting a neighborhood and keeping its ethnic distribution.

Organizational Equivalence (OE) Let $X \in \mathcal{C}$ be a city and let $(T_n^g)_{g \in G}$ be one of its neighborhoods. Let Y be the city that results from dividing $(T_n^g)_{g \in G}$ into two neighborhoods, $(T_{n_1}^g)_{g \in G}$ and $(T_{n_2}^g)_{g \in G}$, with the same ethnic distribution. Namely, $(T_{n_1}^g)_{g \in G} = (\alpha T_n^g)_{g \in G}$ and $(T_{n_2}^g)_{g \in G} = ((1 - \alpha)T_n^g)_{g \in G}$ for some $\alpha \in (0, 1)$. A segregation order \succsim satisfies *organizational equivalence* if for any such cities we have $Y \sim X$.

Organizational equivalence is a very weak requirement and it is in fact satisfied by all segregation indices that we are aware of.

Another well-established requirement is what James and Taeuber [10] call the principle of transfers. It is the analog of the Pigou-Dalton transfers principle of income inequality. It applies only to cities with two ethnic groups and it states that segregation increases whenever members of one group move from a neighborhood in which this group constitutes a relatively lower proportion of its residents to another neighborhood where it constitutes a relatively higher proportion. There is no controversy regarding the desirability of the transfer principle. However, it has no obvious generalization to the multigroup case. Frankel and Volij [7] proposed the following one, which they call

the neighborhood division property. It requires that the segregation of a city increase if one of its neighborhoods is divided into two neighborhoods with different ethnic distributions. Table 2 illustrates such a division.

	X		Y		
	A	B	A_1	A_2	B
Blacks	30	50	20	10	50
Whites	120	50	60	60	50

Table 2: Splitting a neighborhood and changing its ethnic distribution.

Neighborhood Division Property (NDP) Let $X \in \mathcal{C}$ be a city and let $(T_n^g)_{g \in G}$ be a neighborhood of X . Let Y be the city that results from dividing $(T_n^g)_{g \in G}$ into two neighborhoods, $(T_{n_1}^g)_{g \in G}$ and $(T_{n_2}^g)_{g \in G}$, with different ethnic distributions. Namely, $(T_{n_1}^g)_{g \in G} \neq (\alpha T_n^g)_{g \in G}$ for any $\alpha \in [0, 1]$. A segregation order \succsim satisfies the *neighborhood division property* if for any such cities we have $Y \succ X$.

It turns out that when restricted to the two group case, any segregation order that satisfies organizational equivalence satisfies the transfer principle if and only if it satisfies the neighborhood division property. (See Lasso de la Vega and Volij [12, Claim 1]). In other words, for the two-group case and under the assumption of organizational equivalence, the transfer principle and the neighborhood division property are equivalent. Furthermore, all multigroup extensions of indices that comply with the principle of transfers that we are aware of also comply with the neighborhood division property.

The last principle proposed by James and Taeuber [10] is known as *composition invariance*. It holds that segregation is unaffected by proportional changes in the number of residents of a given group. This property is not as uncontroversial as the previous ones, but is widely accepted as a reasonable requirement for indices that aim at capturing the

evenness dimension of segregation.⁶ In fact, it was mentioned as a necessary requirement from a satisfactory measure of segregation as early as in 1947 (see Jahn, Schmidt, and Schrag [11, p. 294]).

Composition Invariance (CI) Let $X = \langle N, ((T_n^g)_{g \in G})_{n \in N} \rangle$ be a city and let $Y = \langle N, ((\alpha_g T_n^g)_{g \in G})_{n \in N} \rangle$ be the city that is obtained from X by multiplying the number of agents of a group g , for $g \in G$, by the same nonzero factor $\alpha_g > 0$ in all neighborhoods. A segregation order \succsim satisfies *composition invariance* if for any such cities we have $Y \sim X$.

Table 3 depicts two cities. One is obtained from the other by multiplying the number of members of one ethnic group by three. Composition invariance states that the two cities are equally segregated.

	X		Y	
	A	B	A	B
Blacks	30	50	90	150
Whites	80	50	80	50

Table 3: Multiplying the number of blacks by three.

Composition invariance requires that only the relative distributions of members of the various ethnic groups across neighborhoods affect segregation. In particular, the city’s ethnic distribution does not affect segregation.

⁶Massey and Denton (1988) define evenness as the “differential *distribution* of two social groups among areal units in a city” and James and Taeuber (1985) see segregation as the “differential distribution of students to schools by race *regardless of the overall racial proportions* in the system concerned.

4 The Blackwell partial order

In this section we investigate the implications of the four basic properties introduced in Section 3. In particular, we want to identify whether any two cities are unanimously ranked by all segregation orders that satisfy the four axioms. In other words, we are interested in the following partial order.

Definition 1 Let $X, Y \in \mathcal{C}$ be two cities. We say that X *segregation-dominates* Y if $X \succcurlyeq Y$ for every segregation order \succcurlyeq that satisfies anonymity, organizational equivalence, the neighborhood division property and composition invariance.

Segregation-dominance is the coarsest partial order on \mathcal{C} that satisfies the above four axioms. If two cities are not related by segregation-dominance then it is possible to find two segregation orders that rank them differently.

We will identify now the segregation-dominance relation more explicitly. It turns out that this relation is one that is derived from the informativeness of the cities' neighborhoods about the ethnicity of a randomly chosen resident. Specifically, we will show that city X segregation-dominates city Y if, and only if, the neighborhoods of the former are more informative about the ethnicity of its residents than the neighborhoods of the latter, where informativeness is measured according to Blackwell's [4] criterion.

Before we introduce the "more informative than" relation we need some definitions taken from the theory of the value of information.

Given a set of states of nature $\Omega = \{1, \dots, I\}$, an *experiment* provides information about the realized state. Specifically, when the realized state is i , the experiment issues a signal with a distribution that depends on i . An experiment on Ω can be described by a Markov matrix (m_{ij}) , whose I rows represent the possible states of nature, and whose columns represent the possible signals, the entry m_{ij} being the probability that the signal j is sent when the realized state is i . Conversely, every Markov matrix with I rows can be interpreted as an experiment on Ω .

Blackwell [4] partially ordered experiments according to their informativeness and showed that the resulting order has a convenient description. Let $X = \langle N, ((T_n^g)_{g \in G})_{n \in N} \rangle$ be a city. Also let $\phi : \{1, 2, \dots, |N|\} \rightarrow N$ be an ordering of the neighborhoods. The *experiment matrix* of X with respect to ϕ is the $|G| \times |N|$ matrix

$$M(X, \phi) = (m_{ij})$$

where $m_{ij} = t_{\phi(j)}^i$ is the proportion of individuals of group i that reside in neighborhood $\phi(j)$. Note that $M(X, \phi)$ is a Markov matrix. It represents an experiment in the sense of Blackwell. Its generic entry m_{ij} is the probability that a randomly chosen individual belongs to ethnic group i given that he resides in neighborhood $\phi(j)$. As an illustration, the experiment matrix of city X described in Table 3 above (and also that of city Y), with respect to the alphabetic ordering of the neighborhoods, is given by

$$\begin{pmatrix} 3/8 & 5/8 \\ 8/13 & 5/13 \end{pmatrix}.$$

Let \mathcal{M} be the set of Markov matrices with $|G|$ rows. These matrices can be partially ordered according to their informativeness (Blackwell [4]). Specifically, given two matrices $A_{|G| \times |N_A|}, B_{|G| \times |N_B|} \in \mathcal{M}$, we say that A is *at least as informative as* B if there is an $|N_A| \times |N_B|$ Markov matrix Π such that $B = A \cdot \Pi$.

Note that if A is at least as informative as B , it will remain so even after we permute each of the matrices' columns in any arbitrary way. Indeed, let P_B be a $|N_B| \times |N_B|$ permutation matrix and let P_A be a $|N_A| \times |N_A|$ permutation matrix. If $B = A \cdot \Pi$ then

$$B \cdot P_B = A \cdot P_A \cdot P_A^T \cdot \Pi \cdot P_B$$

Since $P_A^T \cdot \Pi \cdot P_B$ is a Markov matrix, we conclude that if A is at least as informative as B then $A \cdot P_A$ is at least as informative as $B \cdot P_B$.

We will make use of Blackwell's informativeness order on experiments to define a segregation order on cities. The idea is to consider a city as an experiment where

neighborhoods play the role of signals and ethnic groups the role of states of nature, and say that city X is more segregated than city Y if the neighborhoods of X are more informative about the ethnic group of its residents than the neighborhoods of Y .

Definition 2 Let $X = \langle N_X, ((T_n^g)_{g \in G})_{n \in N_X} \rangle$ and $Y = \langle N_Y, ((T_{n'}^g)_{g \in G})_{n' \in N_Y} \rangle$ be two cities. We say that X is at least as segregated as Y according to Blackwell's criterion, denoted $X \succsim_I Y$, if $M(X, \phi)$ is at least as informative as $M(Y, \psi)$ for some orderings $\phi : \{1, 2, \dots, |N_X|\} \rightarrow N_X$ and $\psi : \{1, 2, \dots, |N_Y|\} \rightarrow N_Y$ of the neighborhoods of X and Y , respectively.

Note that segregation according to Blackwell's criterion is well-defined since the informativeness relation on \mathcal{M} is invariant to permutations of columns. Since for most of the analysis the particular ordering of neighborhoods ϕ that is chosen is not important as long as it remains fixed, in what follows we will keep ϕ tacit and write, with some abuse of notation, $M(X)$ instead of $M(X, \phi)$. Also note that in order to determine whether or not two cities are ranked according to the Blackwell criterion, one only needs to solve a system of linear equations.

As the next result states, the Blackwell segregation order just defined satisfies the axioms introduced in the previous section.

Proposition 1 *The Blackwell segregation order \succsim_I satisfies anonymity, organizational equivalence, the neighborhood division property, and composition invariance.*

Proof. Blackwell's order satisfies anonymity since equivalent cities have the same experiment matrices, up to permutation of columns.

In order to show that it satisfies organizational equivalence, let $X = \langle N, ((T_n^g)_{g \in G})_{n \in N} \rangle$ be a city and consider the city Y that is obtained from X by splitting a particular neighborhood $(T_n^g)_{g \in G}$ into two neighborhoods, n_1 and n_2 , with the same ethnic distribution,

namely, $(T_{n_1}^g)_{g \in G} = (\alpha T_n^g)_{g \in G}$ and $(T_{n_2}^g)_{g \in G} = ((1 - \alpha)T_n^g)_{g \in G}$ for some $\alpha \in (0, 1)$. Then, their experiment matrices satisfy

$$M(Y) = M(X) \cdot I(n, \alpha) \quad (1)$$

where $I(n, \alpha)$ is the splitting matrix that is obtained from the identity matrix $I_{|N|}$ by splitting the column that corresponds to neighborhood n into two columns according to the proportions α and $(1 - \alpha)$. Furthermore,

$$M(X) = M(Y) \cdot I(n_1, n_2) \quad (2)$$

where $I(n_1, n_2)$ is the merging matrix that is obtained from the identity matrix $I_{|N|+1}$ by merging the two columns that correspond to n_1 and n_2 into one. Equations (1) and (2) imply that $M(X)$ and $M(Y)$ are equally informative, and therefore $Y \sim_I X$, which is what we wanted to show.

To show that Blackwell's order satisfies the neighborhood division property, let $X = \langle N, ((T_n^g)_{g \in G})_{n \in N} \rangle$ be a city and consider the city Y that is obtained from X by splitting a particular neighborhood $(T_n^g)_{g \in G}$ into two neighborhoods, n_1 and n_2 , but now with different ethnic distributions. Then

$$M(X) = M(Y) \cdot I(n_1, n_2)$$

where, as before, $I(n_1, n_2)$ is the merging matrix that is obtained from the identity matrix $I_{|N|+1}$ by merging the two columns that correspond to n_1 and n_2 into one. Therefore, $Y \succ_I X$.

On the other hand, as the following lemma states, there is no $|N| \times (|N| + 1)$ Markov matrix Π such that $M(Y) = M(X) \cdot \Pi$. Hence, $Y \not\succ_I X$.

Lemma 1 *Let A be an $n \times m$ Markov matrix and let B be an $n \times (m + 1)$ Markov matrix that is obtained from A by splitting one of A 's columns into two, but not proportionally. Then, there is no Markov matrix Π such that $B = A \cdot \Pi$.*

Proof. See appendix. ■

Finally, to see that Blackwell's order satisfies composition invariance, consider the city $X = \langle N, ((T_n^g)_{g \in G})_{n \in N} \rangle$ and the city $Y = \langle N, ((\alpha_g T_n^g)_{g \in G})_{n \in N} \rangle$ that is obtained from X by multiplying the number of group g individuals by $\alpha_g > 0$, for $g \in G$. Since both cities have the same proportions t_n^g , we have that $M(X) = M(Y)$, and therefore $Y \sim_I X$. ■

Our next result states that all segregation orders that satisfy anonymity, organizational equivalence, the neighborhood division property and composition invariance are consistent with Blackwell's order. Namely, whenever Blackwell's order ranks two cities, any segregation order that satisfies the above four axioms must rank them in the same way. And conversely, any segregation order that is consistent with Blackwell's order must satisfy the four axioms.

Theorem 1 *Let \succsim be a segregation order on \mathcal{C} . It satisfies anonymity, organizational equivalence, the neighborhood division property and composition invariance if and only if for all two cities $X, Y \in \mathcal{C}$,*

$$Y \succsim_I X \Rightarrow Y \succ X \quad (3)$$

$$Y \sim_I X \Rightarrow Y \sim X \quad (4)$$

Proof. Let \succsim be a segregation order that satisfies (3) and (4). We will show that it satisfies the four axioms.

ANON: Let X and Y be two equivalent cities. Then, by Proposition 1, $X \sim_I Y$. By (4), $X \sim Y$.

CI: Let $X = \langle N, ((T_n^g)_{g \in G})_{n \in N} \rangle$ be a city and let $Y = \langle N, ((\alpha_g T_n^g)_{g \in G})_{n \in N} \rangle$ be the city that is obtained by multiplying the number of agents of a group g by the same nonzero factor $\alpha_g > 0$, for $g \in G$ in all neighborhoods. Then, by Proposition 1, $Y \sim_I X$. By (4), $Y \sim X$.

OE: Let $X \in \mathcal{C}$ be a city and let $(T_n^g)_{g \in G}$ be a neighborhood of X . Let Y be the city that results from dividing $(T_n^g)_{g \in G}$ into two neighborhoods, $(T_{n_1}^g)_{g \in G}$ and $(T_{n_2}^g)_{g \in G}$, with the same ethnic distribution. Then, by Proposition 1, $Y \sim_I X$. By (4), $Y \sim X$.

NDP: Let $X \in \mathcal{C}$ be a city and let $(T_n^g)_{g \in G}$ be a neighborhood of X . Let Y be the city that results from dividing $(T_n^g)_{g \in G}$ into two neighborhoods, $(T_{n_1}^g)_{g \in G}$ and $(T_{n_2}^g)_{g \in G}$, with different ethnic distributions. Then, by Proposition 1, $Y \succ_I X$. By (3), $Y \succ X$.

We now show that any partial order that satisfies the four axioms must be consistent with Blackwell's criterion. Let \succsim be a segregation order that satisfies ANON, CI, OE and NDP. Also, let $X = \langle N_X, ((T_n^g)_{g \in G})_{n \in N_Y} \rangle$ and $Y = \langle N_Y, ((T_{n'}^g)_{g \in G})_{n' \in N_Y} \rangle$ be two cities such that $Y \succsim_I X$. We need to show that (3) and (4) hold. Since $Y \succsim_I X$, there is a Markov matrix $\Pi = \left((\pi_{ij})_{i=1}^{|N_Y|} \right)_{j=1}^{|N_X|}$ such that

$$M(X) = M(Y) \cdot \Pi.$$

But Π can be written as a product of two matrices

$$\Pi = \beta \cdot \gamma$$

where

$$\beta = \begin{pmatrix} \pi_{11} & \cdots & \pi_{1|N_X|} & 0 & \cdots & 0 & \cdots & 0 & \cdots & 0 \\ 0 & \cdots & 0 & \pi_{21} & \cdots & \pi_{2|N_X|} & \cdots & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \cdots & 0 & 0 & \cdots & 0 & \cdots & \pi_{|N_Y|1} & \cdots & \pi_{|N_Y||N_X|} \end{pmatrix}$$

and

$$\gamma = \begin{pmatrix} I_{|N_X|} \\ I_{|N_X|} \\ \vdots \\ I_{|N_X|} \end{pmatrix}$$

Therefore,

$$M(X) = M(Y) \cdot \beta \cdot \gamma \tag{5}$$

Note that $M(Y) \cdot \beta$ is the matrix that is obtained from $M(Y)$ by splitting its i th column, $i = 1, \dots, |N_Y|$, into $|N_X|$ columns, in the proportions π_{ij} , $j = 1, \dots, |N_X|$. Also note that $M(Y) \cdot \beta \cdot \gamma$ is obtained from $M(Y) \cdot \beta$ by merging together the columns $i + k|N_X|$ for $k = 0, \dots, |N_Y| - 1$, for $i = 1, \dots, |N_X|$. Therefore, (5) says that $M(X)$ is obtained from $M(Y)$ by successively splitting its columns proportionally and then merging some columns, which may or may not be proportional to each other. Alternatively, matrix $M(Y)$ is obtained from $M(X)$ by splitting its columns, not necessarily in a proportional way, and then merging some proportional columns. Consequently, by OE and NDP,

$$Y \succsim X.$$

An analogous argument shows that if $X \succsim_I Y$ we must also have $X \succsim Y$. Consequently, if $Y \sim_I X$ then $Y \sim X$, which is implication (4).

In order to show implication (3) assume that $Y \succ_I X$. We already know that matrix $M(Y)$ is obtained from $M(X)$ by splitting its columns, not necessarily in a proportional way, and then merging some proportional columns. We now argue that $Y \succ_I X$ implies that at least one of the columns is split *not* proportionally. Indeed, if all the columns of $M(X)$ were split proportionally, we would have

$$M(Y) = M(X) \cdot \beta' \cdot \gamma'$$

for some splitting matrix β' and merging matrix γ' . Since, as a product of Markov matrices, $\beta' \cdot \gamma'$ is a Markov matrix, this would imply that $X \succsim_I Y$, contradicting $Y \succ_I X$. Therefore, Y is obtained from X by splitting some neighborhoods into smaller neighborhoods with different ethnic distributions, and then merging some neighborhoods with the same ethnic distributions. By NDP and OE, $Y \succ X$, which shows the implication in (3). ■

As a corollary we obtain that Blackwell's order is the appropriate criterion to decide whether one city is unambiguously more segregated than another as long as one subscribes to the principles of anonymity, organizational equivalence, the neighborhood division property and composition invariance.

Corollary 1 *The segregation-dominance relation and the Blackwell order are one and the same.*

Proof. Assume that X segregation dominates Y . Then, $X \succcurlyeq Y$ for every segregation order \succcurlyeq that satisfies ANON, OE, NDP and CI. In particular, since \succcurlyeq_I satisfies these axioms, $X \succcurlyeq_I Y$. Assume now that $X \succcurlyeq_I Y$ and let \succcurlyeq be any segregation order that satisfies ANON, OE, NDP and CI. By Theorem 1, $X \succcurlyeq Y$ as well. Consequently, X segregation dominates Y . ■

5 Two groups: The Lorenz partial order

There is another partial order defined on the class of cities with only two groups. It is known as the Lorenz partial order and is based on what is known as segregation curves. See, for instance, Duncan and Duncan [5], James and Taeuber [10, 17] and Hutchens [9].

Let G be a set of two ethnic groups and denote by \mathcal{C}_2 the set of cities with these two groups. For ease of exposition, we refer to the two ethnic groups as blacks and whites. Let $X = \langle N, (B_n, W_n)_{n \in N} \rangle \in \mathcal{C}_2$ be a city where for each neighborhood $n \in N$, B_n and W_n are the numbers of blacks and whites, respectively, that reside in n . For each $n \in N$, denote by p_n the proportion of whites in neighborhood n . That is, $p_n = W_n / (B_n + W_n)$. Also, b_n and w_n denote the proportion of the city's blacks and whites, respectively that reside in neighborhood n . Formally, $b_n = B_n / \sum_{n' \in N} B_{n'}$ and $w_n = W_n / \sum_{n' \in N} W_{n'}$. We will now construct the segregation curve associated with the city X . Segregation curves will allow us to define a partial order on the set of two-group cities. Segregation curves, analogously to experiment matrices, are objects that do not depend on the cities' ethnic distribution. That is, city $X = \langle N, (B_n, W_n)_{n \in N} \rangle$ and city $\hat{X} = \langle N, (b_n, w_n)_{n \in N} \rangle$, which is obtained from X by normalizing the groups' populations so that each group is of size one, will have the same segregation curve. In order to construct the segregation curve, let $\phi : \{1, 2, \dots, |N|\} \rightarrow N$ be an ordering of the neighborhoods such that $i \leq j \Rightarrow p_{\phi(i)} \leq p_{\phi(j)}$. Namely, ϕ orders neighborhoods in a non-decreasing way according to

their proportion whites. Note that

$$p_n \leq p_m \Leftrightarrow w_n/(b_n + w_n) \leq w_m/(b_m + w_m). \quad (6)$$

That is, ordering the neighborhoods in N in non-decreasing order of the proportion of whites in X or in its normalized version \hat{X} results in the same order. Let $\beta_0 = \omega_0 = 0$, and for $i = 1, 2, \dots, |N|$, and let $\beta_i = \beta_{i-1} + b_{\phi(i)}$ and $\omega_i = \omega_{i-1} + w_{\phi(i)}$. That is, β_i is the proportion of blacks that reside in the i neighborhoods with the lowest proportions of whites. Similarly, ω_i is the proportion of whites that reside in these same neighborhoods. *The segregation curve of X* is the graph that is obtained by plotting the points $(\beta_i, \omega_i)_{i=0}^{|N|}$ and connecting the dots. Formally, it is the union of the line segments $\text{seg}[(\beta_{i-1}, \omega_{i-1}), (\beta_i, \omega_i)]$, $i = 1, 2, \dots, |N|$, where for any two points $x, y \in \mathbb{R}^2$, $\text{seg}[x, y] = \{\alpha x + (1 - \alpha)y : \alpha \in [0, 1]\}$. Note that the line segment that connects the points $(\beta_{i-1}, \omega_{i-1})$ and (β_i, ω_i) has a slope of $w_{\phi(i)}/b_{\phi(i)}$. Therefore, given (6), this slope is non-decreasing in i . Furthermore, the segregation curve is invariant to the choice of ordering ϕ as long as it satisfies $i \leq j \Rightarrow p_{\phi(i)} \leq p_{\phi(j)}$. Figure 1 illustrates a Lorenz curve of a three-neighborhood city.

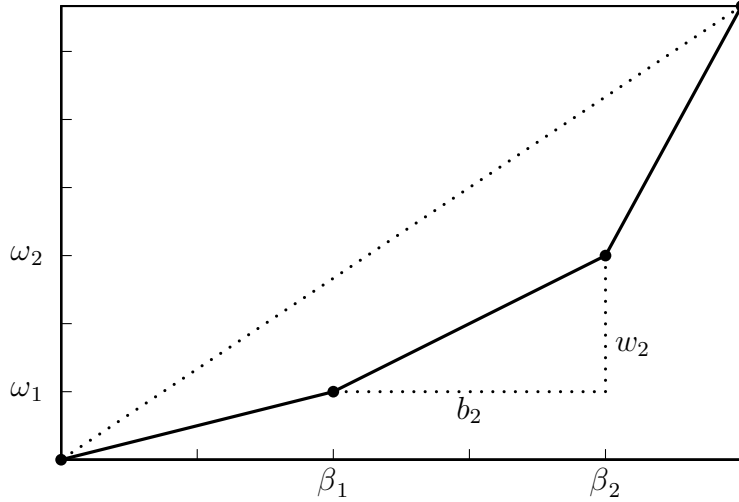


Figure 1: A Lorenz Curve.

We now use the segregation curves to define a segregation order.

Definition 3 Let X and Y be two cities. We say that Y is at least as segregated as X according to the Lorenz criterion, denoted $Y \succcurlyeq_L X$, if the Lorenz curve of Y is nowhere above the Lorenz curve of X .

The Lorenz criterion is generally accepted as an unambiguous benchmark for segregation comparisons. On the other hand, Theorem 1 shows that, as long as one subscribes to the principles of anonymity, organizational equivalence, the neighborhood division property and composition invariance, Blackwell's is the appropriate criterion to determine whether one city is unambiguously more segregated than another. Our next result shows that these two criteria, in fact, coincide.

Theorem 2 *The Blackwell and the Lorenz orders on \mathcal{C}_2 are the same.*

Proof. It can be checked that the Lorenz order \succcurlyeq_L satisfies ANON, CI, OE and NDP. Therefore, by Theorem 1, $Y \succcurlyeq_I X \implies Y \succcurlyeq_L X$.

In order to show the converse implication, let $X = \langle N, (B_n, W_n)_{n \in N} \rangle$ and $Y = \langle N', (B'_{n'}, W'_{n'})_{n' \in N'} \rangle$ be two cities in \mathcal{C}_2 . Since both \succcurlyeq_I and \succcurlyeq_L satisfy CI, we can assume without loss of generality that $\sum_{n \in N} B_n = \sum_{n' \in N'} B'_{n'} = \sum_{n \in N} W_n = \sum_{n' \in N'} W'_{n'} = 1$. Since both \succcurlyeq_I and \succcurlyeq_L satisfy ANON we can also assume that $N = \{1, \dots, I\}$ and $N' = \{1, \dots, I'\}$ and that the neighborhoods are ordered in a non-decreasing order of proportion of whites. Therefore, we can denote X by $(b_n, w_n)_{n=1}^I$ and Y by $(b'_{n'}, w'_{n'})_{n'=1}^{I'}$ with $w_1/(b_1 + w_1) \leq \dots \leq w_I/(b_I + w_I)$ and $w'_1/(b'_1 + w'_1) \leq \dots \leq w'_{I'}/(b'_{I'} + w'_{I'})$. We split the analysis into three cases.

Case 1: For each $n \in N$ and $n' \in N'$, $b_n > 0$ and $b'_{n'} > 0$.

Let us build the following random variables: For each $n \in N$ the random variable x takes the value w_n/b_n with probability b_n . For each $n' \in N'$ the random variable y takes the value $w'_{n'}/b'_{n'}$ with probability $b'_{n'}$. Note that $E[x] = E[y] = 1$.

Denote by F_x the cumulative distribution function of x and by F_y the cumulative distribution function of y . Also denote their generalized inverses by F_x^{-1} and F_y^{-1} , respectively.⁷

By Theorem 3.A.5 of Shaked and Shanthikumar [15],

$$\begin{aligned} \int_0^p F_x^{-1}(t)dt &\geq \int_0^p F_y^{-1}(t)dt && \text{for all } p \in [0, 1] \\ &\iff \\ \sum_{n \in N} b_n \psi(w_n/b_n) &\leq \sum_{n' \in N'} b'_{n'} \psi(w'_{n'}/b'_{n'}) && \text{for all convex functions } \psi : \mathbb{R} \rightarrow \mathbb{R}. \end{aligned} \quad (7)$$

By Sherman's [16] theorem, (7) holds if and only if there is a $I' \times I$ Markov matrix $\Pi = \{\pi_{n'n}\}$ such that

$$\begin{aligned} b_n(w_n/b_n) &= \sum_{n' \in N'} \pi_{n'n} b'_{n'} (w'_{n'}/b'_{n'}) \\ \sum_{n' \in N'} \pi_{n'n} b'_{n'} &= b_n \end{aligned}$$

Or, in matrix notation,

$$M(X) = M(Y) \cdot \Pi$$

Consequently, $\int_0^p F_x^{-1}(t)dt \geq \int_0^p F_y^{-1}(t)dt$ for all $p \in [0, 1]$ if and only if $Y \succcurlyeq_I X$. Since the graphs of $\int_0^p F_x^{-1}(t)dt$ and $\int_0^p F_y^{-1}(t)dt$ are none other than the segregation curves of X and Y respectively, we obtain the desired result, i.e., $Y \succcurlyeq_L X \Leftrightarrow Y \succcurlyeq_I X$.

Case 2: For each $n \in N$, $b_n > 0$, and there is $n' \in N'$ with $b'_{n'} = 0$. Since both \succcurlyeq_I and \succcurlyeq_L satisfy OE, we can assume without loss of generality that in Y , there is only one neighborhood with no blacks. Furthermore, by OE we can assume without loss of generality that $|N'| = |N| + 1$. That is, $I' = I + 1$. Lastly, since both \succcurlyeq_I and \succcurlyeq_L satisfy OE, we can assume without loss of generality that $b_n = b'_n$ for all $n \in \{1, 2, \dots, I\}$. Therefore, we can denote X by $(b_n, w_n)_{n=1}^I$ and Y by $(b_n, w'_n)_{n=1}^{I+1}$ (where $b_{I+1} = 0$ and $w'_{I+1} > 0$). In this case, $X \succcurlyeq_L Y$ is impossible. Assume, therefore, that $Y \succcurlyeq_L X$.

⁷The generalized inverse of a distribution function $F : \mathbb{R}_+ \rightarrow [0, 1]$ is defined as $F^{-1} : [0, 1] \rightarrow \mathbb{R}_+$ such that $F^{-1}(p) = \inf_s \{s > 0 : F(s) > p\}$.

For each $t = 1, \dots$, let $\varepsilon_t = \frac{1}{t} b_I \frac{w'_{I+1}}{w'_I + w'_{I+1}}$, and let $Y_t = (b_n^t, w'_n)_{n=1}^{I+1}$ be the city that is obtained from Y by relocating ε_t blacks from neighborhood I to neighborhood $I + 1$. That is, $b_n^t = b_n$ for $n = 1, \dots, I-1$, $(b_I^t, w'_I) = (b_I - \varepsilon_t, w'_I)$ and $(b_{I+1}^t, w'_{I+1}) = (\varepsilon_t, w'_{I+1})$. See Figure 2. Note that since $\varepsilon_t \leq b_I \frac{w'_{I+1}}{w'_I + w'_{I+1}}$, the proportion of whites in neighborhood

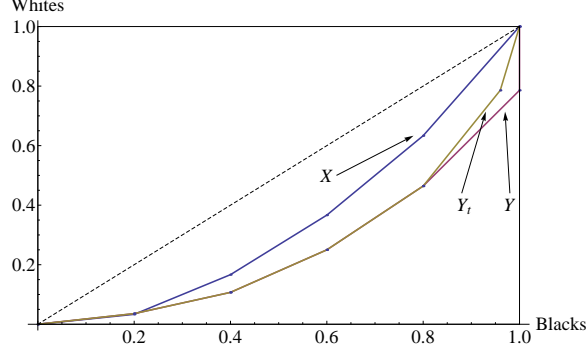


Figure 2: The segregation curves of X , Y and Y_t

I is less than or equal to the proportion of whites in neighborhood $I + 1$. As a result, Y_t 's neighborhoods are ordered in a non-decreasing order of the proportion of whites. Furthermore, it can be seen that $Y_t \succcurlyeq_L X$. By construction, Y_t has no neighborhoods with 0 blacks. By Case 1, $Y_t \succcurlyeq_I X$. That is, there is a $I' \times I$ Markov matrix Π_t such that

$$M(X) = M(Y_t) \cdot \Pi_t$$

Since the set of $I' \times I$ Markov matrices is compact, there is a subsequence $\{\Pi_{t_\ell}\}$ that converges to a Markov matrix Π . Since, $M(Y_t) \rightarrow M(Y)$, we obtain that

$$M(X) = M(Y) \cdot \Pi$$

which means that $Y \succcurlyeq_I X$.

Case 3: There is $n \in N$, and $n' \in N'$ such that $b_n = b'_{n'} = 0$.

Since both \succcurlyeq_I and \succcurlyeq_L satisfy OE, we can also assume without loss of generality that $|N| = |N'| = I$. By OE, we can assume without loss of generality that both in X and in

Y , there is only one neighborhood with no blacks. Lastly, since both \succsim_I and \succsim_L satisfy OE, we can assume without loss of generality that $b_n = b'_n$ for all $n \in \{1, 2, \dots, I\}$.

Assume that $Y \succsim_L X$. For each $t = 1, \dots$, let $\varepsilon_t = \frac{1}{t} b_I \frac{w_I}{w_{I-1} + w_I}$, and let $X_t = (b_n^t, w_n)_{n=1}^I$ be the city that is obtained from X by relocating ε_t blacks from neighborhood $I-1$ to neighborhood I . That is, $b_n^t = b_n$ for $n = 1, \dots, I-2$, $(b_I^t, w_I) = (b_{I-1} - \varepsilon_t, w_{I-1})$ and $(b_I^t, w_I) = (\varepsilon_t, w_I)$. See Figure 3. Note that since $\varepsilon_t \leq b_I \frac{w_I}{w_{I-1} + w_I}$ the proportion

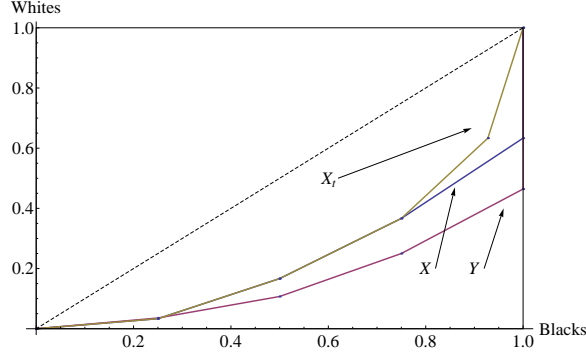


Figure 3: The segregation curves of X , Y and X_t

of whites in neighborhood $I-1$ is less than or equal to the proportion of whites in neighborhood I . As a result, X_t 's neighborhoods are ordered in a non-decreasing order of the proportion of whites. Furthermore, by construction, $Y \succsim_L X_t$. Also by construction, X_t has no neighborhoods with 0 blacks. By Case 2, $Y \succsim_I X_t$. That is, there is a $I' \times I$ Markov matrix Π_t such that

$$M(X_t) = M(Y) \cdot \Pi_t$$

Since the set of $I' \times I$ Markov matrices is compact, there is a subsequence $\{\Pi_{t_\ell}\}$ that converges to a Markov matrix Π . Since, $M(X_t) \rightarrow M(X)$, we obtain that

$$M(X) = M(Y) \cdot \Pi$$

which means that $Y \succsim_I X$. ■

6 Concluding remarks

Measures aimed at capturing the evenness dimension of segregation view it as the differential distribution of residents across neighborhoods by ethnicity, regardless of the overall ethnic distribution. It can be argued that anonymity, organizational equivalence, neighborhood division property and composition invariance are the minimal requirements that any such measure of evenness should satisfy. This paper showed that any order that satisfies these axioms must be consistent with Blackwell's informativeness criterion. That is, all orders that satisfy the above four principles will agree on the ranking of any two cities that are ranked by the Blackwell criterion.

In the case of two ethnic groups, Lorenz's is a well-established criterion for the ranking of cities in terms of their segregation. Since the Lorenz order satisfies anonymity, organizational equivalence, neighborhood division property and composition invariance, it is consistent with the informativeness criterion. More interestingly, we show that Blackwell's order is also consistent with the Lorenz criterion. As a result, The Blackwell and Lorenz orders, restricted to the two-group case, are the same. Moreover, from a computational point of view the implementation of this criterion in the multigroup case is as simple as solving a system of linear equations.

7 Appendix

Proof of Lemma 1: Let A be an $n \times m$ Markov matrix and let B be an $n \times (m + 1)$ Markov matrix that is obtained from A by splitting one of A 's columns into two. Assume that A 's k th column is the one that is split. Alternatively, A is obtained from B by replacing B 's k th and $(k + 1)$ th columns by their sum. Consequently,

$$A = B \cdot S_k \tag{8}$$

where

$$S_k = \begin{pmatrix} I_{k-1} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & I_{m-k} \end{pmatrix}.$$

Let us now assume that there is an $m \times (m+1)$ Markov matrix Π such that

$$B = A \cdot \Pi. \quad (9)$$

We will show that B is necessarily obtained from A by splitting A 's k th column proportionally.

Let Π' be the matrix that is obtained from Π by replacing Π 's k th and $(k+1)$ th columns by their sum. That is,

$$\Pi' = \Pi \cdot S_k. \quad (10)$$

Note that Π' is a square $m \times m$ Markov matrix. Moreover, by (10), (9) and (8),

$$A \cdot \Pi' = A \quad (11)$$

which means that each row of A is an invariant distribution of the matrix Π' .

Since Π' is a square Markov matrix, there exists $r \geq 1$ and a permutation matrix P such that $P^T \cdot \Pi' \cdot P$ can be written in the following (almost block diagonal) form:

$$\left(\begin{array}{cccc|c} R'_1 & & & & 0 \\ & R'_2 & & 0 & 0 \\ & & R'_3 & & 0 \\ & & & \ddots & \vdots \\ & 0 & & & 0 \\ & & & & R'_r & 0 \\ \hline S'_{r+1,1} & S'_{r+1,2} & S'_{r+1,3} & \cdots & S'_{r+1,r} & Q' \end{array} \right)$$

where for all $j = 1, \dots, r$, R'_j are square $(m_j \times m_j)$ irreducible Markov matrices and Q' is an $(n - \sum_{j=1}^r m_j) \times (n - \sum_{j=1}^r m_j)$ reducible matrix. We can assume without loss

of generality that P is the identity matrix and thus that Π' has the above form.⁸

Since R'_j , for $j = 1, \dots, r$, is an irreducible Markov matrix, it has unique invariant distribution $q^j = (q_1^j, \dots, q_{m_j}^j)$, i.e., q^j is the unique probability vector q that satisfies $q = qB'_j$. Furthermore, any invariant distribution of Π' can be written as

$$(\alpha_1 q^1, \alpha_2 q^2, \dots, \alpha_r q^r, \underbrace{0, \dots, 0}_{n - \sum_{j=1}^r m_j})$$

for some $\alpha_1, \dots, \alpha_r \geq 0$ and $\sum_{j=1}^r \alpha_j = 1$ (see, for instance, Lucas and Stokey 1989 (Theorem 11.1, pages 326-330)). Therefore, since each row of A is an invariant distribution of Π' , it can be written as

$$A = \begin{pmatrix} \alpha_{11} q^1 & \alpha_{12} q^2 & \cdots & \alpha_{1r} q^r & 0 & \cdots & 0 \\ \vdots & \vdots & & \vdots & \vdots & & \vdots \\ \alpha_{n1} q^1 & \alpha_{n2} q^2 & \cdots & \alpha_{nr} q^r & 0 & \cdots & 0 \end{pmatrix}, \quad (12)$$

where for each $i = 1, \dots, n$ and $j = 1, \dots, r$, $\alpha_{ij} \geq 0$ and $\sum_{j=1}^r \alpha_{ij} = 1$. If B was obtained from A by splitting column k in a disproportional way, it ought to be the case that this column is one that has at least one positive entry.

Assume that column k corresponds to the h th block of Π' . Therefore we can write

$$R'_h = (R'_{h_1}, v'_{*k}, R'_{h_2})$$

where $v'_{*k} = (v'_{1k}, \dots, v'_{m_h k})^T$ is the column of block B'_h that corresponds to the k th column of Π' . Since Π is obtained from Π' by splitting the k th column into two, Π can

⁸Otherwise, the whole analysis can be done using $A \cdot P$ instead of A , $S_k \cdot P$ instead of S_k , $P^T \cdot \Pi$ instead of Π and $P^T \cdot \Pi' \cdot P$ instead of Π' .

be written as

$$\Pi = \left(\begin{array}{cccccccc|c} R'_1 & 0 & & 0 & 0 & & \cdots & 0 & 0 \\ 0 & \ddots & & 0 & 0 & & \cdots & \vdots & \vdots \\ & & R'_{h_1} & v_{*k} & v_{*k+1} & R'_{h_2} & & 0 & 0 \\ \cdots & \cdots & & 0 & 0 & & \ddots & 0 & 0 \\ 0 & \cdots & & 0 & 0 & & 0 & R'_r & 0 \\ \hline S'_{r+1,1} & \cdots & S'_{r+1,h_1} & s_{*k} & s_{*k+1} & S'_{r+1,h_2} & \cdots & S'_{r+1,r} & Q' \end{array} \right) \quad (13)$$

where v_{*k} and v_{*k+1} are column vectors such that $v_{*k} + v_{*k+1} = v'_{*k}$. Consequently, since $B = A \cdot \Pi$, using (12) and (13) we obtain that B 's k th column is $(\alpha_{1h} q^h v_{*k}, \dots, \alpha_{nh} q^h v_{*k})^T$ and, B 's $(k+1)$ th column is $(\alpha_{1h} q^h v_{*k+1}, \dots, \alpha_{nh} q^h v_{*k+1})^T$, which are proportional to each other (the proportion is $q^h v_{*k} / q^h v_{*k+1}$). ■

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