The Role of Coordination Bias in Platform Competition[*]

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Abstract

This paper considers platform competition in a two-sided market that includes buyers and sellers. One of the platforms benefits from a favorable *coordination bias* in the market, in that the two sides are more likely to join the advantaged platform. We find that the degree of the coordination bias affects the platform's decision regarding the business model (i.e., whether to subsidize buyers or sellers), the access fees and the size of the platform. A slight increase in the coordination bias may induce the advantaged platform to switch from subsidizing sellers to subsidizing buyers, or induce the disadvantaged platform to switch from subsidizing buyers to subsidizing sellers. Moreover, in the former case the advantaged platform switches from oversupplying to undersupplying sellers, while in the later case the disadvantaged platform switches from undersupplying to oversupplying sellers.

Keywords: platform competition, two-sided markets, coordination bias

JEL: L11, L14

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1 Introduction

In platform competition in a two-sided market, a platform's ability to attract consumers depends not only on the consumers' beliefs regarding its quality, but also on consumers' beliefs regarding the platform's ability to attract the other side of the market.

Consider for example the market for smartphones. The recent introductions of Apple's iPhone 4S with the improved operating system, and Samsung's Galaxy II with the improved Android 4, open a new round in the competition between the two platforms. Here, the ability of each platform to attract users depends not only on its perceived quality, but also on users' beliefs regarding the number of application developers that would be willing to develop new applications for the platform. Likewise, the ability to attract application developers to the platform depends on their beliefs regarding the number of users that will join the platform. Similarly, in the battle between HD DVD and BluRay, it mattered not only which format provides better experience of high-definition movies, but also how many movies will be released by the movie studios in a given format.

In many cases one of the competing platforms may benefit from "coordination bias." It may come from a better recognized brand name, or better track record of past success in the marketplace. But the agents choosing which platform to join believe that the market is more likely to coordinate on the advantaged platform, i.e., that both sides will join the advantaged platform with probability higher than $\frac{1}{2}$. Naturally, even if a platform enjoys such favorable coordination bias, it still needs to identify how to translate it into competitive advantage over its rival. Likewise, a platform that suffers from unfavorable coordination bias needs to identify how to choose its prices in order to overcome its competitive disadvantage.

This paper considers platform competition in a two-sided market that includes buyers and sellers. The main feature of our model is that one of the platforms enjoys a favorable coordination bias over the competing platform. The idea of coordination bias is that if buyers and sellers do not know which platform other buyers and sellers are going to join, they will make their individual decisions based on the assumption that all other players will coordinate on joining the advantaged platform with probability $\alpha > \frac{1}{2}$.

Our main research question is the following. Platforms usually compete by setting different prices to the two sides of the market. In particular, a platform may offer a low, perhaps negative price to one of the sides, and then charge a high price to the other side. For example, videogames consoles like Xbox or PlayStation, often sell at a loss in retail, but they

make profits by charging the game developers who sell games to be played on the consoles. We therefore ask how the coordination bias affects the platforms pricing strategies in terms of *(i)* the side to attract and *(ii)* the number of sellers to attract. Notice that we raise this question for both the platform with favorable coordination bias and the platform with unfavorable bias. This is because, as we show, a platform with unfavorable coordination bias can still win the market if it has sufficiently high quality, and if the platform correctly chose its pricing strategies in accordance with the coordination bias against it.

To answer this question, we consider a model with the following features. There are two sides of a market, buyers and sellers. There is large number of potential sellers that can enter the market, but each seller has fixed entry costs. There is an arbitrary number of buyers. Buyers want to buy one unit from each seller, and have a decreasing marginal utility with the number of sellers they buy from. The two sides cannot interact without a platform. Once they join a platform, sellers compete among themselves for buyers. This means that sellers can make positive profit from joining a platform only if buyers indeed joined the same platform and only if not too many other sellers joined the platform, such that competition among sellers fully dissipates their profits.[1]

For example, buyers can represent smartphones users, who wish to buy smartphone applications, or gamers who wish to buy videogames. Sellers can represent developers who can develop, for a given fixed development costs, a smartphone application or a videogame. A platform is then a smartphone operating system or a videogame console.

There are two competing platforms that differ in two respects. First, they may differ in the quality. We allow for cases where either platform is of higher or lower quality than the other. Second, one platform benefits from a favorable coordination bias of the market in that if there are two putative equilibria—one in which buyers and sellers join the advantaged platform, and another in which they join the disadvantaged platform—each side believes that all other players are going to play the first equilibrium with probability $\alpha > \frac{1}{2}$. Since the two platforms do not differ horizontally and since there are positive externalities between sides, we focus on equilibria in which one of the platforms wins the market. The two platforms compete by setting different access fees to sellers and buyers, which can be positive or negative. Setting a negative fee to one side of the market may be a profitable tactics in

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[1]Our results depend on such an asymmetry of the two sides. This asymmetry is common in many real-life two-sided markets. However, there are also two-sided markets to which our model does not apply, e.g., on-line dating.

two-sided markets, because the subsidized side attracts the other side to the platform, from which the platform can recuperate the lost revenue. Each platform needs to choose its business model, in that it needs to decide which side will be the subsidized side, and which will be the side bringing in the revenue. In the context of our model, the business models are characterized by access fees that a platform would charge. In reality, the choice of a business model is related to important decisions on the structure of firm, architecture of supply chain, or investment in marketing. Changing a business model may be costly and time consuming.

We establish the following main results. First, we show that the platforms' pricing strategies are affected by the degree of the coordination bias, α, in two distinct ways. First, if the sellers' fixed costs are very low and neither of the platforms enjoys the coordination bias ($\alpha = \frac{1}{2}$), then both platforms will choose to fully cover the seller's fixed costs and offer a positive access price to buyers. That is, both platforms choose a business model that relies on the revenue from the buyers while subsidizing the sellers. Intuitively, whenever the platforms fully compensate the sellers for their fixed costs, sellers will join the platform regardless of their beliefs, because they know that they can never make loses. If $\alpha = \frac{1}{2}$, then no platform has an advantage in exploiting the two sides' coordination bias, and therefore the identity of the winning platform depends on its quality: the platform with the highest quality wins. We then show that given a same low fixed fees, if α becomes large enough, then the platform with favorable coordination bias prefers to change its business model, by charging higher access fees to sellers, and then using the presence of sellers and the coordination bias to attract the buyers. In this case however, the advantaged platform can win the market even if it offers a lower quality than the disadvantaged platform, because of its ability to exploit the favorable coordination bias. It is also possible for the disadvantaged platform to win, if it offers substantially higher quality.

The platform enjoying favorable coordination bias may want to change its business model when increases under intermediate development costs. As mentioned above, for $\alpha = \frac{1}{2}$ both platforms compete on attracting buyers, and the higher quality platform wins the market. For the same given fixed costs, as α increases, the disadvantaged platform changes its business model to one where it subsidizes sellers and relies on the revenue from the buyers. Again, the advantaged platform wins even if it offers a lower quality. Intuitively, if both platforms compete on the buyers, then they both rely on the buyers' beliefs regarding the probability that sellers join. If $\alpha = \frac{1}{2}$, then these beliefs are the same for both platforms. However, as α increases, one platform gains higher belief advantage over the other. Therefore, it is

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no longer profitable for the disadvantaged platform to subsidize buyers, and will prefer to subsidize sellers.

We also find that whenever a platform subsidizes buyers, it will attract fewer sellers than the trade-maximizing level. However, if a platform subsidizes sellers, it will attract more sellers than the trade-maximizing level. Since an increase in α may alter a platform's decision regarding which side to subsidize, we find that a small increase in α can induce the advantaged platform to substantially decrease the size of its platform, by shifting from attracting more sellers than the trade-maximizing level, to attracting fewer sellers than this level. Likewise, a small increase in α can induce the disadvantaged platform to substantially increase the size of its platform, by shifting from attracting fewer sellers than the trade-maximizing level, to attracting more sellers than this level.

In an extension to our basic model, we consider a multi-period game, in which platforms and agents repeat playing the static game considered in our basic model, but the coordination bias adjusts along time. In each period, the bias slightly increases in favor of the platform that won the previous period. We find that staring from no coordination bias ($\alpha = \frac{1}{2}$), beliefs converge faster, on average, to full coordination bias ($\alpha = 1$) when platforms use the sellers as the main source of revenues, than when platforms subsidize sellers.

The economic literature on competing platforms extends the work of Katz and Shapiro (1985) on competition with network effects. Spiegler (2000) considers an "extractor," such as a platform who can extract positive externalities from two agents. Spulber (1996, 1999) considers intermediary firms that establish a market between buyers and sellers. Caillaud and Jullien (2001, 2003) consider competition between undifferentiated platforms, where one of them benefits from favorable beliefs. Hagiu (2006) considers undifferentiated platform competition in a setting where sellers join the platform first, and only then buyers. Lopez and Rey (2009) consider competition between two telecommunication networks when one of them benefits from "customers' inertia," such that in the case of multiple responses to the networks' prices, consumers choose a response which favors one of the networks. Jullien (2011) considers undifferentiated platform competition in a multi-sided market. Halaburda and Yehezkel (2011) consider undifferentiated competition where the two sides of the market are ex-ante uniformed about their utilities, and are ex-post privately informed. A common feature in the above literature is the assumption that one platform fully benefits from a belief advantage. This is equivalent to assuming full coordination bias ($\alpha = 1$) in

our model. We make two contributions to this literature. First, we consider the case where the advantaged platform benefits from only a partial coordination bias, in that $\frac{1}{2} < \alpha < 1$. As we explained above, this distinction turned out to be important because a platform may choose a different business model depending on whether $\alpha = 1$ or $\frac{1}{2} < \alpha < 1$. The choice of the business model has implication for the access fees, which side is subsidized, and what is the size of the platform (and whether it is below and above trade-maximizing size). The second contribution of our paper is considering endogenous coordination bias, based on the platforms track record of past successes in the marketplace.

Alexandrov, Deltas and Spulber (forthcomming) consider competition between dealers and a market maker, acting as intermediaries between buyers and sellers. They assume that one of the two sides of the market (sellers in their case), is a bottleneck, such that competition is stronger on this side. In our model, it is possible to interpret the buyers as the bottleneck side, because all buyers make the same decision while some sellers join a platform while others do not. Our paper contributes to Alexandrov, Deltas and Spulber (forthcomming) by considering a coordination bias. We find that a platform's profit is more sensitive to its coordination bias if it attracts the bottleneck side (buyers in our model) than the other side. This is because the platform needs to compensate buyers for their alternative payoff from joining the competing platform, which relay on the degree of the coordination bias. To attract sellers, a platform only needs to compensate them for their reservation utility from not trading, which is independent on the coordination bias.

Our paper also contributes to the literature on business models. Ghemawat (1991) and Casadesus-Masanell and Ricart (2010) refer to firms strategy as its choice of a business model: the business model is a set of committed choices that lays the groundwork for the competitive interactions between the firms. The choice of the business model enables or limits particular tactical choices (e.g., prices). Specifically, our paper analyzes the choice of the business model in the context of two-sided platforms. As pointed by Rochet and Tirole (2003) and by Casadesus-Masanell and Zhu (2010), in the context of two-sided markets, one of the most important aspects of the business model is which side of the market is the primary source of the revenue.

2   Characteristics of the Market

We consider an environment with two competing platforms. Each platform needs to serve two groups of customers, the buyers and the sellers. We refer to these two groups as two sides of the market. Each buyer wishes to buy a product that a seller sells. The goods offered by the sellers and demanded by the buyers are homogeneous. However, a buyer and a seller cannot trade unless they have joined the same platform.

Buyers.   There are N_B identical buyers. Buyers can be smartphone users, who have a demand for smartphone applications. Likewise, buyers can be gamers, who have a demand for videogames. The consumption utility of each buyer from buying n products is $u_B(n)$. The number n can represent the number of applications, videogames, etc. This consumption utility is positive for any $n > 0$ and increasing with n, but it reaches a saturation point at $\hat{n}$, i.e., $u_B'(n) > 0$ for $n < \hat{n}$ and $u_B'(n) < 0$ for $n > \hat{n}$. Moreover, $u_B''(n) < 0$. To make sure that the second order conditions are satisfied, we assume that the third derivative is either negative, or positive but not too large. Specifically, $u_B''' < -\frac{u_B''}{n}$. The total buyer's utility also incorporates the cost of purchasing the products. If the price of every product is p, then the total buyers utility is

$$U_B(n) = u_B(n) - p\,n\,.$$

Sellers.   There is a large number, N_S, of identical sellers ready to enter the market, where $N_S > 2\hat{n}$. Sellers can be developers of smartphone applications, developers of videogames, etc. Each seller offers one product (a smartphone app, a videogame for a console, a movie in a given format), but he can sell multiple copies of it to multiple buyers. A seller receives p for every copy of the product he sells. Sellers have a fixed cost of developing the product, $K > 0$, which is the same for all sellers. We normalize marginal production costs to 0.[2]

Buyers and sellers trading on a platform.   If n sellers join a platform, they provide n products and they behave competitively. Hence, products are sold at the price equal to the marginal consumption utility of the n-th product, $u_B'(n)$. If $n > \hat{n}$ such that $u_B'(n) < 0$, buyers will not pay a positive price. As sellers will not sell at a loss, we assume that if $n > \hat{n}$, then only $\hat{n}$ are sold at $p(\hat{n}) = u_B'(\hat{n}) = 0$. Therefore, the equilibrium price is

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[2]Indeed, in most of our examples (a smartphone app, a video game for a console, etc.) involve negligible marginal costs.

$p(n) = \max\{u'_B(n), 0\}$, and it is the same for all products. As a tie-braking rule, we assume that in case sellers are exactly indifferent between joining a platform or not, they will enter as long as they expect to make positive sales (but stay out otherwise). This assumption enables us to eliminate unreasonable equilibria.

After incorporating the development costs, the sellers total payoff is $N_B\, p(n) - K$. Let $k = \frac{K}{N_B}$. Then this payoff can be represented by $N_B\big(p(n) - k\big)$. As $p(n)$ is decreasing with n, we assume that $p(0) > k$, such that k is low enough such that sellers' total payoff is positive for some $n > 0$.

Network effects and the asymmetry between the sides. The above model has two main features that will play an important role in the analysis. First, there are positive network effects between the two sides of the market: buyers (sellers) gain higher utility from joining a platform the more sellers (buyers) join the same platform. In the market for smartphones, for example, a consumer decides which platform to join (iOS, Android, etc.,) based on the expected applications that would be developed to this platform. In the market for videogames, a gamer will buy a console based on the expected games that will be developed to this console. The same argument follows to developers (of smartphone applications or videogames). In the case where there is more than one platform in the market, network effects often lead to tipping of the market towards one of the platforms. This is also the feature of this model. Since both sides want to join the same platform, it creates also a coordination problem. In consequence, it may lead to multiplicity of equilibria.

The second main feature of our model is asymmetry between the two sides of the market. Buyers' participation is non-rivalous. The number of other buyers on the same platform does not affect each buyer's utility. This is not true for the sellers. Larger number of sellers on the same platform increases competition and decreases each seller's payoff. Notice that this asymmetry follows from the competitive environment and no marginal costs to the sellers. A priori all agents on each side are identical. However the market environment forces them to behave differently. We believe that this asymmetry in rivalry reflects many (but not all) two-sided markets: Consumption of smartphone apps is non-rivalous, even if the developers compete for the users. Similar statement is true about video games released for consoles, or movies released for a given format.[3]

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[3] Our model does not apply, for example, to the dating market, where there is competition on both sides.

Trade-maximizing outcome (first-best). To solve for the first-best outcome, notice first that it is cost-reducing for all buyers to join the same platform. This is because the sellers' fixed costs, K, are spread among a large number of buyers. Given that all buyers join the same platform, the number of sellers that maximizes total gains from trade between sellers and buyers, n^*, is the solution to

$$n^* = \arg\max_n \left\{ N_B(U_B(n) + n(p(n) - k)) \right\} . \tag{1}$$

Given our assumptions, above, n^* is unique, with $\hat{n} > n^* > 0$ for any $k > 0$ and $n^* = \hat{n}$ for $k = 0$. Moreover, n^* is decreasing with k.

Platforms. There are two competing platforms, which we call platform A and platform D. They differ in two aspects: in terms of the coordination bias they face in the market, and in that they are vertically differentiated.[4] We discuss the issue of coordination bias later in this section and in Section 3. We measure the vertical differentiation with Q—additional utility that a buyer gains by joining platform A. Variable Q captures, for example, difference in the quality of service between the platforms; or an additional stand-alone service that one platform offers but the other one does not. We allow for both positive and negative Q, i.e., platform A may be perceived as better or worse than platform D. We assume that platforms do not incur costs, so the revenues are equivalent to profits.[5]

Strategies and business models. Platforms compete by setting access fees to buyers and sellers, which can be positive or negative: (F_B^A, F_S^A), (F_B^D, F_S^D). As platforms aim at attracting two groups of "customers, the buyers and the sellers, they may find it optimal to offer lower access fee to one side than the other. In fact, it is well known that it may be optimal for a platform to subsidize one of the sides in order to charge higher fees to the other side. The business model identifies the side which is the primary source of revenue. In the context of our model, we find that platforms' equilibrium pricing strategies involve choosing one of two distinct business models. In one of the business models the sellers are the primary

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[4]In particular, we do not assume horizontal differentiation. In many markets both competing platforms have loyal following resulting from horizontal differentiation. But there always exists a segment of customers that do not have prior preference for one platform or the other, a segment for which the platforms compete. Our model only refers to this segment.

[5]The analysis is very similar (but mathematically significantly more complicated) with positive fixed and marginal costs.

source of revenue, in that the platform fully extracts the sellers' profit, by charging sellers a high access fees, and charge a low—possibly negative access fees—to attract buyers. We call it the Sellers-Revenue-Based (SRB) business model. In the other business model, the buyers are the primary source of revenue, and the sellers may be subsidized. We call it the Buyers-Revenue-Based (BRB) business model.

In our model, the business models are distinguished only by different prices. In reality, the choice of a business model is often related to important choices in infrastructure and supply chain architecture. In the market for smartphones, for example, a platform that chooses a BRB may provide the developers with software development tools, technical training and guidance, and perhaps making the operating system open. A platform that chooses a SRB, however, may need to develop a strong marketing network for selling smartphones, and also incorporate in the operating system elements to control and restrict developers access. Such infrastructure differences require time to build, and may be expensive—or sometimes impossible—to change.[6] Therefore, firms need to decide on the business model before the actual pricing decisions. Moreover, the choice of the business model may constrain the set of prices that the platform can charge.[7] Of course, firms choose their business models considering their expectations about the future prices. In this paper, we investigate how the belief advantage (and disadvantage) affects the business model that the platform chooses.

Timing. The platforms choose their business models (simultaneously). Given the business models, they (simultaneously) decide on the access fees to charge to both sides. The buyers and sellers observe the prices, and then based on the prices make their (simultaneous) decision which platform to join. As they join the platform, the trade takes place according to the competitive environment described earlier.[8]

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[6]The history of videogames (Hagiu and Halaburda, 2009) illustrates the potential difficulty. Atari's original business model was BRB, but when under the competitive threat it wanted to switch to SRB, and charge royalties from the developers, it could not enforce it, for the lack of the restrictions in the system's code.

[7]Casadesus-Masanell and Zhu (2010) provide examples from other industries dominated by platforms: TV channels may choose to be ad-sponsored (most of broadcast channels) or subscription-based (like HBO); most newspapers are sold at positive prices to readers, while there also exist newspapers like *Metro*, which is completely ad-sponsored and free to the readers. As Casadesus-Masanell and Zhu (2010) point out, in both industries firms choosing different business models need to develop different distribution channels (*Metro* is given away at subway stations in large cities, as opposed to shop distribution for other newspapers), and different capabilities (HBO does not need to attract advertisers, but needs to provide samples of its programming to potential subscribers).

[8]Since we focus on the platforms strategies for attracting buyers and sellers, we do not treat trade as a separate stage of the game.

Platforms' choice of business model | Platforms' choice of fees | Agents' choice of platform
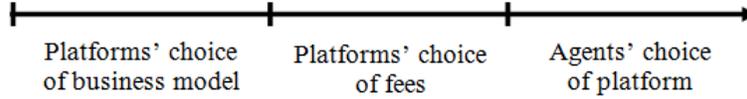
Dominant firm equilibria. We focus on equilibria in which one platform "wins" the market and the competing platform earns zero profit. We ignore equilibria with two active platforms because such equilibria are unstable. This result follows from our assumption that all buyers are identical. Therefore, in an equilibrium with two active platforms, all buyers should be indifferent between joining each platform. However, if a "marginal" buyer switches from one platform to the other, then more sellers will find it optimal to join this platform, which will induce all buyers to switch to this platform as well. Consequently, a "marginal" change eliminates any equilibrium with two active platforms.

In real-life situations, however, most markets for platforms involve more than one active platform. This is because buyers may differ in their preferences for platforms. Consequently, a platform can always focus on attracting buyers that have strong preferences for this specific platform. In order to keep our model tractable, we consider homogenous buyers. We interpret our dominant-platform equilibrium as a reduced form of an equilibrium in which one platform gains all the buyers who do not have strong preferences for a specific platform, while the other platform (that in our model earns zero profit) focuses on serving buyers that have strong preferences for its specific features.

A related assumption is that platforms in our model do not have an initial installed base of existing buyers and sellers that have already joined a platform. We make this assumption because we are interested in examining the net effect of beliefs concerning future participation of agents on the platforms' competitive advantage and choice of a business model. Obviously, if platforms have an initial installed base, the platform with the largest installed base will have a competitive advantage, that may outweighs (or supplement) the competitive advantage coming from beliefs.

While we make this assumption for the sake of simplifying our model and focusing on beliefs, this assumption may still qualitatively hold in several of our examples above. This assumption may hold whenever platforms introduce a completely new product (such as the first iPhone, or Wii). It may also qualitatively hold when platforms introduce a significantly upgraded version of an existing product. The Samsung Galaxy III, for example, can run applications that were initially designed for Samsung Galaxy II and other Android smart-

phones, and therefore have some installed base of potential applications. However, in order to benefit from the new hardware introduced in the Samsung Galaxy III, application developers need to develop new applications. Until such new applications are developed, the Samsung Galaxy III may offer only little new contribution, and cannot justify higher prices. The same argument can apply for a new generation of videogame console, which can operate old videogames, but still requires the development of new videogames in order to exploit their new technologies.

Coordination bias refinement.   We assume that when choosing the platform in the last stage, the two sides of the market play a pure strategy Nash equilibrium. As there are multiple equilibria, we consider a refinement of *coordination bias*, explained in following section.

3   The Concept of Coordination Bias

In the last stage of the game the two platforms charge access fees (F_B^A, F_S^A), (F_B^D, F_S^D), and buyers and sellers simultaneously decide to which platform to join. Since both sides aim to join the same platform, there might be multiple equilibria, resulting from the coordination problem between the sides. We therefore turn to offer a refinement that generates a unique outcome for any (F_B^A, F_S^A), (F_B^D, F_S^D).

In the context of our environment, suppose that there are two putative subgame equilibria in the last stage:

  dominant-A, where all buyers and some sellers join A, and there is no trade on platform D; and

  dominant-D, where all buyers and some sellers join D, and there is no trade on platform A.

When for some access fees both equilibria exist, we apply the concept of coordination bias of α in favor of platform A:

Definition 1 *The market exhibits coordination bias of α in favor of platform A. That is, when both subgame equilibria are possible, all agents believe that the market will coordinate on dominant-A with probability α, and on dominant-D with probability $1 - \alpha$.*

Motivating example. As a motivating example for this concept, consider the battle between BluRay and HD DVD. There was a common agreement that only one of the two formats would survive. Hence, there were two possible equilibria, in one equilibrium everyone adopts BluRay and in the other everyone adopts HD DVD. But it was not clear on which of the two equilibria the market would arrive. Exactly because neither equilibrium was eliminated, either had a positive probability of occurring in the market. We can say that people believed that the market will settle on BluRay with probability α, and on HD DVD with probability $1 - \alpha$.

Discussion of the concept. The concept of coordination bias is the main focus of our paper, as we want to investigate how a platform's advantage or disadvantage in terms of coordination bias affects the platform's business model and competitive advantage. Notice that the concept of coordination bias is a generalized form of the "favorable beliefs" refinement, first introduced by Caillaud and Jullien (2001) and Caillaud and Jullien (2003). In particular, for $\alpha = 1$, platform A has a complete coordination bias in its favor, exactly as in Caillaud and Jullien.[9] Now, however, we can also consider cases where beliefs are not deterministic towards one of the platforms.

In our model we distinguish platform A as the platform with enjoys coordination bias in its favor, that is the platform with α-bias, where $\alpha \geq \frac{1}{2}$. Platform D is the disadvantaged platform, as $1 - \alpha \leq \frac{1}{2}$. The platforms differ vertically by Q. But Q can be positive or negative. Therefore, the only attribute that distinguishes the advantaged platform A from the disadvantaged platform D is the coordination bias, i.e., $\alpha > \frac{1}{2}$. Therefore, if $\alpha \leq \frac{1}{2}$, the analysis is symmetric, with platform D becoming the advantaged platform, and vice versa.

Higher quality (captured by Q) and more favorable position in the market (captured by α) constitute two sources of competitive advantage. We can think of the favorable coordination bias as a better brand name or stronger marketing efforts, for example. A better, more recognizable brand name may, but does not need to, relate with a higher quality. In most of our analysis, we assume that α and Q are independent of each other. In Section 5, we investigate how higher quality may lead to a more favorable coordination bias.

Notice that our concept of coordination bias differs from a mixed strategy equilibrium. For comparison, consider a typical coordination game, where two players, B and S, can either play A or D. They both get a positive payoff if they coordinate on the same decision,

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[9]Caillaud and Jullien (2001, 2003) would call platform A an "incumbent.

and get 0 if they play different strategies. In such a game, there are two pure strategy equilibria, and one mixed strategy equilibrium. In the mixed strategy equilibrium, B plays A with probability β_B, and with the remaining probability, he plays D. Similarly S plays A with probability β_S, and D with the remaining probability. Probabilities β_B and β_S are unique, and such that the other player is indifferent between playing A or D (a necessary condition for a mixed strategy equilibrium). In such a case, the agents coordinate on A with probability $\beta_B\beta_S$, and coordinate on D with probability $(1-\beta_B)(1-\beta_S)$. Under coordination bias, in contrast, with probability α both B and S coordinate on A (i.e., equilibrium A is played), and with probability $(1-\alpha)$ they coordinate on D (i.e., equilibrium D is played). Therefore, this is not equivalent to a mixed strategy equilibrium.

We can think of two explanations for why competing platforms may have a different coordination bias in the market. First, it could be that beliefs adjust along time, such that a platform gains more favorable coordination bias because it managed to win the market in previous periods. For example, a tablet manufacturer that dominated the market in the past, is more likely to enjoy a more favorable coordination bias even when multiple tablet manufacturers lunch a new generation based on a new technology. We model belief adjustment along time in Section 5. Second, it is possible to think of a preliminary stage to our game in which platforms invest in advertising that shifts coordination bias in favor of the platform that invested the most. We leave this alternative explanation for future research.

4   Equilibrium

The objective of each platform is to choose the most profitable business model, given the behavior of the other platform, and the agents in the market. To establish which business model is more profitable, we need to analyze subsequent actions of buyers and sellers. Hence, to solve for the equilibrium outcome, we use standard backward induction. We start by solving for the agents' optimal choice of platforms given (F_B^A, F_S^A), (F_B^D, F_S^D). Then, we solve for the equilibrium access fees and business models that the platforms choose, knowing how (F_B^A, F_S^A), (F_B^D, F_S^D) affects the agents' choices.

4.1 Decisions of Buyers and Sellers

In the last stage, buyers and sellers observe the posted fees, (F_B^A, F_S^A), (F_B^D, F_S^D), and simultaneously decide which platform to join. If a buyer or a seller decides not to join any platform, he gets the total payoff of 0.

The decision of buyers depends on the access fees and on the number of sellers they expect to find in each platform. The payoff of every buyer is the same: joining platform A with n sellers yields buyers payoff $U_B(n) - F_B^A + Q$, and joining platform D with n sellers yields $U_B(n) - F_B^D$. Hence, they all make the same decision:[10] either they all join platform A, they all join platform D.[11]

This is not so for the sellers. Even though sellers are identical, due to competitive forces, it may be optimal for them to make different decisions. The payoff of a seller decreases with the number of other sellers: joining platform i $(i = A, D)$ with all N_B buyers and n sellers yields each seller a payoff of $N_B(p(n) - k) - F_S^i$. Sellers join a platform only until the payoff is 0. Each additional seller would earn negative payoff. Hence, if $n > 0$ join a platform, this number is uniquely characterized by F_S^i: $N_B(p(n) - k) - F_S^i = 0$.

There exists dominant-D equilibrium when following conditions are satisfied:

$$N_B(p(n^D) - k) - F_S^D = 0,\tag{2}$$

$$U_B(n^D) - F_B^D \geq \min\{0, -F_B^A + Q\}.\tag{3}$$

But for some fees conditions for both dominant-D and dominant-A are satisfied. Dominant-A exists when

$$N_B(p(n^A) - k) - F_S^A = 0,\tag{4}$$

$$\text{and}$$

$$U_B(n^A) - F_B^A \geq \min\{Q, -F_B^D\}.\tag{5}$$

When all four conditions (2)–(5) are satisfied, then both equilibria are possible. In such

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[10]The only exception may be when the buyers are indifferent. We consider this case in solution, but platforms always want to avoid the case when the buyers are indifferent. Hence we abstract from it for clarity of exposition.

[11]There is also a third potential equilibrium, in which both sides do not join either platform. We assume that if there is such an equilibrium in addition to the two equilibria above, then agents will not play this equilibrium. Intuitively, we focus on a market in which agents believe that the market eventually will succeed in attracting the two sides, so the only question is which platform is going to be successful.

a case, by coordination bias dominant-A is played with probability $\alpha > \frac{1}{2}$, and dominant-D is played with probability $1 - \alpha$.

4.2 Choice of Business Models and Access Fees

Consider now the stage where the two platforms choose their access fees, (F_B^A, F_S^A), (F_B^D, F_S^D), taking into account that agents are aware of coordination bias. To derive the equilibrium, we first focus on the platform A's best response to (F_B^D, F_S^D). Setting (F_B^A, F_S^A) such that dominant-A is an equilibrium is a necessary condition for platform A to win the market, but it is not sufficient. If dominant-D is also a subgame equilibrium, dominant-A is played only with probabilit α. Instead, at the cost of marginal loss of profit, platform A may assure that under coordination bias dominant-D is never played. Hence, platform A always prefers to "eliminate" dominant-D in this way.

Consider first the situation when both dominant-A and dominant-D are possible. Given coordination bias, a buyer believes that with probability α the market will coordinate on A, i.e., all other agents will play dominant-A. And with probability $1 - \alpha$ the market will coordinate on D, i.e., they play dominant-D. Let $U_B^D(F_B^D, F_S^D, \alpha)$ denote the expected payoff of a buyer from joining platform D when both equilibria are possible, and dominant-D is played with probability $1 - \alpha$. Notice that by our analysis in Section 4.1 this expected payoff depends on the access fees charged by platform D and on α, but it does not depend on the access fees charged by platform A.

In order to eliminate dominant-D, platform A needs to charge such access fees that the buyers strictly prefer to join platform A even if there is probability $1 - \alpha$ that all other agents play dominant-D. Given $U_B^D(F_B^D, F_S^D, \alpha)$, platform A has two options—SRB and BRB business models—which we analyze in turn.

Sellers-Revenue-Based (SRB) business model. The first option for platform A is to set $F_S^A > 0$. If the platform charges positive access fee to the sellers, they find it worthwhile to join the platform only if the buyers are joining as well. Then, in a dominant-A equilibrium, n^A sellers join platform A, where n^A is determined by

$$N_B\, p(n^A) - K - F_S^A = 0\,. \tag{6}$$

Given coordination bias, a buyer believes that those sellers, and all other buyers join platform A with probability α. Consequently, the buyer's expected utility from joining platform A is $Q + \alpha U_B(n^A) + (1 - \alpha)U_B(0) - F_B^A$. As all buyers are the same, platform A can attract all buyers by setting

$$Q + \alpha\, U_B(n^A) - F_B^A > U_B^D(F_B^D, F_S^D, \alpha)\,. \tag{7}$$

Condition (7) eliminates dominant-D as a subgame equilibrium, because it ensures that a buyer strictly prefers to join platform A even if he believes that dominant-D will be played with probability $1 - \alpha$. When condition (7) is satisfied, all buyers play dominant-A with probability 1. Sellers know that, and they will also play dominant-A. Notice that if (7) holds with equality, then coordination bias does not eliminate the possibility that agents will play dominant-D with some probability. Hence, platform A wants to assure that the condition holds with inequality. At the same time, larger inequality in (7) induces lower F_B^A. In the interest of its profit, the platform wants to keep F_B^A as high as possible. Hence, the platform sets F_B^A such that condition (7) holds with *slight* inequality.

With F_S^A and F_B^A identified by (6) and (7), platform A's profit under SRB business model, $\Pi_{SRB}^A = n^A F_S^A + N_B F_B^A$, can be expressed as a function of n^A:

$$\Pi_{SRB}^A(n^A) = N_B \left(\pi_{SRB}^A(n^A) + Q - U_B^D(F_B^D, F_S^D, \alpha) \right) , \tag{8}$$

where

$$\pi_{SRB}^A(n^A) = n^A \left(p(n^A) - k \right) + \alpha U_B(n^A)\,. \tag{9}$$

Let n^{A*} denote the number of sellers that maximizes (9) (and therefore (8)). While we assume that platforms compete by setting access prices, it is more convenient to solve directly for the optimal number of sellers that platform A wishes to attract, n^A.

Equation (9) reveals that when choosing n^A to maximizing its profit, platform A internalizes all the sellers' gains from trade, but only a fraction α of the buyers' gains from trade. In this sense, platform A is oriented towards capturing the revenues from the sellers' side, and as we show below, use F_B^A as the only tool for competing with platform D. Also, because platform A internalizes only fraction α of the benefits that sellers provide to buyers, SRB involves attracting fewer sellers than the first-best (trade-maximizing level).

Buyers'-Revenue-Based (BRB) business model. Next, we turn to platform A's optimal best response given that it chooses any $F_S^A \leq 0$. Notice first that it is never optimal to set $F_S^A < -K$, because sellers will join platform A even when the platform is saturated: even if more than $\hat{n}$ already joined and $p(n) = 0$, just for benefiting from the subsidy that exceeds their development costs. Moreover, notice that it is never optimal for platform A to set $0 \geq F_S^A > -K$. This is because for any $F_S^A > -K$, sellers do not cover their entry costs unless buyers join platform A, which forces platform A to compete in attracting buyers by setting a low F_B^A. Given that it does so, platform A might as well charge a high F_S^A to capture the sellers' profit, by using a SBR. We therefore focus on the case where platform A sets $F_S^A = -K$.

In a dominant-A equilibrium where all buyers join platform A, setting $F_S^A = -K$ attracts $\hat{n}$ sellers join platform A, because for any $n^A < \hat{n}$, $p(n^A) > 0$ and therefore more sellers would like to join.[12] If both dominant-A and dominant-D are possible, then by the α-bias in favor of A, sellers expect all buyers to join platform A with probability α. And since sellers are fully compensated for their fixed costs, $F_S^A = -K$ ensures that $n^A = \hat{n}$ sellers will still find it optimal to join platform A. Intuitively, as sellers are fully compensated for their entry costs, they bear no risk of losing money from joining into an "empty" platform. As there is some probability that the platform will not be empty, sellers will prefer to join it.

Let us turn now to the buyers side under BRB business model. A buyer knows that when both dominant-A and dominant-D are possible, $\hat{n}$ sellers join platform A. Therefore, platform A attracts the buyer as long as

$$Q + U_B(\hat{n}) - F_B^A > U_B^D(F_B^D, F_S^D, \alpha) . \tag{10}$$

Notice that here α does not appear on the left-hand-side, because the buyer knows that by subsidizing sellers, platform A guarantees sellers participation. Given (10) (9), all buyers join platform A and therefore dominant-D is eliminated. As before, in the interest of its profits, platform A sets F_S^A so that (10) holds with slight inequality. Platform A's profit is $\Pi_{BRB}^A = \hat{n} F_S^A + N_B F_B^A$, or

$$\Pi_{BRB}^A = N_B \left(\pi_{BRB}^A + Q - U_B^D(F_B^D, F_S^D, \alpha) \right) , \tag{11}$$

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[12]If more sellers would like to join platform A, the additional sellers would only enter into a "saturated" platform that includes a price $p = 0$, and would not make positive sales. By our tie-braking assumption, these sellers will not enter.

where

$$\pi_{BRB}^A = U_B(\hat{n}) - \hat{n}\,k\,. \tag{12}$$

Equation (12) reveals that now platform A fully internalizes the buyers' gains from trade, but does not provide any gains from trade to the sellers' side (because $p(\hat{n}) = 0$). In this sense, platform A is oriented towards creating maximal value to the buyers side,[13] and capturing all of it. From now onwards we refer to this business model as *Buyers-Revenue-Based* (SRB).

Optimal business model for platform A. By comparing (8) and (11)—maximal profit under each business model—we find that platform A prefers to adopt the SRB business model if $\pi_{SRB}^A(n^{A*}) > \pi_{BRB}^A$, and prefers to adopt the BRB business model if $\pi_{SRB}^A(n^{A*}) < \pi_{BRB}^A$. Notice that both n^{A*} and $\pi_{SRB}^A(n^{A*})$ depend on α, while π_{BRB}^A does not.

Optimal business model for platform D. Platform D's best response is similar to the discussion above, with the exceptions that platform D's coordination bias is $1 - \alpha$ instead of α, and that platform D's quality is 0 instead of Q. Platform D therefore adopts a SRB if $\pi_{SRB}^D(n^{D*}) > \pi_{BRB}^D$ and will adopt the BRB business model if $\pi_{SRB}^D(n^{D*}) < \pi_{BRB}^D$, where:

$$\pi_{SRB}^D(n^{D*}) = (1 - \alpha)U_B(n^{D*}) + n^{D*}\big(p(n^{D*}) - k\big)\,, \tag{13}$$

$$\pi_{BRB}^D = U_B(\hat{n}) - \hat{n}\,k\,, \tag{14}$$

and n^{D*} maximizes (13). It is straightforward to show that if platform D uses a SRB, the optimal number of sellers that platform D attracts, n^{D*}, is lower than n^{A*}, and is decreasing with α. Intuitively, whenever platform D chooses SRB, it has a lower ability to internalize the buyers' utility than platform A, because $\alpha < 1$. Clearly, $\pi_{SRB}^D(n^{D*})$ depends on α. Notice also that if platform D adopts a BRB, than it attracts the same number of sellers, $\hat{n}$, as platform A's BRB business model. From now onwards we can define $\pi_{BRB} \equiv \pi_{BRB}^D = \pi_{BRB}^A$.

Lemmas 1 and 2 summarize the properties of each business model.

Lemma 1 (features of SRB business model) *If platform $i = A, D$ chooses a Sellers-Revenue-Based (SRB) business model, then it charges a positive access fees from the sellers, $F_S^i > 0$, and uses the buyers' access fees as the exclusive tool for competing with the other*

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[13]Within the bounds of controlling the sellers side.

platform. Platform i attracts fewer sellers than the trade-maximizing level, $n^{i*}(\alpha) < n^*$ for all $\alpha < 1$. Moreover,

(i) for platform A, $n^{A*}(\alpha)$ is increasing with α and $n^{A*} = n^*$ for $\alpha = 1$;

(ii) for platform D, $n^{D*}(\alpha) < n^{A*}(\alpha)$ for any α, and $n^{D*}(\alpha)$ is decreasing with α.

Proof. See Appendix.

Lemma 2 (features of BRB business model) *If platform $i = A, D$ chooses a Buyers-Revenue-Base (BRB) business model, then it charges a negative access fees from the sellers, $F_S^i = -K < 0$, and a positive access fees form the buyers, $F_B^i > 0$. The platform attracts $\hat{n}$ sellers, which is more than the trade-maximizing level, $\hat{n} > n^*$ for all α.*

Proof. See Appendix.

The lemmas show that under SRB, a platform attracts fewer sellers than the trade-maximizing level, while under BRB, a platform attracts more sellers than the trade-maximizing level. This result differs from Hagiu (2006) that shows that an incumbent platform which benefits from a full coordination bias advantage (corresponding to platform A in our model) does not distort its level of trade while the competing entrant (corresponding to platform D) distorts the level of trade downwards. These results also differ from Halaburda and Yehezkel (2011) which shows that both platforms distort the level of trade downward regardless of whether they attract the buyer of the seller. For business strategy, Lemmas 1 and 2 show that a platform's business model also indicates whether a platform should "oversell" or "undersell" applications.[14]

Business models in equilibrium. Having analyzed the best responses of each platform, in terms of the access fees charged, and the business model adopted, we now turn to

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[14]For example, Claussen, Kretschmer and Mayrhofer (2011) illustrate this on the example of applications on Facebook: "Facebook encouraged entry of as many developers as possible. The company offered strategic subsidies to third-party developers by providing open and well-documented application programming interfaces, multiple development languages, free test facilities, as well as support for developers through developer forums and conferences" (p.5). In result more than 30,000 applications had been developed within a year of lunching the service. However, not all applications have been adopted by users, and of those that have been installed, not all had been actively used. This suggests that the platform was oversupplying applications.

the business models chosen in equilibrium. Figure 1 shows the business models that platforms A and D choose in the equilibrium, given α and k. The figure reveals that there are three regions: (Ω_{BB}) both platforms adopt a BRB, (Ω_{SS}) both platforms adopt SRB, (Ω_{SB}) platform A adopts SRB and platform D adopts BRB.
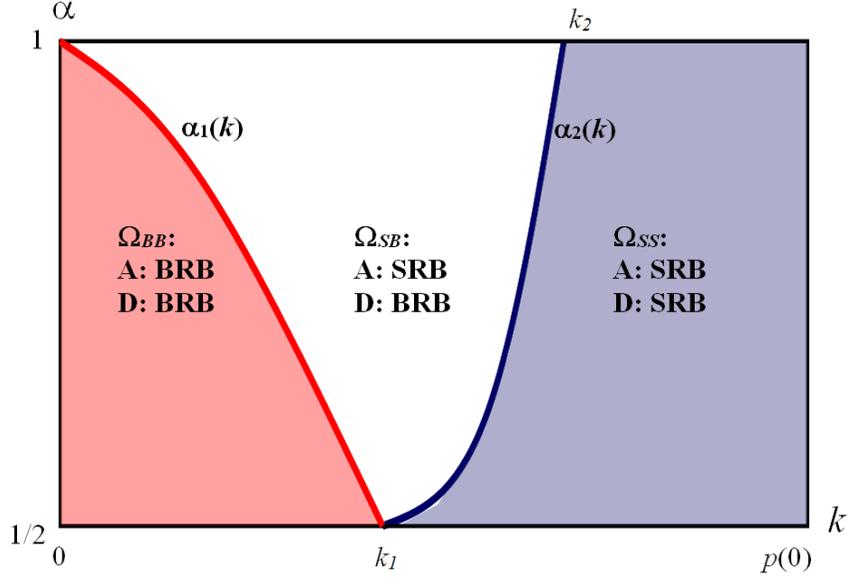
Figure 1: The three subsets, Ω_{BB}, Ω_{SB} and Ω_{SS}

The optimal business model depends on the size of the coordination bias. For a platform i $(i{=}A,D)$ it is optimal to adopt SRB business model when $\pi^i_{SRB}(n^{i^*}) > \pi_{BRB}$, and otherwise it is optimal to adopt the BRB business model. In each case, the direction of inequality depends on α and k. As k increases, π_{BRB} decreases. That is, subsidizing sellers becomes increasingly expensive, and brings lower profits. Therefore it is more likely that a platform adopts SRB business model, where it collects most of the revenue from the sellers while lowering the buyers' access fee to make the platform more attractive.

The cost of subsidizing the sellers changes along k in the same way for both platforms. Therefore, both platforms are likely to switch from BRB to SRB as k increases. The cost of subsidizing the buyers, however, changes along α differently for the two platforms. Hence, the platforms have different response to increasing α. As α increases, attracting the buyers (in order to collect the revenue from the sellers) is increasingly cheaper for platform A, and increasingly more expensive for platform D. Hence, increasing α makes SRB business model more appealing to platform A, while it makes BRB business model more appealing

to platform D.

The two platforms may choose the same business model. If they choose different business models, it is possible that platform A adopts the SRB business model, while platform D adopts BRB, but never the other way around. This is because it is always cheaper for platform A than for platform D to attract the buyers and collect the revenue on the sellers' side.

These results are summarized in the following proposition.

Proposition 1 (equilibrium choice of business model) *For any* $\alpha \in \left[\frac{1}{2}, 1\right]$*, and any* $k \in [0, u_B'(0)]$*, a pair* (α, k) *belongs to exactly one of three regions:*

$\Omega_{SS} = \left\{ (\alpha, k) | \pi_{SRB}^A(n^{A*}) > \pi_{SRB}^D(n^{D*}) > \pi_{BRB} \right\}$*, where both platforms adopt SRB business model;*

$\Omega_{BB} = \left\{ (\alpha, k) | \pi_{BRB} > \pi_{SRB}^A(n^{A*}) > \pi_{SRB}^D(n^{D*}) \right\}$*, where both platforms adopt BRB business model; or*

$\Omega_{SB} = \left\{ (\alpha, k) | \pi_{SRB}^A(n^{A*}) > \pi_{BRB} > \pi_{SRB}^D(n^{D*}) \right\}$*, where platform A adopts SRB and platform D adopts BRB business model.*

The thresholds between those regions are characterized by $\alpha_1(k)$ *and* $\alpha_2(k)$*, where*

$\alpha_1(k)$ *is uniquely defined by* $\pi_{SRB}^A(n^{A*}) = \pi_{BRB}$*; moreover,* $\alpha_1(0) = 1$*,* $\alpha_1' < 0$ *and* $\alpha_1'' > 0$*; also, there is a point* k_1 *such that* $\alpha_1(k_1) = \frac{1}{2}$ *, where* $0 < k_1 < u_B'(0)$*.*

$\alpha_2(k)$ *is uniquely defined by* $\pi_{SRB}^D(n^{D*}) = \pi_{BRB}$*; moreover,* $\alpha_2(k_1) = \frac{1}{2}$*,* $\alpha_2' > 0$ *and* $\alpha_2'' > 0$*; also, there is a point,* k_2*, such that* $\alpha_2(k_2) = 1$*, where* $k_1 < k_2 < u_B'(0)$*.*

Proof. See Appendix.

In Figure 1, we can clearly see that increase in α motivates platform A to switch from BRB business model to SRB. Platform D may be motivated to switch in the opposite direction when α increases. As k increases, both platforms are motivated to switch from BRB to SRB business model.

When $\alpha = \frac{1}{2}$, the platforms are symmetric in the market beliefs, and none has an advantage over the other in attracting the buyers. Hence, both platforms choose the same business model. For k lower than k_1, it is cheaper to subsidize the sellers, and adopt BRB business

model. For k higher than k_1, substituting the sellers becomes more expensive, and both platforms prefer to adopt SRB business model. As α increases and the platforms become more asymmetric in the market beliefs they are facing, they are more likely to adopt different business models.

For higher α, it becomes cheaper for platform A to attract the buyers. This is because the buyers believe that there is better chance that sellers will also join platform A, even though platform A charges sellers a positive access fee. When α increases above α_1 (for $k < k_1$), the effect is strong enough that platform A finds it optimal to switch its business model from BRB to SRB.

For platform D it is the exact opposite: As α increases, it becomes more expensive for platform D to attract the buyers. Hence, if for $k < k_1$ platform D subsidized the sellers in BRB business model for $\alpha = \frac{1}{2}$, it continues to do so for any larger α. However, for $k \in (k_1, k_2)$ when α increases above α_2, attracting buyers becomes more expensive for platform D then subsidizing sellers, and the platform finds it optimal to switch its business model from SRB to BRB. However, for $k > k_2$ subsidizing sellers is too expensive for both platforms. Even the disadvantaged platform, D, for such high k prefers to bear the cost of attracting the buyers and sticks to SRB for any α. The corollary below summarizes this discussion.

Corollary 1 *Small change in α may lead a platform to choose a different business model:*

  (i) *For $k < k_1$, an increase in α motivates platform A to switch from BRB to SRB business model, and switch from oversupplying to undersupplying sellers.*

  (ii) *For $k_1 < k < k_2$, an increase in α motivates platform D to switch from SRB to BRB business model, and switch from undersupplying to oversupplying sellers.*

 (iii) *For $k > k_2$, an increase in α has no effect on platform's optimal business models.*

The results above show that the relative position in the market—as measured by the strength of the coordination bias—has a significant effect on the choice of a business model. The respective choices of the business models in turn affect the platforms' pricing decisions, and whether the oversupply or undersupply sellers, in comparison with the trade-maximizing number of sellers. We illustrate parts (i) and (ii) of Corollary 1 in Figures 2 and 3.

Figure 2 shows the equilibrium number of sellers that platforms attract, as a function of α, for $k < k_1$. The figure reveals that while platform D always oversupply sellers, platform A
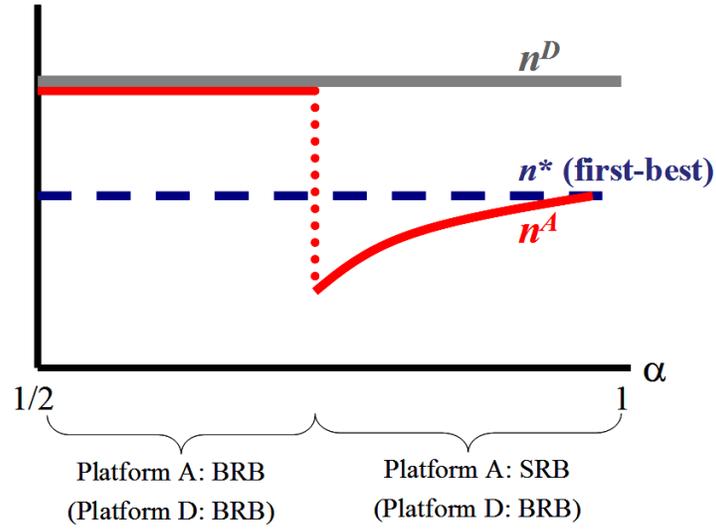
Figure 2: The equilibrium number of sellers as a function of α, for $k < k_1$
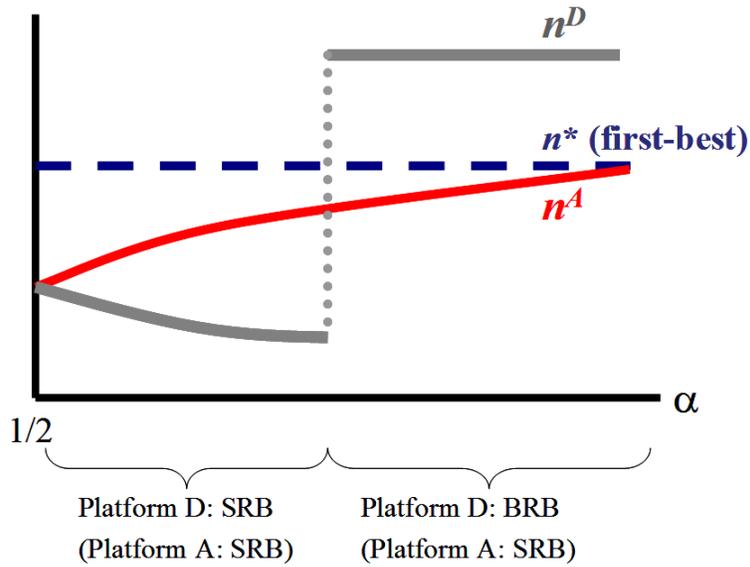

Figure 3: The equilibrium number of sellers as a function of α, for $k_1 < k < k_2$

first oversupply, and then undersupply when an increase in α motivate the platform to switch from BRB to SRB business models. Figure 3 shows the equilibrium number of sellers that platforms attract, as a function of α, for $k_1 < k < k_2$. Now, platform A always undersupply, but platform D first undersupply, and then oversupply when an increase in α motivate the platforms to switch from SRB to BRB.

Winning platform. We now turn to showing which platform wins the market. Again, it is important to emphasize that in real-life situation, more than one platform can gain positive market share because of horizontal product differentiation that we didn't incorporate into our model. We therefore interpret the question of who wins the market as who wins the indifferent consumers: the consumers who do not have a strong preference towards one of the platforms. In real-life situation, this "wining" platform is the one to gain the higher, though not exclusive, market share.

In competing with one another, each platform reduces its access fees to the buyers in order to attract them. Eventually, one platform cannot reduce its access fees any longer without making negative profits, while the competing platform still makes positive profit and therefore can win the market.

From the discussion above, we know that platform D wins the market only if its quality is sufficiently higher than the quality of platform A. To identify the winning platform, in Lemma 3 we define the cutoff level Q^C, such that in equilibrium, platform A wins the market for $Q > Q^C$, and platform D wins the market otherwise. The threshold indicates extend to which higher quality relates to winning the market.

Lemma 3 (winning platform) *Let*

$$
Q^C = \begin{cases}
\pi_{SRB}^D(n^{D*}) - \pi_{SRB}^A(n^{A*}); & (\alpha, k) \in \Omega_{SS}; \\
\pi_{BRB} - \pi_{SRB}^A(n^{A*}); & (\alpha, k) \in \Omega_{SB}; \\
0; & (\alpha, k) \in \Omega_{BB}.
\end{cases}
$$

Then, platform A wins the market if and only if $Q > Q^C$, and earns $\Pi^A = (Q - Q^C)N_B$. Platform D wins the market if and only if $Q < Q^C$, and earns $\Pi^D = (Q^C - Q)N_B$.

Notice that the threshold Q^C depends on α and k. Moreover, $Q^C \leq 0$, which means that

platform A can win the market even when it is of lower quality than platform D.[15] This is because the coordination bias favors platform A. The following proposition describes how Q^C depends on α and k.

Proposition 2 (comparative statics on Q^C)

(i) ***The effect of the belief advantage,*** α***:*** *For all regions, if $\alpha = \frac{1}{2}$ then $Q^C = 0$. For regions Ω_{SS} and Ω_{SB}, $Q^C < 0$; Q^C and Π^D are decreasing with α and Π^A is increasing with α. For Ω_{BB}, $Q^C = 0$ and Π^A and Π^D are independent of α.*

(ii) ***The effect of the seller's fixed costs,*** k***:*** *For regions Ω_{SS} (Ω_{SB}), Q^C and Π^D are increasing (decreasing) with k, while Π^A is decreasing (increasing) with k. For region Ω_{BB}, $Q^C = 0$ and Π^A and Π^D are independent of k.*

Proof. See Appendix.

The first part of Proposition 2 shows that when there is no coordination bias (i.e., $\alpha = \frac{1}{2}$), then the platform with higher quality wins the market, regardless of the business model that each platform adopted. Intuitively, without coordination bias, the platforms are symmetric except for their qualities. Hence the quality is the only source of competitive advantage, and it determines the identity of the winning platform.

For region Ω_{BB}, quality alone determines the identity of the winning platform for all $\alpha > \frac{1}{2}$. Both platforms adopt BRB business model and subsidize sellers to collect highest possible revenue from the buyers. Given the subsidy to the sellers, coordination bias does not play a role, as each platform assures $\hat{n}$ sellers. Consequently, the platform with the highest quality wins the market. In this region the profits are also determined solely by Q.

In the other two regions, i.e., when at least one platform adopts SRB business model, the coordination bias plays a role in determining which platform wins, and how large are the profits of the winning platform. Larger coordination bias gives larger competitive advantage to platform A. Therefore, this platform can win the market even if it offers lower quality ($Q^C < 0$). This is because with larger α, it can command higher access fee (or lower subsidy) from the buyers. However, if it offers lower quality, it limits the access fee it can command from the buyers. Therefore, platform D can profitably win the market if its quality advantage

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[15]Recall that we treat α and Q as independent of each other. In Section 5 we investigate how higher quality may lead to larger belief advantage.

is sufficiently large. The higher is α, the larger quality difference platform D needs to win the market, in order to compensate for coordination bias in favor of platform A (Q^C is decreasing in α). We also find that the negative effect of α on Q^C is stronger when both platforms adopt SRB business models, than when only platform A does so. Intuitively, in Ω_{SS}, both platforms need to offer a discount on buyers to collect the revenue from the sellers; and an increase in α increases platform A's ability to attract the buyers without too large of a discount, while the same increase forces platform D to offer a larger discount to attract the buyers. Thus, a larger quality advantage is needed to overcome both effects. In Ω_{SB}, α only has effect on platform A's discount, since platform D subsidizes sellers, and that subsidy does not depend on α.

Next consider the effect of k. As $k = K/N_B$, k is increasing with the sellers' fixed costs and decreasing with the number of buyers. An increase in k makes it more costly for both platforms to attract sellers into the platform, either because the fixed entry costs increased, or because there are fewer potential buyers to buy from each seller. Now, as Lemma 1 showed, in Ω_{SS}, platform A's business model involves attracting more sellers than platform D ($n^{A*} > n^{D*}$). Consequently, as k increases, platform A's ability to win the market decreases, i.e., Q^C decreases. Moreover, platform A's profit in case it does win the market decreases, while platform D's profit increases. In contrast, in Ω_{SB}, platform A's business model involves attracting fewer sellers than platform D, because $n^{A*} > \hat{n}$. Therefore, as k increases, platform A's ability to win the market increases, i.e., Q^C decreases; and platform A's profit in case it does win increases, while platform D's profit decreases. Finally, in Ω_{BB} both platforms attract the same amount of sellers, $\hat{n}$, and therefore an increase in k does not change their comparative competitive advantage; hence in this case Q^C, Π^A and Π^D are independent of k.

4.3 Example

Suppose for simplicity that $N_B = 1$, and that the buyer's utility is

$$u_B(n) = \Lambda\, n - \frac{n^2}{2}\,,$$

where Λ is a demand parameter, with $K = k < \Lambda$. Solving the above model given this functional form, we obtain that $n^* = \Lambda - k$, $n^{D*} = (\Lambda - k)/(1 + \alpha)$, $n^{A*} = (\Lambda - k)/(2 - \alpha)$ and $\hat{n} = \Lambda$. Notice that n^*, n^{D*}, n^{A*} and $\hat{n}$ indeed satisfy all of the findings of Propostition 1.

In this case, $\alpha_1(k)$ and $\alpha_2(k)$ are:

$$\alpha_1(k) = \begin{cases} \frac{k^2}{\Lambda(\Lambda - 2k)}; & \text{if} \quad k < k_1 = \frac{1}{2}(\sqrt{3} - 1)\Lambda; \\ \frac{1}{2}; & \text{if} \quad k > k_1; \end{cases}$$

$$\alpha_2(k) = \begin{cases} \frac{1}{2}; & \text{if} \quad k < k_1 = \frac{1}{2}(\sqrt{3} - 1)\Lambda; \\ \frac{\Lambda^2 - 2\Lambda k - k^2}{\Lambda(\Lambda - 2k)}; & \text{if} \quad k_1 < k < k_2 = (\sqrt{2} - 1)\Lambda; \\ 1 & \text{if} \quad k_2 < k. \end{cases}$$

Moreover, drawing $\alpha_1(k)$ and $\alpha_2(k)$ obtains a figure with the same characteristics as Figure 1.

5 Extension: Belief adjustment along time

One potential explanation for coordination bias is that the advantaged platform enjoys the advantage because it was successful in attracting buyers and sellers in previous rounds. Given the history of successes, each agent expects that the two sides are more likely to continue joining the platform.

In this extension we consider the case where coordination bias can endogenously adjust along time in that it is positively affected by the platform's winning history. We ask how α-bias can converge over time to the full coordination bias (i.e., $\alpha = 1$), and how the rate of convergence depends on the business model that each platform chooses. We find that convergence to the full coordination bias is faster, on average, when both platforms adopt a SRB, than in the case where one or both platforms adopt a BRB.

To this end, suppose that the two sides and the two platforms play repetitively the static model described above, where coordination bias adjusts along time. In particular, suppose that in time t, the coordination bias is

$$\alpha_t = \begin{cases} \min\{\alpha_{t-1} + \varepsilon, 1\}, & \text{if } A \text{ won in } t - 1, \\ \max\{\alpha_{t-1} - \varepsilon, 0\}, & \text{if } D \text{ won in } t - 1. \end{cases}$$

That is, if a platform won in the previous period, the coordination bias is increasing in its favor by some $\varepsilon > 0$. A notable limitation of our analysis is that we assume that platforms are static players: platforms do not internalize the long-term effect of increasing their belief

advantage while setting their current-period strategies. This assumption enables us to obtain close-form solutions and to provide general predictions regarding the adjustment process of beliefs. We discuss the robustness of our results to this limitation at the end of this section.

Suppose that in each period, platform A's quality advantage, Q, is drawn independently from the uniform distribution with support $[-q, q]$. Let Q_t^C denote the cutoff Q^C (as defined in Lemma 3), evaluated at α_t, such that platform A wins in period t if $Q > Q_t^C$. This means that each platform has a positive probability of winning the market in period t if $-q < Q_t^C < q$.

Suppose that the market starts at the symmetric case of $\alpha_0 = \frac{1}{2}$, such that $Q_0^C = 0$. As the two platforms are initially symmetric, we can define platform A as the platform that enjoys coordination bias in its favor at time t: $\alpha_t > \frac{1}{2}$. The identity of platform A can therefore change along time. For simplicity, we focus on the case where $\alpha t > \frac{1}{2}$ (the opposite case is symmetric).

Suppose that evaluated at $\alpha_t = 1$, $Q_t^C < -q$. This assumption implies that there is a cutoff of α, α^C, such that once α_t crosses this cutoff, platform A is going to win all coming periods with probability 1, and coordination bias converges to a steady state of a full coordination bias, $\alpha_t = 1$. Intuitively, this assumption implies that the stochastic variations of Q are sufficiently small such that even with the worst realization of $Q = -q$, platform D cannot overcome platform A's full coordination bias.

By the law of large numbers, coordination bias is going to adjust to a steady state as long as the number of periods is large enough. We therefore ask how the rate of adjusting to $\alpha_t = 1$ depends on the equilibrium business models. Let $p_t = \Pr(Q > Q_t^C)$ denote the probability that platform A wins in period t. As p_t increases, coordination bias adjusts faster, because platform A has a higher probability to win and consequently increase the coordination bias in its favor.

The following corollary follows directly from Proposition 2.

Corollary 2

(i) If $k < k_1$, such that at $\alpha_0 = \frac{1}{2}$ both platforms adopt BRB (region Ω_{BB}), then $p_t = \frac{1}{2}$ for all $\alpha_t > \frac{1}{2}$ within region Ω_{BB}.

(ii) If $k > k_1$ such that at $\alpha_0 = \frac{1}{2}$ both platforms adopt SRB (region Ω_{SS}), then for all $\alpha_t > \frac{1}{2}$, $p_t > \frac{1}{2}$, and p_t is increasing with α_t. Moreover, if q is sufficiently low, there

is a cutoff, α^C, such that coordination bias adjusts to the study state if $\alpha_t > \alpha^C$. The same results apply to $k < k_1$, if α_t is high enough such that $(\alpha_t, k) \in \Omega_{SB}$.

Corollary 2 indicates that the process of adjusting coordination bias along time is faster, on average, for $k > k_1$, then for $k < k_1$. For $k < k_1$, even if platform A benefits from a certain degree of coordination bias such that $\alpha_t > \frac{1}{2}$, this advantage is fragile because in period t each platform has equal probability of winning the market and coordination bias can adjust upward or downward with equal probabilities. This implies that at $k < k_1$, when coordination bias starts at $\alpha_0 = \frac{1}{2}$, they are likely to fluctuate around $\alpha_t = \frac{1}{2}$ with a different platform winning each period, until finally one of the platforms obtains a sufficiently large coordination bias to shift α_t to region Ω_{SB}. For $k > k_1$, or for $k < k_1$ but with a high enough coordination bias such that $(\alpha_t, k) \in \Omega_{SB}$, platform A has a higher probability of winning than platform D, which implies a higher probability of coordination bias adjustment in favor of platform A. Moreover, the probability that platform A wins is increasing with α_t, and eventually there is a cutoff, α^C, such that once $\alpha_t > \alpha^C$, platform A is going to win all periods and coordination bias will converge to a study state of a full coordination bias, with platform A being the dominant platform for all realizations of Q.

These results indicate that the adjustment process of coordination bias is faster, on average, for high values of k, when both platforms initially adopt a SRB, than for low values of k, when both platforms adopt a BRB. In the former case, however, coordination bias can start to adjust faster whenever the accumulated bias motivates platform A to switch to a SRB.

We obtain the results of this section with the simplifying assumption that platforms do not take into account the effect of winning the market has on future coordination bias. This assumption is suitable in two cases. First, when the process of adjusting coordination bias is very slow: ε is sufficiently small. In this case, winning in period t has only a marginal effect on coordination bias in period $t + 1$, implying that platforms will not be willing to sacrifice profits in period t, just so they can affect future coordination bias. Second, when the platforms' discount factor is sufficiently high, such that platforms do not place a high weight on future profits. If these two conditions do not hold, platforms will have an inventive to compete more aggressively and sacrifice current profits for obtaining coordination bias in the future. This in turn will result in lower profits in the early periods than the profits predicted by our model, and a faster convergence to a steady state. However, notice that our result that $p_t = \frac{1}{2}$ whenever both platforms adopt BRB, while $p_t > \frac{1}{2}$ and increasing

with α_t whenever both platforms adopt SRB, is independent of dynamic considerations by platforms.

6   Conclusion

Our paper considers platform competition in a two sided market that includes buyers and sellers. One of the platforms, the advantaged platform, benefits from coordination bias, in that agents, buyers and sellers, believe that it is more likely that other agents will join this platform. The two platforms may also differ in their qualities. We study how platforms compete in the face of coordination bias. This research question is important for an advantaged platform, for deciding how to translate its advantage in agents' coordination into a competitive advantage, and for the disadvantage platform, for deciding how to overcome the unfavorable coordination bias.

We establish the following main results. First, we find that each platform will choose between two distinctive business models. It is important to emphasize that we do not restrict the platforms to choose one of these two business models. Instead, we find that this is what platforms will choose as an equilibrium outcome. The first business model is a sellers' revenue based (SRB), in which the platform competes aggressively in attracting buyers and use the sellers as its main source of revenues. A platform that chooses this business model will have an incentive to increase the sellers' gross revenues and will therefore attract fewer sellers than is socially optimal. This way, the platform restricts competition between sellers and increases their gross profits, which it can then collect through its access price. The second business model is a buyers' revenue based (BRB), in which the platform subsidizes the sellers and collects revenues from buyers. Under this business model, the platform has an incentive to increase the buyers' gross utility and will therefore attract more sellers than is socially optimal.

We then turn to show which business model each platform will choose. We find that the equilibrium choice of a business model depends on the strength of the coordination bias towards the advantaged platform and on the sellers' development costs. If development costs are small and no platform benefits from a substantial coordination bias, then both platforms adopt a buyers' revenue based. Intuitively, both platforms find it more profitable to attract sellers by subsidizing the sellers' small development costs, which does not rely on the coordination bias. If the coordination bias increases, the advantaged platform take

advantage of the favorable bias by switching from the buyers' revenue based to the sellers' revenue based business model. Intuitively, increase in the favorable coordination bias makes it easier for the advantaged platform to attract buyers, as buyers expect sellers to join the advantaged platform even when the platform does not subsidize the sellers. This implies that the advantaged platform undersupply sellers for small coordination bias, and oversupply for large coordination bias. If the sellers' development costs are high and the coordination bias is small, then both platforms adopt the sellers' revenue based business model. If the coordination bias increases, it is the disadvantaged platform that will switch to the buyers' revenue based business model, in which the platform attracts sellers without relying on beliefs. In this case, the disadvantaged platform switches from undersupplying sellers to oversupplying them.

In our model, we assume that the two platforms differ in their qualities, and therefore each platform can win the market if its quality is high enough. However, the advantaged platform can win the market even if its quality is inferior to that of the disadvantaged, depending on the equilibrium business model that each platform chooses.

In an extension to our basic model, we consider the case where the coordination bias adjusts along time in favor of the platform that won the market in the previous period. We find that beliefs can converge to a full coordination bias, in which the disadvantaged platform cannot win the market. The convergence process is, on average, faster when both platforms adopt the sellers' revenue base, then in the case where they adopt the buyers' revenue base.

In real-life situations, it is possible to think of other factors that might affect a coordination bias. One such factor, that we do not model, is advertising. Platforms can advertise in order to enhance coordination in favor of their platform. For example, when Apple advertises "There is an app for that," Apple signals to buyers that application developers are likely to join Apple, and therefore users should also join. This in turn convinces developers to join as they expect users to join as well, resulting in a stronger coordination bias in favor of Apple. The results of our model suggest that platforms will have an incentive to invest in such advertising. However, the effect of such advertising on welfare might be inconclusive, as it may increase or decrease the number of sellers a platform attracts. Moreover, it is unclear whether a platforms' ability to advertise for enhancing coordination increases or decreases the coordination bias. This is because both platforms can potentially advertise, making it unclear whether competing advertising campaigns affect the coordination bias or just cancel each other. Another factor that may affect the coordination bias are certifiers, that provide

reviews and recommendations as to which platform is more likely to be successful. For example, PC magazines can provide a recommendation that a certain tablet or smarthphone is more likely to attract more users and application developers in the future than others. Such a recommendation may enhance a platform's coordination bias. As in the previous example, our model suggest that certifies may increase or decrease welfare, depending on how they affect the platforms' business models. We leave these issues for future research.

Appendix

Proof of Lemma 1.

Since $U_B(n) = u_B(n) - n\, u'_B(n)$ and $p(n) = u'_B(n)$, we can write the first-order conditions that determine n^*, n^{A*} and n^{D*}, respectively, as:

$$u'_B(n^*) - k = 0 \tag{15}$$

$$u'_B(n^{A*}) - k + (1-\alpha)\left[n^{A*} \cdot u''_B(n^{A*})\right] = 0 \tag{16}$$

$$u'_B(n^{D*}) - k + \alpha\left[n^{D*} \cdot u''_B(n^{D*})\right] = 0 \tag{17}$$

(15) follows directly from maximization problem in (1). (16) follows from equation (9), and (17) follows from (13).

Since by assumption, $u''_B < 0$, the terms in the squared brackets in (16) and (17) are negative. Since $\frac{1}{2} < \alpha < 1$, it follows from the above equations that $n^* > n^{A*} > n^{D*}$. Moreover, (16) and (17) imply that as α increases, n^{A*} increases and n^{D*} decreases, with $n^{A*} = n^*$ for $\alpha = 1$. The second order conditions are:

$$u''_B(n^*) < 0\,,$$

$$u''_B(n^{A*}) + (1-\alpha)\left[n^{A*} \cdot u'''_B(n^{A*}) + u''_B(n^{A*})\right] < 0\,,$$

$$u''_B(n^{D*}) + \alpha\left[n^{D*} \cdot u'''_B(n^{D*}) + u''_B(n^{D*})\right] < 0\,,$$

which are satisfied by assumptions of $u''_B(n) < 0$ and $u'''_B(n) < -\frac{u''_B(n)}{n}$.

Proof of Lemma 2.

Recall that $\hat{n}$ is the solution to $u'_B(\hat{n}) = 0$. Comparing $u'_B(\hat{n}) = 0$ with (15) yields that $\hat{n} > n^*$ for $k > 0$. (If we allowed for $k = 0$, then $\hat{n} = n^*$.)

Proof of Proposition 1.

To prove the characteristics of $\alpha_1(k)$ and $\alpha_2(k)$, we use the following claims

Claim: There is at most one α, $\alpha_1(k)$, that solves $\pi^A_{SRB}(n^{A^*}) = \pi_{BRB}$, such that if $\frac{1}{2} \leq \alpha_1(k) \leq 1$. Moreover, $\pi^A_{SRB}(n^{A^*}) > (<) \pi_{BRB}$ for $\alpha > (<) \alpha_1(k)$.
Proof: $\pi^A_{SRB}(n^{A^*})$ is strictly decreasing with α while π_{BRB} is independent of α, thus $\pi^A_{SRB}(n^{A^*})$ can intersect π_{BRB} only once, and $\pi^A_{SRB}(n^{A^*}) > (<) \pi_{BRB}$ for $\alpha > (<) \alpha_1(k)$.

Claim: $\alpha_1(0) = 1$.
Proof: To prove the claim we need to show that evaluated at $(\alpha, k) = (1, 0)$, $\pi^A_{SRB}(n^{A^*}) = \pi_{BRB}$. From Lemmas 1 and 2 show that at $(\alpha, k) = (1, 0)$, $n^{A^*} = \hat{n} = n^*$. Substituting $n^{A^*} = \hat{n}$ and $\alpha = 1$ into $\pi^A_{SRB}(n^{A^*})$ yields:

$$\pi^A_{SRB} = U_B(\hat{n}) + \hat{n}\left(p(\hat{n}) - k\right) = U_B(\hat{n}) - k\,\hat{n} = \pi_{BRB},$$

where the second equality follows because, $p(\hat{n}) = u'_B(\hat{n}) = 0$, and the last equality follows from the definition of π_{BRB}.

Claim: $\alpha'_1(0) = 0$, $\alpha'_1(k) < 0$ and $\alpha''_1(k) > 0$.
Proof: Since $\alpha_1(k)$ is the solution to $\pi^A_{SRB}(n^{A^*})\pi_{BRB}$, we have:

$$\frac{d\,\alpha_1(k)}{d\,k} = -\frac{\frac{d\left(\pi^A_{SRB}(n^{A^*}) - \pi_{BRB}\right)}{d\,k}}{\frac{d\left(\pi^A_{SRB}(n^{A^*}) - \pi_{BRB}\right)}{d\,\alpha}}. \tag{18}$$

The nominator of (18) is:
$$\frac{d\left(\pi^A_{SRB}\left(n^{A^*}\right) - \pi_{BRB}\right)}{d\,k} = \frac{\partial\,\pi^A_{SRB}\left(n^{A^*}\right)}{d\,k} + \frac{\partial\,\pi^A_{SRB}\left(n^{A^*}\right)}{\partial\,n}\frac{\partial\,n^{A^*}}{\partial\,k} - \frac{d\,\pi_{BRB}}{d\,k} = -(n^{A^*} - \hat{n}), \tag{19}$$

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where the last equality follows from the envelope theorem and from the definitions of $\pi_{SRB}^A(n^{A*})$ and π_{BRB}. The denominator of (18) is:

$$\frac{d\left(\pi_{SRB}^A\left(n^{A*}\right)-\pi_{BRB}\right)}{d\alpha}=\frac{\partial\pi_{SRB}^A\left(n^{A*}\right)}{d\alpha}+\frac{\partial\pi_{SRB}^A\left(n^{A*}\right)}{\partial n}\frac{\partial n^{A*}}{\partial\alpha}-\frac{d\pi_{BRB}}{d\alpha}=U_B(n^{A*}),\ (20)$$

where the equality follows from the envelope theorem and from the definitions of $\pi_{SRB}^A(n^{A*})$ and π_{BRB}. Substituting (19) and (20) back into (18) yields:

$$\frac{d\alpha_1(k)}{dk}=-\frac{\hat{n}-n^{A*}}{U_B(n^{A*})}.$$

Now, for $(\alpha,k)=(1,0)$, $n^{A*}=\hat{n}$, implying that $\alpha_1'(0)=0$. As k increases, $\hat{n}$ remains constant but n^{A*} decreases, implying that $\alpha_1'(k)<0$ and $\alpha_1''(k)>0$.

Remark: Since $\alpha_1'(k)<0$ and $\alpha_1''(k)>0$, it has be that there is a k such that $\alpha_1(k)=\frac{1}{2}$. We define the solution to $\alpha_1(k)=\frac{1}{2}$ as k_1. As $\alpha_1(0)=1$, it has to be that $k_1>0$, but we still need to prove that $k_1<u_B'(0)$. It would be convenient for us to do this for the subsequent proof of characteristics of $\alpha_2(k)$.

Claim: There is at most one α, $\alpha_2(k)$, that solves $\pi_{SRB}^D(n^{D*})=\pi_{BRB}$, such that if $\frac{1}{2}\leq\alpha_2(k)\leq1$. Moreover, $\pi_{SRB}^D(n^{D*})>\ (<)\ \pi_{BRB}$ for $\alpha<\ (>)\ \alpha_2(k)$.
Proof: $\pi_{SRB}^D(n^{D*})$ is strictly decreasing with α while π_{BRB} is independent of α, thus $\pi_{SRB}^D(n^{D*})$ can intersect π_{BRB} only once, with $\pi_{SRB}^D(n^{D*})>\ (<)\ \pi_{BRB}$ for $\alpha<\ (>)\ \alpha_2(k)$.

Claim: $\alpha_2(k_1)=\frac{1}{2}$.
Proof: Recall that k_1 is the solution to $\alpha_1(k)=\frac{1}{2}$, thus evaluated at $(\alpha,k)=(\frac{1}{2},k_1)$, $\pi_{SRB}^A(n^{A*})=\pi_{BRB}$. To prove that it is also the solution to $\alpha_2(k_1)=\frac{1}{2}$, we need to show that evaluated at $(\alpha,k)=(\frac{1}{2},k_1)$, $\pi_{SRB}^D(n^{D*})=\pi_{BRB}$, which holds if $\pi_{SRB}^D(n^{D*})=\pi_{SRB}^A(n^{A*})$. To see that, notice that (16) and (17) imply that at $\alpha=\frac{1}{2}$, $\pi_{SRB}^D(n^{D*})=\pi_{SRB}^A(n^{A*})$.

Claim: $\alpha_2'(k)>0$ and $\alpha_2''(k)>0$.
Proof: Using the envelope theorem, and applying similar calculations as in (18), (19) and (20) yields:

$$\frac{d\alpha_2(k)}{dk}=\frac{\hat{n}-n^{D*}}{U_B(n^{D*})}. \tag{21}$$

From Lemmas 1 and 2, $n^{D*} < \hat{n}$, hence $\alpha'_2(k) > 0$. Moreover, as α increases, $\hat{n}$ remains constant, while n^{D*} decreases, thus (21) increases.

Claim: There is a point, k_2, such that $\alpha_2(k_2) = 1$, where $0 < k_1 < k_2$.

Proof: Since $\alpha'_2(k) > 0$ and $\alpha''_2(k) > 0$, there is a k such that $\alpha_2(k) = 1$. Also, as $\alpha_1(0) = 1$ and $\alpha'_1(k) < 0$, it has to be that $0 < k_1$, while as $\alpha'_2(k) > 0$, it has to be that $k_1 < k_2$.

Claim: $k_2 < u'_B(0)$.

Proof: To show that $k_2 < u'_B(0)$, it is sufficient to show that evaluated at $k = u'_B(0)$, it is always the case that $\pi^D_{SRB}(n^{D*}) > \pi_{BRB}$. This is because if at $k = u'_B(0)$, $\pi^D_{SRB}(n^{D*}) > \pi_{BRB}$ for all α, it has to be that $\alpha_2(k)$ is always to the left-hand side of the vertical line defined by $k = u'_B(0)$. To show that, notice that (17) implies that if $k = u'_B(0)$, then $n^{D*} = 0$. This in turn implies that at $k = u'_B(0)$, $\pi^D_{SRB}(n^{D*}) = 0$. Turning to π_{BRB}, evaluating π_{BRB} at $k = u'_B(0)$ yields:

$$\pi_{BRB} = u_B(\hat{n}) - \hat{n}\, u'_B(\hat{n}) - \hat{n}\, k$$
$$= u_B(\hat{n}) - \hat{n} \cdot 0 - \hat{n}\, u'_B(0)$$
$$= \int_0^{\hat{n}} \left(u'_B(n) - u'_B(0) \right) dn \ < 0\,,$$

where the first equality follows because $u'_B(\hat{n}) = 0$ and $k = u'_B(0)$, the second equality follows because $u_B(0) = 0$, and the last inequality follows because $u'_B(n)$ is decreasing in n. We therefore have that evaluated at $k = u'_B(0)$, $\pi^D_{SRB}(n^{D*}) = 0 < \pi_{BRB}$.

Proof of Proposition 2.

Consider first the effects of α. From Lemma 1, if $\alpha = \frac{1}{2}$ then $n^{A*} = n^{D*}$, implying that $\pi^D_{SRB}(n^{D*}) = \pi^A_{SRB}(n^{A*})$ and therefore $Q^C = 0$. Using the envelope theorem, the derivatives of Π^A, Π^D and Q^C with respect to α, in region Ω_{SS}, are

$$\frac{d\,\Pi^A}{d\,\alpha} = U_B(n^{A*}) + U_B(n^{D*}) > 0, \qquad \frac{d\,\Pi^D}{d\,\alpha} = \frac{d\,Q^C}{d\,\alpha} = -\left(U_B(n^{A*}) + U_B(n^{D*}) \right) < 0.$$

In region Ω_{SB}, the derivatives are:

$$\frac{d\,\Pi^A}{d\,\alpha} = U_B(n^{A*}) > 0, \qquad \frac{d\,\Pi^D}{d\,\alpha} = \frac{d\,Q^C}{d\,\alpha} = -U_B(n^{D*}) < 0.$$

In region Ω_{BB}, it is straightforward to see that α does not affect Π^A, Π^D and Q^C.

Next, we turn to the effects of k. Using the envelope theorem, the derivatives of Π^A, Π^D and Q^C with respect to k, in region Ω_{SS} are:

$$\frac{d\,\Pi^A}{d\,k} = -n^{A*} + n^{D*} < 0, \qquad \frac{d\,\Pi^D}{d\,\alpha} = \frac{d\,Q^C}{d\,\alpha} = n^{A*} - n^{D*} > 0,$$

where the inequalities follow because $n^{D*} < n^{A*}$. In region Ω_{SB}, the derivatives are:

$$\frac{d\,\Pi^A}{d\,k} = -n^{A*} + \hat{n} > 0, \qquad \frac{d\,\Pi^D}{d\,\alpha} = \frac{d\,Q^C}{d\,\alpha} = n^{D*} - \hat{n} < 0,$$

where the inequalities follow because $\hat{n} > n^{A*}$. Finally, it is straightforward to see that in region Ω_{BB}, k does not affect Π^A, Π^D and Q^C.

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