Co-location and Incentives*

Eyal Winter[†]

March 17, 2006

Abstract

The paper studies the effect of the information agents have about their peers' effort on the principal's cost of providing incentives. We use the analysis to address the issue of co-location in organizations, i.e., moving workers from private offices to open spaces or "war room-s".Using directed graphs to represent peer information we show how the pricipal can gain from more transparancy among peers. We also use the analysis to argue that process-based teams are more effective than function-based teams.

1 Introduction

A noticeable recent trend in the evolution of the workplace is the move away from private offices or cubicles to open-space environments or "war rooms." This strategy, which is being increasingly adopted by different types of organizations is often referred to as "co-location." While the trend is there, the debate over the net advantage of this strategy has not been concluded. Among the obvious downside arguments are concerns about workers' privacy and the potential distraction when concentration is needed to perform individual tasks. As for the gains, it is mainly argued that co-location allows workers to coordinate their activity and offers them the possibility to learn

*I'm grateful to Ed Lazear and Ilya Segal for helpful suggestions. Comments and suggestions made by seminar participants at Boston U, Cornell, Ohio State, Nottingham and York are also gratefully acknowledged.

[†]The Center for the Study of Rationality and the Economics Department. The Hebrew Univesrity of Jerusalem, mseyal@pluto.huji.ac.il

from each other. The purpose of this paper is to address yet another role of co-location, namely, its effect on workers' incentives to exert effort. "Open" offices and war rooms provide agents with information about their peers that is not observable when they work in closed offices. The internal information about peers affects agents' incentives and thus also the principal's cost of providing them.

To address this issue we will present a moral hazard model in which agents' effort decisions are mapped into a probability that their joint project will succeed. The principal offers agents rewards that are contingent only on the final outcome of the project. Agents' effort while unobservable by the principal may be observable by peers. The internal information about peers will be given in a rather general form. Specifically, we will represent the information structure by a directed graph where an arc from agent i to agent j means that i is informed about $j's$ effort. We will interpret these graphs as emerging from the workplace structure where a movement towards co-location corresponds to richer graphs, i.e., more transparency among peers. Our analysis will compare different information structures in terms of the cost of the optimal incentive mechanism.

Our first result asserts that transparency among peers works in favor of the principal as it is easier to generate incentives under more transparent structures. Roughly, with more transparency among peers the implicit threat against shirking is stronger. Agents who are observed by many of their peers will be reluctant to shirk for fear that doing so will trigger the shirking of other peers, which will substantially reduce the probability that the project will succeed. This observation and the intuition behind it seem consistent with various recent empirical studies about co-location and team work. Teasley et al. (2002) study the effect of co-location in software development teams. The authors evaluated the workers' productivity using measures commonly used in software development. Comparing the war room teams' scores with those in traditionally arranged offices, they conclude that teams in war rooms are twice as productive as similar teams working in closed offices. A related evidence is reported by Heywood and Jirjahn (2004) who show that blue-collar workers who work jointly in small teams have a lower absentee rate than other similar workers who work alone.

While more transparency among peers can never harm incentives, not every information arc between two agents contributes to incentives. Proposition 2 asserts that the principal's cost of inducing all agents to exert effort depends on the graph (of information among peers) only through its transitive closure. Put differently, it does not matter whether agent i observes the effort of agent j directly or through observing other agents, i.e., observing an agent that observes an agent ... that observes agent j. In both cases the cost of inducing agent i to exert effort will be the same. This result implies that workplace structures that are rich in sequentiality are desirable from the point of view of incentives. This result will also allow us to compare workplace architectures on a wider domain than that of Proposition 1.

Architectural and organizational changes in the workplace affect the internal information peers have about each other. But in many environments this effect is not deterministic but rather probabilistic. A move towards co-location merely affects the probability that an agent will observe the effort of his peers. Section 4 is meant to address this issue with a model of random graphs. In this framework agent i observing agent $j's$ effort is a random event that occurs with an exogenous probability. Our result in this section will show that the principal's cost of inducing effort by all agents is a decreasing function of this probability. Hence, even measures that only slightly increase the probability of transparency among peers will have a strict positive effect on incentives.

The second part of the paper is devoted to studying the role of the organizational technology in attaining the incentive advantages from co-location. We will show that transparency among peers is more effective when agents' tasks are complementary (i.e., when the technology is convex) than when there is substitution across tasks. Specifically, under complementarity the gains from more transparency will occur also when the implementation of effort is carried out with respect to more stringent solution concepts than Nash equilibrium (such as perfect Bayesian or Nash with weakly undominated strategies). The basic intuition behind this result is that under complementarity each agent's incentives to exert effort grow the more others exert effort. This intuition is consistent with a recent empirical study. Using a panel data on the performance of baseball players Gould and Winter (2005) show that the nature of externalities among players depend on the degree of complementarity and substitutability between players. Complementary players induce positive externalities on each other, i.e., the effort of a player increases with the degree of effort by his peer. On the other hand players who are substitutes generate negative peer effects.

The distinction we will make between complementarity and substitutability will alow us to address a related important issue: Should team formation be process based or function-based[1]? In a process-based team or war room each member is in charge of a different stage in the production process of a single product. In contrast, a function-based team accommodates agents who all work on the same stage of the production process (i.e., performing identical tasks on different products). We shall argue that incentive considerations should favor process-based teams since agents' effort in such teams involves complementarity.

This paper is part of the extensive literature on multi-agent incentive mechanisms that started with Holmstrom (1982). Some of the papers in this literature, such as Holmstrom and Milgrom (1990) and Itoh (1991), have pointed out that a principal can gain from collusion or coordination among his agents. Che and Yoo (2001) even made this collusion explicit by considering a model in which agents interact repeatedly, while Baliga and Sjoestrom (1998) consider side contracting among agents and messge games. The current paper complements this literature by focusing on the role information among peers in sustaining collusion. An important message our results add to this literature is that the mere fact that agents are informed about each other's effort is sufficient for the principal to be better off even when the organizational environment does not allow for real communication among agents or side contracting.

Also related to the current work are two of my former papers Winter (2004) and Winter (2005). These papers use a similar moral hazard model to derive optimal incentive mechanisms. Winter (2004) argues that even when agents are identical and act simultaneously (i.e., with no information about peers) the principal may gain by discriminating among them. Winter (2005) considers a sequential production and addresses the issue of how to allocate optimally tasks and agents with different attributes across different production slots. Like almost all the related literature, both papers deal with a fixed structure of information about peers. In sharp contrast, however, our main objective here is to compare between different information structures and study their effect on agents' incentives. Finally, we note that this paper, in certain respects, also relates to the recent literature on networks in games, which is extensively surveyed by Jackson (2005). Links in our networks stand for information channels between peers. However, in contrast to the networks literature, in which the game is in forming the network, here the network is

[1] I would like to thank Ilya Segal for pointing out the relevance of my model to this issue.

designed by the principal and the interaction among the agents relies on the network rather than forms it.

2 A Simple Two-Agent Example

Before introducing the formal model in its generality it would be instructive to demonstrate the incentive gains from transparency among peers with the simplest possible example. Consider an organization of two agents, each of which is in charge of one task on which he can either exert effort or shirk. The cost of effort is $c > 0$ for both agents. If an agent does not exert effort on his task the task will end successfully with probability $\alpha > 0$. If the agent exerts effort the task will succeed with probability 1. The two tasks together form a project that will succeed if and only if both tasks are successful. The principal cannot monitor agents' effort or the outcome of individual tasks. He can only find out whether the entire project is successful. A mechanism pays the agents rewards v_1, v_2 if the projects succeeds and zero if it fails. We would like to find the the mechanism with a minimal total reward that will induce both agents to exert effort in equilibrium.

We first assume that the two agents act simultaneously; i.e., none of them is informed about the effort decision of the other one. For effort by both players to be an equilibrium it must be that effort by i is his best response to effort by j. Suppose that agent i exerts effort, then agent j's payoff will be $v_j - c$ if he exerts effort as well and it will be αv_j if he shirks. We therefore must have $v_j - c \geq \alpha v_j$. Hence the principal must pay $c/(1-\alpha)$ to each agent in order to sustain effort by both.

Consider now the case in which the agents act sequentially; i.e., agent 1 makes his effort decision. Agent 2 observes that decision (but not the outcome of the task) and decides himself whether to exert effort or to shirk. To generate an equilibrium in which both agents exert effort player 2 must be paid $c/(1-\alpha)$ (as in the simultaneous case). If he is paid less his best response to seeing agent 1 exerting effort would be to shirk. How much then should agent 1 be paid to sustain an effort equilibrium? Suppose that agent 1 is promised $v_1 = c/(1-\alpha^2)$ if the project succeeds and player 2 is promised $c/(1-\alpha)$. We shall argue that there exists an equilibrium in which both agents exert effort under these rewards. Consider the strategy profile under which agent 1 exerts effort and agent 2 exerts effort if and only if agent 1 does so. If agent 1 deviates from his strategy by shirking agent 2 will shirk as well and agent 1 will get $\alpha^2 v_1$. If agent 1 instead exerts effort the project will succeed with probability 1 and he will get $v_1 - c$. Since $\alpha^2 v_1 = v_1 - c$ it is optimal for agent 1 to exert effort, and given that agent 1 exerts effort it is optimal for agent 2 to do likewise. In fact, for an arbitrarily small increase of the rewards for the two agents the strategy profile specified above is also the unique subgame perfect equilibrium of the game.

Because $v_1 = c/(1-\alpha^2) < c/(1-\alpha)$ the principal needs to pay less if the agents move sequentially. Put differently, the fact that agent 2 is informed about agent 1's effort decision makes it easier for the principal to provide incentives for effort. Roughly, the implicit threat that agent 1 faces not to shirk (because agent 2 will do likewise), which allows the principal to reduce rewards for agent 1, applies only when they move sequentially. We will build on this example in constructing a general model of an organization with internal information about effort.

3 The Model

The organizational project involves a set N of n agents that collectively manage a project. Each agent has to decide whether to exert effort towards the performance of his tasks or not. The cost of effort is c and is constant across all players. Henceforth we interchangeably use the term investment for the action of exerting effort. The technology of the organization maps a profile of investment decisions into a probability of the project's success. For a group $S \subseteq N$ of investing agents the probability that the project will succeed is $p(S)$. The principal who cannot monitor the agents for their effort but knows only if the project succeeds sets up a mechanism $v = (v_1, ..., v_n)$ by which agent i receives the payoff v_i if the project succeeds and zero otherwise.

Agents' decisions about whether to invest or not depend on the internal information about other agents' investment decisions.

An Internal Information about Effort (IIE) is defined to be a partial order "k" over the set of agents N, where $i\ k\ j$ stands for agent i knowing the effort decision of agent j before making his own decision. We shall also refer to this relation by saying that i sees j.

We impose that k is acyclic; i.e., for any sequence $i_1\ k\ i_2\ k, ..., k\ i_r$ with $r \leqslant n$ we must have that $i_1, ..., i_r$ are distinct. This condition simply reflects the fact that there can be no mutual knowledge about effort. Indeed $i\ k\ j$ implies that j has taken his effort decision before i, which precludes $j\ k\ m$ for some m with $m\ k\ i$. The acyclicity property of k also implies that there is a timing structure according to which agents make their investment decisions. We denote by K_i the set of agents that i sees, i.e., $K_i = \{j|\ i\ k\ j\}$.

Given a vector of rewards $v = (v_1, ..., v_n)$ that are to be paid if the project succeeds, and an IIE k, we can now consider the following normal form game $G(k, v)$:

A strategy for player i is a function s_i: $2^{K_i} \longrightarrow \{0, 1\}$ specifying to each player whether to invest (choose 1) or to shirk as a function of the information he possesses on other agents' decisions. For every strategy profile $s = (s_1, ..., s_n)$ we denote by $M(s)$ the set of agents who exert effort under the profiles. In the Appendix we argue that $M(s)$ is well defined and show how $M(s)$ is constructed from the strategy profile s. Finally, the payoff for player i under $s = (s_1, ..., s_n)$ is given by $f_i(s) = v_i p(M(s)) - c$ if $i \in M(s)$ and $f_i(s)= v_i p(M(s))$ if $i \notin M(s)$.

We say that the vector of rewards v is an investment-inducing (INI) mechanism with respect to k if it induces all agents to invest in equilibrium. Formally, v is an INI mechanism if there exists a Nash equilibrium s for the game $G(k, v)$ with $M(s) = N$.

We say that v is an optimal INI mechanism if it yields minimal total payoff among all INI mechanisms.

For an IIE k we denote by $v^*(k)$ the total reward in an optimal INI mechanism with respect to k. We point out that assumption that the principal provides incentives to *all* agents is merely for the sake of simplicity. An alternative model in which the principal attempts to maximize the net profit of the project (taking into account the value of the project) makes the analysis more cumbersome and offers no additional insight. The results will basically remain the same.

As indicated earlier the binary relation "k" represents the prevailing information structure about peers' effort in the organization. Our objective is to compare such information structures in terms of the cost of inducing effort by all agents. To this end we shall define a binary relation over IIEs by which we shall be able to say that the one contains more information about effort than the other.

Let K be the set of all IIEs. For k_1 and k_2 in K we say that k_1 is richer than k_2 if for all i, j in N we have $i\ k_2\ j$ implies $i\ k_1\ j$.

Proposition 1 argues that it is easier to provide incentives under more transparency within the organization, i.e., more information about peers' effort.

Proposition 1: If k_1 is richer than k_2, then $v^*(k_1) \leqslant v^*(k_2)$.

We will omit the proof of Proposition 1 here as we shall later state and prove a stronger result. However, the example in Section 2 suggests the intuition. In a richer IIE effort is more transparent, which allows more agents to make their effort decision dependent on that of more of their peers. For each of these agents shirking will result in a more detrimental effect on the project's success probability and thus also on the expected reward this agent receives. Hence the exposure of an agent's effort increases his/her incentive to exert effort and thus decreases the principal's cost of incentivizing that agent.

4 Indirect Information about Effort

As we have seen in Proposition 1 the network of information about peer effort affects the principal's cost of providing incentives. We would now like to go a step further in understanding how the information structure affects incentives by extending Proposition 1 to the case where IIEs are not necessarily comparable in terms of "richness." This analysis relies on the observation that agent $i's$ shirking may also affect the behavior of agents who do not observe $i's$ shirking directly. Suppose for example that agent 1's action is observed only by agent 2, while agent 2's action is observed only by agent 3. Assuming that the three agents adopt strategies by which they shirk if and only if they observe one of their peers shirking, we shall see agent 1 affecting agent 3's behavior without agent 3 being directly informed about what agent 1 did. In designing the optimal mechanism for a given IIE the principal will attempt to sustain as an equilibrium the behavior described above by which agents exert effort unless they detect one of their peers shirking. This in particular means that enriching the information structure described in the above example by letting agent 3 directly observe agent 1's effort choice does not allow the principal to reduce the cost of incentivizing agent 1. This all hints at the fact that in the IIE only those arcs $i\ k\ j$ matter for which there is no way for i to learn indirectly about the action of j through the actions of other peers. This leads us to the graph-theoretic notion of Transitive Closure.

For an IIE k we denote by $t(k)$ the IIE obtained from the transitive closure of k. Specifically, in the IIE $t(k)$ we have $i\ t(k)\ j$ if and only if there exists a sequence of agents $i_1, i_2, ..., i_r$ with $i_1 = i$ and $i_r = j$ and $i_m\ k\ i_{m+1}$ $(m = 1, 2, ..., r-1)$. Proposition 2 asserts that from the point of view of incentives only the transitive closure of an IIE matters.

Proposition 2: If $t(k_1)$ is richer than $t(k_2)$, then $v^*(k_1) \leqslant v^*(k_2)$, with strict inequality if and only if $t(k_1) \neq t(k_2)$.

Proof: For each $i \in N$ let $C(i,k)$ be the set of agents seeing i according to $t(k)$, i.e., $C(i,k) = \{j \in N| \ j \ t(k) \ i\}$. We will show that $v_i = \frac{c}{p(N)-p(N\backslash[C(i,k)\cup\{i\}])}$ is the optimal incentive-inducing mechanism. Note that the payoff v_i makes i indifferent between shirking and investing provided that his shirking triggers the shirking of all j in $C(i,k)$ and no more; i.e., v_i solves the following:

$p(N)v_i - c = v_i p(N\backslash[C(i,k) \cup \{i\}])$. (*)

If $t(k_1)$ is richer than $t(k_2)$, then $C(i,k_2)$ is a subset of $C(i,k_1)$ for all i. Furthermore, if $t(k_1) \neq t(k_2)$, then for some i this inclusion is strict. Hence the mechanism proposed above satisfies $v^*(k_1) < v^*(k_2)$. It thus remains to show that the mechanism described above is the optimal incentive-inducing mechanism. We first argue that with the above mechanism an equilibrium exists in which all agents invest. Consider the strategy combination in which each agent invests unless he sees (directly with respect to k) someone shirking. Under this profile all agents invest. Furthermore, any deviation by player i will trigger (sequentially) all players in $C(i,k)$ to shirk. Hence, if i shirks his expected payoff will be according to the RHS of equation (*). This means that i cannot increase his payoff by deviating. It remains to show that any mechanism that pays less than v_i to some player cannot admit an equilibrium in which all agents invest. Indeed, consider by way of contradiction an equilibrium of the underlying game in which all agents invest. Consider such a player i. If $C(i,k)$ is empty, then i is affecting no player's behavior. Since i is indifferent between investing and shirking in v_i, a lesser payoff will make shirking strictly preferable. Assume now that $C(i,k)$ is not empty, then shirking by i will trigger at most the set of players $C(i,k)$ to shirk as well (possibly less). Since v_i makes i indifferent between investing and shirking, a lesser payoff will make shirking strictly better, which contradicts the equilibrium assumption. Q.E.D.

An almost immediate consequence of Proposition 2 is that an information structure corresponding to a chain is optimal for the principal. Specifically, an IIE k is said to be a chain if there exists an order of the agents $i_1, i_2, ..., i_n$ such that the set of arcs i k is given by $i_1 k \ i_2, ..., k \ i_n$. Such an IIE can be interpreted as a sequential production in which each agent observes the effort decision of his immediate predecessor. Furthermore, it follows trivially from Proposition 2 that the empty IIE, which corresponds to no internal information about effort at all (as if all agents decides about their effort simultaneously), is the least attractive for the principal.

Corollary 1: Among all IIEs an IIE k_s corresponding to a chain yields the minimal cost for the principal, i.e., $v^*(k_s) \leqslant v^*(k)$ for all k.

Proof: The result follows from the following graph-theoretic claim: If the graph g is a chain, then the transitive closure of g is a maximal graph (richest) among all acyclic graphs. This can be shown as follows: Take an ordered sequence of nodes $1, 2, ..., n$, and consider the graph $g^* = \bigcup_{i=1}^{n-1} \{(i,k)| \ k > i\}$, where (i, j) stands for an arc from i to j. It is easy to note that g is maximal among all acyclic graphs. Indeed, if we add another arc to g it must be of the form (i, j), where $j < i$ and $2 \leq i \leq n$. But upon addition of such an arc we create the cycle $(i, j), (j, i)$. Hence g is maximal among all acyclic graphs. Consider now the chain graph given by $g = \{(1, 2), (2, 3), ..., (n-1, n)\}$. It is easy to see that the transitive closure of g is $g^*$and the claim follows. The corollary now follows directly from the claim together with Proposition 2. Q.E.D.

Corollary 2: Among all IIEs the IIE k_e that corresponds to the empty graph yields the maximal cost for the principal, i.e., $v^*(k_e) > v^*(k)$ for all k.

5 Random Peer Monitoring

The fact that the internal information about effort among peers affects the principal's cost of incentivizing his agents suggests that the principal might want to influence this information in order to reduce his cost. Since Propositions 1 and 2 assert that the principal can only gain from sustaining more transparency among his peers he might want to "enrich" the IIE so that more agents would be informed about more of their peers. This can be done, for example, by forming teams and co-locating workers in one office. It can also be achieved by organizing more frequent recreational activities within the organization through which the information about effort is transmitted more effectively. Finally, it can even be promoted by simply redesigning the architecture of the workplace so that there is more visibility across workers. Of course, when promoting the objective of more visibility among peers the principal will be sensitive to other managerial constraints that are not incentive-based (such as the cost of reorganization). But regardless of the method the principal chooses to adopt in order to facilitate more transparency among peers it is reasonable to assume that whether agent i is informed about agent $j's$ effort remains a random event. The tools and means by which the principal can affect the transparency among his peers can only influence the probability that this event would take place. We would therefore like to move on now to a model in which agents' monitoring opportunities are random. Specifically, we would like to model a situation where agents are uncertain about which of their peers can observe their effort decisions. Roughly speaking, our objective in this section is to argue that the principal's cost of incentivizing his agents is monotonically decreasing with the probability with which an agent can observe his peers. Thus any measure taken by the principal that can increase even slightly the probability of the availability of information will result in a reduction in the principal's cost of providing incentives.

To be able to address this issue we will need to separate between the chronology of the order of moves and the other factors that determine the information about peers. This will be done by making the order of moves explicit.

A *random* IIE is defined by a pair (w, k_q), where w is an order of the agents and k_q is a random directed graph on the set of agents with arcs emerging randomly according to an IID Bernoulli distribution with probability $0 < q < 1$ (i.e., every arc forms with probability q independently of the other arcs). We interpret w as the order in which agents take their effort decisions, and k_q represents the random structure of the information about peer effort. An arc from i to j represents $i's$ technical or formal ability to monitor agent $j's$ effort decision. More specifically, agent i is informed about agent's $j's$ effort decision before making his own if and only if i appears after j in the order w and the arc (i, j) from i to j realizes in k_q. When agent i takes his turn to act (in the order w) a realization takes place to determine which of the arcs (i, j) with $w(j) < w(i)$ forms, i.e., whose effort action does i see among those who preceded him? However, at this stage i is uncertain about who is going to observe his own effort decision. We assume that each agent is informed about the arcs formed among his predecessors but as we shall see this assumption is inessential to the analysis. Of course the pair (w, k_q) is commonly known to all agents. The principal himself has to design the incentive mechanism ex ante before any realization. He is only informed about q and w. As before, given a reward mechanism v and a random IIE (w, k_q) agents are facing a game (now involving Nature's random moves). The definition of an optimal incentive-inducing mechanism remains unchanged.

For a random IIE (w, k_q) we denote by $v^*(w, k_q)$ the total reward of the optimal incentive-inducing mechanism under k_q. In the sequel and with a slight abuse of notation we will use $v^*(q)$ for $v^*(w, k_q)$. The result by which the principal should favor transparency among peers now reads as follows:

Proposition 3: If $q' > q$, then $v^*(q) > v^*(q')$.

Proof of Proposition 3: We start with some definitions and notations: For an agent i and an order w we denote by $W(i)$ the set of agents following i including i himself, i.e., $W(i) = \{j \in N; w(j) \geq w(i)\}$. Fix a probability q and let Θ be the sample space that defines the realizations of k_q. For each $\theta \in \Theta$ we denote by $k_q(\theta)$ the deterministic graph that emerges at θ. Fix a probability $q, i \in N, \theta \in \Theta$ and an order w. Denote by $k_q(\theta)|_{W(i)}$ the graph $k_q(\theta)$ restricted to the set of nodes $W(i)$; i.e., $k_q(\theta)|_{W(i)}$ is obtained from $k_q(\theta)$ by deleting all nodes in $N \backslash W(i)$ with all the arcs in and out of these nodes. $k_q(\theta)|_{W(i)}$ represents the monitoring opportunities among $i's$ successors. For each j in $W(i)$ we denote by $j \to i$ if there exists a path $j = i_1, ..., i_r = i$ from j to i in the graph $k_q(\theta)|_{W(i)}$ such that $w(i_j) > w(i_{j+1})$. The intuition behind $j \to i$ is that there is a channel of messages from i to j based on the realized arcs such that the chain across which the messages pass involves only $i's$ successors and is consistent with the order of moves. In another words, if $j \to i$, then agent j who succeeds i can indirectly monitor (through a sequence of other agents) whether i has exerted effort. We next denote by $S_q(i, w, \theta)$ the set of agents that can indirectly monitor i, i.e., $S_q(i, w, \theta) = \{j \in N; j \to i\}$. If all agents adopt the strategy by which they decide to shirk if and only if they (directly) detected the shirking of at least one agent, then the random set $S_q(i, w, \theta)$ represents the set of agents that will be triggered to shirk as a consequence of $i's$ shirking. For a technology p and given the strategies specified above the probability that the project will succeed following $i's$ shirking (and assuming all his predecessors are exerting effort) is $p(N \backslash S_q(i, w, \theta))$, which is a random variable.

Lemma: If $q\prime > q$, then $p(N \backslash S_q(i, w, \theta)) \geq p(N \backslash S_{q'}(i, w, \theta))$ with a strict inequality for some θ.

Proof: We can embed the two random graphs k_q and $k_{q'}$ in one sample space such that the event "an arc exists from i to j in k_q" is a subset of the same event for the graph $k_{q'}$, and such that these two events have probability q and q' respectively. Hence, $k_q(\theta) \subset k_{q'}(\theta)$ for all θ, with some θ for which this inclusion is strict. Therefore $k_q(\theta)|_{W(i)} \subset k_{q'}(\theta)|_{W(i)}$ for all θ, with some θ for which this inclusion is strict, and hence $S_q(i, w, \theta) \subset S_{q'}(i, w, \theta)$ for all θ, with some θ for which this inclusion is strict. Because p is strictly increasing we have $p(N \backslash S_q(i, w, \theta)) \geq p(N \backslash S_{q'}(i, w, \theta))$ for all θ, with some θ for which this inequality is strict.

We now continue with the proof of Proposition 3. Let $p_q(i)$ be the expectation of the random variable $p(N \backslash S_q(i, w, \theta))$, i.e., $p_q(i) = \int_\theta p(N \backslash S_q(i, w, \theta))dF(\theta)$. If all agents adopt the strategy by which they shirk if and only if they encounter at least one agent shirking, then $p_q(i)$ is the ex-ante probability that agent i attributes to the project succeeding if he shirks. By the lemma above we have $p_q(i) > p_{q'}(i)$. We can now define $v_i^*(q) = c/[p(N) - p_q(i)]$. Since $p_q(i) > p_{q'}(i)$ the proof will be completed if we show that $v_i^*(q)$ is the optimal INI mechanism under the probability q.

Consider again the strategy profile defined earlier in the proof; i.e., an agent shirks if and only if he observes at least on of his predecessors shirking. We first show that this profile is a Nash equilibrium under the mechanism $v_i^*(q)$. By the definition of $p_q(i)$ and $v_i^*(q)$ we have $p(N)v_i^*(q) - c = v_i^*(q)p_i(q)$ and each player i is indifferent between investing and shirking if all other players adopt the strategy specified above. We now show that if some player i receives a reward $w_i < v_i^*(q)$ and all other players j receive $v_j^*(q)$ or more, then there exists no Nash equilibrium in which all players invest with probability 1. Assume by way of contradiction that such a Nash equilibrium exists. It must be the case that the strategy specifies that a player invests if he observes no shirking by others. Consider now player $i's$ deviation in which he shirks regardless of the information he receives. Along the equilibrium path, if i shirks under the realization θ he will trigger at most the set of players $N \backslash S_q(i, w, \theta)$ to shirk (possibly a subset depending on the strategies of others), so with a reward $v_i^*(q)$ player i is either indifferent between investing and shirking or strictly prefers to shirk, and with a reward w_i he strictly prefers to shirk. Q.E.D.

6 Substitution vs. Complementarity

As we saw earlier the optimal mechanisms sustain investment by all agents through a strategy profile in which every agent exerts effort unless observing one of his peers shirking. The optimal mechanism is tailored so that this type of "imitation" behavior is a Nash equilibrium. This applies regardless of the technology and regardless of the information structure. But a closer look at Example 1 shows that this strategy profile is not only a Nash equilibrium but it is also one using weakly undominated strategies.[2] In this section we shall argue that the proposed mechanisms, while valid for all technologies, are expected to be more effective for technologies in which agents' tasks satisfy complementarity. For such technologies our mechanism will implement the desired outcome as a Nash equilibrium with undominated strategies (which will also be a perfect Bayesian equilibrium). This will not be the case for technologies with substitution across tasks. We shall also show that the optimal mechanisms implementing full effort as an equilibrium with undominated strategies are different for technologies with substitution, and are invariant with respect to the information structure about peer effort. The intuition runs as follows: In the complementarity case each agent's incentives to exert effort are increasing the more the others exert effort. This means that the effort-implementing strategies by which agents are instructed to invest if and only if they observe no shirking are optimal whether on or off the equilibrium path. In particular, agents who observe shirking are better off shirking as well. Under substitution this is not the case. Agents observing others shirking are better off exerting effort. This means that implicit incentives generated by the strategies described above are more effective under complementarity.

We will now provide a formal argument in the deterministic case.

We say that a technology p satisfies complementarity across tasks if for every two sets of agents (coalitions) S, T with $T \subset S$ and every agent $i \notin S$ we have $p(S \cup \{i\}) - p(S) > p(T \cup \{i\}) - p(T)$, i.e., $i's$ effort is more effective the more the other agents exert effort. In contrast, we say that p has substitutability across tasks if $p(S \cup \{i\}) - p(S) \leq p(T \cup \{i\}) - p(T)$. Note that that the example in Section 2 is one of complementarity. Specifically, $p(S) = \alpha^{2-|S|}$.

Proposition 4: Suppose that the principal wants to implement full effort as a Nash equilibrium with undominated strategies. Then the following holds:

(1) If p satisfies complementarity across tasks, then the optimal mechanism derived in Proposition 2 remains the optimal mechanism also within this framework.

(2) If p satisfies substitutability across tasks, then the optimal incentive-inducing mechanism is given by $v_i^* = c/[p(N) - p(N \backslash \{i\})]$ and is identical for all IIEs.

Proof: We first show (1). Consider the mechanism $v_i = \frac{c}{p(N) - p(N \backslash [C(i,k) \cup \{i\}])}$

[2] In the sequential game this strategy profile also forms a subgame perfect equilibrium.

as defined in the proof of Proposition 2. Let us denote by s_i the equilibrium strategy of agent i by which he shirks if and only if he detects at least one of his peers shirking. We have already seen that a profile of these strategies forms a Nash equilibrium in the underlying game for any technology p and, in particular, for ones that satisfy complementarity across tasks. We thus need to show that this strategy in not weakly dominated. Consider a different strategy s_i'. Assume that this strategy instructs agent i to exert effort under some scenario in which he observed one of his peers shirking. Let R be the set of agents that i observes shirking under such a scenario. Assume a strategy profile for $N \backslash \{i\}$ in which all agents other than those in R use the strategy by which they shirk if and only if observing at least some other agent shirking. Denote by $R^* = \{j \in N | j\ t(k)\ m\ for\ some\ m \in R\}$; i.e., R^* is the set of agents that see someone that sees someone ... that is in R. Hence the shirking of the agents in R will trigger all the agents in R^* to shirk even if i himself is exerting effort. The expected payoff of agent i in this case will be $v_i p(N \backslash R^*) - c$. On the other hand, if i shirks his payoff is $v_i p((N \backslash R^*) \backslash (C(i,k) \cup \{i\}))$ (i.e., in addition to R^* the coalition $C(i,k) \cup \{i\}$ will also shirk). At this stage we need a bit of set theory algebra:

Lemma: If p satisfies complementarity across tasks, then for any two non-empty coalitions of agents $R, C \subset N$ we have $p(N) - p(N \backslash C) > p(N \backslash R) - p((N \backslash R) \backslash C)$.

Proof: We first note that if p satisfies complementarity, then the inequalities defining it also apply to coalitions, i.e., for any two coalitions S, T with $T \subset S$ and a coalition Q with $Q \cap S = \emptyset$ we have

$p(S \cup Q) - p(S) > p(T \cup Q) - p(T)$ (**).

This is proved by induction on the number of members in Q using the definition of complementarity agent by agent. Let now $T = (N \backslash R) \backslash C$, $S = N \backslash C$ and $Q = C$. Because $C \neq R$ we have $T \subset S$ and we can use (**) to establish the claim of the lemma.

By the lemma above we conclude that $p(N) - p(N \backslash [C(i,k) \cup \{i\}]) > p(N \backslash R^*) - p((N \backslash R^*) \backslash (C(i,k) \cup \{i\}))$ (i.e., setting $R = R^*$ and $C = C(i,k) \cup \{i\}$) and thus $v_i p((N \backslash R^*) \backslash (C(i,k) \cup \{i\})) > v_i p(N \backslash R^*) - c$. Hence no strategy of the type s_i' weakly dominates the strategy s_i. Clearly, the strategy by which i is instructed to shirk when observing no agent shirking does not weakly dominate s_i as well. This is because for any arbitrary increase of $i's$ payoff he is better off investing (because he is indifferent between shirking and investing when he is paid v_i he is strictly better off investing when promised more). With this we conclude that the profile in which each player is using

s_i forms a Nash equilibrium with undominated strategies.

To show (2), assume that p satisfies substitution across tasks. For each agent i let $v_i^* = c/[p(N) - p(N\backslash\{i\})]$. We argue that v_i^* is the optimal incentive- inducing mechanism regardless of the information structure about peers' efforts. Indeed, consider the strategy combination in which each agent exerts effort regardless of the information that he/she observes about other agents' effort. We argue that if agents rewards are given by v_i^* then this strategy profile is a Nash equilibrium with undominated strategies regardless of the information structure under which the game is played. To see that it is an equilibrium note that under v_i^* each agent is indifferent between exerting effort and shirking given that all the other players are investing (i.e., v_i^*solves $vp(N) - c = vp(N\backslash\{i\})$). We now argue that the strategy described above is weakly undominated. For this it is enough to argue that under the above mechanism the best response to observing some agents shirking is to exert effort. Indeed, for any set S of shirking agents $i's$ expected payoff when he exerts effort is $v_i^* p((N\backslash S) \cup \{i\}) - c$ and is $v_i^* p(N\backslash S)$ when he shirks. But since p satisfies substitution across tasks we have $p(N) - p(N\backslash\{i\}) \leq p((N\backslash S) \cup \{i\}) - p(N\backslash S)$ and i is better off exerting effort. Finally, we note that the mechanism v_i^* is optimal. Consider some agent i and assume a different mechanism that pays $v_i' < v_i^*$ to player i. Suppose by way of contradiction that an equilibrium exists in the corresponding game with all players exerting effort. It must be the case that this equilibrium specifies that each player exerts effort if he encounters no shirking. Otherwise, along the equilibrium path there will be at least one agent shirking. But because all agents are indifferent between investing and shirking under v_i^*, they will be induced to shirk under v_i'. Q.E.D.

We note here that the equilibrium by which effort is implemented under the mechanisms of Proposition 4 is also perfect Bayesian in the extensive form game. Given what an agent observed transpire before making his effort decision the specified action is optimal for each of his information sets and independenly of his beliefs about those agents whose actions are unobservable.

7 Function-Based vs. Process-Based Teams

The distinction between complementarity and substitution that we made in Proposition 4 not only identifies the environments in which co-location is expected to be more effective, but it also has an important implication for optimal structure of teams, i.e., which agents should be located together? While our model suggests that, from the point of view of incentives, the more peer information the better, there may be other managerial considerations that favor keeping teams small. This brings us to the issue of how to design teams optimally given a size constraint. We shall consider an environment in which there are several products each of which is produced through a process in which different agents ressume different functions. In a function-based team all agents perform the same function on different products. A process-based team involves agents dealing with the entire process of a single product, each agent assuming a different function. Suppose for example that a project involves the preparation of nine turkey sandwiches each of which has to undergo a process of cutting the bread, spreading the mayonnaise, and slicing the turkey. There are nine workers of which three are bread cutters, three are mayonnaise spreaders, and three are turkey slicers. A process-based structure will have three teams each consisting of one bread cutter, one mayonnaise spreader, and one turkey slicer. A function-based structure will again have three teams one consisting of all the bread cutters, one of only mayonnaise spreaders, and one of only turkey slicers.

Our objective in this section is to argue that incentive considerations should favor the process-based structure to the function-based one.

The intuition is roughly as follows: Agents who assume different functions within the production of the same product are involved in tasks that are complementary, whereas the relation between agents performing the same function is that of substitution. As we argued earlier with sequential rationality, agents with complementarity can generate implicit incentives when possessing information about peer effort, while agents with substitution cannot. This means that in utilizing the implicit incentives optimally the principal should co-locate agents among which there is complementarity.

We will now provide a formal argument to our intuition above, which, for simplicity, will be presented in the framework of two-function, two-product organizations. We first need to set up the properties of complementarity and substitution as a binary relation among agents. This may be achieved in a couple of definitions.

We say that agents i and j are complementary with respect to the technology p if the investment of each of these agents is more effective when the other agent invests than when he does not. Formally: For each coalition of agents S such that $i, j \notin S$ we have $p(S \cup \{i,j\}) - p(S \cup \{j\}) >$

$p(S \cup \{i\}) - p(S)$. We say that agents i and j are substitutable if each agent's investment (weakly) decreases the marginal contribution of the other agent, i.e., $p(S \cup \{i,j\}) - p(S \cup \{j\}) \leq p(S \cup \{i\}) - p(S)$.[3]

Consider now the following four-agent organization involving the production of two products A and B. The production of each product involves a two-stage process: an upstream stage and a downstream stage. Each agent is in charge of one production stage of one of the products. We denote by a_d, a_u, b_d, b_u the four agents according to the tasks they are in charge of (i.e., a_d is in charge of the downstream task of product A, etc.). We assume that there exists complementarity across different stages of the same product and substitution across different products at the same stage. Specifically, there exists complementarity between a_d and a_u as well as between b_d and b_u, whereas a_d and b_d are substitutes and so are a_u and b_u. An example of a technology satisfying these conditions can be given as follows: Assume that each stage of production succeeds with probability α if effort is not exerted and with probability β with $\beta > \alpha$ if effort is exerted. Each product succeeds if and only if the two stages of its production end successfully. Define now the project's goal to be the successful production of at least one of the two goods. The resulting technology p satisfies precisely the conditions imposed above.

Suppose now that teams are constrained to contain no more than two agents. The process-based structure involves two teams $\{a_d, a_u\}$ and $\{b_d, b_u\}$, while the teams in the function-based structure are $\{a_d, b_d\}$ and $\{a_u, b_u\}$. To introduce the effect of co-location on agents' information about peers' effort we assume that in both structures and in each of the teams one agent observes the action of the other agent in his team before performing his own (i.e., an a-type acts before a b-type, and a d-type before a u-type). No effort information is revealed between the teams. We can now assert the following:

Proposition 5: Suppose that the principal wants to sustain full effort as a Nash equilibrium with undominated strategies. Then the optimal mechanism in the process-based structure costs less than the optimal mechanism in the function-based structure.

Lemma 1: The optimal mechanism under the process-based structure pays the four agents the following rewards: $a_d : \frac{c}{p(N)-p(b_d,b_u)}$, $a_u : \frac{c}{p(N)-p(a_d,b_d,b_u)}$, $b_d : \frac{c}{p(N)-p(a_s,a_u)}$, $b_u : \frac{c}{p(N)-p(a_d,b_d,a_u)}$.

[3] See Segal (2003) who discusses similar properties to study integration among agents in a cooperative game using a Shapley value framework.

Lemma 2: The optimal mechanism under the function-based structure pays the four agents the following rewards: $a_d : \frac{c}{p(N)-p(b_d,a_u,b_u)}$, $a_u : \frac{c}{p(N)-p(a_d,b_d,b_u)}$, $b_d : \frac{c}{p(N)-p(a_d,a_u,b_u)}$, $b_u : \frac{c}{p(N)-p(a_d,b_d,a_u)}$

Proof of Lemma 1: Consider the following strategy profile in the game induced by the process-based structure: Players a_d and b_d exert effort, and player x_u exerts effort if and only if player x_d exerts effort, where $x \in \{a, b\}$. We first argue that this strategy profile is an equilibrium with undominated strategies. Assume first that a_d exerts effort. Then under this profile player a_u gets $p(N)v_{a_u} - c$ if he invests and $p(a_d, b_d, b_u)v_{a_u}$ if he shirks (where v_{a_u} is the reward specified in the lemma). Hence a_u is indifferent between his two actions when a_d invests. We now show that it is optimal for a_u to shirk if a_d shirks. If a_u invests after a_d shirks he receives $p(a_u, b_d, b_u)v_{a_u} - c$ and he gets $p(b_d, b_u)v_{a_u}$ if he shirks. But because a_u and a_d are complementary we have $p(N)-p(a_d, b_d, b_u) > p(a_u, b_d, b_u)-p(b_d, b_u)$ and therefore $p(a_u, b_d, b_u)v_{a_u}-c < p(b_d, b_u)v_{a_u}$. This implies that a_u is better off shirking. The same claims established above apply also with respect to b_d and b_u. To verify that the proposed strategy profile specifies optimal action for a_d and b_d we note that if, say, a_d shirks then a_u will shirk as well and the expected payoff for a_u is $p(b_d, b_u)v_{a_d}$, which is identical to $p(N)v_{a_d}-c$; hence it is optimal for a_d to exert effort and also for b_d to do so. We conclude that the strategy profile describe above specifies optimal behavior for all agents both on and off the equilibrium path, and is therefore an equilibrium with undominated strategies. To verify that the rewards specified in the proposition are optimal consider a different reward vector for which some agent gets less and consider a strategy profile in which all agents exert effort. It is easy to see that such a profile cannot be an equilibrium. For d-type agents this is straightforward because they are already indifferent between shirking and investing under the rewards specified in the lemma. For u-type agents the argument is as follows. First, the strategy profile specified above cannot be an equilibrium because of the indifference. So consider for example the strategy for a_u in which he exerts effort regardless of the action taken by a_d. It can easily be shown that this strategy cannot be part of an equilibrium because of complementarity (we have shown that under the reward specified in lemma 1 a_u is better off shirking if a_d shirks- all the more so for a lower reward). Hence, there exists no equilibrium in which all agents invest under the alternative reward scheme. Q.E.D.

Proof of Lemma 2: Consider the following strategy profile in the game induced by the function-based structure: All agents exert effort regardless of the information they obtain about the others. It is easy to show that given the rewards specified in Lemma 2 each agent is indifferent between exerting effort and shirking given that the rest are exerting effort. Hence the strategy profile above is an equilibrium. To show that the profile specifies optimal action also off the equilibrium path we note that each agent who observes his peer shirking is weakly better off investing than shirking. Consider agent a_u who observed a_d shirking. If a_u invests his payoff is $p(a_u, b_d, b_u)v_{a_u} - c$ and it is $p(b_d, b_u)v_{a_u}$ if he shirks. But because of substitution we have $p(N) - p(a_d, b_d, b_u) \leq p(a_u, b_d, b_u) - p(b_d, b_u)$ and hence $p(a_u, b_d, b_u)v_{a_u} - c \geq p(b_d, b_u)v_{a_u}$, which establishes the claim. Showing that this mechanism is optimal is proved similarly as in Lemma 1. Q.E.D.

Proof of Proposition 5: Let v^1 and v^2 be the total reward paid by the principal under Lemmas 1 and 2 respectively. Then $v^1 - v^2 = \frac{c}{p(N)-p(b_d,b_u)} + \frac{c}{p(N)-p(a_s,a_u)} - \frac{c}{p(N)-p(a_d,b_d,b_u)} - \frac{c}{p(N)-p(a_d,a_u,b_u)}$. But because of strict monotonicity of p we have $p(N) - p(b_d, b_u) > p(N) - p(a_d, b_d, b_u)$ and $p(N) - p(a_s, a_u) > p(N) - p(a_d, a_u, b_u)$ or $\frac{c}{p(N)-p(b_d,b_u)} < \frac{c}{p(N)-p(a_d,b_d,b_u)}$ and $\frac{c}{p(N)-p(a_s,a_u)} < \frac{c}{p(N)-p(a_d,a_u,b_u)}$, implying that $v^1 - v^2 < 0$. Hence the optimal mechanism for the process-based structure is less expensive. Q.E.D.

8 Discussion

The purpose of this paper is to study in isolation the role peer information plays in providing incentives in organizations. In almost any organization, with or without co-location, agents are exposed to some information about their peers' effort. Our main finding is that the more dispersed this information, the easier it is to provide incentives. Co-location and teaming is one way in which a better dispersion of this information can be achieved . Needless to say, teams are not created for this sole purpose; they have other advantages which may even be more prominent such as facilitating coordination between agents and allowing them to learn from each other.

The role of peer information in teams is particularly important in environments in which: (1) peers can easily monitor each other and are better informed about each other's effort than the principal is, and (2) agents' tasks involve complementarities. Environments in which agents work on professionally similar tasks and for which the project's output depends on the success of the weakest link seem to have both these features. Software development, R&D projects, and large architecture or engineering projects are some of the relevant examples. Further empirical studies into the functioning of these types of environments are likely to shed additional light on the role of peer information in providing incentives.

9 Appendix

Proposition: For every strategy profile $s = (s_1, ..., s_n)$ the set $M(s)$ is well defined and unique.

Proof: Define by $k_i = \{j \in N | j \ k \ i\}$, i.e., the set of agents seeing i. We say that a player is a core player with respect to k if k_i is empty. Since k is acyclic it has a non-empty core and we denote it by $C(N)$. Given the strategy profile s, actions are uniquely determined for all players in the core. Consider now the binary relation k restricted to the set of players $N \backslash C(N)$. This binary relation is again acyclic and has a non-empty core which we denote by $C(N \backslash C(N))$. Any player j in $C(N \backslash C(N))$ is informed only about actions taken by players in $C(N)$. Hence, given that the actions taken by players in $C(N)$ are well defined and unique, so are the actions of players in $C(N \backslash C(N))$. We can now proceed by induction. At each stage we eliminate players for which the actions have already been determined and we remain with an acyclic graph on the remaining players, which has a non-empty core. The process terminates when no players are left, at which stage we attain the vector of actions consistent with the profile s, and $M(s)$ is simply the set of players who choose to exert effort. Q.E.D.

References

[1] Baliga, S. and T. Sjostrom (1998) "Decentralization and Collusion," *Journal of Economic Theory* 83,162-232.

[2] Che, Y.K. and Yoo, S.W. (2001) "Optimal Incentives for Teams," *American Economic Review*, 91, 525-541.

[3] Heywood, J.S. and Jirjahn, U. (2004), "Teams, Teamwork and Absence," *Scandinavian Journal of Economics* 106, 765-782.

[4] Holmstrom, B. (1982) "Moral Hazard in Teams," *Bell Journal of Economics*, 13, 324-340.

[5] Holmstrom, B. and Milgrom, P. (1990) "Regulating Trade among Agents," *Journal of Institutional and Theoretical Economics*, 146, 85-105.

[6] Jackson, M. (2005) "A Survey of Models of Network Formation: Stability and Efficiency," Forthcoming in Group Formation in Economics; Networks, Clubs and Coalitions, edited by Gabrielle Demange and Myrna Wooders, Cambridge University Press.

[7] Gould, E. D. and E. Winter (2005) "Rational Peer Effects Between Workers: Evidence from Professional Baseball" Mimeo, The Hebrew University of Jerusalem, The Department of Economics.

[8] Itoh, H. (1991) "Incentives to Help Multi-Agent Situations," *Econometrica*, 59, 611-636.

[9] Segal, I. (2003) "Collusion, Exclusion, and Inclusion in Random-Order Bargaining," *Review of Economic Studies,* 70, 439-460.

[10] Teasley, S. D., Covi, L., Krishnan, M.S., and Olson, J. S. (2002) "Rapid Software Development through Team Co-location. *IEEE Transactions on Software Engineering*, 28, 671-683.

[11] Winter, E. (2004) "Incentives and Discrimination," *American Economic Review*, 94, 764-773.

[12] Winter, E. (2005) "Optimal Incentives for Sequential Production Processes," forthcoming in *Rand Journal of Economics.*