Women's Lifetime Labor Supply and Labor Market Experience*

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Abstract

The pattern of joining the labor force only at an advanced stage of the life-cycle was widespread among American women in the 1960s and 1970s, but not since the 1980s. To explain this change we conduct a theoretical analysis of the interrelation between women's lifetime labor supply choices and the dynamic macroeconomic environment. In our model women choose the late-entry pattern only at early stages of the growth process when wages are sufficiently low and grow sufficiently rapidly. As the economy grows, this lifetime labor profile vanishes and women either join the labor force either early in life or not at all.

Keywords: Experience, Labor Force Participation.

JEL Classification Numbers: J16, J21, J22, J31.

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1. Introduction

The steady increase in women Labor Force Participation (LFP) rates in the U.S. since World War II masks a dramatic change that took place during that time in the lifetime labor supply of American women. Initially a large number of new entrants were married women past their child-rearing age with low skills and little labor market experience. Since the late 1970s, however, entering the labor market at an advanced stage of the life-cycle has become much less widespread and the dominant factor in the continuing rise of women's LFP has become the surge in the participation rates of women who entered the labor market at a young age and did not leave it before retirement age. *Figure 1* exemplifies this change in the pattern of women's lifetime labor supply choices using two cohorts: the one born between 1921 and 1930 and the one born between 1951 and 1960.

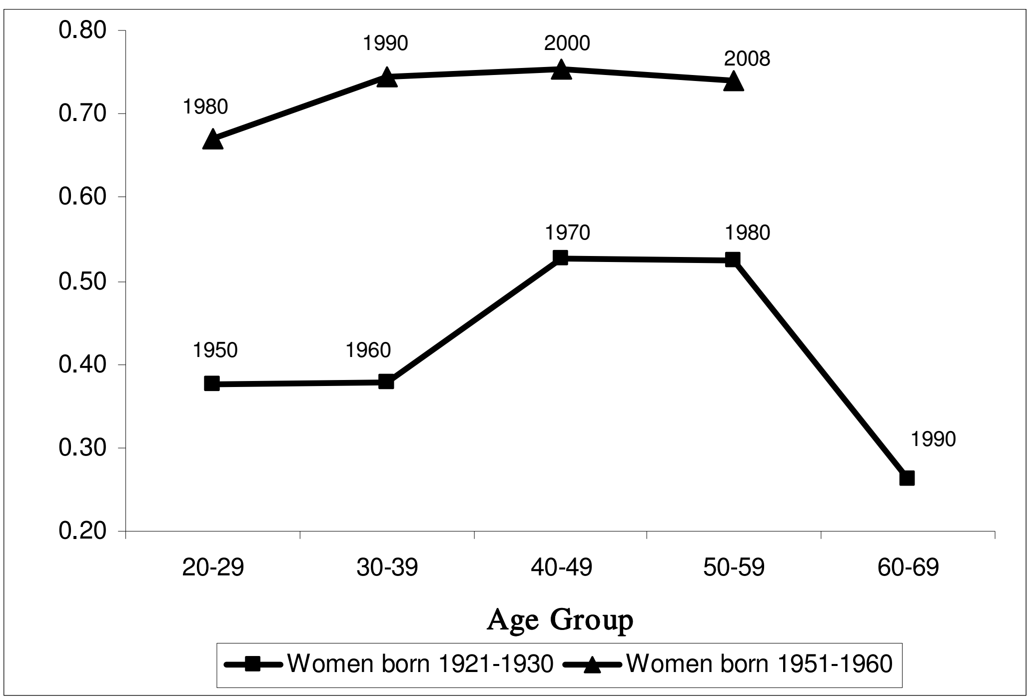

Figure 1: Women's labor force participation rates by age-groups of the cohorts born 1921-30 and 1951-60. The labels next to the points show the years when each cohort was in each age group. Source: authors' calculations of IPMUS-USA and IPMUS-CPS data.

The most important effect of this change is was on the dynamics of the ratio of women's average earnings to men's. As the empirical works of Smith and Ward (1989), Goldin (1989, 1990) and O'Neil and Polachek (1993) show, during the 1960s and 1970s the massive entry of relatively old women with low skills and little labor market experience has lowered women's average labor market experience and women's average labor market skills, relative to men's. This has prevented the ratio of average earnings from rising, rendering it constant at about 0.6 throughout these two decades. Then, as entering the labor market at an advanced stage of the life-cycle has become much less widespread, the ratio of average earnings took-off sharply, going from 0.6 in 1981 to 0.74 in 1996.[1]

While the above mentioned studies provide a thorough quantitative account of how the economy was affected by the different patterns of women's lifetime labor supply, they refrain from studying how these patterns have emerged and treat them as exogenous. The purpose of the current paper therefore is to add to the literature a theoretical analysis of how these lifetime labor supply patterns emerge endogenously as the outcome of individual choices. Specifically, we focus on the following two questions: First, what are the conditions that make entering the labor market in a late stage of the life-cycle a widespread choice of women? Second, is it indeed possible that along the economy's equilibrium dynamic path these conditions should initially prevail and later on cease to prevail?

Regarding the first question, we find that a massive entry of women in a relatively advanced stage of their life-cycle into the labor force occurs in times in which wages are sufficiently low and grow sufficiently rapidly. In such times the rapid wage growth

[1] The data on the ratio of average earnings is from O'Neil (2003).

makes the wages a woman faces when she is young and when she is older sufficiently different from one another to induce a difference in her LFP choices for these different stages of her life. In contrast, in times in which wages grow sufficiently slow, the similarity of the wages the woman faces in the different stages of her life-cycle makes her LFP decisions in those stages similar to one another too.

We also show, regarding our second question, that along the economy's dynamic equilibrium path, periods of rapid wage growth indeed appear before those of the second type. Based on the standard assumption of decreasing marginal productivity of capital, this is in fact a common result in macroeconomic dynamic models. Moving from theory to reality, U.S. data for the post World War II period suggest that this was indeed the case. Specifically, Table 14 in McGrattan and Rogerson (1998) shows that during 1950-1960, and during 1960-1970, real compensation per hour in the business sector has grown by 3.4 percent and 2.9 percent per annum, respectively. In contrast, during 1970-1980, and during 1980-1990, real compensation per hour in the business sector has grown by merely 0.9 percent and 0.4 percent per annum, respectively.

Following the Mincer (1962) approach, we model the LFP expansion as the outcome of a gradual increase in the wages women face. Following Galor and Weil (1996), we model this wage growth as the result of gradual capital accumulation. The individuals are organized in overlapping generations where each generation lives and works for two periods. The economy comprises three production sectors: home production, a physical sector and a modern sector. At home and in the physical sector labor is the sole input, while the modern sector production utilizes both labor and capital. We refer to employment in the physical and the modern sectors as labor market participation. Men and women have the same distributions of the abilities that are relevant to the modern sector. The only difference assumed between women and men's abilities is that men have higher productivity in the physical sector. A key assumption in the model is that by entering the labor force in the first period of life, individuals acquire labor market experience that increases their labor market productivity in their second period of life. Several simplifying assumptions regarding productivity at home and in the physical sector ensure that women never choose to work in the physical sector and men never choose home production.

Due to these assumptions, in the initial stage of the economy's growth, women labor supply can follow three alternative dynamic labor profiles: The least able women work at home in both their lives' periods; abler women work at home in the first period of their life and in the modern sector in the second period; the ablest women work in the modern sector in both periods. The "middle group" exists because in this initial stage wages grow rapidly, attracting women in their second period of life to the modern sector. As the wages growth decelerates, the middle group disappears.

This paper is also related to a strand in the literature that explains the expansion in female LFP via channels other than wage growth. Greenwood, Seshadri and Yorukoglu (2005) and Albanesi and Olivetti (2007) have done so using models in which technological improvements in the production at home promote women's LFP. By enabling the same production at home with more time at the labor market, the technological improvements lower the alternative cost attached to LFP. Hazan and Maoz (2002) argue that the erosion of the social cost associated with female LFP can generate an *S*-shape dynamics in female LFP. Fernández, Fogli, and Olivetti (2004) too focus on such costs. They argue that growing up to a working mother reduces these costs and generates thus an increase in female LFP. Finally, Fernández (2007) and Fogli and Veldkamp (2007) present models in which the costs associated with female LFP are uncertain and a process of learning these costs generates an *S*-shape dynamics in female LFP, similar to Hazan and Maoz (2002).

The article that comes closest to ours is that of Olivetti (2006) who argues that married women's hours of market work increased significantly in the U.S. during the past few decades and that young married mothers are responsible for this change. She argues that this happened in response to an increase in the return to experience.

The paper is organized as follows. In section 2 we present the basic structure of the model. In section 3 we analyze the individuals' labor supply decisions. In section 4 we analyze the equilibrium and the dynamics of the economy portrayed by the model. In section 5 we offer concluding remarks.

2. The Structure of the Economy

Consider a closed overlapping-generations economy that operates in a perfectly competitive environment. Time is discrete and infinite. In every period the economy produces a single good that can be used for consumption or investment.

2.1 Production

Production can take place at home or in the market. There are two production sectors in the market: the physical sector and the modern sector. Working in the market, in either sector, in life's first period rewards the individual with labor market experience that increases her or his productivity in the market production in life's second period.

The marginal productivity of labor at home is the constant *H*, regardless of gender and experience. In contrast, the marginal productivity of labor in the physical sector differs across the genders. In the first period of their lives, each man who works in the physical sector produces the constant amount *P*, and each woman who works in the physical sector produces the constant amount *P'*, where *P* and *P'* are constants satisfying *P*>*P'*. In life's second period, each individual *j*'s productivity in the physical sector is multiplied by e^j which is a function of *j*'s labor market experience, as defined in the next sub-section.

The production function in the modern sector is:

$$Q_t = K_t^{0.5} L_t^{0.5} \tag{1}$$

Where Q_t is output, K_t is the amount of capital and L_t is the amount of efficiency units of labor in this sector in period *t*.[2]

Markets are assumed to be competitive. Firms are assumed to be unable to charge their young employees for the experience these employees acquire on the job. Hence the labor demand of the firms is based merely on the marginal productivity of labor in the production of goods. Due to these assumptions the wage of one unit of efficiency labor in period *t* and the return to one unit of capital in period *t* are respectively:

$$w_t = 0.5 K_t^{0.5} L_t^{-0.5} \tag{2}$$

[2] The specific value of 0.5 in the exponent of the production function is chosen to enable closed form solution for the variables of the model.

$$R_t = 0.5 K_t^{-0.5} L_t^{0.5} \tag{3}$$

Thus:

$$R_t = \frac{1}{4 w_t} \tag{4}$$

2.2 Individuals

In each period, a generation of measure 2 joins the economy where the measures of the women and the men in each generation are normalized to 1. All individuals live and work for two periods. We assume that individuals' preferences are defined over consumption in both periods of life. Since there is no distinction between market and home good, maximization of utility is equivalent to maximizing the net present value of earnings. We assume that individuals cannot operate in more than one sector during a certain period.[3]

Individuals differ in the amount of efficiency units that they can supply to the modern sector. Let a^j be the amount of efficiency units that individual j has in his or her first life period. We assume a^j~U[0,1] regardless of j's gender. In life's second period, j's amount of efficiency units is $e^j a^j$ where e^j is a function of the labor market experience j acquired in j's life's first period. The function e^j takes three values: 1, θ_1 and θ_2. It equals 1 if j has no market experience due to working at home in j's life's first period, θ_1 if j

[3] This assumption is consistent with the heterogeneity of the U.S. female labor force participation during the past 50 years, as observed by Heckman and Willis (1977) and Goldin (1989). "Heterogeneous participation" means that a woman either participates in the LFP full-time year round, or not at all.

works in the same market sector in both periods of j's life or θ_2 if j moves from one market sector to the other, where $\theta_1 \geq \theta_2 > 1$. In order to focus efficiently on the dynamics of the women's choice between working at home and working in the market sectors we simplify the analysis of the choice between the two market sectors by assuming from now on that $\theta_1 = \theta_2 \equiv \theta$.[4] Finally, we assume that $P > H$, an assumption which assures that men do not work at home.

3. Labor Supply

In this section we analyze the labor supply of each of the following four groups of individuals that exist in the economy at each period:

A - women in their life's first period
B - women in their life's second period
C - men in their life's first period
D - men in their life's second period.

We denote the number of group z members that work in the modern sector in period t by x_t^z, where z is a group index satisfying $z \in \{A, B, C, D\}$. In a similar way we denote the amount of efficiency units of labor supplied to the modern sector by members of group z by L_t^z. The total number of efficiency units of labor supplied to the modern sector in period t satisfies therefore:

$$L_t = L_t^A + L_t^B + L_t^C + L_t^D . \tag{5}$$

[4] Angrist (1990) shows a strong impact of labor market experience on lifetime earnings.

3.1 Men's Labor Choice

In each period t man j chooses to work in the modern sector if $a^j w_t \geq P$. Note that in that case $a^j \theta w_t \geq \theta P$ holds too and thus j prefers the modern sector regardless of his experience. We can define therefore an ability threshold for men denoted by a_t^Y, where if $a^j \geq a_t^Y$ then j works in the modern sector, and if $a^j < a_t^Y$ he works in the physical sector. The threshold a_t^Y satisfies:

$$a_t^Y = \begin{cases} \frac{P}{w_t} & \text{if } \frac{P}{w_t} < 1 \\ 1 & \text{otherwise} \end{cases} \tag{6}$$

Due to the uniform distribution of a^j:

$$x_t^C = x_t^D = 1 - a_t^Y = \begin{cases} 1 - \frac{P}{w_t} & \text{if } \frac{P}{w_t} < 1 \\ 0 & \text{if } \frac{P}{w_t} \geq 1 \end{cases} \tag{7}$$

Given w_t, the amount of efficiency units supplied to the modern sector in period t by men born in period t is:

$$L_t^C(w_t) = \int_{a_t^Y}^{1} a da = \frac{1-(a_t^Y)^2}{2} = \begin{cases} \dfrac{1-\left(\dfrac{P}{w_t}\right)^2}{2} & \text{if } \dfrac{P}{w_t} < 1 \\ 0 & \text{otherwise} \end{cases} \tag{8}$$

and the amount of efficiency units supplied to the modern sector in period *t* by men born in period *t*-1 is:

$$L_t^D(w_t) = \theta \int_{a_t^Y}^{1} a da = \theta L_t^C(w_t) \tag{9}$$

3.2 The Labor Supply of Women in their Life's First Period

In contrast to men, the period *t* occupational choice of a woman born in period *t* depends not only on current wage, w_t, but also on w_{t+1}.

As discussed in the introduction, this article focuses on the dynamics of the number of women who choose to work at home in their life's first period and join the modern sector in their life's second period. To do so more efficiently we take simplifying assumptions which ensure that women either work at home or in the modern sector, but not in the physical sector. Specifically, we assume for that purpose that P'=0.[5] Given this assumption, each woman born in period *t* has to choose in that period one of the following dynamic labor profiles:

[5] If P' = 0, then working in the modern sector, which yields a positive income and an experience premium, is always better than working in the physical sector for each woman in each period of her life.

Profile 1: Home production in period t and in period t+1.

Profile 2: Home production in period t and work in the modern sector in period t+1.

Profile 3: Work in the modern sector in period t and home production in period t+1.

Profile 4: Work in the modern sector in period t and in period t+1.

We define V^i as the present value of earnings under each of the profiles $i \in \{1,2,3,4\}$. Given w_t, w_{t+1} and R_{t+1}, V^i satisfies:

$$V^1 = H + \frac{H}{R_{t+1}} \qquad V^2 = H + \frac{aw_{t+1}}{R_{t+1}}$$

$$V^3 = aw_t + \frac{H}{R_{t+1}} \qquad V^4 = aw_t + \frac{a\theta w_{t+1}}{R_{t+1}}$$

We define $a_{mn,t}$ as follows: all the women to whom $a > a_{mn,t}$ prefer profile m to profile n, where $m > n$ and $m,n \in \{1,2,3,4\}$. For each of the possible (m,n) combinations, $a_{mn,t}$ is the value of a for which $V^m(a, w_t, w_{t+1}) = V^n(a, w_t, w_{t+1})$. Thus, by (4), the different possibilities of $a_{mn,t}$ are the following functions of w_t and w_{t+1}:

$$a_{21,t} = \frac{H}{w_{t+1}} \qquad a_{31,t} = \frac{H}{w_t} \qquad a_{41,t} = \frac{H + 4Hw_{t+1}}{w_t + 4\theta w_{t+1}{}^2}$$

$$a_{43,t} = \frac{H}{\theta w_{t+1}} \qquad a_{42,t} = \frac{H}{w_t + 4(\theta - 1)w_{t+1}{}^2} \tag{10}$$

A threshold for the decision between profile 2 and 3 is unnecessary because, as the following proposition shows, no woman chooses profile 3 when wages are increasing over time and no woman chooses profile 2 when wages are decreasing over time.

Proposition 1:

(a) If $w_{t+1}>w_t$, working in her life's first period in the market and in her life's second period at home (profile 3) cannot be optimal for any woman.

(b) If $w_{t+1}<w_t$, working in her life's first period at home and in her life's second period in the market (profile 2) cannot be optimal for any woman.

Proof: (a) Profile 3 is optimal for some women only if V^3 exceeds V^1, V^2 and V^4 for some a. However, if $V^3>V^1$ for a certain a, then $aw_t>H$, implying that $a\theta w_{t+1}>H$ (since $w_{t+1}>w_t$) and therefore that $V^4>V^3$ for that a. Thus, profile 3 cannot be optimal for any woman.

(b) For profile 2 to be optimal for some women, V^2 must exceed V^1, V^3 and V^4 for some a. However, if $V^2>V^1$ for a certain a, then $aw_{t+1}>H$, implying that $aw_t>H$ (since $w_t>w_{t+1}$) and therefore that $V^4>V^2$ for that a. Thus, profile 2 cannot be optimal for any woman. □

Figure 2 shows how the (w_t, w_{t+1}) plain can be divided to three distinct ranges and the following *Proposition 2* determines the order of the relevant labor profiles thresholds in each range. Those three ranges are as follows:

$$\mathbf{E} \equiv \left\{ (w_t, w_{t+1}): \ \ w_t \leq w_{t+1} - 4(\theta - 1) w_{t+1}{}^2 \right\}$$

$$\mathbf{F} \equiv \left\{ (w_t, w_{t+1}): \ \ w_{t+1} - 4(\theta - 1) w_{t+1}{}^2 < w_t \leq \theta w_{t+1} \right\}$$

$$\mathbf{G} \equiv \left\{ (w_t, w_{t+1}): \ \ \theta w_{t+1} < w_t \right\}.$$

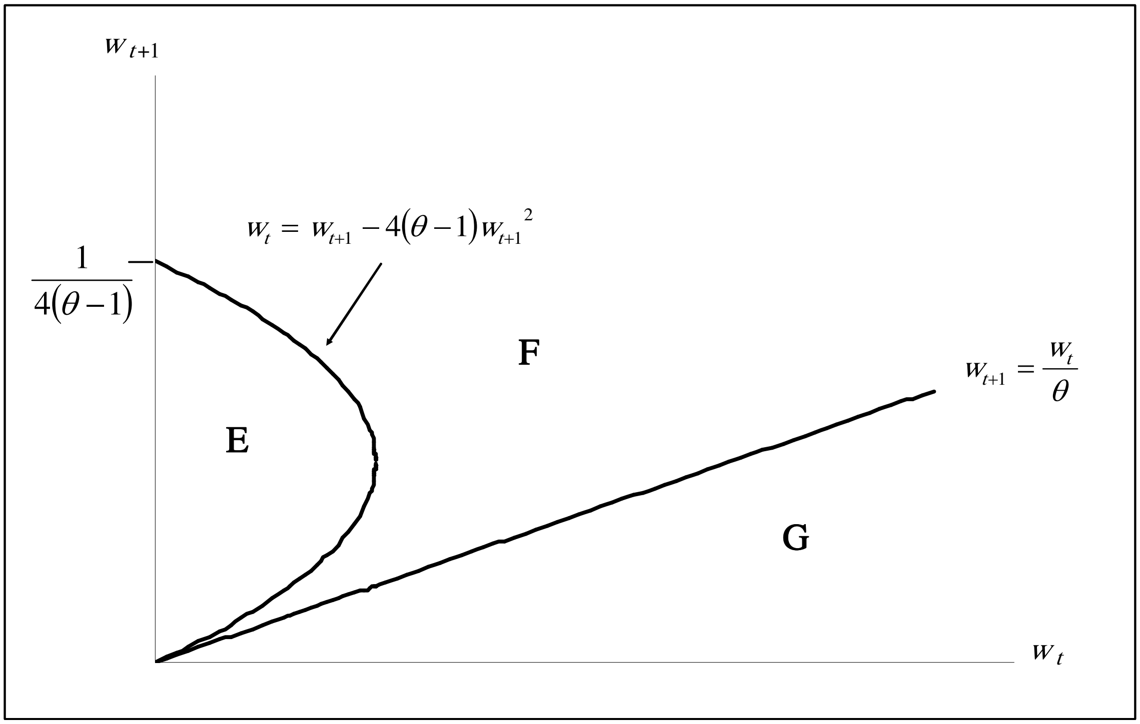

Figure 2: Dividing the (w_t, w_{t+1}) plain to three distinct ranges using the lines $w_{t+1} = \frac{w_t}{\theta}$ and $w_t = w_{t+1} - 4(\theta - 1) w_{t+1}{}^2$.

Proposition 2:

(a) If $(w_t, w_{t+1}) \in \mathbf{E}$ then $a_{21} < a_{41} < a_{42}$

(b) If $(w_t, w_{t+1}) \in \mathbf{F}$ then $a_{42} < a_{41} < a_{21}$ and $a_{43} < a_{41} < a_{31}$

(c) If $(w_t, w_{t+1}) \in \mathbf{G}$ then $a_{31} < a_{41} < a_{43}$.

Proof: The proof follows directly from (10). □

Figure 3 shows the distribution of labor profiles according to abilities as follows from *proposition 2* for the case when $w_t < w_{t+1}$. Following *Proposition 1*, profile 3 is not considered here. The figure refers to the case where the three thresholds are below unity.

As *figure 3.a* shows, when $(w_t, w_{t+1}) \in \mathbf{E}$ the most able women choose profile 4, the least able women choose profile 1 and there are also women that choose profile 2, women whose abilities are between those of the women in the other two groups.

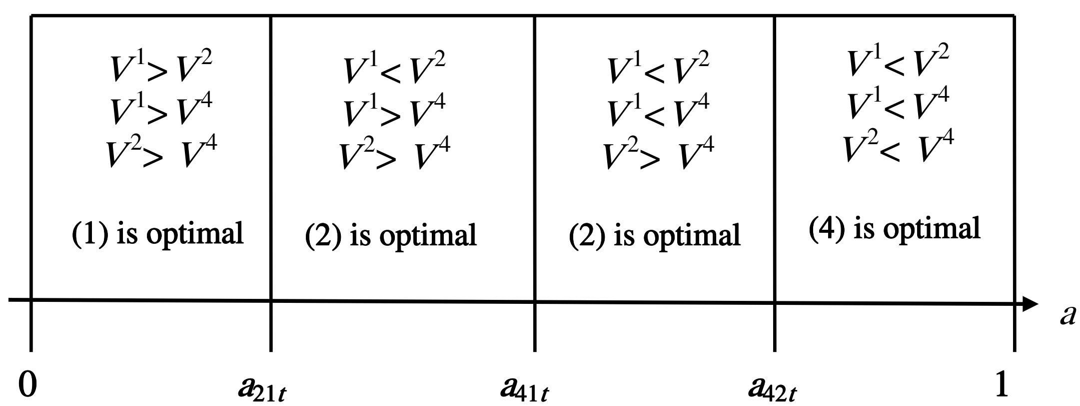

Figure 3.a: The order of the ability thresholds when $w_t \le w_{t+1} - 4(\theta - 1) w_{t+1}^2$

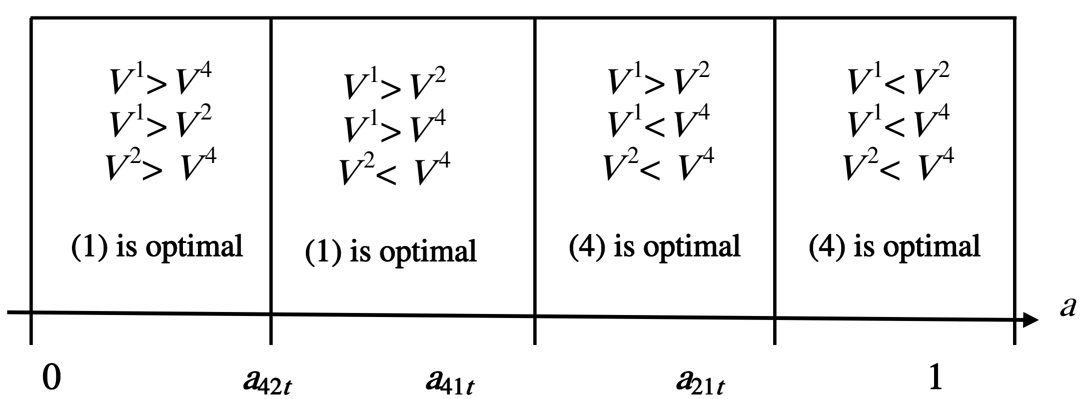

Figure 3.b: The order of the ability thresholds when $w_{t+1} - 4(\theta - 1) w_{t+1}^2 < w_t \le w_{t+1}$

Sticking with the case of $w_t < w_{t+1}$, *figure 3.b* shows the distribution of the relevant labor profiles according to abilities as follows from *proposition 2* when $(w_t, w_{t+1}) \in \mathbf{F}$. In this case no woman chooses profile 2, only profile 4 and 1 are chosen.

The rationale behind the result that profile 2 is chosen only in range **E** can be explained by looking at *figure 2*. As the figure illustrates, a pair (w_t, w_{t+1}) belongs to that range if w_{t+1} is sufficiently above w_t, yet not too much above it. w_{t+1} being sufficiently above w_t is required to make some women take a different choice in each period: stay at home in period t and work in the market in period t+1. By doing so these women do not enjoy the experience premium, a loss that is proportional to w_{t+1}. Thus, this behavior also requires that w_{t+1} is not too high, otherwise those women would prefer to work in the market already in period t in order to acquire experience.

Summing up this analysis the following equation shows the proportion of women born in period t who work in the modern sector in that period for the case where $w_{t+1} \geq w_t$:

$$x_t^A = \begin{cases} 1 - \tilde{a}_{42,t} & if \quad w_t < w_{t+1} - 4(\theta - 1) w_{t+1}{}^2 \\ 1 - \tilde{a}_{41,t} & if \quad w_{t+1} - 4(\theta - 1) w_{t+1}{}^2 < w_t < w_{t+1} \end{cases} \tag{11}$$

where $\tilde{a}_{mn,t} = \min(a_{mn,t}, 1)$.

A similar analysis to the one taken for the case where wages are increasing over time yields that (assuming again that the relevant thresholds are below unity) when wages are decreasing over time there are always women who choose profiles 1 and 4 and no woman who chooses profile 2. Some women choose profile 3 but this occurs only if the decrease in wages is sufficiently rapid. The women who choose profile 4 are more able than those who choose profile 3 and the women who choose profile 3 are more able than those who choose profile 1. For this case in which $w_t \geq w_{t+1}$ the proportion of women born in period t who work in the modern sector in that period is:

$$x_t^A = \begin{cases} 1-\tilde{a}_{41,t} & if \quad w_{t+1} \leq w_t < \theta w_{t+1} \\ 1-\tilde{a}_{31,t} & if \quad \theta w_{t+1} < w_t \end{cases} \tag{12}$$

Combining both cases, $w_t < w_{t+1}$ and $w_t \geq w_{t+1}$, we get:

$$x_t^A(w_t, w_{t+1}) = \begin{cases} 1-\tilde{a}_{42,t}(w_t, w_{t+1}) & if \quad w_t \leq w_{t+1} - 4(\theta-1)w_{t+1}^{\;2} \\ 1-\tilde{a}_{41,t}(w_t, w_{t+1}) & if \quad w_{t+1} - 4(\theta-1)w_{t+1}^{\;2} \leq w_t < \theta w_{t+1} \\ 1-\tilde{a}_{31,t}(w_t, w_{t+1}) & if \quad \theta w_{t+1} < w_t \end{cases} \tag{13}$$

One result that shall become important when the dynamics of the economy will be analyzed is that if $H > \frac{1}{4(\theta-1)}$ then profile 2 is not chosen by any woman even if $(w_t, w_{t+1}) \in \mathbf{E}$. The reason is that if $H > \frac{1}{4(\theta-1)}$ and $(w_t, w_{t+1}) \in \mathbf{E}$ then $1 < a_{21} < a_{41} < a_{42}$ and no woman chooses to work in the market, as follows from (10).

It follows from the analysis in this sub-section that the women who are born in period t and choose to work in the market are those whose abilities satisfy $1 - x_t^A \leq a \leq 1$. Based on the [0, 1] uniform distribution of abilities, the amount of efficiency units of labor supplied to the modern sector in period t by this group is therefore:

$$L_t^A(x_t^A) = \int_{1-x_t^A}^{1} a\,da = \frac{1-(1-x_t^A)^2}{2} \tag{14}$$

3.3 The Labor Supply of Women in their Life's Second Period

As was established in section 3.2 there exists an ability threshold, which we denote in this sub-section by a_{t-1}^X, such that in period t-1 the young women with 0<a< a_{t-1}^X work at home while the young women with a_{t-1}^X <a<1 work in the modern sector.

There are three possible cases. In the first case $a_{t-1}^X < \frac{H}{\theta w_t} < \frac{H}{w_t}$ and the women who did not acquire experience in period t-1, those with 0<a< a_{t-1}^X, do not work in the market in period t since for them a< $\frac{H}{w_t}$ implying that aw_t<H. Women who did acquire experience in period t-1 and their amount of efficiency units is in the range a_{t-1}^X<a<$\frac{H}{\theta w_t}$ also do not work in the market since for them $a\theta w_t < H$. The only women born in period t-1 who work in period t are those with $\frac{H}{\theta w_t}$ <a<1. Thus, $x_t^B = 1 - \frac{H}{\theta w_t}$, due to the uniform distribution of a. The period t labor supply of these women in this case is $L_t^B = \int_{\frac{H}{\theta w_t}}^1 ada$.

In the second case, $\frac{H}{\theta w_t} < a_{t-1}^X < \frac{H}{w_t}$. As in the previous case, those who stayed at home in period t-1, do so also in period t since for them a<$\frac{H}{w_t}$. On the other hand, all the women who did acquire experience in period t-1 work in the market in period t, since for them a> a_{t-1}^X and therefore a>$\frac{H}{\theta w_t}$. Thus, in this case, $x_t^B = x_{t-1}^A$. The period t labor supply of women born in period t-1 in this case is $L_t^B = \theta \int_{a_{t-1}^X}^1 ada$.

In the third case, $\frac{H}{\theta w_t} < \frac{H}{w_t} < a_{t-1}^X$. As in the second case, all the women who did acquire experience in period t-1 work in the market in period t, for the same reason as in the second case. In addition, some women born in period t-1 who worked at home in period t-1 work in the market in period t too. These women are the ones with $\frac{H}{w_t} < a < a_{t-1}^X$. The rest of the women who were born in period t-1 are those with $0<a<\frac{H}{w_t}$. Since for them $a < a_{t-1}^X$, they worked at home in t-1 and have no market experience. $a<\frac{H}{w_t}$ makes them work therefore at home in period t too. The period t labor supply of women born in period t-1 in this case is $L_t^B = \theta\int_{a_{t-1}^X}^1 ada + \int_{\frac{H}{w_t}}^{a_{t-1}^X} ada$.

Equation (15) summarizes this sub-section by showing L_t^B as a function of w_t and x_{t-1}^A:

$$L_t^B\left(x_{t-1}^A, w_t\right) = \begin{cases} \theta L_{t-1}^A\left(x_{t-1}^A\right) + \dfrac{\left(1-x_{t-1}^A\right)^2 - \left(\frac{H}{w_t}\right)^2}{2} & \text{if} \quad x_{t-1}^A < 1 - \dfrac{H}{w_t} \\ \theta L_{t-1}^A\left(x_{t-1}^A\right) & \text{if} \quad 1 - \dfrac{H}{w_t} < x_{t-1}^A < 1 - \dfrac{H}{\theta w_t} \\ \theta \dfrac{1 - \left(\frac{H}{\theta w_t}\right)^2}{2} & \text{if} \quad x_{t-1}^A > 1 - \dfrac{H}{\theta w_t} \end{cases} \tag{15}$$

4. Equilibrium and Dynamics

After the previous section has focused on the individuals' period t labor decisions given w_t and w_{t+1}, the current section analyzes the general equilibrium dynamics of the economy. We start with the dynamics of the physical capital which, according to the assumptions on individuals' behavior specified in the previous section, satisfies:

$$K_{t+1}\left(w_t, x_t^A\right) = L_t^A w_t + L_t^C w_t + \left(1 - x_t^A\right)H + \left(1 - x_t^C\right)P \tag{16}$$

Note that x_t^C and L_t^C are functions of w_t by (7) and (8). L_t^A is a function of x_t^A by (14).

Another stock that is created in period t and transferred to period t+1 is the stock of women that in period t+1 would be in their life's second period with modern sector work experience, x_t^A. In this section we show that given the initial values of these two stocks, denoted by K_0 and x_{-1}^A, a unique Perfect Foresight Equilibrium (PFE) exists. We first define the PFE and then turn to its determination and properties.

Definition 1: A PFE is a set of allocations $\{x_t^A, x_t^B, x_t^C, x_t^D, K_t\}_{t=0}^{\infty}$ and a set of prices $\{w_t, R_t\}_{t=0}^{\infty}$ that satisfy (2), (3), (5), (7), (8), (9), (10), (13), (14), (15), (16) for all t, where t=0,…, ∞, given the initial stocks K_0 and x_{-1}^A.

In the next section we show that the entire PFE is obtained if its subset $\{w_t, x_t^A\}_{t=0}^{\infty}$ is given. We therefore focus in the first part of this section on the properties of this subset. Specifically, we show that (w_t, x_t^A) is a two-dimensional dynamical system characterized by a unique steady state and a unique saddle path leading to it. We focus on the case where the equilibrium dynamics are characterized by a monotonic movement along this saddle and show that the initial stocks, K_0 and x_{-1}^A determine the exact course that the economy takes along this saddle path. Some of the more technical parts of the analysis of the properties of the (w_t, x_t^A) system are relegated to two appendices.[6]

[6] Appendix B is available from authors. It offers formal presentations and proofs of general properties that can be noticed merely by applying specific parameter values in equations presented in the article itself.

4.1 The System (w_t, x_t^A)

In this sub-section we first show, using *lemma 1*, that if the subset $\{w_t, x_t^A\}_{t=0}^{\infty}$ of the PFE is given then, the entire PFE can be obtained. In the following two sub-sections, we show that (w_{t+1}, x_{t+1}^A) is uniquely determined by (w_t, x_t^A). This implies that (w_t, x_t^A) is a two-dimensional dynamic system that fully describes the evolution of the economy.

Lemma 1: Given the sub-set $\{w_t, x_t^A\}_{t=0}^{\infty}$ of the PFE, the entire PFE can be obtained.

Proof: Given w_t, (7) yields x_t^C and x_t^D. Then, (8) and (9) yield L_t^C and L_t^D. Given w_t and x_{t-1}^A we can obtain x_t^B and L_t^B by (15). Given x_t^A we can obtain L_t^A by (14). This yields L_t by (5) and leads to K_t by using L_t and w_t in (2). Finally, applying w_t in (4) yields R_t. □

4.1.1 The function $w_{t+1}(w_t, x_t^A)$

The function $w_{t+1}(w_t, x_t^A)$ is based on the relation between these three variables, as captured by (13). Focusing on the case where $x_t^A > 0$, substituting (10) into (13) yields:

$$x_t^A = \begin{cases} 1 - \dfrac{H}{w_t + (\theta - 1)4\,w_{t+1}^{\;2}} & \text{if} \quad w_t < w_{t+1} - 4(\theta-1)w_{t+1}^{\;2} \\[2ex] 1 - \dfrac{H(1 + 4\,w_{t+1})}{w_t + 4\theta w_{t+1}^{\;2}} & \text{if} \quad w_{t+1} - 4(\theta-1)w_{t+1}^{\;2} < w_t < \theta w_{t+1} \\[2ex] 1 - \dfrac{H}{w_t} & \text{if} \quad \theta w_{t+1} < w_t \end{cases} \tag{17}$$

Note that x_t^A is continuous in w_t and w_{t+1}. Differentiation of the first two lines of (17) shows that $\frac{\partial x_t^A}{\partial w_{t+1}} > 0$ and therefore that $x_t^A \geq 1 - \frac{H}{w_t}$ for all w_{t+1}.

Manipulating the first line in (17) yields $w_{t+1} = \sqrt{\frac{\frac{H}{1-x_t^A} - w_t}{4(\theta - 1)}}$. Applying this in $w_t < w_{t+1} - 4(\theta - 1)w_{t+1}{}^2$ shows that this case is relevant in the range $\tilde{E} \equiv \left\{ \left(w_t, x_t^A\right) \,:\, w_t < \frac{H}{1-x_t^A} - 4(\theta - 1)(\frac{H}{1-x_t^A})^2 \right\}$. Likewise, isolating w_{t+1} in the second line of (17) yields $w_{t+1} = \frac{\frac{H}{1-x_t^A} + \sqrt{(\frac{H}{1-x_t^A})^2 + \theta(\frac{H}{1-x_t^A} - w_t)}}{2\theta}$ and the relevant range becomes $\tilde{F} \equiv \left\{ \left(w_t, x_t^A\right) : \frac{H}{1-x_t^A} - 4(\theta - 1)(\frac{H}{1-x_t^A})^2 \leq w_t < \frac{H}{1-x_t^A} \right\}$. In the third line of (17) $w_t > \theta w_{t+1}$, implying a rapid decline in wages over time. Note that in this range there is a set of values of w_{t+1}, rather than a single value, that corresponds to a given pair. As shall be shown later, $w_t > \theta w_{t+1}$ cannot be part of the PFE. Equation (18) summarizes this analysis:

$$w_{t+1}\left(w_t, x_t^A\right) = \begin{cases} \sqrt{\dfrac{\frac{H}{1-x_t^A} - w_t}{4(\theta - 1)}} & if \quad \left(w_t, x_t^A\right) \in \tilde{E} \\ \dfrac{\frac{H}{1-x_t^A} + \sqrt{\left(\frac{H}{1-x_t^A}\right)^2 + \theta\left(\frac{H}{1-x_t^A} - w_t\right)}}{2\theta} & if \quad \left(w_t, x_t^A\right) \in \tilde{F} \end{cases} \quad (18)$$

It follows directly from (18), that $\frac{\partial w_{t+1}\left(x_t^A, w_t\right)}{\partial w_t} < 0$ and $\frac{\partial w_{t+1}\left(x_t^A, w_t\right)}{\partial x_t^A} > 0$ in both $\tilde{E}$ and $\tilde{F}$.

Figure 4 shows the division of the plain $\left(w_t, x_t^A\right)$ to different ranges based on the function $w_{t+1}\left(w_t, x_t^A\right)$. The individuals' choices, captured by (17) eliminate equilibrium below the $x_t^A = 1 - \frac{H}{w_t}$ line. By the definition of $\tilde{E}$ and $\tilde{F}$, these sets are distinguished by the line formed by the pairs (w_t, x_t^A) satisfying:

$$x_t^A = 1 - \frac{8(\theta-1)H}{1 \pm \sqrt{1-16(\theta-1)w_t}} \equiv b(w_t) \tag{19}$$

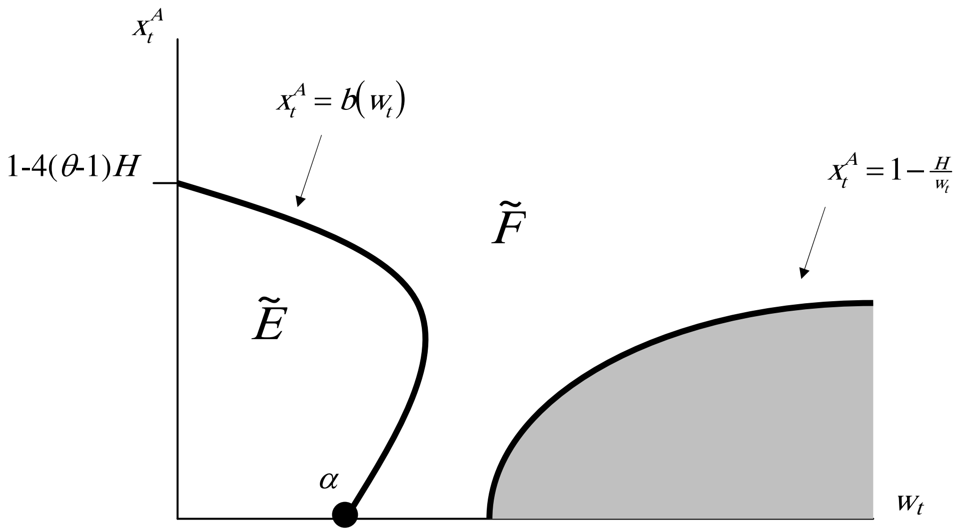

Figure 4: The division of the plain (w_t, x_t^A) to different ranges based on $w_{t+1}(w_t, x_t^A)$.

By (19), the function $b(w_t)$ returns two values of x_t^A for each $0 \le w_t < \frac{1}{16(\theta-1)}$. When w_t approaches 0 one of those values approaches $-\infty$ and the other approaches $1-4(\theta-1)H$. This implies that if $H > \frac{1}{4(\theta-1)}$ then the entire range $\tilde{E}$ is located over negative values of x_t^A and that labor profile 2 therefore is not consistent with a positive labor supply of young women, a result that was already derived in section 3.2. If $H < \frac{1}{4(\theta-1)}$ then $b(0)>0$

implying that part of range $\tilde{E}$ is located over positive values of x_t^A enabling the equilibrium existence of profile 2. By (17), in points along the $b(w_t)$ line $w_{t+1}>w_t$ while the $x_t^A = 1 - \frac{H}{w_t}$ is based on $w_{t+1}<w_t$. Combining that with $\frac{\partial x_t^A}{\partial w_{t+1}} > 0$, which also follows from (17), ensures that $b(w_t)$ is above the $x_t^A = 1 - \frac{H}{w_t}$ line for each $w_t>0$ in the definition range of $b(w_t)$.

4.1.2 The function $x_{t+1}^A\left(w_t, x_t^A\right)$

In this section we present x_{t+1}^A as a function of w_t and x_t^A and analyze some of the properties of this function. We start by manipulating (2) in order to present L_{t+1} as a function of w_{t+1} and K_{t+1} and therefore as the following function of x_t^A and w_t:

$$L_{t+1}\left(x_t^A, w_t\right) = \frac{K_{t+1}\left(w_t, x_t^A\right)}{4\, w_{t+1}\left(w_t, x_t^A\right)^2} \tag{20}$$

While (20) shows the demand for labor, (5) shows its supply and combining the two yields the period t+1 labor market clearing condition. Note from (8) and (9) that L_{t+1}^C and L_{t+1}^D are functions of w_{t+1} and therefore, through (18), of x_t^A and w_t. Equation (15) shows L_{t+1}^B, as a function of x_t^A and w_{t+1}. Thus, L_{t+1}^A is the following function of x_t^A and w_t:

$$L_{t+1}^A\left(w_t, x_t^A\right) = L_{t+1}\left(w_t, x_t^A\right) - L_{t+1}^B\left[w_{t+1}\left(w_t, x_t^A\right), x_t^A\right] - L_{t+1}^C\left[w_{t+1}\left(w_t, x_t^A\right)\right] - L_{t+1}^D\left[w_{t+1}\left(w_t, x_t^A\right)\right] \tag{21}$$

Finally, by (21) and (14), x_{t+1}^A can be shown as a function of x_t^A and w_t:

$$x_{t+1}^A\left(w_t, x_t^A\right) = 1 - \sqrt{1 - 2L_{t+1}^A\left(w_t, x_t^A\right)} \tag{22}$$

In part A of the appendix we prove that $\frac{\partial x_{t+1}^A\left(w_t, x_t^A\right)}{\partial w_t} > 0$ and $\frac{\partial x_{t+1}^A\left(w_t, x_t^A\right)}{\partial x_t^A} > 0$.

4.1.3 The steady state of the system $\left(w_t, x_t^A\right)$

In this sub-section we show, using the following *Proposition 3*, that the $\left(w_t, x_t^A\right)$ system has a unique steady state equilibrium. Our focus is on the case where in the steady state men too work in the modern sector and we show in *proposition 3* that there is a range of parameter values satisfying that.

Proposition 3: There exist a range of parameter values for which:

(a) The dynamical system $\left(w_t, x_t^A\right)$ has a unique steady state point denoted by $\left(\overline{w}, \overline{x}^A\right)$.

(b) $H \leq P < \overline{w}$

(c) $\overline{w}$ is increasing in H and P

Proof: See part (***ii***) of Appendix A. □

4.1.4 The *ww* curve

The *ww* curve is defined as the set of pairs of $\left(w_t, x_t^A\right)$ for which $w_{t+1} = w_t$. The *ww* curve cannot be part of range $\widetilde{E}$ because in that range $w_t < w_{t+1} - 4(\theta - 1)w_{t+1}{}^2$, which implies

$w_t < w_{t+1}$. The ww curve also cannot intersect with the $x_t^A = 1 - \frac{H}{w_t}$ line since by (13) $w_t > \theta w_{t+1}$ along this line. Thus the ww curve is restricted to range $\tilde{F}$. In that range the possible women's labor profiles are either 1 or 4 and therefore $x_t^A = 1 - a_{41,t}$, implying by (10) that along ww:

$$x_t^A = 1 - H \frac{1 + 4w_t}{w_t + 4\theta w_t^2} \tag{23}$$

Straightforward differentiation of (23) shows that the ww curve is a concave increasing line in the (w_t, x_t^A) plain. The ww curve is located above the $x_t^A = 1 - \frac{H}{w_t}$ line since along the ww curve $w_{t+1} = w_t$ while along the $x_t^A = 1 - \frac{H}{w_t}$ line $w_{t+1} < \frac{w_t}{\theta}$ and the higher w_{t+1} implies a higher x_t^A by (13). The ww curve is also located to the right of the $b(w_t)$ frontier function. To see this, note that along the $b(w_t)$ line $w_t < w_{t+1} - 4(\theta - 1)w_{t+1}{}^2 < w_{t+1}$ while along the ww curve $w_{t+1} = w_t$ and combine that with $\frac{\partial x_t^A}{\partial w_{t+1}} > 0$, which follows from (17). *Figure 5* shows the ww curve.

By the definition of $\tilde{E}$, as long as (w_t, x_t^A) is in that range w increases over time. In range $\tilde{F}$ however, w increases over time if (w_t, x_t^A) is above the ww curve and vice versa, due to $\frac{\partial w_{t+1}}{\partial x_t^A} > 0$. The horizontal arrows in *Figure 5* show these dynamics.

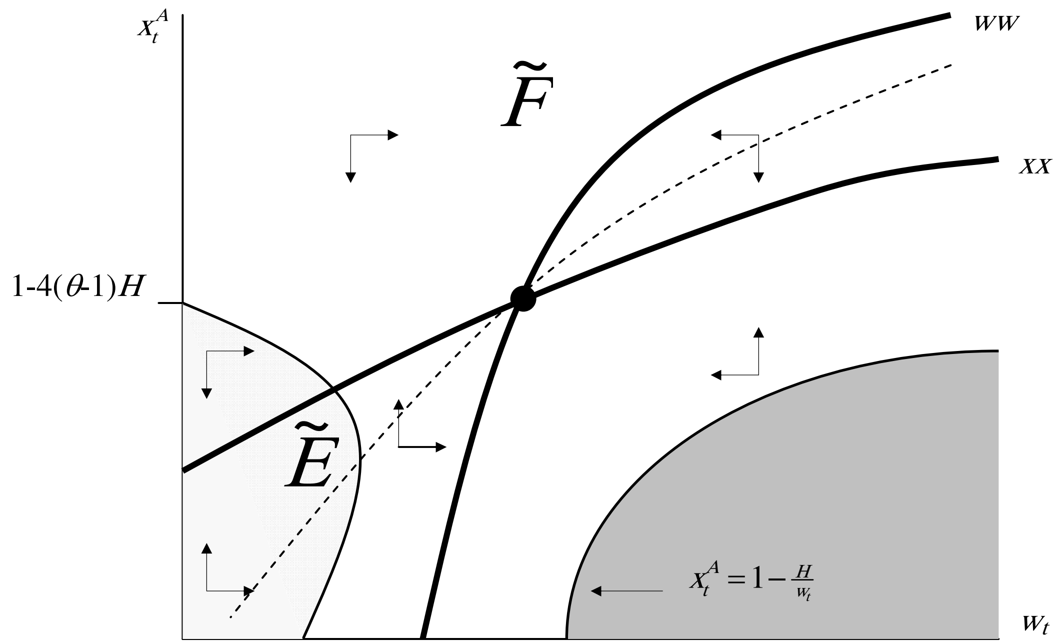

Figure 5: The *ww* and the *xx* curves and the dynamics in the $\left(w_t, x_t^A\right)$ system

4.1.5 The *xx* curve

The *xx* curve is defined as the set of all the $\left(w_t, x_t^A\right)$ pairs for which $x_{t+1}^A = x_t^A$. In the $\left(w_t, x_t^A\right)$ plain this curve is an upward sloping line since implicit derivation of:

$$x_t^A - x_{t+1}^A\left(w_t, x_t^A\right) = 0 \tag{24}$$

shows that along the *xx* curve:

$$\frac{dx_t^A}{dw_t} = -\frac{-\dfrac{\partial x_{t+1}^A\left(w_t, x_t^A\right)}{\partial w_t}}{1 - \dfrac{\partial x_{t+1}^A\left(w_t, x_t^A\right)}{\partial x_t^A}} > 0, \tag{25}$$

where the inequality follows from $\frac{\partial x_{t+1}^A\left(w_t, x_t^A\right)}{\partial w_t}>0$ and $\frac{\partial x_{t+1}^A\left(w_t, x_t^A\right)}{\partial x_t^A}<0$ that are established in part (*i*) of Appendix A. This curve too is shown in *Figure 5*.

Since $\frac{\partial x_{t+1}^A\left(w_t, x_t^A\right)}{\partial w_t} > 0$, at points above the xx curve x^A falls over time and vice versa. The vertical arrows in *Figure 5* show these dynamics.

The xx line can either be crossing the x_t^A axis (as in *Figure 5*) or the w_t axis, depending on parameter values. Nonetheless, even if it crosses the w_t axis, then this cross is to the left of the $b(w_t)$ line which hedges range $\tilde{E}$.[7] This implies that at least part of range $\tilde{E}$ is under xx. As the vertical arrows of motion in *Figure 5* reveal, it is crucial for enabling equilibrium dynamics in which labor profile 2 is part of a convergence process characterized by increasing w_t and x_t^A.

4.1.6 Dynamics in the system $\left(w_t, x_t^A\right)$

The dynamics in the $\left(w_t, x_t^A\right)$ system are rather interesting. Although the arrows of motion in *Figure 5* all point at the direction of the steady state point – the system does not display global convergence. In fact, convergence to the steady state can occur only along a unique upward sloping saddle path located in the lower-left and the upper-right parts of the four parts to which the ww and xx curves divide the $\left(w_t, x_t^A\right)$ plain. Starting at a point not along this saddle sets the economy on a path of oscillatory divergence until (21) and (22) yield values of x_t^A that are either above 1, below zero or below $1-\frac{H}{w_t}$, values that are

[7] This result is formalized and proven in Appendix B. It is also presented, for a particular set of parameters, in part (*iii*) of Appendix A

not compatible with a PFE.[8]

The saddle path result is deduced by analyzing the properties of the eigenvalues of the $\left(w_t, x_t^A\right)$ system at the vicinity of its steady state. The analysis, carried out in part (*iii*) of Appendix A, shows that the smallest among the two eigenvalues of this dynamical system is smaller than -1, implying that convergence to the steady state can occur only along a certain saddle path. It is also shown in that appendix that there can be parameter values for which the larger eigenvalue is a positive number smaller than 1 and therefore the convergence along the saddle path towards the steady state is monotonic. The dashed line in *Figure 5* shows the saddle path in such a case. Given other parameter values the larger eigenvalue can be actually a negative number larger than -1 and therefore the convergence along the saddle path to the steady state is oscillatory.[9]

4.2 Perfect Foresight Equilibrium

In this section we show how the initial stocks, K_0 and x_{-1}^A, determine the dynamic equilibrium path the economy takes. Specifically we show that K_0 and x_{-1}^A determine the period 0 values of $\left(w_t, x_t^A\right)$ and therefore determine the values of $\left(w_t, x_t^A\right)$ in all subsequent periods too, as established in sections 4.1.1 and 4.1.2. By that, K_0 and x_{-1}^A, also determine the values of all the elements of the PFE of the economy for each period t where $t\geq 0$, as shown by *lemma 1*.

We start by showing how K_0 and x_{-1}^A constrain the values of $\left(w_0, x_0^A\right)$. Given K_0

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[8] In Appendix B we formally establish the existence of these oscillatory non-equilibrium dynamics.
[9] See Galor (2007) for an analysis of how the eigenvalues determine the convergence manner.

and x_{-1}^A, the following condition, based on (2), (5), (8), (9), (14) and (15), and describing the period 0 labor market clearing, must hold in equilibrium:

$$L_0^A\left(x_0^A\right) = L_0\left(K_0, w_0\right) - L_0^B\left(w_0, x_{-1}^A\right) - L_0^C\left(w_0\right) - L_0^D\left(w_0\right) \tag{27}$$

We define the xw curve as the collection of all the $\left(w_0, x_0^A\right)$ pairs that satisfy (27), $0 \leq x_0^A \leq 1$ and $w_t > 0$, given K_0 and x_{-1}^A. The xw curve is a downward sloping line since, by (14), L_0^A is increasing in x_0^A, the labor demand $L_0\left(K_0, w_0\right)$ is decreasing in w_0, and L_0^B, L_0^C and L_0^D are increasing in w_0. At w_0=0 the xw is not well defined because, due to (2), the firms' demand for labor, L_0, is infinite while L_0^B, L_0^C and L_0^D are 0. Thus, the value of x_0^A returned by (27) approaches ∞ in that case, in contrast to the constraint of $x_0^A \leq 1$. When w_0 approaches infinity the value of x_0^A that clears (27) is negative because, facing the infinite wage, the firms' want to employ no labor while L_0^B, L_0^C and L_0^D are at their maximal values due to full participation in the modern sector. Thus, the xw curve is a downward sloping line defined only over values of w_0 in the range [w^d, w^u], where w^d and w^u both have strictly positive finite values. x_0^A has the values of 1 and 0 at the points on the xw curve in which w_0 equals w^u or w^d respectively. *Figure 6* shows the xw curve.

As described in section 4.1.6 our focus rests on the case where the $\left(w_t, x_t^A\right)$ system is characterized by an upward sloping saddle path that leads to its steady state with a monotonic convergence of w_t and x_t^A along this path. Since the xw curve is a downward sloping line that spans over all the possible values of x_0^A it crosses the saddle path at a certain point, and only at that point. This point is the only possibility for a pair $\left(w_0, x_0^A\right)$ that is consistent with the definition of the PFE.

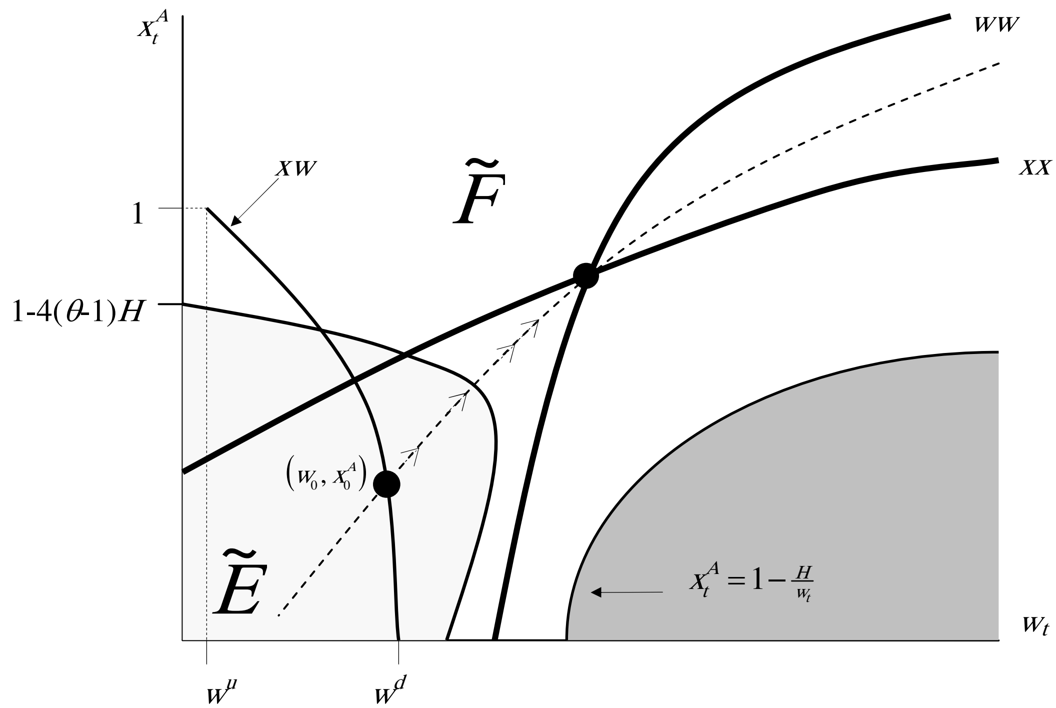

Figure 6: The ww and the xx curves and the dynamics in the $\left(w_t, x_t^A\right)$ system

The higher the economy's period 0 capital stock, K_0, the more to the right the location of the xw curve, as follows from (2) and (27). Thus, an equilibrium path along which labor profile 2 exists at some periods is possible only if K_0 is sufficiently small so that the xw curve crosses range $\tilde{E}$, rather than located above it. In addition, the higher K_0 the higher are the values of both w_0 and x_0^A.

The effect of x_{-1}^A on the location of xw and therefore on the evolution of the economy is opposite to that of K_0. As (15) and (27) show, the higher x_{-1}^A the more to the left the location of xw and the lower are w_0 and x_0^A. The rationale underlying these results is that, ceteris paribus, a larger quantity of physical capital raises the demand for labor in general and for young women labor in particular. On the other hand, a higher x_{-1}^A implies a larger amount of women in their life's second period with labor market experience and that would, ceteris paribus, increase the supply of labor and therefore lower wages, crowding thus young women out of the labor market.

5. Concluding Remarks

In this paper we have studied the considerations underlying women's choices of a lifetime labor profile and have embedded the analysis within a dynamic macroeconomic framework. Our focus has been on the labor profile in which a woman enters the labor market only in a relatively late stage of her life with little labor market experience after avoiding the labor market in the earlier stage of her life. We have found that this labor profile is chosen during the early stages of the growth process, because then wages are sufficiently low and grow sufficiently rapidly. Later, as wages become higher and their growth rate declines, this labor profile become less common until it completely vanishes. This dynamic pattern fits the U.S. women LFP of the post World War II era.

For simplicity, we have assumed that productivity at home is constant. Greenwood, Seshadri and Yorukoglu (2005) and Albanesi and Olivetti (2007) have shown that time-saving technological improvements in home production play an important role in explaining female LFP dynamics in the past century in the U.S. Incorporating such progress in home production in our model requires replacing the assumption that individuals either works at home or in the market in each period by the assumption that in each period individuals optimally allocate part of their time to home production and the rest to LFP. Such technological improvements in home production should have the same positive effect on individuals LFP decision as an increase in wages.

Finally, based on the women LFP data in Kao, Polachek, and Wunnava (1994), Bauer (2001), Okunishi (2001), and Pissarides, Garibaldi, Olivetti, Petrongolo, and Wasmer (2005), we hypothesize that our model also fits the growth in employment in several East Asian and European countries. However, since to the best of our knowledge, a formal analysis along the lines of Smith and Ward (1989), Goldin (1989, 1990) and O'Neil and Polachek (1993) has not been conducted, neither for Asian countries, not for European countries, we can only speculate on the fit of our model to these countries.[10]

Appendix A

(i) The signs of the derivatives of $x_{t+1}^A\left(w_t, x_t^A\right)$

In this part of the appendix we show that $\frac{\partial x_{t+1}^A}{\partial x_t^A} < 0$ and $\frac{\partial x_{t+1}^A}{\partial w_t} > 0$. Since x_{t+1}^A is positively related to L_{t+1}^A through (14), we do so by showing that $\frac{\partial L_{t+1}^A}{\partial x_t^A} < 0$ and $\frac{\partial L_{t+1}^A}{\partial w_t} > 0$ where $L_{t+1}^A\left(w_t, x_t^A\right)$ is based on (21). We start with the derivatives of K_{t+1} with respect to x_t^A and w_t. Based on (16) and (14):

[10] The levels of employment in Europe display large variation across Europe. The analysis in Pissarides, Garibaldi, Olivetti, Petrongolo, and Wasmer (2005) suggests that institutional differences play a major role in accounting for this variation. Clearly our model is silent with respect to labor market institutions and therefore can account for the growth in employment at best.

$$\frac{\partial K_{t+1}\left(w_t, x_t^A\right)}{\partial x_t^A} = \left(1 - x_t^A\right)w_t - H \leq 0 \tag{A.1}$$

where the inequality follows from $x_t^A \geq 1 - \frac{H}{w_t}$, established by (13). For the partial derivative of K_{t+1} with respect to w_t we return to (16) and look at two cases: First, based on (7) and (8), if $w_t \leq P$ then both x_t^C and L_t^C are 0 and the derivative is equal to L_t^A and therefore positive. To see that it is also positive if $w_t > P$ note that in that case, based on (7), (8), (14) and (16):

$$K_{t+1}\left(w_t, x_t^A\right) = \frac{1-\left(1-x_t^A\right)^2}{2} w_t + \frac{1-\left(\frac{P}{w_t}\right)^2}{2} w_t + \left(1 - x_t^A\right)H + \left(\frac{P}{w_t}\right)P \tag{A.2}$$

And therefore:

$$\frac{\partial K_{t+1}\left(w_t, x_t^A\right)}{\partial w_t} = 1 - \frac{\left(1-x_t^A\right)^2}{2} - \frac{P^2}{2\,w_t^2} > 1 - \frac{1}{2} - \frac{1}{2} = 0\,, \tag{A.3}$$

where the inequality springs from $0 \leq x_t^A \leq 1$ and $w_t > P$. We now turn to look at L_{t+1}. Based on (20), L_{t+1} is decreasing in x_t^A since K_{t+1} is decreasing in x_t^A and w_{t+1} is increasing in x_t^A as follows from (18). L_{t+1} is also increasing in w_t because K_{t+1} is increasing in w_t while, as (18) shows, w_{t+1} is decreasing in w_t.

With these results about L_{t+1} at hand we turn to (21) and deduce that when x_t^A

increases L^A_{t+1} falls because L_{t+1} falls while L^B_{t+1}, L^C_{t+1} and L^D_{t+1} are all increasing. The latter three increase because of their positive relation with w_{t+1} which increases in x^A_t. L^B_{t+1} increases also because x^A_t has an additional, direct, positive effect on it, as (15) shows. A similar analysis shows that when w_t increases L^A_{t+1} increases too.

(*ii*) Properties of the steady state - the proof of *proposition 3*

Since this proof deals with the steady state of the economy, time indexes are omitted. Initially in this proof we merely assume that the parameter values are such that $\overline{w}>P$ and some men work therefore in the modern sector. Later, we show that such parameter values indeed exist.

In the steady state, since w is constant over time, only profiles 1 and 4 are chosen by women, implying that $x^A = x^B = 1 - a_{41}$. The men supply labor to the modern sector according to $x^C = x^D = 1 - \frac{P}{\overline{w}}$. Using (5), (8), (9) and (10) we can calculate the steady state amount of labor supplied to the modern sector:

$$\overline{L} = (1+\theta)\frac{1-\left(\frac{P}{\overline{w}}\right)^2}{2} + (1+\theta)\frac{1-(a_{41})^2}{2} = (1+\theta)\left[1 - \frac{P^2 + H^2\left(\frac{1+4\overline{w}}{1+4\theta\overline{w}}\right)^2}{2\overline{w}^2}\right] \tag{A.4}$$

Applying $x^A = 1 - a_{41}$, $L^A = \frac{1 - a_{41}{}^2}{2}$, (7), (8) and (10) in (16) and simplifying, yields that the stock of physical capital in the steady state satisfies:

$$\overline{K}=\frac{2\overline{w}^2+P^2+\dfrac{2H^2\left(1+4\overline{w}\right)}{1+4\theta\overline{w}}-\dfrac{H^2\left(1+4\overline{w}\right)^2}{\left(1+4\theta\overline{w}\right)^2}}{2\overline{w}} \tag{A.5}$$

Applying (A.4) and (A.5) in (20) and simplifying yields:

$$\begin{aligned}&128(1+\theta)\theta^2\overline{w}^5+32\theta(2+\theta)\overline{w}^4-8\left[\theta-1+8\theta^2P^2(1+\theta)+8(1+\theta)H^2\right]\overline{w}^3\\&\quad-2\left[1+8\theta(2+3\theta)P^2+8(1+4\theta)H^2\right]\overline{w}^2-4(1+3\theta)\left(P^2+H^2\right)\overline{w}-P^2-H^2=0\end{aligned} \tag{A.6}$$

Define the function $g(w, \theta, H, P)$ as the LHS of this equation with w replacing $\overline{w}$. This function is a polynomial of the form:

$$g(w, \theta, H, P) = \alpha_5 w^5+\alpha_4 w^4+\alpha_3 w^3+\alpha_2 w^2+\alpha_1 w+\alpha_0$$

where $\alpha_0<0$ and $\alpha_5>0$. Thus, $g(0, \theta, H, P) < 0$ and $\lim_{w\to\infty} g(w,\theta,H,P)=\infty$ which ensures, by continuity, the existence of a positive root to this equation. According to Descartes' rule of sign, this is the only positive root, because α_5, $\alpha_4 > 0$ while α_3, α_2, α_1, $\alpha_0<0$.

$g(0, \theta, H, P) < 0$, $\lim_{w\to\infty} g(w,\theta,H,P)=\infty$, and the uniqueness of the root $\overline{w}$ ensure that $g_w(\overline{w})>0$. Partial derivation of $g(w, \theta, H, P)$ yields that $g_H<0$ and that $g_P<0$ for all values of w. This implies, by implicit derivations of (A.6), that $\overline{w}$ is positively affected by the magnitudes of H and P. *Figure 7* shows $g(w, \theta, H, P)$ as a function of w.

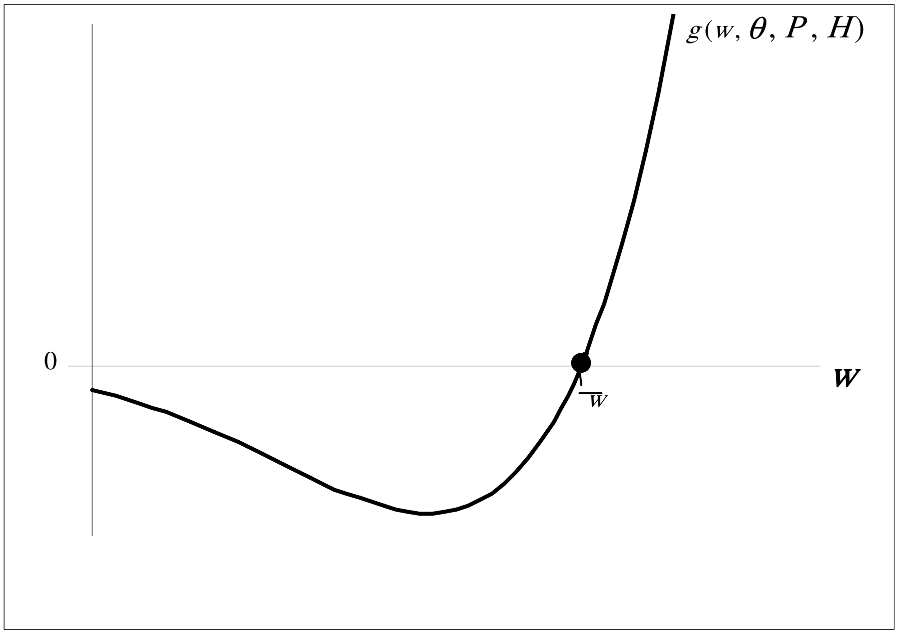

Figure 7: The function $g(w, \theta, H, P)$.

Throughout the proof of *proposition 3* it was assumed that the parameter values are such that $P<\overline{w}$ and that therefore at least some men work in the modern sector. To see that indeed a range of such parameter values exists, notice that, when evaluated at $H=P=0$, (A.6) becomes:

$$128(1+\theta)\theta^2\overline{w}^3 + 32\theta(2+\theta)\overline{w}^2 - 8(\theta-1)\overline{w} - 2 = 0\,, \qquad \text{(A.7)}$$

which has a single positive root at $\frac{1}{4(\theta+1)}$ due to Descartes' rule of sign. Combining this result with H and P positively affecting $\overline{w}$, ensures that there exist a range of sufficiently small positive values of H and P for which (A.6) yields a value of $\overline{w}$ that exceeds P.

(*iii*) **Properties of the saddle path**

The stability of the (w_t, x_t^A) dynamical system is determined by the values of its two eigenvalues which are the two roots of the following characteristic equation:

$$\lambda^2 - (e_{22} + e_{11})\lambda + e_{22}e_{11} - e_{12}e_{21} = 0 \tag{A.8}$$

where e_{11}, e_{12}, e_{21} and e_{22} respectively denote the values of $\frac{\partial x_{t+1}^A(w_t, x_t^A)}{\partial x_t^A}$, $\frac{\partial x_{t+1}^A(w_t, x_t^A)}{\partial w_t}$, $\frac{\partial w_{t+1}(w_t, x_t^A)}{\partial x_t^A}$ and $\frac{\partial w_{t+1}(w_t, x_t^A)}{\partial w_t}$ in the steady state. We start the analysis by showing that the smaller root of this equation is below -1. This implies that convergence to the steady state is not global and can only occur along a saddle path. Standard analysis of the function on the LHS of (A.8) shows that at its minimum point its value is:

$$-\tfrac{1}{4}(e_{22} - e_{11})^2 - e_{12}e_{21} < 0$$

where the inequality follows from $e_{12}e_{21} > 0$ which follow from $\frac{\partial x_{t+1}^A(w_t, x_t^A)}{\partial w_t} > 0$, as established in part (*i*) of this appendix, and from $\frac{\partial w_{t+1}(w_t, x_t^A)}{\partial x_t^A} > 0$, that follows from (18). The result that the minimum of the LHS of (A.8) is negative implies that (A.8) indeed has two roots. To establish that the smaller of these two roots is smaller than -1, we apply (7) and (8) in (16) and then apply (8), (9), (16), (20) and $L_{t+1}^B = \theta L_t^A$ (that holds in the steady state) in (21) and simplify. This yields:

$$L_{t+1}^A\left(w_t, x_t^A\right)=\frac{\left(1-x_t^A\right)H+\frac{P^2}{2\,w_t}+L_t^A w_t+\frac{w_t}{2}+2(1+\theta)P^2}{4\,w_{t+1}\left(w_t, x_t^A\right)^2}-\theta L_t^A-\frac{1+\theta}{2} \tag{A.9}$$

From (14) it follows that $\frac{dL_t^A}{dx_t^A}=1-x_t^A$ and therefore:

$$\frac{\partial L_{t+1}^A\left(w_t, x_t^A\right)}{\partial x_t^A}= B-\left(1-x_t^A\right)\theta \tag{A.10}$$

where:

$$B\equiv\frac{\left[\left(1-x_t^A\right)w_t-H\right]4\,w_{t+1}{}^2-8\,w_{t+1}\frac{\partial w_{t+1}}{\partial x_t^A}\left[\left(L_{t+1}^A+\theta L_t^A+\frac{1+\theta}{2}\right)4\,w_{t+1}{}^2\right]}{16\,w_{t+1}{}^4} \tag{A.11}$$

The term in the left square brackets is based on (A.9). Note that B<0 since $\frac{\partial w_{t+1}\left(w_t, x_t^A\right)}{\partial x_t^A}$>0 and since $x_t^A>1-\frac{H}{w_t}$. Based on B<0 and on (14):

$$\frac{\partial x_{t+1}^A\left(w_t, x_t^A\right)}{\partial x_t^A}=\frac{\partial x_{t+1}^A\left(w_t, x_t^A\right)}{\partial L_{t+1}^A\left(w_t, x_t^A\right)}\frac{\partial L_{t+1}^A\left(w_t, x_t^A\right)}{\partial x_t^A} = \frac{1}{1-x_{t+1}^A}\left[B-\left(1-x_t^A\right)\theta\right]<-\frac{1-x_t^A}{1-x_{t-1}^A}\theta\,. \tag{A.12}$$

Noting that in the steady state $x_t^A = x_{t+1}^A$ yields that $e_{11}<-\theta<-1$.

$e_{11}<-1$ and $e_{22}<0$, which follows from (18), help showing that the smaller root of (A.8) is below -1. We do so by looking at two different cases. First, if $e_{22}<-1$, then the smaller root of this equation is below -1 because the quadratic at the LHS of (A.8) has a minimum at $\frac{e_{11}+e_{22}}{2}<-1$ since e_{11}<-1 too, as established above. Looking now at the case where -1≤e_{22}<0 we define the LHS of (A.8) as the function $f(\lambda)$ and evaluate it at -1:

$$f(-1) = 1+\left(e_{22}+e_{11}\right)+\left(e_{22}e_{11}-e_{12}e_{21}\right) = \left(1+e_{22}\right)\left(1+e_{11}\right)-e_{12}e_{21}<0 \qquad \text{(A.13)}$$

where the inequality follows from e_{11}<-1, e_{22}≥-1 and e_{12}, e_{21}>0. Finding that f(-1)<0 yields that in that case too the smaller root of (A.8) is below -1.

In order to find the exact values of the two eigenvalues the formulas for e_{12}, e_{21} and e_{22} should be obtained too. A similar procedure to the one that led to (A.8) yields:

$$e_{12} = \frac{\frac{1}{2}-\frac{P^2}{2w^2}-8we_{22}\left(L^A+\theta L^A+\frac{1+\theta}{2}\right)}{4\left(1-x^A\right)w^2}, \qquad \text{(A.14)}$$

where time indexes are removed because it is a steady state. The steady state is in range $\tilde{F}$ and therefore e_{21} and e_{22} are based on differentiating the second row of (18), yielding:

$$e_{22} = \frac{\partial w_{t+1}}{\partial w_t} = \frac{-\left(1-x^A\right)}{8\theta\left(1-x^A\right)w-4H} \qquad \text{(A.15)}$$

and:

$$e_{21} = \frac{\partial w_{t+1}}{\partial x_t^A} = \frac{H\left(1+4w\right)}{8\theta\left(1-x^A\right)^2 w-4H} \qquad \text{(A.16)}$$

Finding the steady state value of w by (A.6) and applying it in (13) yields the steady state x^A and enables calculating e_{11}, e_{12}, e_{21} and e_{22} and therefore finding the actual eigenvalues of the $\left(w_t, x_t^A\right)$ system. For example, the set (H=0.015, P=0.025, θ=1.3) leads to a steady state wage of $\overline{w} = 0.1157$ and to a value of 0.454 for the larger eigenvalue, which implies a monotonic convergence along the saddle path towards the economy's steady state. Note that under this parameter set $\overline{w} > P > H$ and note also that $H < \frac{1}{4(\theta-1)}$, which implies that the upper part of range $\tilde{E}$ is indeed located over positive values of x_t^A. Numerically solving (21) shows that under these parameter values the Saddle path passes through range $\tilde{E}$, as it is numerically found to start at (w_t=0.008907036, $x_t^A = 0$) while (19) yields that $b(w_t)$=0 at w_t=0.0147.

Different parameter values lead to different dynamics. The set (H=0.5, P=0.6, θ=1.3) leads to steady state wage of $\overline{w} = 0.621$ and to a value of -0.004 for the larger eigenvalue. This means that the movement of the economy along its saddle path is oscillatory, where the oscillations from one side of the steady state point to the other are becoming smaller and smaller. In this case once again $\overline{w} > P > H$ and $H < \frac{1}{4(\theta-1)}$.

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