

Real Estate Transactions in Ancient Israel: Excavating Imbedded Options Utilizing Modern Finance

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Abstract:

Certain real estate transactions in ancient Israel involved financial options that seem to be overlooked by the commentaries and the literature in general. The goal of this paper is to investigate the prices of land, and other assets with imbedded options, in Israel during the time these rules were in effect. This paper uses modern financial theory to value these assets (land and houses). It demonstrates the complexity of the pricing system that would have been needed in order to capture true market prices of these assets.

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Comments are welcomed**

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I: Introduction

The rules of selling a piece of land (a field) in Israel are specified in the Hebrew Bible¹ in Leviticus chapter 25 verses 23-28. The original Hebrew text is quoted¹ in the endnote.

23. The land shall not be sold permanently, for the land belongs to Me, for you are strangers and [temporary] residents with Me. 24. Therefore, throughout the land of your possession, you shall give redemption for the land. 25. If your brother becomes destitute and sells some of his inherited property, his redeemer who is related to him shall come forth and redeem his brother's sale. 26. And if a man does not have a redeemer, but he gains enough means to afford its redemption, 27. he shall calculate the years for which the land has been sold, and return the remainder to the man to whom he sold it, and [then] he may return to his inheritance. 28. But if he cannot afford enough to repay him, his sale shall remain in the possession of the one who has purchased it, until the Jubilee year. And then, in the Jubilee year, it shall go out and revert to his inheritance.

These rules were followed after the settlement of the twelve tribes in Israel and when the Jubileeⁱⁱ was celebrated. A land at that time was not sold in perpetuity but rather the land (agricultural field) was returnedⁱⁱⁱ to its original owner in the Jubilee year (every 50 years). The effect of this rule (among others) was that the distributions of land between the tribes returned to their original state every 50 years. Thus, in fact, the selling of a piece of land was not the transfer of the land itself, but the right to work it and own its produce until the Jubilee. Accordingly, the price of the land was essentially^{iv} the present value of the stream of income from the land during the years until the Jubilee. Leviticus chapter 25 verses 13-16 explain the calculation of the price (the original Hebrew text is quoted in the endnote^v):

13. During this Jubilee year, you shall return, each man to his property.
14. And when you make a sale to your fellow Jew or make a purchase from the hand of your fellow Jew, you shall not wrong one another.
15. According to the number of years after the Jubilee, you shall purchase from your fellow Jew; according to the number of years of crops, he shall sell to you.
16. The more [the remaining] years, you shall increase its purchase [price], and the fewer the [remaining] years, you shall decrease its purchase [price], because he is selling you a number of crops.

The original owner of the land was given some buy back rights, referred to as redemption ("geula" in Hebrew), of the field. The original owner could force a future owner to sell the field back to the original owner. In the terminology of modern finance

¹ The Hebrew Bible also referred to as the Torah or Old Testament and includes the books (English Names): Genesis, Exodus, Leviticus, Numbers, Deuteronomy, Joshua, Judges, Ruth, Samuel, Kings, Kings, Chronicles, Chronicles, Ezra, Nehemiah, Esther, Job, Psalms, Proverbs, Ecclesiastes, Song of Solomon, Isaiah, Jeremiah, Lamentations, Ezekiel, Daniel, Hosea.

this right is a "call option". A call option is a financial contract that gives its holder the right, not the obligation, to buy a certain asset (the underlying asset) for a certain price (called the strike price or the exercise price) on or up to a certain date (called the expiration or the maturity date). The right to buy back an asset at a certain price has a monetary value as it may allow its holder to purchase the asset for less than its market value.

Thus when a land owner sells land and receives the rights to buy it back, the land owner in fact is buying a call option from the buyer. The money that is paid by the buyer to the owner is therefore the value of the field minus the value of the call option. The selling of a land was therefore composed of two transactions: selling the land and buying the option. Such a transaction that is in fact composed of a few transactions is referred to in modern finance as a "structured product". The transaction of selling land is a structured product due to the imbedded option (the redemption right) that must be a part of the deal as commanded in the Bible.

The Bible, its commentators and other sources (such as the Mishnah² and the Talmud³) do not seem to acknowledge the value of such an option or to address⁴ it in a discussion of the price the redeemer should pay for the field. Leviticus chapter 25 verse 27 addresses the price (the options' exercise price) the redeemer should pay for the field. The Bible instructs the calculation of the exercise price in verse 27 by saying "He shall calculate the number of years for which he sold the land and return the remainder (excess) to the man to whom he had sold it, and he shall return to his ancestral land".

However, the commentaries in examples of these calculations do not acknowledge the existence of the imbedded option and its effect on the price of the field. Furthermore, the Bible, while explaining the price of the field (Leviticus chapter 25 verse

² The Mishnah (Hebrew: משנה, "repetition"), redacted *circa* 200 CE by Yehudah Ha-Nasi (יהודה הנשיא / "President Judah"), is the first written recording of the Oral Torah of the Jewish people, as championed by the Pharisees, and as debated between 70-200 CE by the group of rabbinic sages known as the Tannaim. It is considered the first important work of Rabbinic Judaism and is a major source of Rabbinic Judaism's religious texts: Rabbinic commentaries on the Mishnah over the three centuries after its composition were then redacted as the Gemara (Aramaic: "learning by tradition"). This explanation is taken from <http://en.wikipedia.org>.

³ The **Talmud** (Hebrew: תלמוד) is a record of rabbinic discussions pertaining to Jewish law, ethics, customs, and history. The Talmud has two components: the Mishnah (c. 200 CE), the first written compendium of Judaism's Oral Law; and the Gemara (c. 500 CE), a discussion of the Mishnah and related Tannaitic writings that often ventures onto other subjects and expounds broadly on the Tanakh. The terms Talmud and Gemara are often used interchangeably. The Gemara is the basis for all codes of rabbinic law and is much quoted in other rabbinic literature. This explanation is taken from <http://en.wikipedia.org>.

⁴ Knhall (2004) points out that the sages of the Talmud were aware of a relation between different types of options (the put call parity). In his opinion the relation is acknowledged in the Talmud in the discussion of redemption of a house. See also Chance (1995) which in his paper about the history of derivatives site the first option to the patriarch Jacob "To start we need to go back to the Bible. In Genesis Chapter 29, believed to be about the year 1700 B.C., Jacob purchased an option costing him seven years of labor that granted him the right to marry Laban's daughter Rachel. ... Some argue that Jacob really had forward contracts, which obligated him to the marriages but that does not matter. Jacob did derivatives, one way or the other. Around 580 B.C., Thales the Milesian purchased options on olive presses and made a fortune off of a bumper crop in olives. So derivatives were around before the time of Christ."

17) states: "...you shall not wrong one another ". This warning seems out of place in this context given in the middle of the stipulation of selling fields. Perhaps this was bothering Rashi⁵ and thus his explanation^{vi} that one should know the number of years until the Jubilee. The buyer and seller should make each other aware of it, so the price will be fair to both.

The effects and subtleties of the imbedded option, as we shall soon see, are much more esoteric than the number of years to the Jubilee which is public knowledge. Being aware of the intricacies of the option imbedded in the deal, it would make sense that the Bible warning "you shall not wrong each other" should be interpreted as a warning of the buyer and the seller to be aware of the option details. Some of these details, as we shall see, are specific to each deal and not common knowledge. While it might be possible to interpret the commentaries figure of speech as if they were aware of the option, it seems a bit of a stretch.

Some aspects of these field transactions are discussed in Buchholz, (1988) and Westbrook (1971). These papers however are completely silent on the imbedded options. In fact, we are not aware of any study that deals with these options and their price. The goal of this paper is to investigate the prices of land in Israel during the time these rules were in effect. The paper uses modern financial theory to value these assets (lands and houses). As we will see, there are a few details involved in the option imbedded in such a transaction.

The rest of the paper is organized as follows. Section II introduces the rules governing a land transaction, in particular, the "buy back" option and under what conditions the buy back is possible. Section II stipulates the contract of the option and also refers to the commentary of the Torah and the Talmud and specifies, to a certain extent, how the conditions for these rules are arrived at. The model used to price the imbedded option is explained in section III. Data on the prices of fields in ancient Israel are not readily available. Thus in lieu of an empirical study, a numerical analysis of the value of the hidden option and its effect on the prices of fields is taken in section IV. This section demonstrates the complexity of the pricing system that would have been needed in order to capture true market prices of these assets in this period. Conclusions and some remarks are offered in section V.

II: Contract Specifications and Attributes

The sale of land in ancient Israel, when the Jubilee was observed, included the provision for the owner or a relative to have the right to buy the land back (redeem) after two years⁶

⁵ Rabbi Solomon ben Isaac, (Hebrew: רבי שלמה יצחקי), better known by the acronym **Rashi** (Hebrew: רש"י), (February 22, 1040 – July 13, 1105), was a rabbi from France, famed as the author of the first comprehensive commentaries on the Talmud, Torah and Tanakh (Hebrew Bible). Acclaimed for his ability to present the basic meaning of the text in a concise yet lucid fashion, Rashi appeals to both learned scholars and beginning students, and his works remain a centerpiece of contemporary Jewish study. His commentaries, which appear in many printed editions of the Talmud and Torah (notably the Chumash), are an indispensable companion to both casual and serious students of Judaism's primary texts. This explanation is taken from <http://en.wikipedia.org>.

⁶ This is deducted from the plural use of "years" in Leviticus chapter 25 verses 15. See vi for the Hebrew text.

(of crops) had elapsed since the sale. The buyer could not have prevented^{vii} the owner (or the owner's relatives) from buying it back. The period of two years, however, is contingent on none of these years being a drought year, so in fact the right to buy back is only after two rainy^{viii} years have elapsed since the sale.

Consequently, at the same time that the seller is selling the land, the seller is also buying from the purchaser a call option. When the land transaction takes place, the money that is being transferred from the buyer to the seller is the price of the field less the price of the option. The buyer in fact is writing (selling) a call option to the owner of the land in which a commitment is given to sell the field back to the owner for a certain price (the exercise price) during the period specified above.

The underlying asset of this option is the land's produce value until the Jubilee. This call option is of an American type (can be exercised during a period of time and not only on one day). However, it can be exercised only after two years ("a delayed option") and up to the Jubilee (at which time the land was returned to its original owner). The provision requiring these two years being rainy years is kind of a weather derivative since the exercise period depends on the weather.

It is important to note that the land and the call option are not separable^{ix}. If a secondary buyer buys the field, this buyer in fact writes a call option to the original owner of the land. The original owner can force the secondary buyer to sell the field back to the original owner. The option that the first buyers wrote the owner is no longer "alive". The process is therefore that the first buyer sells the field to the secondary buyer and at the same time the secondary buyer (essentially) assumes the first buyer's commitment to sell, upon request, the field to the original owner. Therefore, the money that is being transferred from the buyer to the seller is the price of the field less the price of an option. This option, as we shall soon see, may have a different exercise price than the original option.

The exercise price of the option also has a few provisions. The exercise price is calculated based on the number of years until the Jubilee at the time the field was sold and the number of years until the Jubilee from the exercise time. The calculation is mentioned in the Mishnah^x, based on verse 27 in Leviticus chapter 25, and both the Rambam⁷ and Rashi^{xi} elaborate on it using an example like the following:

⁷ **Moses Maimonides**, (March 30, 1135 Córdoba, Spain – December 13, 1204 Fostat, Egypt), was a Jewish rabbi, physician, and philosopher in Andalusia, Morocco and Egypt during the Middle Ages. He was one of the various medieval Jewish philosophers who also influenced the non-Jewish world. Although his copious works on Jewish law and ethics were initially met with opposition during his lifetime, he was posthumously acknowledged to be one of the foremost rabbinical arbiters and philosophers in Jewish history. Today, his works and his views are considered a cornerstone of Jewish thought and study. Maimonides' full Hebrew name was *Moshe ben Maimon* (Hebrew: משה בן מימון) and his Arabic name was أبو عمران موسى بن ميمون بن عبد الله القرطبي الإسراييلي (*Abu Imran Mussa bin Maimun ibn Abdallah al-Qurtubi al-Israili*). However, he is most commonly known by his Greek name, **Moses Maimonides** (Μωσῆς Μαΐμωνίδης), which literally means, "Moses, son of Maimon", like his name in Hebrew and Arabic. Several Jewish works call him Maimoni, מימוני. However, more Jewish works refer to him by the Hebrew acronym of his title and name — **Rabbi Moshe ben Maimon** — calling him the **RaMBaM** or the **Rambam** (רמב"ם). This explanation is taken from <http://en.wikipedia.org>.

If the field was sold at say 1000 and there are 10 years to the Jubilee it means that the product of each year was valued at 100. Hence, if the option is exercised when there are 3 years to the Jubilee the exercise price⁸ will be 300. That is, the original owner should give back the money that was paid to him initially, assuming the sale was for 10 years, for the years that the field will not be with the buyer.

In this example the land is redeemed (the option was exercised) 3 years prior to the Jubilee and the exercise price was 300. The example assumes that the time value of money is zero (no interest is allowed to be charged by the Jewish code of law) and that the uncertainty of the value of the produce, does not increase in the number of the years. Consequently the present value of future crops is as its value at the time of sale. Furthermore, both the Rambam and Rashi do not mention the option's value. To have their examples consistent with the existence of the option one must interpret "If the field was sold at say 1000" (or the Hebrew, Rashi, כגון אם מכרה קודם היובל עשר שנים בעשר ליטרין) as if the field sold refers to the net price of the field (less the option value) .

The fact that produce of further years have greater risk and also a lower present value is ignored in the example of Rashi and Rambam. In fact, they treat the value of each year of produce as being deterministic and not subject to any risk at all and assume that the market price of the product is not changing. Under this assumption of course there is no value to the imbedded option.

Yet, the Mishna does address the case of the field being sold to a third party at a price (per the annual product) different from that of the original transaction. In this case, the uncertainty of market prices is acknowledged, which, of course, means that the option does have a value. The exercise price can also be affected by the price of the field in the resale transaction that had taken place between the original buyer and a new buyer. If the value of the produce from the exercise time until the Jubilee, based on such a transaction, is smaller than the value of the produce based on the original price, the lower exercise price will be used.

The exercise price is therefore the minimum between; the value of the produce based on the original price and the value of the produce based on a secondary transaction that was done from the original sale until the exercise time. The guideline here, as summarized by the Rambam^{xii}, is that the original owner is always being put in an advantageous position.

The price of the option can be implicit in the prices of fields in the market. Consider a field that was sold four years prior to the Jubilee and observe its price two years after it was sold (assume these years were rainy years). Suppose that at the same time (two years prior to the Jubilee) another field is being sold in the market and assume the fields are about the same quality. The field that is sold two years prior^{xiii} to the Jubilee is sold without the rights to buy it back, and will be returned to the original owner in the Jubilee. The other field, that was sold four years prior to the jubilee, can be bought back. The difference between their prices is thus the price of the option.

⁸ We will come back to this calculation in view of the point raised by this paper of the imbedded option. Also this example assumes that the time value of money is zero (no interest is allowed to be charged by the Jewish code) and that the uncertainty of the value of the produce, does not increase in the number of the years. Consequently the present value of future crops is as its value at the time of sale.

The features of the options are such that there could be in the market, two fields that are of equal quality both eligible to be redeemed but with different prices. Consider two fields that were originally sold at different dates where the market price of the crops was different. The exercise price of the options that correspond to the two fields will be different with the exercise price of the field with the lower historical price being lower than the other. The option with the lower exercise price has a higher value. Consequently, the market price (net price) of the field with the lower exercise price will be lower. This price differential can occur also between two fields that originally sold at the same time but one of which was sold again later at a lower price.

Hence, the historical price at which a field was sold or rather the minimum of these prices, if it was sold a few times, affects the current price of the field. This is a hidden attribute of the field. It should be part of the field description and be disclosed to potential buyers. We would like to suggest that the phrase "...you shall not wrong one another" in Leviticus chapter 25 verse 17 might refer to these hidden esoteric attributes. Perhaps this better settles Rashi's difficulties with the placement of this warning in the context of selling a field. After all, these attributes are not exposed to the buyers, while the number of years until the Jubilee is common knowledge.

Each time the field is sold, the buyer, as an integral part of purchasing the field, writes an option to the original owner. Thus the exercise price of the option that is always held by the original owner may be reduced. The current buyer, however, only worries about the price at which they can be forced to sell the field, i.e., about the exercise price of the option they write to the original owner. The price they will be willing to pay for the field is therefore affected only by the exercise price of the option they write.

The next section suggests a pricing model for the option imbedded in a sale of a land. Within this model the phrase (Leviticus chapter 25) "he shall calculate the years for which the land has been sold, and return the remainder to the man to whom he sold it" is explained in a realistic way. In this interpretation the risk of future crops is not assumed away but it is captured by the model. The risk of future crops increases with time and its present value decreases with time.

III: Modeling

There are a few features of the option imbedded in the field transaction that we will relax somewhat in order to simplify matters. We start by assuming that the buy back option can be exercised starting two years after the transaction time, regardless of these years being rainy or drought years. This assumption obviously overestimates⁹ the value of the option.

The option as illustrated above gives the seller the right to purchase back the land. However, since the item sold is actually the stream of income from the field, "according to the number of years of crops, he shall sell to you" and not the field itself, the option is to *purchase back* the remaining *stream of income* (until the Jubilee). In the terminology

⁹ The drought year however should have been not only in the location of the field but all over (see endnote viii) the world. Hence the probability of such an occurrence is low and its affect on the pricing will not be significant. On the other hand incorporating this feature will require us to use pricing methods which are beyond arbitrage pricing and necessitates some inputs about the risk tolerance of the agent in the economy.

of option pricing the underlying asset is the *stream of income*. Financially, it makes sense that the crops of further years have greater uncertainty relative to one of a closer year. We have already alluded to the fact that the commentaries (at least Rashi and the Rambam) in their examples of "he shall calculate the years for which the land has been sold, and return the remainder" do not address the increasing uncertainty of further years of crops. Consequently, in their examples they assign the same present value to crops of different years.

We therefore suggest the following frame of analysis. Assume for a moment that the field could be sold permanently, its price in that case will be the present value of the infinite sequence of the value of the crops. The crops in such an analysis are the counter part of dividends in the case of a dividend-paying stock. Dividends are usually modeled as being paid continuously by a (deterministic) dividend yield which is a percentage of the value of the stock. The redemption option is an option to purchase the stream of dividends from the time the option is exercised until the Jubilee. This way of modeling provides us with a framework that will recognize the increasing uncertainty of crops of further years. Furthermore, it also facilitates, with a slight modification, the use of the classic Black-Scholes model of option pricing. We therefore assume that if the current price of the field is S its price in t years will be $S(t)e^y$ where y follows the normal distribution with expected value of μt and standard deviation of $\sigma\sqrt{t}$. The price of the field, $S(t)$, therefore is a lognormal random variable.

It is indeed the case that, at the ancient time, the field cannot be sold permanently and thus its price cannot be observed¹⁰. However, as we pointed out in endnote iii there are cases where the field can be sold for a very long period. That is the case where the contract specified the number of years to which the field is sold. For example, if the contract specified that the field is sold for 5000 years it will not be returned to the original owner in the Jubilee but after 5000 years. The price of a field in such a contract will be close to the value of a field that is sold permanently. We can also think about a field owner that decides about a strategy of reselling the field for 50 years after each Jubilee. Under this strategy the value of the field will be the present value of the infinite sequences of income streams - the value of the crops.

The crops, like the dividend yield, are assumed to generate a continuous stream of income which is $divS(t)$ at time t , where div is a deterministic constant representing the crops yield. This model therefore captures the risk of crops in future years since $divS(t)$ is a random variable. It also encompasses¹¹ the fact that viewed from the current time, time 0 given $t_1 < t_2$ the dividend at time t_2 possesses a larger volatility ($\sigma\sqrt{t_2}$) than the

¹⁰ Since there is not much hope in conducting an empirical study of market prices of fields (due to the unavailability of these prices we will resort to some numerical examples. The ease of modeling gained is worth the difficulties in an empirical work that is unlikely to be conducted.

¹¹ It is true that we ignore the possibility of a drought year and the sabbatical year which is not counted in the calculation of the value of the crop. However when two rainy years passed from the time of the sell sale, our calculation is correct. We decided not to incorporate the rainy years provision as otherwise the option valuation could not be done only by arbitrage arguments. Hence we settled for this approach which can be done without reference to utility and risk attitude.

volatility ($\sigma\sqrt{t_1}$) of the dividends at time t_1 . The volatility in our model is thus an increasing function of time.

The analysis is done from the point of view of the time of sale which will be denoted as 0. The time until the Jubilee will be denoted as T . Hence at future time t the time to the Jubilee will be $T - t$. The present value of the field at time t , as of time 0, not including the crops (dividends) that are obtained during the time interval $[0, t]$ is¹² $e^{-div(t)}S(0)$. The present value of the perpetual stream of the dividends (crops) is of course $S(0)$.

Thus the present value of the stream of crops from time 0 to t is $S(0) - e^{-div(t)}S(0)$. The present value of the crops from time 0 to $t + 1$ is $S(0) - e^{-div(t+1)}S(0)$ and therefore the present value of the crops from time t to $t + 1$ is

$$S(0) - e^{-div(t+1)}S(0) - (S(0) - e^{-div(t)}S(0)) = S(0)(e^{-div(t)} - e^{-div(t+1)}) \quad \text{Equation 1}$$

It is easy to verify that

$$\left(\frac{d}{dt}\right)S(0)(e^{-div(t)} - e^{-div(t+1)}) = S(0)div(e^{-div(t+1)} - e^{-div(t)}) \quad \text{Equation 2}$$

is negative. Hence the present value of the crops from time t to $t + 1$, as of time 0, is a decreasing function of t . Thus, we see that this model captures the time dimension.

The value of the field at time T , as of time t , not including the crops (dividends) that are obtained during the time interval $[t, T]$ is $e^{-div(T-t)}S(t)$. Applying the same argument as above, the value of the dividends (crops) from time t to T is

$$S(t) - S(t)e^{div(T-t)} = S(t)(1 - e^{-div(T-t)}) \quad \text{Equation 3}$$

which is what the value received is when the option is exercised at time t . This expression is a decreasing function of t and approaches zero, as one expects, when t approaches T .

If the option is exercised, say at time t , the original owner also has to pay a certain amount (the exercise price). The issue at hand now is how to interpret the phrase in Leviticus chapter 27 verse 15 "... he shall calculate the years for which the land has been sold, and return the remainder to the man to whom he sold it, and [then] he may return to his inheritance." If one takes the simplistic approach ignoring uncertainty of the value of the crops then the interpretation is as we saw above in the examples of Rashi and Rambam.

Within the model presented here if the field was originally sold, at time 0 where its market price was $S(0)$, then the original owner sold the crops between time t to T for $S(0)(e^{-div(t)} - e^{-div(T)})$. Therefore, we suggest that the **remainder** that is needed to be returned by the original owner is

$$S(0)(e^{-div(t)} - e^{-div(T)}) \quad \text{Equation 4}$$

¹² This is a standard argument by which the value of an option on a dividend paying stock is calculated.

In our opinion the expression in $S(0)(e^{-div(t)} - e^{-div(T)})$ Equation 4 fits better the Biblical text of

'return the **remainder** to the man to whom he sold it'

or in Hebrew

וְהָשִׁיב אֶת הַדָּף לְאִישׁ אֲשֶׁר מָכַר לוֹ.

The other alternative is to define the **remainder** based on $S(0)(1 - e^{-div(T-t)})$ which is the crops' price¹³ of the next $T - t$ years (as of the redemption time, but based on the price that prevailed at the time of the original sale. The latter expression better suits the interpretation of the original owner buying back the next $T - t$ years of crops based on the price of the field that prevailed at the original time of sale. Thus if the field is redeemed at time t , the original owner return to the buyer the price of the crops paid to the original owner for the last $T - t$ years of crops at the time of the original sale.

By the same argument if the field has been sold again between the original time and the redemption time when its market price was $S < S(0)$, in keeping with the advantage given to the original owner, the remainder is defined by $S(e^{-div(t)} - e^{-div(T)})$.

Based on the above model we can revisit the above example, given in the sprit of the Rambam and Rashi, of a field that was sold ten years prior to the Jubilee for 1000 shekel. If we assume that the field value at that time was 3962.04 and that the dividend yield was about 0.03 the value of the crops for year i is given by $3962.04(e^{-0.03(i-1)} - e^{-0.03(i)})$. The numerical value is stipulated in the table below:

1	2	3	4	5	6	7	8	9	10
117.096	110.2768	107.0177	103.8548	100.7854	97.80678	94.91616	92.11096	89.38867	86.74683

The payoff from a standard call option is $Max(S(t) - K, 0)$ where K is the exercise price and $S(t)$ the price of the underlying asset at the exercising time t . In our case the maximum price the redeemer pays for the field at time t , is what the original owner received for these years. Thus the expression in $S(0)(e^{-div(t)} - e^{-div(T)})$ Equation 4 is the exercise price of this option. The market price of the crops from time t to time T , is $S(t) - S(t)e^{div(T-t)} = S(t)(1 - e^{-div(T-t)})$ Equation 3, $S(t)(1 - e^{-div(T-t)})$. Thus the payoff from the call option, when the field is redeemed, is

$$Max(S(t)(1 - e^{-div(T-t)}) - S(0)(e^{-div(t)} - e^{-div(T)}), 0) \text{ Equation 5}$$

This is not a standard option as its exercise price, $S(0)(e^{-div(t)} - e^{-div(T)})$, is a function of time, and the underlying asset is a random variable (the price of the field) but multiplied by a function of time, e.g., by $(1 - e^{-div(T-t)})$.

This option can be exercised over an interval of time and not just at a point of time. That is, it is an American option not a European option. Moreover, in some instances (when the field is sold originally), the option could only be exercised at least

¹³ Adapting this alternative for the remainder will of course alter the numerical result of the model but not its essence.

two years after the date of sale. For this reason a numerical procedure, such as the Binomial Tree, must be used to value the option. The next section investigates some properties of this option including its relation to a European option and its price sensitivity to certain parameters.

IV: Numerical Results

Assume for a moment that the option described by Equation 4, would have been a European option. This will allow us to find an analytical solution of its value which of course will only serve as a lower bound of the true value of the option. Nevertheless, it facilitates gauging the magnitude of the value of the option as a percentage of the price of the underlying asset.

To this end assume that the option could have been exercised only at time v . An examination of $Max(S(t)(1 - e^{-div(T-t)}) - S(0)(e^{-div(t)} - e^{-div(T)}), 0)$ Equation 5 reveals that in this case the t that appears in the equation is a fixed number v and consequently the payoff of the European option in this case is

$$Max\left(\left(1 - e^{-div(T-v)}\right)S(t) - S(0)(e^{-div(v)} - e^{-div(T)}), 0\right) \text{ Equation 6}$$

As v increases the time to maturity of this option decreases. The value of a regular option decreases as a result of a decrease in the time to maturity. However in this case the exercise price also decreases as the time to maturity decreases which causes an increase in the value of the option. Hence as the time to maturity decreases, even though the value of the underlying asset in this case also decreases, the value of the option may increase.

In order to investigate this affect numerically we first note that $Max\left(\left(1 - e^{-div(T-v)}\right)S(t) - S(0)(e^{-div(v)} - e^{-div(T)}), 0\right)$ Equation 6 can be written as

$$\left(1 - e^{-div(T-v)}\right)Max\left(S(t) - S(0)\frac{e^{-div(v)} - e^{-div(T)}}{1 - e^{-div(T-v)}}, 0\right) \text{ Equation 7}$$

$\left(1 - e^{-div(T-v)}\right)Max\left(S(t) - S(0)\frac{e^{-div(v)} - e^{-div(T)}}{1 - e^{-div(T-v)}}, 0\right)$ Equation 7 stipulates a payoff of a European call option, where the underlying asset is the field *not the crops*, with an exercise price of $S(0)\frac{e^{-div(v)} - e^{-div(T)}}{1 - e^{-div(T-v)}}$, multiplied by $(1 - e^{-div(T-v)})$. Consequently, the price of this European option is obtained by applying the Black-Scholes formula to a European call option, written on the field, with an exercise price of $S(0)\frac{e^{-div(t)} - e^{-div(T)}}{1 - e^{-div(T-t)}}$, and multiplying it by $(1 - e^{-div(T-t)})$.

In Jeremiah^{xiv} Chapter 32 verses 1-9 we are told that the field of Hanamel was redeemed by his uncle Jeremiah. Michelson (2006) calculates that the field was redeemed 14 years eight months and 18 days prior to the Jubilee. That is 14.715 years, assuming 365 days in a year. Michelson also deducted that the price paid for the field (crops) was

185 Shekel and that the area of the field was about 138240 square meters. Assume that the option to redeem this field was not in the money¹⁴ at that time. Thus the price paid is the (pure) crops' price for the period, as once the field is redeemed, by the owner or a relative on his behalf, no option is involved in the process. Hence the price paid satisfies, in our terminology, the next equation

$$185 = S(1 - e^{-div(T-t)}) = S(1 - e^{-div*14.715}) \text{ Equation 8}$$

where S is the market price of the field. Therefore we have the relation

$$div = -0.06779 \ln\left(\frac{-185 + S}{S}\right) \text{ Equation 9}$$

or

$$S = \frac{-185}{-1 + e^{-14.715div}} \text{ Equation 10}$$

Let us assume for example that $div = 0.03$ and hence $S = 518.357$. If the field would have been sold to a third party and not redeemed by Jeremiah than the original owner would still have the option to buy it back. Let us see what would have been the value of such an option.

To this end we need to also assume the time from the original sale to the Jubilee and the price of the field at that time. Let us assume that the field was sold the first time (at time 0) V years prior to the Jubilee and that the field's price at that time was $S(0)$. The current time is therefore $V - 14.715$ the field will return to its original owner in 14.715 years from the current time. If this option would have been of a European type allowing it to be exercised only in t years ($t < 14.715$) i.e., at time $V - 14.715 + t$, its payoff would have been¹⁵

$$(1 - e^{-0.03(14.715-t)}) \text{Max}\left(S(t) - S(0) \frac{e^{-0.03(V-14.715+t)} - e^{-0.03V}}{1 - e^{-0.03(14.715-t)}}, 0\right) \text{ Equation 11}$$

¹⁴ Indeed one can conjecture that if the option was not in the money it was not optimal to exercise it at that time. However, if it is assumed that the option was in the money, generating the relation in

$S = \frac{-185}{-1 + e^{-14.715div}}$ Equation 10 would have necessitated an assumption about the original price of the field at the time it was first sold and . See also the discussion in the conclusion about the timing of the exercising.

¹⁵ Note that the time of the original sale (or rather the length of time to the Jubilee at that time) is known to the original owner (the redeemer). The price that should be used to calculate the exercise price is based on the minimum between the price of the field when it was sold originally and the price(s) of subsequent transactions. The latter is known only to the current owner. Thus each party knows a piece of private information (not publicly available) and it may explain the bible warning of "you shall not wrong one another". It seems that Rashi tries to explain this phrase by the fact that if the price of the field is calculated based on too many years to the Jubilee the buyer is not paying the fair price and vice versa.

Finally to calculate the value of the option we need to assume the value of r - the risk free rate, and the value of sigma σ - the volatility of the continuously compounded rate of return of the price of the field, i.e., the standard deviation of y in $S(t) = S(0)e^y$.

If these parameters are assume to be $r = 0.01$ and $\sigma = 0.025$ the value of the call is a function of V , $S(0)$ and t as stipulated below:

$$\text{Call}(t, S_0, V) = (1 - e^{-0.03(14.715-t)}) (PN(d_1) - e^{-rt}KN(d_2)) \quad \text{Equation 12}$$

where P is the current price of the field and N , d_1 and d_2 are as defined below

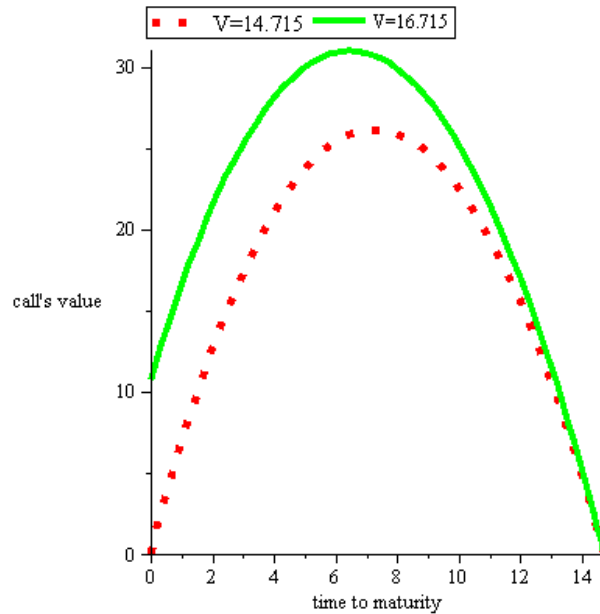
$$N(z) = \int_{-\infty}^z \frac{e^{-\frac{x^2}{2}}}{\sqrt{2\pi}} dx$$

$$d_1 = \frac{\ln \left(\frac{P}{\frac{e^{-0.03(V-14.715+t)} - e^{-0.03V}}{1 - e^{-0.03(14.715-t)}}} \right) + (r + \frac{\sigma^2}{2})t}{\sigma\sqrt{t}}$$

$$d_2 = d_1 - \sigma\sqrt{t}.$$

The graphs below demonstrate the value of the option as a function of t - the time to maturity¹⁶ for $V = 14.715$ and $V = 16.715$ where $r = 0.01$ and $\sigma = 0.025$ and in both cases $S_0 = 518.357$.

¹⁶ Indeed the case of $V = 14.715$ is not consistent with the story of Hanamel as it means that field was first sold while it was sold before and redeemed when $V = 14.715$. Furthermore for this value of V the field could have not been redeemed for at least two years after the sale.



The dotted graph corresponds to $V = 14.715$ and it is apparent that for every t the dotted graph is lower than the solid graph. This is a result of the lower exercise price that corresponds to the option with $V = 16.715$ and thus the higher value of this option. The lower exercise price is a result of the calculation of the value of the *remainder* (in Hebrew *הע'ר*) which decreases as V increases. (See also the above discussion of the interpretation of the *remainder*.)

Furthermore, as indicated, indeed the value of the option does not increase with its time to maturity. Rather the value of the option as a function of its time to maturity possesses a maximum. The maximum values of the options are obtained by solving numerically the following:

$$\text{Max Call}(t, 518.357, 14.715) \quad \text{and} \quad \text{Max Call}(t, 518.357, 16.715)$$

The solutions are:

$$t = 7.233 \quad \text{and} \quad \text{Call}(7.233, 518.357, 14.715) = 26.182, \quad \text{and} \\ t = 6.4577 \quad \text{and} \quad \text{Call}(6.4577, 518.357, 16.715) = 31.00 \quad \text{respectively.}$$

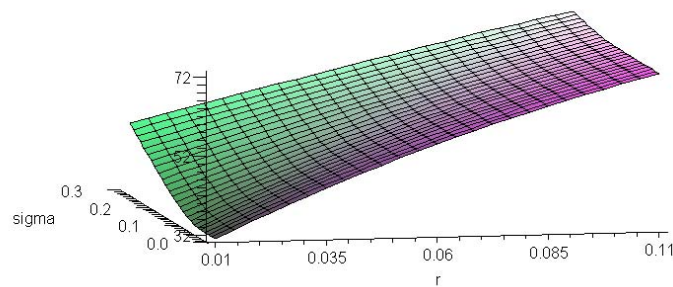
The option that is granted to the original owner is however American. Hence if the field would have been sold to a third party, instead of being redeemed by Jeremiah, the original owner could have exercised the option any time during the next 14.715 years until the jubilee. Hence the value of the American option is higher than the value of a European option that allows exercising only t years prior to the jubilee for every $0 < t < 14.715$. Thus the American option's value is higher than the maximum value of the European option. Hence for the case of $V = 16.715$ the value of the American would have been at least 31.00, that is approximately 16.75% of the price of the crops (6% of the price field). That is the money (net) that would have been transferred from the buyer to the owner, if the field (crops) would have been sold and not redeemed, would have been

at most $185 - 31 = 154$. Yet if this would have been the first time the field was sold the exercise price¹⁷ of the option should be calculated based on the price of the crop for each year: That is based on a price of $518.357(e^{-0.03(i-1)} - e^{-0.03(i)})$ for the i th year as stipulated in the table below

1	2	3	4	5	6	7	8	9	10	11	12	13	14	14.75
15.29	14.84	14.40	13.97	13.56	13.16	12.77	12.39	12.03	11.67	11.33	10.99	10.67	10.35	7.56

and not on a price of the crop per year of $\frac{154.55}{14.75} = 10.44$.

The value of the European option is of course sensitive to the assumption of the parameters value. The figure below demonstrates the value of the option that corresponds to $V = 16.471$ as a function of sigma and r .



Equations (4) and (5) are equivalent only if the option is of a European type. Furthermore, the case at hand is different than the regular call option as both the exercise price and the underlying asset depend directly on t . For these reasons a numerical procedure is utilized to value the option. We utilized the Binomial Tree methods with 470 nodes to solve for the value of the American option for the case $V = 16.471$. The value of the option was 36.105.

One must remember however that when the field is sold the first time the option can be exercised only after two years (as we assume away the prevision of drought years). The numeric calculation of such an option is slightly different than the option above. One must calculate the value of the option in two years, for each state of nature and then use the risk neutral probability to calculate the expected value of the option in two years and discount it with the risk free rate.

V: Conclusions

¹⁷ Since we assumed that the price of the field when redeemed by Jeremiah was as its price when it was first sold the exercise price of the new option would be the same as the exercise price of the original option written by the first buyer to the owner.

There were a few assets in ancient Israel where imbedded options were part and parcel of the deal so that the real estate transaction was in fact a "structured product". This paper focused on land transactions as they are more complex than others. The pricing methods employed in this paper assumed that exercising the option is done at an optimal time. Since redeeming the land is considered to be a deed this assumption may not necessarily describe the behavior of the redeemer.

It is apparent from the discussion above that the prices of fields were dependent on some attributes that were not readily available. The exercise price of the option depends on the original time of the sale as well as the price at which the field was sold in the secondary market. Consequently there could be, in the market, two fields that are identical but their prices would be different since the exercise price of the imbedded option is different. Furthermore some of these hidden attributes of the options are known to the current holder of the field (e.g., the price at which it was last sold) and some to the original owner e.g., the price of the original sale and its time. Both attributes effect the price of the option and hence the price of the field. It might therefore explain why in the middle of the paragraphs in which the rules of the Jubilee are stipulated the bible states: you shall not wrong one another.

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i ויקרא פרק כה

(כג) וְהָאָרֶץ לֹא תִמְכַּר לְצַמְתָּת כִּי לִי הָאָרֶץ כִּי גֵרִים וְתוֹשְׁבִים אִתְּם עִמְדִי:
(כד) וְכִכֹּל אֶרֶץ אֲחֻזְתְּכֶם גְּאֻלָּה תִתְּנוּ לְאָרֶץ:
(כה) כִּי יִמְוֶן אֲחִיךָ וּמָכַר מְאֻחָזְתוֹ וְכָא גֵ' אֵלוֹ הִקְרַב אֵלָיו וְגָאֵל אֶת מִמְכָר אֲחִיו:
(כו) וְאִישׁ כִּי לֹא יִהְיֶה לוֹ גֵ' אֵל וְהִשְׁיִגָה יָדוֹ וּמָצָא פְדִי גְאֻלָּתוֹ:
(כז) וְחָשַׁב אֶת שְׁנֵי מִמְכָרוֹ וְהִשִּׁיב אֶת הַעֲדָה לְאִישׁ אֲשֶׁר מָכַר לוֹ וְשָׁב לְאֲחֻזָּתוֹ
(כח) וְאִם לֹא מָצָאָה יָדוֹ דֵּי הִשִּׁיב לוֹ וְהִנֵּה מִמְכָרוֹ בְּיַד הַקֹּנֵה אִתּוֹ עַד שְׁנַת הַיּוֹבֵל וְנָצָא בִּי כָל נֶשֶׁב לְאֲחֻזָּתוֹ:

ⁱⁱ For an investigation on when and how the counting of the Jubilee was in affect see

אנציקלופדיה תלמודית כרך כב, יובל [טור קיב]

.. אין היובל נוהג אלא בזמן שכל יושבי הארץ נמצאים עליה, שנאמר: וקראתם דרור בארץ לכל ישיביה (ויקרא כה י) שבעה עשר יובלות מנו ישראל משנכנסו לארץ עד שיצאו, בחורבן בית ראשון (ערכין יב ב; רמב"ם שמו"י פ"י ה"ג. ועי' ערכין שם ובהשגות הרשב"י ור"י קורקוס וכ"מ שם אם הכונה שעשו כל היובלות או שהכונה למנין בלבד)...

ⁱⁱⁱ There are cases in which the field is returned after the Jubilee. Such is the case if the contract specifies literally that the field is sold for a period that exceeds the Jubilee. See

תלמוד בבלי מסכת בבא מציעא דף עט עמוד א

.... דאמר רב חסדא אמר רב קטינא: מנין למוכר שדהו לששים שנה שאינה חוזרת ביובל - שנאמר +ויקרא כ"ה+ והארץ לא תמכר לצמיתות - מי שאין שם יובל - נצמתת, יש שם יובל - אינה נצמתת, יצתה זו שאף על פי שאין שם יובל - אינה נצמתת.
and the RAMBAM

רמב"ם הלכות שמיטה ויובל פרק יא הלכה ב

המוכר שדהו לס' שנה אינה יוצאה ביובל שאין חוזר ביובל אלא דבר הנמכר סתם או הנמכר לצמיתות.

^{iv} Legally the purchaser of the field had the rights to use the field as if he\she owned it e.g. to build on it etc. However, the value of these rights might be negligible.

v ויקרא פרק כה

(יד) נְכִי תִמְכְּרוּ מִמְכָר לְעִמִּיתְךָ אוֹ קִנְיָה מִיַּד עִמִּיתְךָ אֶל תּוֹנוּ אִישׁ אֶת אֲחִיו:
(טו) בְּמִסְפָּר שָׁנִים אַחַר הַיּוֹבֵל תִּקְנֶה מֵאֵת עִמִּיתְךָ בְּמִסְפָּר שָׁנֵי תְבוּאֹת יִמְכָר לְךָ:
(טז) לְפִי רַב הַשָּׁנִים תִּרְבֶּה מְקָנְתוֹ וּלְפִי מַעֲט הַשָּׁנִים תִּמְעִיט מְקָנְתוֹ כִּי מִסְפָּר תְּבוּאֹת הוּא מִכֹּר לְךָ:

vi רש"י ויקרא פרק כה

אל תונו - זו אונאת ממון:

(טו) במספר שנים אחר היובל תקנה - זהו פשוטו ליישב מקרא על אופניו על האונאה בא להזהיר, כשתמכור או תקנה קרקע דע כמה שנים יש עד היובל. ולפי השנים ותבואות השדה שהיא ראויה לעשות ימכור המוכר ויקנה הקונה, שהרי סופו להחזירה לו בשנת היובל. ואם יש שנים מועטות וזה מוכרה בדמים יקרים הרי נתאנה לוקח, ואם יש שנים מרובות ואכל ממנה תבואות הרבה ולקחה בדמים מועטים הרי נתאנה מוכר, לפיכך צריך לקנותה לפי הזמן. וזהו שנאמר במספר שני תבואות ימכר לך, לפי מנין שני התבואות שתהא עומדת ביד הלוקח תמכור לו.

This rule is deducted from ^{vii}

vii רש"י מסכת ערכין דף כט עמוד ב

המוכר, בשעת היובל - בזמן שהיובל נוהג.

פחות משתי שנים - אבל לאחר שתי שנים אם רוצה לפדותה פודה בעל כרחו של לוקח ונותן לו לפי מה שמכרה

^{viii} This is deducted from

ויקרא פרק כה פסוק טו

במספר שנים אחר היובל תקנה מאת עמיתך במספר שני תבואות ימכר לך:

The drought year however should have been not only in the location of the field but all over. This is examined and deducted in

תלמוד בבלי מסכת בבא מציעא פרק ט - המקבל שדה מחבירו [דף קו עמוד א]

... מיתבי: היתה שנת שדפון וירקון, או שביעית, או שהיו שנים כשני אליהו - אינו עולה לו מן המנין. קתני שדפון וירקון דומיא דשנים כשני אליהו, מה שני אליהו - דלא הוי תבואה כלל, אף הכא נמי - דלא הוי תבואה כלל. אבל דאיכא תבואה - סלקא ליה, ולא קאמרינן מכת מדינה היא. - אמר רב נחמן בר יצחק: שאני התם, דאמר קרא +ויקרא כה+ במספר שני תבואות ימכר לך - שנים שיש

בהן תבואה בעולם. אמר ליה רב אשי לרב כהנא: אלא מעתה שביעית תעלה לו מן המנין, דהא איכא תבואה בחוצה לארץ! - אמר ליה: שביעית אפקעתא דמלכא היא. אמר ליה מר זוטרא בריה דרב מרי לרבינא: אלא מעתה שביעית לא תעלה לו מן הגירוע,....
It is also dealt with in

תלמוד בבלי מסכת ערכין דף כט עמוד ב

...מי לא תניא: אכלה שנה אחת לפני היובל - משלימין לו שנה אחרת אחר היובל!

See also the Rambam

רמב"ם הלכות שמיטה ויובל פרק יא הלכה י

וצריך שיאכל הלוקח שתי תבואות בשתי שנים ואח"כ יגאל שנאמר שני תבואות, לפיכך אם היתה אחת משתי השנים שביעית או שנת שדפון או ירקון אינה עולה מן המנין

רמב"ם הלכות שמיטה ויובל פרק יא הלכה טו

מכר שדהו לראשון וראשון מכר לשני ושני לשלישי אפילו מאה זה אחר זה בשנת היובל תחזור לאדון הראשון, שנאמר בשנת היובל ישוב השדה לאשר קנהו מאתו לאשר לו אחוזת הארץ.

משנה מסכת ערכין פרק ט משנה א

המוכר את שדהו בשעת היובל אינו מותר לגאול פחות משתי שנים שנאמר (ויקרא כ"ה) במספר שני תבואות ימכר לך היתה שנת שדפון וירקון או שביעית אינה עולה לו מן המנין נרה או הובירה עולה לו מן המנין רבי אלעזר אומר מכרה לו לפני ראש השנה והיא מלאה פירות הרי זה אוכל ממנה שלש תבואות לשתי שנים:

רש"י מסכת ערכין דף כט עמוד ב

המוכר, בשעת היובל - בזמן שהיובל נוהג.

פחות משתי שנים - אבל לאחר שתי שנים אם רוצה לפדותה פודה בעל כרחו של לוקח ונותן לו לפי מה שמכרה כדכתיב (ויקרא כה) וחשב את שני ממכרו שמחשב כמה שנים משמכרה עד היובל ומחלק הדמים לפי השנים כגון אם מכרה קודם היובל עשר שנים בעשר ליטרין נמצא שמכר פירות של כל שנה ושנה בליטרא שהרי סתם מכירה אינה אלא עד היובל הלכך אם שהתה ביד לוקח ה' שנים ואח"כ בא מוכר לגאולה מנכה לו לוקח ה' ליטרין ליטרא לכל שנה שאכלה שכך עלה חשבון כשיוצא מתחילה.

אינו עולה מן המנין - השתי שנים דהא שני תבואות כתיב שתי שנים הראויין לתבואה תשהה ביד לוקח אבל היתה שנה הראויה לתבואה ונרה ולא זרעה או הובירה שהניחה בורה שאפ"ל ניר לא עשה בה איהו אפסיד אנפשיה ועולה לו במנין שתי השנים.

The Rambam also explains it very similarly:

פירוש המשנה לרמב"ם מסכת ערכין פרק ט משנה א

דין מוכר שדה אחוזה כפי שנתבאר בכתוב הוא כמו שאסדיר לך, והוא, אם מכר האדם משדה אחוזתו במחיר מסויים מתחלקים אותם הדמים לפי מנין השנים שנשתיירו עד היובל, ויודע כמה ראוי לכל שנה, ולפי מנין מה שנשארה ביד הקונה מנכין לו מן הקרן, המשל בזה, אם מכר שמעון לראובן קרקע במאה דינרין, והיה הנשאר עד היובל מיום המכירה עשר שנים, ודר בה ראובן ואכלה ארבע שנים, ואחר כך רצה שמעון לפדות את שדהו, הרי זה מחזיר לראובן ששים דינר וזה הוא גרעון כסף, ועל דרך זו תדון, ומחשב עמו בעת שרוצה לפדות את שדהו על כל החדשים והימים שאכל כפי שמראה החשבון.

רמב"ם הלכות שמיטה ויובל פרק יא הלכה טז

מכרה לראשון במאה דינר וראשון לשני במאתים ורצה האדון לגאול אינו מחשב אלא עם הראשון שנאמר לאיש אשר מכר לו, מכרה לראשון במאתים וראשון לשני במאה ה"ז מחשב עם האחרון, וכן אם מכר במאה והשביחה ביד הלוקח והרי היא ראויה להמכר במאתים מחשב לפי מה שמכר, ואם מכרה במאתים והכסיפה והרי היא ראויה להמכר במאה מחשב לפי מה שהיא, ולעולם מיפיים כח מוכר שדה אחוזה ומריעין כח הלוקח

^{xiii} If the field is sold one year prior to the Jubilee the buyer gets the produce of the second year and not the original owner. The field is then returned to the original owner, a year after the jubilee at no cost. This is mentioned in:

תלמוד בבלי מסכת ערכין דף כט עמוד ב

מי לא תניא: אכלה שנה אחת לפני היובל - משלימין לו שנה אחרת אחר היובל!

and the Rambam also says:

רמב"ם הלכות שמיטה ויובל פרק יא הלכה יב

מכרה שנה אחת לפני היובל הרי הלוקח אוכל אותה שנה שניה אחר היובל שנאמר שני תבואות.

There is also a case where the buy back rights can be forced prior to the elapsed time of two years from the transaction time. This is the case of an unproductive field, see the RAMBAM says:

רמב"ם הלכות שמיטה ויובל פרק יא הלכה יג

מכר נקעים מלאים מים או סלעים שאינם ראויין לזריעה ה"ז פודה בפחות משתי שנים שנאמר במספר שני תבואות, שדה הראוי לתבואה הוא שאינה נגאלת אלא אחר שתי שנים, ואם לא גאלה אע"פ שאינה ראויה לזריעה חוזרת לבעלים ביובל.

(א) הדבר אשר היה אל ירמיהו מאת יק' נק' <בשנת> בשנה העשירית לצדקיהו מלך יהודה היא השנה שמה נה עשרה שנה לנבוכדנאצר:

(ו) וי' אמר ירמיהו היה דבר יק' נק' אלי לאמר:

(ז) הנה חנמאל בן שלם ד' דב' בא אליך לאמר קנה לך את שדי אשר בענתות פי לך משפט הגאולה לקנות:

(ח) ונב' א' אלי חנמאל בן ד' די כדבר יק' נק' אל חצר המטרה וי' אמר אלי קנה נא את שדי אשר בענתות אשר בארץ בנגמיו פי לך משפט הירשה ולך הגאולה קנה לך נאדע פי דבר יק' נק' הוא:

(ט) ואקנה את השדה מאת חנמאל בן ד' די אשר בענתות ואשקלה לו את הכסף שבעה שקלים ועשרה הכסף:

(י) ואכת' ב' בספר נאחת' מ' ואעד עדים ואשק' ל' הכסף במ' אזנים:

(יא) ואקח את ספר המקנה את החתום המצנה והחקים ואת הגלוי

6. And Jeremiah said: The word of the Lord came to me, saying:

7. Behold, Hanamel, the son of Shallum your uncle, is coming to you, saying: Buy for yourself my field that is in Anathoth, for the right of redemption is yours to buy it.

8. Then Hanamel, my uncle's son, came to me in the prison yard, according to the word of the Lord, and said to me; Please buy my field that is in Anathoth, that is in the country of Benjamin, for the right of inheritance is yours, and you have the right of redemption; buy it for yourself." And I knew that this was the word of the Lord.

9. So I bought the field from Hanamel, my uncle's son, which was in Anathoth, and weighed him the money, seven shekels and ten pieces of silver.

10. And I wrote a bill of sale and signed it and took witnesses, and weighed the silver on a scale.