Strategic Disclosure with Fake and Real News*

Sivan Frenkel
Tel Aviv University†

Ilan Guttman
New York University‡

Ilan Kremer
Hebrew University and
University of Warwick§

September 29, 2024

Abstract

We develop a model in which information about a firm's value can be obtained from two sources: (i) voluntary disclosure by a firm's manager, if she is informed, and (ii) an exogenous source - news - with uncertain accuracy, i.e., who may be real or fake. We focus on the case where the accuracy of the news is positively correlated with the manager's information endowment, and the manager makes the disclosure decision without knowing the news. In contrast to the existing theoretical literature, our model does not admit a pure-strategy disclosure equilibrium. Instead, the equilibrium is characterized by two thresholds: an informed manager never discloses values below the lower threshold, always discloses values above the higher threshold, and employs a mixed strategy with a monotonically increasing probability of disclosure for values between the two thresholds. We show that the presence of news crowds-out managerial disclosure.

---

*We thank...and numerous seminar and conference participants.

†Email: frenkels@tauex.tau.ac.il. Frenkel acknowledges financial support from the Israel Science Foundation (grant No. 2426/21), the Henry Crown Institute of Business Research in Israel and the Jeremy Coller Foundation.

‡Email: iguttman@stern.nyu.edu.

§Email: ikremer@mail.huji.ac.il.

1 Introduction

The theoretical literature on corporate voluntary disclosure focuses on settings in which all value-relevant information regarding the firm is held by a "sender" - often interpreted as the firm's manager. The manager chooses whether to disclose her information to a "receiver" (capital market) or conceal it. Financial markets, however, operate within a multifaceted information environment characterized by numerous information sources, including analysts, media outlets, and social media platforms, among others. This outside information may encompass accurate and pertinent data, yet it can also encompass hearsay and unverified rumors. In the contemporary landscape of social media and sensationalist news platforms, distinguishing between the two can prove challenging.

In this paper, we consider the implications of incorporating such additional source of unverified information to a standard voluntary disclosure setting with uncertainty about information endowment (Dye, 1985, and Jung and Kwon, 1988 - hereafter DJK). The innovation of our setting is that this additional source of information, which we label as "news," may be "real" and give a precise reflection of the firm's value, or may be "fake," that is, independent of this value. In our basic setting, we assume that this additional source of information is always present, but the capital market cannot distinguish between real and fake news.[1] Our basic setting assumes that when the manager is uninformed news must be fake, while when the manager is informed news can be either fake or real.[2] This property seems natural for information that is produced within the firm (e.g., internal accounting data, clinical trial outcomes, geological survey results, R&D outcome).

We show that disclosure equilibrium in the presence of news is qualitatively different from the one without it. In particular, we find that the standard pure-strategy threshold disclosure equilibrium does not exist in this case. In the equilibrium of our setting, informed managers withhold significantly negative information, disclose significantly positive information, and use a mixed disclosure strategy for intermediate levels of information. Within this strategy, the probability of disclosure monotonically increases in the realized value of the manager's

[1] In Section 5.2 we show that the results hold qualitatively also for the case where news does not always arrives.

[2] Section 5.1 shows that our qualitative results are unchanged even when news may be real when the manager is uninformed, as long as news is sufficiently more likely to be real when the manager is informed.

private information. The key to our findings is that in equilibrium, the absence of managerial disclosure informs the market not only about the firm's value but also sheds light on the *quality* of the additional signal, i.e., the probability that news is real. Consequently, pure-strategy equilibrium does not exist as long as the accuracy of news is significantly higher when the manager is informed compared to when she is uninformed.

The paper's contribution to the theoretical literature on voluntary disclosure is two-fold. First, we study the implications of multiple information sources, particularly the ability to evaluate the quality of one source based on the content of another. When presented with unverified information from a specific source, it is natural to question why, if this information is authentic, it was not divulged by other sources, thereby assessing its credibility. Our analysis reveals that these intuitive considerations about additional information (news) has both qualitative and quantitative effects on voluntary disclosure. Qualitatively, they give rise to a mixed-strategy equilibrium, while quantitatively, they reduce the amount of managerial voluntary disclosure. Second, our model is the first to demonstrate that the pure-strategy equilibrium, which was the only equilibrium identified in existing literature on static voluntary disclosure settings, does not exist in some realistic settings. In particular, augmenting DJK's standard disclosure setting, with uncertainty about information endowment, with an additional source of information, whose precision is correlated with the manager's information endowment, precludes pure-strategy equilibrium.

The exogenous signal in our model resembles the nature of social media platforms, where users continuously generate a stream of opinions and information, but the veracity of this information remains uncertain, as it's challenging to distinguish between genuine insights and false content online.[3] Under such interpretation, our paper also contributes to the literature on social media and financial markets.[4] Existing literature has shown that social media has become an effective information source in financial markets.[5] However, the literature

[3]Kakhbod et al. (2023) examine financial influencers on the StockTwits social media platform, and find that while 28% of them are skilled, generating a positive abnormal return, 56% have negative skill, generating negative abnormal return. Influencers with negative skill actually have more followers and influence on retail trading than skilled influencers..

[4]For a recent survey, see Cookson et al. (2024).

[5]For example, Chen et al. (2014) and Avery et al. (2015) show that stock recommendations on social platforms are informative about future stock prices. Renault (2017) shows that investor sentiment on such platforms can predict intraday returns, and Bartov et al. (2018) show that such sentiment predicts earning announcement returns .

on the equilibrium effect of social media on market quality and price efficiency is scarce. Previous literature has shown that social media might inform the management, and thus create a feedback effect (Cookson et al., ming), and that it might have an ambiguous effect on price efficiency through its effect on information acquisition (Dessaint et al., ming). To the best of our knowledge, however, this paper is the first to analyze theoretically how the presence of social media news affects information dissemination by anther market participant. Specifically, we show that the presence of an additional signal decreases corporate disclosure.

The distinct disclosure strategy that we present in this paper, not only offers novel theoretical insights into disclosure equilibria but may also help in reconciling theoretical predictions with observed disclosure practices. Specifically, corporate voluntary disclosure exhibits significant variation concerning the values disclosed and the probability of disclosure, both across different firms and over time by a single firm.[6] This variation seems to be too large to be explained only by different firm characteristics or changes in the economic environment over time, as suggested by the existing theoretical literature. A mixed disclosure strategy is an alternative source of such observed variation, and our model is the first to present such equilibrium pattern in a setting with uncertainty about information endowment.

We next describe the setting of our model, present the main results, and elucidate the intuition for these results. Our model's starting point is the DJK framework, in which a firm's manager obtains value-relevant information with a commonly known probability. An informed manager has the option to disclose this information to the market credibly and costlessly. The manager's objective is to maximize the (expected) firm price.[7]

We extend this framework by introducing an additional source of information: subsequent to the manager's disclosure or lack thereof, the market receives a noisy public signal, termed "news," regarding the realized value. With some probability news is "real," and gives an accurate reflection of the true value; with some probability it is "fake," constituting of noise . Market participants cannot tell whether the news is real or fake. Disclosure is credible, and hence if the manager discloses the firm's price is set at the disclosed value. In the absence of

[6]For a relevant survey of the nature and determinants of voluntary disclosure see Section 3.2 of Beyer et al. (2010)

[7]For simplicity, we assume that the manger cares only about the price at the end of the game. Assuming that the manager also cares about the price following the disclosure stage and before the arrival of public news does not change the nature of the equilibrium: see Section 5.2.

managerial disclosure, the price is set by the risk-neutral market to the expected value given the news and the fact that the manager did not disclose.

In our base model, we posit that when the manager is uninformed, news is invariably fake. When the manager is informed, news can be either real or fake, with some probability. We first establish that in such settings, the standard pure-strategy threshold disclosure policy is never an equilibrium. To gain an intuition for this result, suppose (by contradiction) that the manager follows a pure-strategy threshold disclosure strategy, in which she discloses (when informed) only when the realized value is greater than a threshold y. Consider two informed managers with values $y - \epsilon$ and $y + \epsilon$ where ϵ is positive and $\epsilon \to 0$. If the two types disclose then their payoff is equal to their value, which in the limit is the same value, y. If they withhold, then with some probability the public signal is imprecise (fake news) and with some probability precise (real news). The payoff of the manager following a fake signal is independent of her type, implying that any potential difference in the payoff following no-disclosure between types $y - \epsilon$ and $y + \epsilon$ is due to the pricing following a precise public signal.

Consider the manager's payoff from no disclosure when the public signal is precise. Under an assumed threshold disclosure policy, a public signal that is greater than y cannot be true if the manager is informed (since if she were informed she would have disclosed it). Recall that (by assumption) the public signal cannot be true also if the manager is uninformed. Therefore, observing no managerial disclosure and a public signal that is higher than the disclosure threshold implies that the public signal is false. A false public signal, in turn, implies that the manager is more likely to be uninformed (due to the positive correlation between the manager's information endowment and the signal's accuracy). In the paper we show that this discrete increase in the probability that the manager is uninformed around the threshold value leads to a discrete jump in the price following no disclosure and a signal s around the signal $s = y$. This, in turn, implies that a manager of type $y + \epsilon$ enjoys a strictly higher expected payoff from no disclosure compared to a manager of type $y - \epsilon$. This contradicts the assumption of a threshold equilibrium in which type $y + \epsilon$ discloses and $y - \epsilon$ withholds.

We next show that the equilibrium is always characterized by two thresholds of the manager's private information, where an informed manager never discloses if her type is below the low threshold and always discloses if it is above the high threshold. The intuition for this is as follows: due to the additional public signal, the expected payoff of the manager from no-disclosure is increasing in her type. However, because news may be false, this expected payoff is less sensitive to the manager's type than her payoff from disclosure. Clearly, an informed manager with sufficiently low type strictly prefers to withhold information and be pooled with uninformed managers and an informed manager with sufficiently high type strictly prefers to disclose.

The intuition for the mixing region is more involved, and builds on the same rationale we used to demonstrate that a pure strategy threshold disclosure cannot be part of an equilibrium – following no disclosure, there cannot be a discrete jump in the price of two adjacent types, as such a change would create a positive jump in the manager's payoff from no-disclosure. The mixing probabilities are determined so that the manager remains indifferent between disclosing and withholding throughout the mixing interval. Since an informed manager's payoff from disclosure increases in her type at a rate of one, indifference requires that her expected payoff from no disclosure also increases at a rate of one. Recall that the manager's payoff from no disclosure conditional on the public signal being fake is independent of the manager's type. In order for the manager's expected payoff from no disclosure to increase at a rate of one, her payoff from no-disclosure conditional on the public signal being true should increase at a rate greater than one (in particular, a rate of one over the probability of the news being true). For that to hold, the market's beliefs that the manager is uninformed should be monotonically increasing in the news, which necessitates the manager's probability of disclosing to be monotonically increasing in her type over the mixing region.Analysis of the equilibrium reveals that for any parameter values and any distribution, the lower disclosure threshold is always higher than the disclosure threshold in the case where the public signal is always pure noise – which is equivalent to the setting of DJK. This implies that the introduction of a partially-informative public signal crowds-out managerial voluntary disclosure. This however, does not imply that the overall information available to the market is lower in the presence of unverifiable public signal, as the public signal adds more information beyond the manager's disclosure (and we cannot use Blackwell informativeness criterion).

In an extension of the base model we allow for the public signal to be precise even in cases that the manager is uninformed. While the assumption that any information about the firm must also be known to the manager in advance may fit many types of internal information (accounting, operational, technological, clinical trials, firm's exploration, strategy information), there are additional types of information, such as market conditions, information about competitors or suppliers, which may be known to outsiders, and reported in the news, even when they are unknown to the manager. This extension allows us to consider also these types of information. We show that our results and equilibrium characteristics extend to such settings, provided the probability of a precise public signal when the manager is uninformed is not too high. In another extension, we show that equilibrium characteristics are the same in a setup where the public signal arrives only with a partial probability.

Finally, for completeness, we also study an extension in which the manager knows the public signal's realization when making her disclosure decision. Unlike our main setting, the manager faces no price uncertainty in this case. We find that an informed manager discloses if an only if the firm value exceeds a threshold that depends on the public signal. For public signals below a certain threshold, the manager's disclosure threshold is increasing in the public signal. However, for sufficiently high public signals, the manager's disclosure threshold becomes independent of the public signal. The latter effect arises due to the same intuition as before: high public signals that are not disclosed must be false, and are therefore ignored by the market.

The end of this section includes a short literature review. The outline of the remaining sections is as follows. In the next section we present the model's setup. Section 3 presents several definitions needed for the analysis of the model. It also analyzes two useful benchmarks, in which news is either completely uninformative, or uninformative only when the manager is uninformed. Section 4 presents the main results of the paper: it shows that there is no pure-strategy threshold equilibrium, and characterizes the two-threshold equilibrium that contains a mixing interval. Section 5 analyzes three extensions of the model: a model where news may be real even if the manager is uninformed, a model in which news may arrive or not, and a model in which the manager observes the realization of news before her disclosure decision.

Related theoretical literature. Much of the literature on voluntary disclosure presents models with a unique equilibrium characterized by a single threshold, whereby an informed sender discloses her private information ("type") only if it is above this threshold and withholds information otherwise. One of the most notable frictions that prevent unraveling and full disclosure pioneered by DJK, is that the capital market is uncertain regarding whether the manager possesses private information or not. In the DJK setting, the only equilibrium has a pure-strategy disclosure strategy, characterized by a single disclosure threshold. There are a relatively small number of papers that consider additional frictions, and identify pure-strategy disclosure equilibria in which the type space is partitioned to several connected intervals, such that in each interval an informed sender always discloses or always withholds her information, but never uses a mixed strategy (see, for example, Dutta and Trueman, 2002; Suijs, 2007; Beyer and Guttman, 2012; Bond and Zeng, 2022).[8] One contribution of our paper is to show that a mixed-strategy equilibrium may be introduced to the above literature by adding an additional source of information, whose precision is correlated with the manager's information endowment

Our paper is also related to models that analyze voluntary disclosure with additional information. In two related papers, Langberg and Sivaramakrishnan (2008, 2010) analyze voluntary disclosure models in which an analyst also provides information about the firm. In contrast to our model, in their papers the analyst's information is completely orthogonal to the manager's information. Einhorn (2018) explores the effect of additional information sources on voluntary disclosure, but in her model the manager's strategic considerations are regarding a noise term and not the fundamental value of the firm. Banerjee et al. (2024) offer a model that combines informed trading and corporate voluntary disclosure. Quigley and Walther (2024) present a model of costly disclosure with an additional signal designed by a regulator ("stress test").

Fischer and Stocken (2023) and Bertomeu et al. (2024) offer communication models in which receivers observe a message that is precise with some probability and distorted with another, similarly to our exogenous signal. Fischer and Stocken (2023) present a cheap talk

[8] In addition, some papers have considered settings where the sender has several pieces of information (signals), and thus its type-space is multidimensional (Pae, 2005; Song Shin, 2003, 2006; Guttman et al., 2014). In these models, the sender discloses a subset of her available signals, but still uses a pure strategy.

model in which the receiver does not know whether the sender attempts to inform or misinform. Bertomeu et al. (2024) present a disclosure model that follows the DJK framework, but in which disclosed information may be pure noise (miscommunicated) with an exogenous probability.

Grubb (2011) presents a two-period DJK model where the sender possesses private information about her persistent probability of acquiring information. The sender has an incentive to develop a reputation for having a low probability of information acquisition to maximize the price following no-disclosure in the second period. Grubb (2011) finds that in equilibrium the "weak" type, who has a high probability of information endowment, uses a continuous and increasing mixed strategy over a specific range of values. While the setup and takeaways of Grubb (2011) and our papers differ significantly, both present mixed strategies equilibria resulting from an additional public signal that informs the receiver about multiple dimensions of the sender.

The closest papers to this one are Frenkel et al. (2020) and Libgober et al. (2023). Both papers offer disclosure models that follow the DJK framework with an additional exogenous signal, and examine the effect of such signal on voluntary disclosure. In Frenkel et al. (2020), in contrast to the present paper, the exogenous signal arrives with some probability, but when it arrives it is always precise. The arrival of the signal may be correlated with the information endowment of the manager. In Frenkel et al. (2020) there is no learning about the accuracy of the exogenous signal, which is the main driver behind our results. Frenkel et al. (2020) find a pure-strategy disclosure equilibrium, and focus on the effect of the signal's arrival probability on the overall information that is available to the market.

Libgober et al. (2023), in a contemporaneous paper, offer an exogenous signal that may be precise or pure noise, as in this paper. They present a dynamic disclosure model, in which the manager can disclose information both before and after the signal was realized. In their model, the probability that the exogenous signal is precise is independent of the manager's information endowment (similar to particular case of $\ell = 1$ in Section 5.1). In their setting (and as we show in Section 5.1), the equilibrium is always a pure-strategy, single threshold equilibrium .

2 Model

Our model builds on a standard DJK framework (Dye, 1985; Jung and Kwon, 1988), and adds a public signal with uncertain precision.

Fundamentals. There is a single firm, whose fundamental value X is unknown. It is common knowledge that X is distributed according to some CDF F. For ease of exposition, we assume that (1) $X \in \mathbb{R}$ – it is easy to accommodate upper and lower bounds – and that (2) F is continuous and twice differentiable and denote its PDF by f.

Information. There are two potential sources of information in the model: corporate voluntary disclosure and news. With probability q there is an "information event," and the manager of the firm becomes informed about X. An informed manager can choose whether to disclose it or not. As in DJK, any disclosure is truthful and costless, and an uninformed manager cannot credibly disclose the fact that she is uninformed.

The main innovation of this model is that the market observes an exogenous signal about X, denoted by S, which can be either precise or false. We refer to this public signal as "news" but it can also be other sources of information, for example, a result of an analyst's report.[9] In the base version of the model, we assume that in the absence of an information event, no information about X can be learned, and hence the signal S is pure noise. If an information event does occur, implying that the manager knows X, the signal S is precise with probability p ("real news"), and with probability $1-p$ the signal is pure noise ("fake news"). This information environment isnatural for information that is produced within the firm, such as internal accounting data, clinical trial outcomes and geological survey results. Formally, define a "false" random variable Z, such that $Z \propto X$ but independent of X, and a probability $p \in [0,1]$. The signal S is as follows:

$$S \mid \text{Manager Informed} = \begin{cases} X & \text{w.p. } p \\ Z & \text{w.p. } 1-p \end{cases} \qquad S \mid \text{Manager Uninformed} = Z \tag{1}$$

[9] We abstract from strategic aspects in the dissemination of the public signal, e.g., due to conflict of interests of media outlets, analysts, etc., which are beyond the scope of this paper.

We explore other possible dependencies between the manager's information endowment and the quality of news in Section 5.1.

Informed Manager's Decision. The manager's objective is to maximize the expected price after her disclosure (or lack of) and the arrival of the news. In Section 5.2 we analyze the cases where news may not arrive, and where the manager cares also about the price following disclosure but before the arrival of news. When making the disclosure decision, the manager does not know the realization of the signal S.****Consider removing the footnote*****[10] In section 5.3 we complement the analysis by analyzing the case where the manager knows the value of S at the time of her disclosure decision.

Pricing. The market, or investors, observe the manager's disclosure (or lack of) and the news, and set the price of the firm. We assume that investors are Bayesian and risk-neutral, and thus the price of the firm equals its expected value given all available information.

Timeline.

1. X, Z, and S are randomly drawn by nature. An information event occurs with probability q.
2. If an information event occurs, the manager observes X and chooses whether to disclose it or not.
3. The public signal S (news) is publicly revealed.
4. The firm's price P is set and the manager's payoff is determined according to P.

[10] Another possible interpretation is that investors have additional private information sources, such as a buy-side analyst, and the manager does not know the content of this information when making her disclosure decision.

3 Equilibrium - Preliminaries

3.1 Notation and Definitions

Events. Denote the event of "no-disclosure" by ND. Denote the event where an informed manager deliberately chooses not to disclose (i.e., withholds) as NDI.

Strategies. The manager's disclosure strategy depends on her private information only. Denote the (mixed) strategy of a type-x manager by $\alpha(x) : [0,1] \to [0,1]$, where

$$\alpha(x) \equiv \Pr\left(\text{type } x \text{ withholds} \mid \text{ manager informed}\right).$$

An informed type x discloses with probability $1 - \alpha(x)$. Denote the manager's equilibrium strategy by $\alpha^*(x)$.

A particular type of strategy is a "threshold strategy," which is characterized by a threshold x^* and denoted by θ_{x^*} such that

$$\theta_{x^*} : \alpha(x) = \begin{cases} 1 & x < x^* \\ [0,1] & x = x^* \\ 0 & x > x^*. \end{cases} \tag{2}$$

Observe that θ_{x^*} does specify the strategy of the threshold type x^* - $\alpha(x^*)$. We will specify the strategy of the threshold type only when that matters for the results, and in all other cases θ_{x^*} refers to the unique strategy up to $\alpha(x^*)$.

Expected Values. The ex-ante probability that an informed manager who uses strategy α withholds information is

$$W(\alpha) \equiv \Pr(\text{NDI}; \alpha) = \int_{-\infty}^{\infty} \alpha(x) \cdot f(x)\, \mathrm{d}x. \tag{3}$$

Suppose the market does not observe the public signal. Then the price (expected value) conditional on no-disclosure (without an additional information) is

$$\begin{aligned} E\left[X \mid \mathrm{ND};\alpha\right] &= \frac{(1-q)E\left[X\right] + q\cdot W(\alpha)\cdot E\left[X \mid \mathrm{NDI};\alpha\right]}{1-q+q\cdot W(\alpha)} \\ &= \frac{(1-q)E\left[X\right] + q\cdot \int_{-\infty}^{\infty} x\cdot\alpha(x)\cdot f(x)\,\mathrm{d}x}{1-q+q\cdot W(\alpha)}. \end{aligned} \tag{4}$$

As we shall see in the next section, pricing following no-disclosure and specific news will depend on the market belief about the authenticity of news. It is therefore useful to define the expected value conditional on no-disclosure and a belief that the news is fake. This price does not depend on s, but is also different than $E\left[X \mid \mathrm{ND};\alpha\right]$; News is less likely to be fake if the manager is informed, and hence fake news imply that the manager is informed with probability that is less than q. Formally,

$$\Pr(\text{Manager Informed} \mid S = Z) = q\frac{1-p}{1-p\cdot q} < q,$$

and the price conditional on no-disclosure and fake news, denoted by $P_f^{\mathrm{ND}}(\alpha)$, is

$$P_f^{\mathrm{ND}}(\alpha) \equiv E\left[X \mid \mathrm{ND}, S = Z;\alpha\right] = \frac{(1-q)E\left[X\right] + q(1-p)W(\alpha)\cdot E\left[X \mid \mathrm{NDI};\alpha\right]}{1-q+q(1-p)W(\alpha)}. \tag{5}$$

Note that $P_f^{\mathrm{ND}}(\alpha) > E\left[X \mid \mathrm{ND};\alpha\right]$ if and only if no-disclosure is bad news, that is, $E\left[X \mid \mathrm{ND};\alpha\right] < E[X]$.

3.2 Risk-Neutral Pricing

This section describes how risk-neutral Bayesian investors determine the firm's price given the news and their beliefs about the manager's disclosure strategy. If the manager discloses x then $P = x$ and investors ignore the news. When there is no disclosure, the price depends on the news. After observing no-disclosure, investors update their belief regarding the value of the firm X as well as the accuracy of S. Pricing of the firm is affected by both. Specifically,

$$P^{\mathrm{ND}}\left(s,\alpha\right) = \hat{\rho}(s)\cdot s + \left(1-\hat{\rho}(s)\right) P_f^{\mathrm{ND}}(\alpha), \tag{6}$$

where $\hat{\rho}(s)$ is the updated (posterior) probability that the news is accurate.

For a given disclosure strategy, some public signals are more likely to be accurate than others; for example, when a type y manager discloses with probability one (i. e., $\alpha(y) = 0$), then $s = y$ together with no-disclosure occur only if the news is fake, i.e. $\hat{\rho}(y) = 0$ (remember that if the manager is uninformed the news must be fake). In general,

$$\hat{\rho}(s) \equiv \Pr\left(X = s \mid \mathrm{ND}, S = s\right) = \frac{q \cdot p \cdot \alpha(s)}{1 - q + q(1-p)W(\alpha) + q \cdot p \cdot \alpha(s)}. \tag{7}$$

Note that $\hat{\rho}(s)$ is increasing in $\alpha(s)$ (and, as described above, $\hat{\rho}(s) = 0$ iff $\alpha(s) = 0$).[11]

3.3 Disclosure Strategy – Two Benchmarks

Before analyzing the full model, we briefly analyze the extreme cases where $p = 0$ and $p = 1$ as benchmarks. When $p = 0$ the signal is uninformative and therefore we are back to canonical model of Dye (1985) where there are no exogenous news. When $p = 1$ news is always real if there is an information event, and fake news spread only if there is nothing to report.

3.3.1 Equilibrium with $p = 0$ (Dye Model)

Since the market sole source of information is the manager's disclosure, there is a single price following no-disclosure, which equals $E\left[X \mid \mathrm{ND}; \alpha\right]$ as defined in Equation (4). The manager discloses if and only if $x \geq E\left[X \mid \mathrm{ND}; \alpha\right]$. The equilibrium strategy is therefore a threshold strategy, as defined in (2), that we denote θ_{x^0}. The threshold is defined implicitly using the equality

$$x^0 = E\left[X \mid \mathrm{ND}; \theta_{x^0}\right].$$

It is well known that the equality above has a unique solution and thus α^* is unique (Jung and Kwon, 1988). Moreover, Acharya et al. (2011) have shown that the solution to

[11] Equation (7) can be easily calculated using Bayes' rule. Observe that

$$\Pr\left(\mathrm{ND} \cap S = s\right) = (1-q)f(s) + qf(s)\left[p \cdot \alpha(s) + (1-p)\, W(\alpha)\right]$$

and

$$\Pr\left(X = s \cap \mathrm{ND} \cap S = s\right) = q \cdot p \cdot f(s) \cdot \alpha(s) = \rho \cdot f(s) \cdot \alpha(s).$$

the equality above also satisfies the property

$$x^0 = \min_y E\left[X \mid \text{ND}; \theta_y\right].$$

It is easy to generalize this property to general strategies and not just those who involve a threshold, that is,

$$x^0 = \min_\alpha E\left[X \mid \text{ND}; \alpha\right] \tag{8}$$

– the proof remains similar to that in Acharya et al. (2011). Henceforth we refer to Equation 8 as the "minimum principle".

3.3.2 Equilibrium with $p = 1$ (Most Informative News)

When $p = 1$, news is "conditionally perfect": $S = X$ if the manager is informed and $S = Z$ otherwise. Investors do not know whether news is real, but they knows that S is fake news only if the sender is uninformed, and therefore the price given no-disclosure and the belief that news is fake, as defined in Equation (5), is $P_f^{\text{ND}}(\alpha) = E\left[X\right]$ for any disclosure strategy α.

An informed manager of type y knows that investors will observe $S = y$ and thus expects to obtain, in case she does not disclose (Equation (6))

$$P^{\text{ND}}\left(y\right) = \hat{\rho}(y) \cdot y + (1 - \hat{\rho}(y))\, E\left[X\right].$$

Observe that: (i) $y \leq P^{\text{ND}}\left(y\right)$ iff $y < E[X]$, and $y \geq P^{\text{ND}}\left(y\right)$ iff $y > E[X]$; (ii) by Equation (7), $\hat{\rho}(y) > 0$ iff $\alpha(y) > 0$. This implies that the manager will withhold information if $y < E\left[X\right]$ and disclose if $y > E\left[X\right]$. Thus, the unique equilibrium strategy is the threshold strategy $\theta_{E[X]}$.

Note that when we compare the extreme case of $p = 0$ and $p = 1$ we find that there is less voluntary disclosure when the market is more informed. Investors know that if news is not real, the probability that the manager is actively withholding information is lower (in the case of $p = 1$, it is zero), and therefore prices following no disclosure are higher. This, in turn, reduces the incentive of the manager to disclose information. This, however, does not entail that in general the market obtains less information when $p = 1$, as one has to take into account the combined effect of the exogenous signal with the endogenous disclosure.

4 Equilibrium

We now analyze the disclosure decision of the manager in a model with general $p \in (0, 1)$. In this case, an informed manager is uncertain about the news that investors observe, and thus is uncertain about the price following no-disclosure. However, because the news S is correlated with X, the expected price depends on the manager's type. A manager of type x expects to obtain, in case she withholds information, a payoff of

$$\begin{aligned} U^{\mathrm{ND}}(x) &\equiv (1-p)E_z\left[P^{\mathrm{ND}}(z)\right] + p \cdot P^{\mathrm{ND}}(x) \\ &= (1-p)E_z\left[P^{\mathrm{ND}}(z)\right] + p \cdot \left[\hat{\rho}(x) \cdot x + (1-\hat{\rho}(x))\, P_f^{\mathrm{ND}}\right], \end{aligned} \tag{9}$$

where the second line uses Equation (6). Remember $\hat{\rho}(x)$ (Equation (7)) is the probability that the news is accurate given a signal $S = x$ and no disclosure. Note that $E_z\left[P^{\mathrm{ND}}(z)\right]$ is the expected price if the market has a false signal. Thus it averages prices that are conditional on no-disclosure, but uses the unconditional prior distribution F, which is also the distribution of Z. In equilibrium, (i) if $x < U^{\mathrm{ND}}(x)$ then $\alpha^*(x) = 1$, (ii) if $x > U^{\mathrm{ND}}(x)$ then $\alpha^*(x) = 0$, and (iii) if $x = U^{\mathrm{ND}}(x)$ then $\alpha^*(x) \in [0, 1]$.

4.1 No Threshold Equilibrium

We have seen that in the benchmark cases of $p = 0$ and $p = 1$ the equilibrium features a threshold disclosure strategy. We now show that such a threshold equilibrium, which is the common equilibrium in DJK models, does not exist in our setting for any $p \in (0, 1)$.

Proposition 1. *If $p \in (0, 1)$ a threshold disclosure policy cannot be an equilibrium strategy.*

Proof. Assume (by contradiction) that the equilibrium strategy is a threshold disclosure policy with threshold y, denoted by θ_y. We prove the proposition in several stages. First, we show that a threshold equilibrium implies a discontinuity in the beliefs of investors that the signal is accurate, $\hat{\rho}(s)$, and therefore a discontinuity in the price following no-disclosure $P^{\text{ND}}(s)$, around $s = y$. Then, we show that this discontinuity implies a discontinuity in the manager's expected payoff from no-disclosure, $U^{\text{ND}}(x)$. Finally, we show that there is a discrete and positive "jump" in $U^{\text{ND}}(x)$ around $x = y$, implying that a manager of type $y - \epsilon$ wishes to disclose information and/or a manager of type $y + \epsilon$ wishes to withhold information, which is in contradiction to a threshold equilibrium.

Discrete negative jump in the probability that the signal is true around y. By the definition of a threshold strategy (Equation (2)), $\alpha(x) = 1$ if $x < y$, and $\alpha(x) = 0$ if $x > y$. We can use Equation (7) to obtain the posterior probabilities after observing a signal s and no-disclosure. If $s < y$ then[12]

$$\overline{\rho_y} \equiv \frac{\rho}{1 - q(1-p)\left[1 - F(y)\right]} > \rho, \tag{10}$$

and the posterior if $s > y$ is $\hat{\rho} = 0$. At the threshold $\hat{\rho}(y) \in [0, \overline{\rho_y}]$, depending on $\alpha(y)$. If investors observe a signal $s < y$ and no disclosure then they believe news is more likely to be accurate (because an informed manager would not disclose this information); if they observe $s > y$ then they believe news is fake: if they were accurate, they would have been disclosed (remember news is always fake if the manager is uninformed).

Discontinuity in price and payoff following no-disclosure around y. For a given threshold strategy θ_y we can use (6) and the posteriors above to write the price following no disclosure and signal s as

$$P^{\text{ND}}\left(s, \theta_y\right) = \begin{cases} \overline{\rho_y} \cdot s + \left(1 - \overline{\rho_y}\right) P_f^{\text{ND}}\left(\theta_y\right) & s < y \\ P_f^{\text{ND}}\left(\theta_y\right) & s \geq y. \end{cases} \tag{11}$$

Note there is discontinuity in $P^{\text{ND}}\left(s, \theta_y\right)$ if $y \neq P_f^{\text{ND}}\left(\theta_y\right)$.

Now consider types $x = y + \epsilon$ and $x = y - \epsilon$, where $\epsilon > 0$ is arbitrarily small. Using

[12] To obtain $\overline{\rho_y}$ substitute $\alpha(s) = 1$ in (7). Also note that, by (3), $W(\theta_y) = F(y)$, and, by definition, $q \cdot p = \rho$.

Equation (9), we can write the difference in the expected payoff of these two types if they do not disclose:

$$\lim_{\epsilon\to 0}\left[U^{\mathrm{ND}}\left(y+\epsilon\right)-U^{\mathrm{ND}}\left(y-\epsilon\right)\right]=\lim_{\epsilon\to 0}p\left[P^{\mathrm{ND}}\left(y+\epsilon\right)-P^{\mathrm{ND}}\left(y-\epsilon\right)\right],$$

and substituting (11) we obtain

$$\lim_{\epsilon\to 0}\left[U^{\mathrm{ND}}\left(y+\epsilon\right)-U^{\mathrm{ND}}\left(y-\epsilon\right)\right]=p\cdot\overline{\rho_y}\left[P_f^{\mathrm{ND}}\left(\theta_y\right)-y\right]. \tag{12}$$

Thus, there is a discontinuity in $U^{\mathrm{ND}}(x)$ if $y\neq P_f^{\mathrm{ND}}\left(\theta_y\right)$.

The threshold y must be less than $P_f^{\mathrm{ND}}\left(\theta_y\right)$. Using (9), we can rewrite the indifference condition $y=U^{\mathrm{ND}}(y)$ as

$$y=\frac{1-p}{1-p\cdot\hat{\rho}(y)}E_z\left[P^{\mathrm{ND}}\left(z,\theta_y\right)\right]+\frac{p-p\cdot\hat{\rho}(y)}{1-p\cdot\hat{\rho}(y)}P_f^{\mathrm{ND}}\left(\theta_y\right). \tag{13}$$

That is, the threshold y is a weighted average of $E_z\left[P^{\mathrm{ND}}\left(z,\theta_y\right)\right]$ and $P_f^{\mathrm{ND}}\left(\theta_y\right)$.

Now substitute (11) to write $E_z\left[P^{\mathrm{ND}}\left(z,\theta_y\right)\right]$ explicitly and obtain

$$E_z\left[P^{\mathrm{ND}}\left(z,\theta_y\right)\right]=F(y)\overline{\rho_y}E_z\left[Z\mid Z<y\right]+\left(1-F(y)\overline{\rho_y}\right)P_f^{\mathrm{ND}}\left(\theta_y\right).$$

Thus, $E_z\left[P^{\mathrm{ND}}\left(z,\theta_y\right)\right]$ is a weighted average of $E\left[Z\mid Z<y\right]$ and $P_f^{\mathrm{ND}}\left(\theta_y\right)$. Now use Equation (5) to write $P_f^{\mathrm{ND}}\left(\theta_y\right)$ explicitly in a threshold equilibrium:

$$P_f^{\mathrm{ND}}\left(\theta_y\right)=\frac{(1-q)E\left[X\right]+q(1-p)F(y)E\left[X\mid X<y\right]}{1-q+q(1-p)F(y)}.$$

$P_f^{\mathrm{ND}}\left(\theta_y\right)$ is a weighted average of $E\left[X\right]$ and $E\left[X\mid X<y\right]$, and is therefore greater than $E\left[Z\mid Z<y\right]=E\left[X\mid X<y\right]$. We can therefore conclude that $E_z\left[P^{\mathrm{ND}}\left(z,\theta_y\right)\right]<P_f^{\mathrm{ND}}\left(\theta_y\right)$ for any y. By Equation (13), this implies that any equilibrium threshold satisfies $y<P_f^{\mathrm{ND}}\left(\theta_y\right)$.

Positive jump in $U^{\mathrm{ND}}(x)$ around y and contradiction. Equation (12) together with the fact that $y < P_f^{\mathrm{ND}}(\theta_y)$ imply that there is a positive discontinuity jump between $U^{\mathrm{ND}}(y-\epsilon)$ and $U^{\mathrm{ND}}(y+\epsilon)$. Observe also that $y = U^{\mathrm{ND}}(y) \in \left[U^{\mathrm{ND}}(y-\epsilon), U^{\mathrm{ND}}(y+\epsilon)\right]$, depending on $\alpha(y)$, and therefore

$$\lim_{\epsilon\to 0} U^{\mathrm{ND}}(y-\epsilon) < y \text{ and/or } y < \lim_{\epsilon\to 0} U^{\mathrm{ND}}(y+\epsilon).$$

But, since the payoff from disclosure is continuous in x (and equals x) then this implies that type $y-\epsilon$ prefers to disclose, and/or type $y+\epsilon$ prefers to withhold - a contradiction to the assumption of a threshold equilibrium. □

4.2 Characterizing a Continuous Two Threshold Equilibrium

We now characterize the equilibrium disclosure strategy $\alpha^*(x)$. We first establish the fact that any equilibrium must contain two thresholds, such that the manager does not disclose if her type is below the low threshold, and always discloses if her value is higher than the high threshold.

Lemma 1. *In any equilibrium $\alpha^*(x)$ there are two critical values, denoted by x_l and x_h such that: (1) $\alpha^*(x) = 1$ iff $x < x_l$; (2) $\alpha^*(x) = 0$ iff $x > x_h$.*

The sketch of the proof is as follows: suppose that the market believes that the manager uses a given disclosure strategy $\alpha(x)$, and prices the firm accordingly. We show that, for any $\alpha(x)$, types that are low enough find it optimal to withhold information ($x < U^{\mathrm{ND}}(x)$), and types that are high enough find it optimal to disclose information ($x > U^{\mathrm{ND}}(x)$). Note that while we assume above that $x \in \mathbb{R}$, this result holds also if the value of the firm is bounded from above and/or below.

We now present our first main result. We show that in any equilibrium of the model the probability of disclosure increases continuously in the manager's type, and so there is a continuum of types that mix between disclosing and withholding.

First, for a given strategy $\alpha(x)$, let

$$\overline{\rho_\alpha} \equiv \hat{\rho}(s)\left(\alpha(s)=1\right) = \frac{q\cdot p}{1-q+q(1-p)W(\alpha)+q\cdot p} > \rho \tag{14}$$

be the posterior probability that the signal is true, $\hat{\rho}(s)$, after observing no-disclosure and a signal that is never disclosed according to α. $\overline{\rho_\alpha}$ is obtained by substituting $\alpha(s)=1$ in (7). Similarly, $\hat{\rho}(s)\,(\alpha(s)=0)=0$. We now present the proposition:

Proposition 2. *The equilibrium of the game has the following continuous and decreasing disclosure strategy:*

$$\alpha^*(x)=\begin{cases}1 & x\le x_l\\ \frac{[1-q+q(1-p)W(\alpha^*)](x_h-x)}{q\cdot p(1-p)(x-E_z[P^{\mathrm{ND}}(z,\alpha^*)])} & x\in(x_l,x_h)\\ 0 & x\ge x_h,\end{cases}\tag{15}$$

where

$$x_l=\frac{1-p}{1-p\cdot\overline{\rho_{\alpha^*}}}E_z\left[P^{\mathrm{ND}}\left(z,\alpha^*\right)\right]+\frac{p-p\cdot\overline{\rho_{\alpha^*}}}{1-p\cdot\overline{\rho_{\alpha^*}}}P_f^{\mathrm{ND}}\left(\alpha^*\right),\ \text{and}$$
$$x_h=(1-p)E_z\left[P^{\mathrm{ND}}\left(z,\alpha^*\right)\right]+pP_f^{\mathrm{ND}}\left(\alpha^*\right).\tag{16}$$

From the proof of Proposition 1 we know that the equilibrium disclosure strategy must be continuous, because discrete changes imply discrete "jumps" in the no-disclosure payoff, and thus $\alpha(x_l)=1$ and $\alpha(x_h)=0$. The proof shows that the threshold types must be indifferent in equilibrium, and that in equilibrium $E_z\left[P^{\mathrm{ND}}\left(z,\alpha^*\right)\right]<P_f^{\mathrm{ND}}\left(\alpha^*\right)$. This, together with the indifference condition $x=U^{\mathrm{ND}}(x)$, implies that the thresholds are as in (16). The proof then uses the indifference condition $x=U^{\mathrm{ND}}(x)$ to characterize the equilibrium strategy between the thresholds, showing it is as in (15).

Following the equilibrium strategy described above, we can use some algebra on (6) and (9) to obtain directly the market price of the firm following no-disclosure and a signal s:

$$P^{\mathrm{ND}}(s)=\begin{cases}P_f^{\mathrm{ND}}-\overline{\rho_{\alpha^*}}\left(P_f^{\mathrm{ND}}-s\right) & s\le x_l\\ P_f^{\mathrm{ND}}-\frac{1}{p}\left(x_h-s\right) & s\in(x_l,x_h)\\ P_f^{\mathrm{ND}} & s\ge x_h.\end{cases}\tag{17}$$

Note that $P^{\mathrm{ND}}(s)$ is continuous, and thus $U^{\mathrm{ND}}(x)$ is also continuous.

4.3 Disclosure with and without News ($p > 0$ vs. $p = 0$)

One of our main results is comparing the model with news $p \in (0, 1)$ to the benchmark when there is no news ($p = 0$) of Section 3.3.1. Maybe surprisingly, the firm discloses less information when the market is partially informed. Formally, let $W^*(p)$ be the probability of withholding W in equilibrium when the informativeness of the market's signal s is $p \in (0, 1)$. Remember x^0 is the disclosure threshold in a model with $p = 0$, as described in Section 3.3.1). We prove the following:

Lemma 2. *The probability of withholding is higher when the market observes a noisy signal, that is,* $W^*(p) > F(x^0)$

Proof. First observe that in a model with $p > 0$, $E_x \left[P^{\text{ND}}(x, \alpha^*) \mid \text{ND}\right] = E\left[x \mid \text{ND}, \alpha^*\right]$. This is because pricing is rational, so on average the market prices right the information included in the event of no-disclosure. Because higher types disclose with higher probability, that is, $\alpha^*(x)$ is a weakly decreasing function (Equation (15)), then

$$E_z \left[P^{\text{ND}}(z, \alpha^*) \mid \text{ND}\right] < E_z \left[P^{\text{ND}}(z, \alpha^*)\right].$$

Second, observe that $E_z \left[P^{\text{ND}}(z, \alpha^*)\right] < x_l < x_h$ (Equation (16)). Third, from the minimum principle, $x^0 \leq E\left[x \mid \text{ND}, \alpha^*\right]$ (Equation 8). Thus $x^0 < x_l < x_h$, which entails the desired result. □

However, since the market obtains an additional signal, the fact that there is less disclosure does not mean that the market is less informed.

5 Robustness and Extensions

5.1 Accurate News with Uninformed Manager

An important assumption in our model is that the manager is always informed if there is an event that changes the value of the firm (an "information event") and therefore news is always fake if the manager is not informed (there is nothing real to report). This conditionality between news quality and the manager's information endowment (summarized by Equation (1)) generates many of our results.

One might argue that although the manager is informed about many events that change the value of the firm, she must not be informed about *all* these events, and thus news may be real even if the manager is uninformed. For example, consider a news report that exposes a serious flaw in the product of a major competitor of the firm. These are good news about the value of the firm, as it can expect to gain market share over its competitor. It seems reasonable to assume, though, that the firm's manager must not necessarily know such information, and therefore may not disclose it in advance of the public news report.

The above example is not possible in our base model. Could it be that the introduction of such events to our model changes the nature of equilibrium? Does our results rely on the specific information environment? In this section we show that our results are robust to some changes in the information environment. We extend the model and allow news to be real even if the manager is uninformed, and consider general dependency between news quality and manager information endowment.

Our main result is that if news is likely enough to be real when the manager is informed compared to when she is uninformed, then the equilibrium of the game continues to have two thresholds and a mixing region, as the one described in Proposition 2. Technically, we prove this by exploring the effect of discontinuity in the belief that the manager is informed around the possible threshold. In the proof of Proposition 1 we show that in any potential equilibrium with a threshold y (1) there is a discrete jump in the belief that the manager is informed following no-disclosure and a public signal s around $s = y$, and (2) this implies a positive discrete jump in the payoff of the informed manager from no disclosure around $x = y$, which prevents it from being an equilibrium. Below, we show that the same reasoning holds even when news can be real when the manager is uninformed. If, however, the probability that news is real is independent of the manager's information endowment, then there is a discrete jump in the beliefs as in (1), but (2) no longer holds: the discrete jump in beliefs implies a *negative* discrete jump in the no-disclosure payoff of the informed manager around $x = y$, supporting a threshold equilibrium. A pure-strategy threshold equilibrium is therefore possible if the public signal cannot tell us anything about the information endowment of the manager.

We now describe the extended model in length and describe the result.

Extended Information environment. We want to analyze how a change in the dependency between the news quality and the manager's information endowment affects the results, all else equals. We therefore continue to assume, as in the base model of Section 2, that there is an exogenous signal, which is precise ($S = X$, "real/accurate news") with some probability, which we label by ρ, and complete noise with probability $1 - \rho$ ($S = Z$, "fake news"). Through this section we keep ρ constant, to make sure that our results in this section are driven by the changing nature of news, and not by a change in the expected accuracy of news.

We now, however allow general dependency between the quality of news and the manager's information endowment, captured by the likelihood ratio

$$\ell \equiv \frac{\Pr(S = X \mid \text{Manager Uninformed})}{\Pr(S = X \mid \text{Manager Informed})}.$$

Specifically, instead of Equation (1), the probabilities that the news is real are now

$$\Pr(S = X \mid \text{Manager Informed}) \equiv p(q, \rho, \ell) = \frac{\rho}{q + (1 - q)\ell} \tag{18}$$
$$\Pr(S = X \mid \text{Manager Uninformed}) = \ell \cdot p(q, \rho, \ell).$$

If $\ell = 0$, the signal is always false if the manager is uninformed, as in the base model. $\ell > 0$ implies news can be real even if the manager is uninformed. If $\ell = 1$, then the quality of news is independent of the manager's information endowment. This fits, for example, the case where the manager and a reporter/analyst learn the same information with independent probabilities. We make the economically plausible assumption that $\ell \in [0, 1]$, that is, the probability that news is real is always weakly higher if the manager is informed.

Under this setup, the probability p is not a parameter but a function of ρ and ℓ (and, of course, q) that is determined so that the overall expected accuracy of news is ρ. Again, the purpose of that is to analyze the effect of different levels of ℓ when the precision of both signals is fixed. Note also that the set of feasible parameter values is constrained by the condition $p(q, \rho, \ell) \leq 1$, that is, $\rho \leq q + (1-q)\ell$.

Pricing. The probability that the manager is informed given fake news is now

$$\Pr(\text{Manager Informed} \mid S = Z) = q\frac{1 - p(\rho, \ell)}{1 - \rho} \leq q.$$

Because $p(\rho, \ell) \geq \rho$ and decreasing in ℓ, this probability is weakly lower than q and is increasing in ℓ. Stronger dependency between news accuracy and managerial information endowment, which is captured by lower ℓ, implies that the manager is less likely to be informed conditional on fake news.

The price conditional on no-disclosure and a fake news, denoted by $P_f^{\text{ND}}(\alpha)$, is now

$$P_f^{\text{ND}}(\alpha) = \frac{(1-q)\,(1-\ell p(\cdot))\,E\,[X] + q\,(1-p(\cdot))\,W(\alpha)E\,[X \mid \text{NDI}; \alpha]}{(1-q)\,(1-\ell p(\cdot)) + q\,(1-p(\cdot))\,W(\alpha)}. \tag{19}$$

The price following no-disclosure and a signal s is still as in (6), just with the new definition of P_f^{ND} ((19) replaces (5)). The belief that a signal s is true following no disclosure is now

$$\hat{\rho}(s) \equiv \Pr\,(S = X \mid \text{ND}, s) = \frac{p(\cdot)\,[(1-q)\ell + q\alpha(s)]}{1 - q + q\,(1-p(\cdot))\,W(\alpha) + q \cdot p(\cdot) \cdot \alpha(s)}. \tag{20}$$

This belief is increasing in ℓ, as weaker association between news quality and managerial information endowment implies that we can learn less about the quality of news from no-disclosure.

Conditions for a Threshold Equilibrium. Proposition 1 show that in the base model, that is, when $\ell = 0$, there is no threshold equilibrium. We now extend this result and show that it holds in general for "low" values of ℓ. The proof of Proposition 1 shows that a threshold strategy leads to a discrete positive jump in the expected payoff from no-disclosure (U^{ND}) around the threshold, which implies such strategy is never optimal. As a first step, we show that in the extended model that such discrete positive jump in U^{ND} happens if and only if a threshold strategy θ_y implies $P_f^{\text{ND}}(\theta_y) > E_z\left[P^{\text{ND}}(z, \theta_y)\right]$. Thus

Lemma 3. *Any threshold equilibrium y must satisfy $P_f^{\text{ND}}(\theta_y) \leq E_z\left[P^{\text{ND}}(z, \theta_y)\right]$.*

Using Lemma 3 we can prove that if ℓ is low enough then any potential equilibrium strategy θ_y implies a positive discrete jump in the payoff from no disclosure ($U^{\mathrm{ND}}(x)$) around y, which implies no threshold equilibrium exists.

Proposition 3. *There exists a constant $\overline{\ell} \in (0,1)$, such that if $\ell \leq \overline{\ell}$, then the game does not admit a threshold equilibrium.*

As in the base model, the key is the association between the quality of news and the manager's information endowment. Proposition 3 shows that any threshold strategy cannot be an equilibrium strategy if this association is high enough (remember that *lower* ℓ implies *higher* positive association).

5.2 News Does Not Always Arrive

In the base model we assume that a public signal always arrives, and therefore pricing always depends on at least one piece of information. Clearly, this assumption is somewhat strong. In this section we show that our results hold (qualitatively) also in the case where sometimes neither news nor corporate disclosure is available to the market.

Formally, consider the base model as described in Section 2 with one change: the public signal S arrives with a known probability $r \in [0,1]$ and does not arrive with probability $1-r$. If the public signal arrives, it has the same properties as in the base model (Equation (1)). Observe that the case of $r = 1$ is simply the base model, while when $r = 0$ we are back to the DJK framework. We therefore focus naturally of the cases where $r \in (0,1)$. The arrival or non-arrival of the public signal is independent of any other random variable in the model, and specifically do not inform the market about the information endowment of the manager.[13]

Note that another interpretation of this model is that news always arrives, but the manager also cares about the price before their arrival, just following the disclosure stage. In this case, $1-r$ is interpreted as the weight that the manager gives to the price following disclosure but before news, and r is the weight given to the price following the arrival of news. The

[13]See Frenkel et al. (2020) for a model where the arrival of the signal may be correlated with the manager's information endowment.

analysis in mathematically identical because the price in the event that news does not arrive is exactly the same as the price following possible disclosure but before news arrival.

As in the base model, if the manager discloses x then $P = x$ and the public signal, if observed, is ignored. The arrival of news matters only if there is no disclosure. The price following no disclosure and a signal s is as in the base model, $P = P^{\text{ND}}(s, \alpha)$ as defined in (6). If there is no disclosure and no news then the only information that can be conditioned on is the fact that the manager did not disclose, and therefore $P = E[X \mid \text{ND}; \alpha]$ as defined in (4).

in the proposition below we show that the equilibrium of this extended model has the same properties as the equilibrium of the base model.

Proposition 4. *If $r \in (0, 1]$, an equilibrium strategy $\alpha_r^*(x)$ in characterized by two thresholds, $\hat{x}_l$ and $\hat{x}_h$ such that $\hat{x}_l < \hat{x}_h$ and (1) $\alpha^*(x) = 1$ iff $x \leq \hat{x}_l$; (2) $\alpha^*(x) \in (0, 1)$ iff $x \in (\hat{x}_l, \hat{x}_h)$; (3) $\alpha^*(x) = 0$ iff $x \geq \hat{x}_h$.*

Therefore, for any positive probability that a public signal arrives, there is no pure-strategy threshold equilibrium, and an equilibrium is characterized by two threshold and a mixing region between them.

5.3 Predisclosure Public News

The purpose of this paper is to analyze voluntary disclosure decisions in a complex information environments, in which the manager faces uncertainty about additional sources of information. Thus, we have assumed so far that the manager's disclosure decision is taken without knowing S. To complement our analysis, we analyze also a model that is similar to the original model, except that the manager observes S before making her disclosure decision. The correlation between the manager's information endowment and the probability that the news is real separates this section from previous research that analyzed voluntary disclosure with other predisclosure information sources. Specifically, the "irrelevance result" of Acharya et al. (2011), that the probability of disclosure does not depend on the content of the ex-ante news, does not hold in this case.

First, observe that in if $p = 0$ or $p = 1$, it does not matter whether the manager knows or don't know the value of S. If $p=0$ this is obvious, as all players ignore the signal; if $p=1$, an informed manager of type x knows that the signal is $S=x$, even if she does not observe it directly. Thus, equilibrium of the benchmark cases that were analyzed in Section 3.3 are relevant also for this model. We now analyze the general case where $p\in(0,1)$.

In this "known news" scenario, the manager's strategy depends also on the public signal S, and we use a similar definition of $\alpha(x,s)$ to denote the probability that an informed manager of type x discloses following a signal s.

When the value of news is known, the manager has no uncertainty about the equilibrium price following no disclosure, $P^{\mathrm{ND}}(s,\alpha)$. Thus, as in the DJK framework, the equilibrium strategy is a threshold strategy – the manager discloses if $x>P^{\mathrm{ND}}(s,\alpha)$ and withholds if $x<P^{\mathrm{ND}}(s,\alpha)$. The threshold following a signal s, $x^*(s)$, is the solution to the fixed-point operator $x^*(s)=P^{\mathrm{ND}}\left(s,\theta_{x^*(s)}\right)$.

We can use the price function (11) to find the equilibrium threshold $x^*(s)$ for any signal. The result is as follows.

Proposition 5. *Define*

$$\underline{x}\equiv\min_{\alpha}P_f^{\mathrm{ND}}(\alpha). \tag{21}$$

When the signal s is known to the manager before the disclosure decision, the manager has a unique signal-dependent threshold strategy defined using the following weakly-increasing threshold function $x^*(s)$:

1. *If $s<\underline{x}$, the threshold is the fixed-point solution of*

$$x^*(s)=\overline{\rho_{x^*(s)}}\cdot s+\left(1-\overline{\rho_{x^*(s)}}\right)P_f^{\mathrm{ND}}\left(\theta_{x^*(s)}\right).$$

   where $\overline{\rho_{x^*(s)}}$ is defined in (10). Such a threshold exists and is unique for a given s.

2. *If $s\geq\underline{x}$, the threshold is $x^*(s)=\underline{x}$.*

Proposition 5 shows that when signals are low, higher signal implies less disclosure. The obvious intuition is that better news make investors more optimistic about the value of the firm even if there is no disclosure, thus reducing incentives to disclose information. This effect, however, does not hold for higher signals, above a certain cutoff level. The reason is that above this level, investors infer from no-disclosure that news must be fake (if it was real, the manager would have disclosed it), and therefore ignore the news following no disclosure. Comparing the results of Proposition 5 to the benchmark with no news in Section 3.3.1, we can see that positive news (high values of S) can result in less disclosure compared to no news, because $\underline{x} > x_0$ (obviously, low values of S will result in a threshold that is less than x_0).

To conclude this section, we provide two empirical predictions that arise from Proposition 5. First, the disclosure behavior of better firms is less affected by news compared to lesser firms; if the value of the firm is $x > \underline{x}$, it would disclose information regardless of the news. Below that value, however, disclosure decision depends on the news. Second, firm's disclosure behavior is less sensitive to public news the better this news is - $x^*(s)$ is increasing and concave in s if $s < \underline{x}$, and constant if $s \geq \underline{x}$.

References

Acharya, V. V., P. DeMarzo, and I. Kremer (2011, December). Endogenous information flows and the clustering of announcements. *American Economic Review 101*(7), 2955–2979.

Avery, C. N., J. A. Chevalier, and R. J. Zeckhauser (2015, 09). The "CAPS" Prediction System and Stock Market Returns. *Review of Finance 20*(4), 1363–1381.

Banerjee, S., I. Marinovic, and K. Smith (2024). Disclosing to informed traders. *The Journal of Finance 79*(2), 1513–1578.

Bartov, E., L. Faurel, and P. S. Mohanram (2018, 05). Can Twitter Help Predict Firm-Level Earnings and Stock Returns? *The Accounting Review 93*(3), 25–57.

Bertomeu, J., E. Cheynel, and K. Hu (2024). Miscommunication. Working paper.

Beyer, A., D. A. Cohen, T. Z. Lys, and B. R. Walther (2010). The financial reporting environment: Review of the recent literature. *Journal of Accounting and Economics 50*(2), 296–343.

Beyer, A. and I. Guttman (2012). Voluntary disclosure, manipulation, and real effects. *Journal of Accounting Research 50*(5), 1141–1177.

Bond, P. and Y. Zeng (2022). Silence is safest: Information disclosure when the audience's preferences are uncertain. *Journal of Financial Economics 145*(1), 178–193.

Chen, H., P. De, Y. J. Hu, and B.-H. Hwang (2014, 02). Wisdom of Crowds: The Value of Stock Opinions Transmitted Through Social Media. *The Review of Financial Studies 27*(5), 1367–1403.

Cookson, J. A., W. Mullins, and M. Niessner (2024, May). Social media and finance. *Available at SSRN 4806692*.

Cookson, J. A., M. Niessner, and C. Schiller (forthcoming). Can social media inform corporate decisions? evidence from merger withdrawals. *Journal of Finance*.

Dessaint, O., T. Foucault, and L. Frésard (forthcoming). Does alternative data improve financial forecasting? the horizon effect. *Journal of Finance*.

Dutta, S. and B. Trueman (2002). The interpretation of information and corporate disclosure strategies. *Review of Accounting Studies 7*(1), 75–96.

Dye, R. A. (1985). Disclosure of nonproprietary information. *Journal of Accounting Research 23*(1), 123–145.

Einhorn, E. (2018, 07). Competing Information Sources. *The Accounting Review 93*(4), 151–176.

Fischer, P. E. and P. C. Stocken (2023, May). Disinformation to persuade or confuse. Working paper.

Frenkel, S., I. Guttman, and I. Kremer (2020). The effect of exogenous information on voluntary disclosure and market quality. *Journal of Financial Economics 138*(1), 176–192.

Grubb, M. D. (2011). Developing a reputation for reticence. *Journal of Economics & Management Strategy 20*(1), 225–268.

Guttman, I., I. Kremer, and A. Skrzypacz (2014, August). Not only what but also when: A theory of dynamic voluntary disclosure. *American Economic Review 104*(8), 2400–2420.

Jung, W.-O. and Y. K. Kwon (1988). Disclosure when the market is unsure of information endowment of managers. *Journal of Accounting Research 26*(1), 146–153.

Kakhbod, A., S. M. Kazempour, D. Livdan, and N. Schuerhoff (2023, July). Finfluencers. Technical Report 23-30, Swiss Finance Institute Research Paper.

Langberg, N. and K. Sivaramakrishnan (2008). Voluntary disclosures and information production by analysts. *Journal of Accounting and Economics 46*(1), 78–100.

Langberg, N. and K. Sivaramakrishnan (2010). Voluntary disclosures and analyst feedback. *Journal of Accounting Research 48*(3), 603–646.

Libgober, J., B. Michaeli, and E. Wiedman (2023). With a grain of salt: Uncertain veracity of external news and firm disclosures. Working paper.

Pae, S. (2005). Selective disclosures in the presence of uncertainty about information endowment. *Journal of Accounting and Economics 39*(3), 383–409.

Quigley, D. and A. Walther (2024). Inside and outside information. *The Journal of Finance 79*(4), 2667–2714.

Renault, T. (2017). Intraday online investor sentiment and return patterns in the u.s. stock market. *Journal of Banking & Finance 84*, 25–40.

Song Shin, H. (2003). Disclosures and asset returns. *Econometrica 71*(1), 105–133.

Song Shin, H. (2006). Disclosure risk and price drift. *Journal of Accounting Research 44*(2), 351–379.

Suijs, J. (2007). Voluntary disclosure of information when firms are uncertain of investor response. *Journal of Accounting and Economics 43*(2), 391–410.

A Proofs

A.1 Proof of Lemma 1

Proof. Fix an arbitrary strategy $\alpha(x)$. Given this strategy, there is a general probability that an informed manager does not disclose, denoted by $W(\alpha)$ (Equation (3)). Note that $\hat{\rho}(x)$, the posterior probability that the signal is true, also depends on the market's belief about $\alpha(x)$.

From (9) it is immediate that in equilibrium, a certain value x satisfies $x \lesseqqgtr U^{\mathrm{ND}}(x)$ if and only if

$$x \lesseqqgtr \frac{1-p}{1-p\cdot\hat{\rho}(x)} E_z\left[P^{\mathrm{ND}}\left(z,\alpha\right)\right] + \frac{p-p\cdot\hat{\rho}(x)}{1-p\cdot\hat{\rho}(x)} P_f^{\mathrm{ND}}(\alpha),$$

that is, the manager chooses to disclose if x is greater than a certain average of $E_z\left[P^{\mathrm{ND}}\left(z\right)\right]$ and P_f^{ND}, where the weights and the prices are a function of $\alpha(x)$. In equilibrium, $\hat{\rho}(x)$ correctly describes the behavior of the manager $\alpha(x)$. Thus, an equilibrium strategy $\alpha^*(x)$ must satisfy

$$\begin{aligned}
&\alpha^*(x) = 1 \iff x \leq \frac{1-p}{1-p\cdot\overline{\rho_{\alpha^*}}} E_z\left[P^{\mathrm{ND}}\left(z,\alpha^*\right)\right] + \frac{p-p\cdot\overline{\rho_{\alpha^*}}}{1-p\cdot\overline{\rho_{\alpha^*}}} P_f^{\mathrm{ND}}(\alpha^*), \text{ and}\\
&\alpha^*(x) = 0 \iff x \geq (1-p)E_z\left[P^{\mathrm{ND}}\left(z,\alpha^*\right)\right] + pP_f^{\mathrm{ND}}(\alpha^*).
\end{aligned}$$

Observe that in this stage we do not know whether $E_z\left[P^{\mathrm{ND}}\left(z\right)\right]$ is greater or less than P_f^{ND}. However, denote

$$\begin{aligned}
x_l &\equiv \min\left\{(1-p)E_z\left[P^{\mathrm{ND}}\left(z\right)\right] + pP_f^{\mathrm{ND}}, \frac{1-p}{1-p\cdot\overline{\rho_\alpha}} E_z\left[P^{\mathrm{ND}}\left(z\right)\right] + \frac{p-p\cdot\overline{\rho_\alpha}}{1-p\cdot\overline{\rho_\alpha}} P_f^{\mathrm{ND}}\right\}\\
x_h &\equiv \max\left\{(1-p)E_z\left[P^{\mathrm{ND}}\left(z\right)\right] + pP_f^{\mathrm{ND}}, \frac{1-p}{1-p\cdot\overline{\rho_\alpha}} E_z\left[P^{\mathrm{ND}}\left(z\right)\right] + \frac{p-p\cdot\overline{\rho_\alpha}}{1-p\cdot\overline{\rho_\alpha}} P_f^{\mathrm{ND}}\right\};
\end{aligned}$$

It is clear that: (1) $0 < x_l < x_h < 1$ for any $p \in (0,1)$; (2) if $\alpha(x) < 1$ for $x < x_l$ or $\alpha(x) > 0$ for $x > x_h$ then $\alpha(x)$ is not an equilibrium; (3) by continuity of the type-space, x_l and x_h exist. □

A.2 Proof of Proposition 2

Proof. The proof is composed of two parts. First we show that in equilibrium $E_z\left[P^{\mathrm{ND}}\left(z,\alpha^*\right)\right] < P_f^{\mathrm{ND}}\left(\alpha^*\right)$. Following the proof of Lemma 1 this entails that the thresholds are as in (16). Then we characterize the resulting equilibrium strategy, showing it is as in (15).

Proving that $E_z\left[P^{\mathrm{ND}}\left(z,\alpha^*\right)\right] < P_f^{\mathrm{ND}}\left(\alpha^*\right)$. Suppose, in contrast, there is an equilibrium strategy α^* that imply $E_z\left[P^{\mathrm{ND}}\left(z,\alpha^*\right)\right] > P_f^{\mathrm{ND}}\left(\alpha^*\right)$.[14] Then

$$
\begin{aligned}
x_l &= (1-p)E_z\left[P^{\mathrm{ND}}\left(z,\alpha^*\right)\right] + pP_f^{\mathrm{ND}}\left(\alpha^*\right)\\
x_h &= \frac{1-p}{1-p\cdot\overline{\rho_{\alpha^*}}}E_z\left[P^{\mathrm{ND}}\left(z,\alpha^*\right)\right] + \frac{p-p\cdot\overline{\rho_{\alpha^*}}}{1-p\cdot\overline{\rho_{\alpha^*}}}P_f^{\mathrm{ND}}\left(\alpha^*\right).
\end{aligned}
$$

Using (6) we can write $E_z\left[P^{\mathrm{ND}}\left(z\right)\right]$ explicitly, taking into account Lemma 1:

$$
\begin{aligned}
E_z\left[P^{\mathrm{ND}}\left(z,\alpha^*\right)\right] \;=\;& F\left(x_l\right)\left[\overline{\rho_{\alpha^*}}E\left[Z \mid Z \le x_l\right] + \left(1-\overline{\rho_{\alpha^*}}\right)P_f^{\mathrm{ND}}\left(\alpha^*\right)\right]\\
&+\int_{x_l}^{x_h}\left[\hat{\rho}(z)\cdot z + \left(1-\hat{\rho}(z)\right)P_f^{\mathrm{ND}}\left(\alpha^*\right)\right]f(z)\,\mathrm{d}z + \left(1-F(x_h)\right)P_f^{\mathrm{ND}}\left(\alpha^*\right),
\end{aligned}
$$

where $\hat{\rho}(z) \in [0,\overline{\rho_{\alpha^*}}]$ is calculated using (7) and $\alpha^*(z)$, and $\overline{\rho_{\alpha^*}}$ is defined in (14). Because $P_f^{\mathrm{ND}}\left(\alpha^*\right) < x_l < x_h$ then for any $z \in [x_l, x_h]$,

$$
\hat{\rho}(z)\cdot z + \left(1-\hat{\rho}(z)\right)P_f^{\mathrm{ND}}\left(\alpha^*\right) \le \overline{\rho_{\alpha^*}}\cdot z + \left(1-\overline{\rho_{\alpha^*}}\right)P_f^{\mathrm{ND}}\left(\alpha^*\right),
$$

and therefore

$$
\begin{aligned}
E_z\left[P^{\mathrm{ND}}\left(z,\alpha^*\right)\right] <& F\left(x_h\right)\left[\overline{\rho_{\alpha^*}}E\left[Z \mid Z \le x_h\right] + \left(1-\overline{\rho_{\alpha^*}}\right)P_f^{\mathrm{ND}}\left(\alpha^*\right)\right] + \left(1-F(x_h)\right)P_f^{\mathrm{ND}}\left(\alpha^*\right)\\
&= F\left(x_h\right)\overline{\rho_{\alpha^*}}E\left[Z \mid Z \le x_h\right] + \left(1-F\left(x_h\right)\overline{\rho_{\alpha^*}}\right)P_f^{\mathrm{ND}}\left(\alpha^*\right).
\end{aligned}
$$

Because $x_h < E_z\left[P^{\mathrm{ND}}\left(z,\alpha^*\right)\right]$ then $E\left[Z \mid Z \le x_h\right] < E_z\left[P^{\mathrm{ND}}\left(z,\alpha^*\right)\right]$, implying that

$$
F\left(x_h\right)\overline{\rho_{\alpha^*}}E\left[Z \mid Z \le x_h\right] + \left(1-F\left(x_h\right)\overline{\rho_{\alpha^*}}\right)P_f^{\mathrm{ND}}\left(\alpha^*\right) < E_z\left[P^{\mathrm{ND}}\left(z,\alpha^*\right)\right]
$$

[14] When $E_z\left[P^{\mathrm{ND}}\left(z,\alpha^*\right)\right] = P_f^{\mathrm{ND}}\left(\alpha^*\right)$ we are back to a threshold equilibrium, which cannot be an equilibrium by Proposition 1.

– a contradiction.

Equilibrium Strategy. Using the proof of Lemma (1) and the fact that $E_z\left[P^{\mathrm{ND}}\left(z,\alpha^*\right)\right] < P_f^{\mathrm{ND}}\left(\alpha^*\right)$, we conclude the thresholds are as in (16). Substituting (7) into (9) we can verify the following for $x \in [x_l,\, x_h]$:

- x_l: in equilibrium, if $\alpha^*(x_l) < 1$ then $x_l < U^{\mathrm{ND}}(x_l)$ - a contradiction. Thus, $\alpha^*(x_l) = 1$.
- x_h: in equilibrium, if $\alpha^*(x_h) > 0$ then $x_h > U^{\mathrm{ND}}(x_l)$ - a contradiction. Thus, $\alpha^*(x_h) = 0$.
- For any $y \in (x_l, x_h)$: in equilibrium, if $\alpha^*(y) = 1$ then $y > U^{\mathrm{ND}}(y)$ - a contradiction; if $\alpha^*(y) = 0$ then $y < U^{\mathrm{ND}}(y)$ - a contradiction. Thus, $\alpha^*(y) \in (0,1)$ - intermediate types mix.

Note all types $x \in [x_l, x_h]$ are indifferent in equilibrium.

For types $x \in (x_l, x_h)$ we can substitute (9) in the indifference condition $x = U^{\mathrm{ND}}(x)$ to back-out $\hat{\rho}(x)$,

$$\hat{\rho}(x) = \frac{(1-p)E_z\left[P^{\mathrm{ND}}\left(z\right)\right] + pP_f^{\mathrm{ND}} - x}{p\left(P_f^{\mathrm{ND}} - x\right)} = \frac{x_h - x}{p\left(P_f^{\mathrm{ND}} - x\right)}. \tag{22}$$

Observe that $\hat{\rho}(x)$ is decreasing in x on (x_l, x_h) . This also implies that $\alpha^*(x)$ is decreasing in x on (x_l, x_h), that is, higher types disclose with greater probability. Doing some algebra on (7) we obtain that

$$\alpha^*(x) = \frac{\hat{\rho}(x)}{1-\hat{\rho}(x)} \cdot \frac{(1-q) + q(1-p)W}{q \cdot p},$$

and after substituting (22) we obtain the mixed strategy as in 15 $\square$

A.3 Proof of Lemma 3

Proof. We prove by contradiction. Assume a threshold equilibrium strategy with threshold y. Substitute the equilibrium strategy (2) in the general posterior equation (20), and observe that the probability of informed withholding is $W(\theta_y) = F(y)$, to obtain the probability that a signal is true, $\hat{\rho}(s)$:

$$\hat{\rho}(s)=\begin{cases}\overline{\rho_y}\equiv\frac{p(\cdot)[(1-q)\ell+q]}{1-q+q(1-p(\cdot))F(y)+q\cdot p(\cdot)} & s<y\\ \underline{\rho_y}\equiv\frac{p(\cdot)(1-q)\ell}{1-q+q(1-p(\cdot))F(y)} & s>y,\end{cases}$$

and $\hat{\rho}(y)\in\left[\underline{\rho_y},\overline{\rho_y}\right]$, depending on the strategy $\alpha(y)$.

Now consider types $y+\epsilon$ and $y-\epsilon$, where $\epsilon>0$ is arbitrarily small. Using Equation (9), observe that

$$\lim_{\epsilon\to 0}\left[U^{\mathrm{ND}}(y+\epsilon)-U^{\mathrm{ND}}(y-\epsilon)\right]=\lim_{\epsilon\to 0}p(\cdot)\left[P^{\mathrm{ND}}(y+\epsilon)-P^{\mathrm{ND}}(y-\epsilon)\right].$$

Using the definition of prices in (6) and $\hat{\rho}(s)$ above we can calculate the prices following no disclosure of both types:

$$\begin{aligned}P^{\mathrm{ND}}(y-\epsilon)&=\overline{\rho_y}(y-\epsilon)+(1-\overline{\rho_y})P_f^{\mathrm{ND}}\\ P^{\mathrm{ND}}(y+\epsilon)&=\underline{\rho_y}(y+\epsilon)+\left(1-\underline{\rho_y}\right)P_f^{\mathrm{ND}}.\end{aligned}$$

Using these prices we can rewrite the limit above as

$$\begin{aligned}\lim_{\epsilon\to 0}\left[U^{\mathrm{ND}}(y+\epsilon)-U^{\mathrm{ND}}(y-\epsilon)\right]&=p(\cdot)\left(\overline{\rho_y}-\underline{\rho_y}\right)\left(P_f^{\mathrm{ND}}-y\right)\\ &=p(\cdot)\left(\overline{\rho_y}-\underline{\rho_y}\right)\frac{1-p(\cdot)}{1-p(\cdot)\hat{\rho}(y)}\left(P_f^{\mathrm{ND}}-E_z\left[P^{\mathrm{ND}}(z)\right]\right),\end{aligned}\tag{23}$$

where the second inequality is obtained by substituting (13) (this equation is independent of ℓ and therefore holds also in the general model).

If $P_f^{\mathrm{ND}}>E_z\left[P^{\mathrm{ND}}(z)\right]$ then Equation (23) implies there is a discrete jump in $U^{\mathrm{ND}}(x)$ around y. Thus, one can find ϵ such that $y-\epsilon>U^{\mathrm{ND}}(y-\epsilon)$ and/or $y+\epsilon<U^{\mathrm{ND}}(y+\epsilon)$ – a contradiction to the assumption that a threshold equilibrium exists.

□

A.4 Proof of Proposition 3

Proof. The proof of Proposition 1 shows that if $\ell = 0$ then $E_z\left[P^{\mathrm{ND}}\left(z, \theta_y; \ell = 0\right)\right] < P_f^{\mathrm{ND}}\left(\theta_y; \ell = 0\right)$ for any y. From the continuity of $E_z\left[P^{\mathrm{ND}}\left(z, \theta_y; \ell\right)\right]$ and $P_f^{\mathrm{ND}}\left(\theta_y; \ell\right)$ w.r.t ℓ, this implies that there exists a constant $\bar{\ell} > 0$ such that $E_z\left[P^{\mathrm{ND}}\left(z, \theta_y; \ell\right)\right] < P_f^{\mathrm{ND}}\left(\theta_y; \ell\right)$ for any y if $\ell < \bar{\ell}$. Together with Lemma 3 this implies the desired result.

We are left to show that $\bar{\ell} < 1$. We do so by showing that a threshold equilibrium exists if $\ell = 1$. This can be done using the following three steps:

1. $P_f^{\mathrm{ND}}\left(\theta_y; \ell = 1\right) = E\left[X \mid \mathrm{ND}; \theta_y\right]$ – to see this first use 19 to write P_f^{ND} for a given ℓ and a threshold strategy y as

$$P_f^{\mathrm{ND}}\left(\theta_y; \ell\right) = \frac{(1-q)\left[1 - \ell p(q, \rho, \ell)\right] E\left[X\right] + q\left[1 - p(q, \rho, \ell)\right] F(y) \cdot E\left[X \mid X \leq y\right]}{(1-q)\left[1 - \ell p(q, \rho, \ell)\right] + q\left[1 - p(q, \rho, \ell)\right] F(y)}. \tag{24}$$

   Now use Equation (4) to write $E\left[X \mid \mathrm{ND}; \alpha\right]$ in the case of $\alpha = \theta_y$ as

$$E\left[X \mid \mathrm{ND}; \theta_y\right] = \frac{(1-q)E\left[X\right] + qF(y) \cdot E\left[X \mid X < y\right]}{1 - q + qF(y)}.$$

   It is easy to see that the result is the same as substituting $\ell = 1$ in (24) (if $\ell = 1$ then $S = Z$ provides no information on the price).

2. $E_z\left[P^{\mathrm{ND}}\left(z, \theta_y; \ell\right)\right] > E\left[X \mid \mathrm{ND}; \theta_y, \ell\right]$ for any ℓ – to see why remember that $P^{\mathrm{ND}}(z) = E\left[X \mid \mathrm{ND}, S = z\right]$. Thus, by the Law of Total Expectation,

$$E_z\left[P^{\mathrm{ND}}(z, \alpha) \mid \mathrm{ND}; \alpha\right] = E_z\left[E_x\left[X \mid \mathrm{ND}, S = z; \alpha\right] \mid \mathrm{ND}; \alpha\right] = E_x\left[X \mid \mathrm{ND}; \alpha\right]$$

   for any α. In the specific case of a threshold equilibrium, no-disclosure (ND) always implies "bad news," that is

$$E_z\left[P^{\mathrm{ND}}\left(z, \theta_y\right)\right] > E_z\left[P^{\mathrm{ND}}\left(z, \theta_y\right) \mid \mathrm{ND}; \theta_y\right] = E_x\left[X \mid \mathrm{ND}; \theta_y\right].$$

3. $E_z\left[P^{\mathrm{ND}}\left(z, \theta_y\right)\right] < E[X]$ for any ℓ – to see why simply use 6 and the fact that $Z \sim X$

to write $E_z\left[P^{\mathrm{ND}}\left(z,\theta_y\right)\right]$ explicitly in a threshold equilibrium:

$$\begin{aligned}E_z\left[P^{\mathrm{ND}}\left(z\right)\right]=&E\left[\hat{\rho}(x)\cdot X+\left(1-\hat{\rho}(x)\right)P_f^{\mathrm{ND}}\right]\\=&F(y)\overline{\rho_y}\cdot E\left[X\mid X\leq y\right]+\left(1-F(y)\right)\underline{\rho_y}\cdot E\left[X\mid X>y\right]\\&+\left[F(y)\left(1-\overline{\rho_y}\right)+\left(1-F(y)\right)\left(1-\underline{\rho_y}\right)\right]P_f^{\mathrm{ND}}.\end{aligned}$$

Observe that

$$E_z\left[P^{\mathrm{ND}}\left(z\right)\right]<E\left[\underline{\rho_y}\cdot X+\left(1-\underline{\rho_y}\right)P_f^{\mathrm{ND}}\right]=\underline{\rho_y}E[X]+\left(1-\underline{\rho_y}\right)P_f^{\mathrm{ND}}.$$

Because $P_f^{\mathrm{ND}}<E[X]$ (Eq. (24)) we obtain the desired result.

From these three steps it is evident that if $\ell=1$ then

$$P_f^{\mathrm{ND}}\left(\theta_y\right)<E_z\left[P^{\mathrm{ND}}\left(z,\theta_y\right)\right]<E[X].$$

Observe from Eq. (24) that

$$\lim_{y\to-\infty}P_f^{\mathrm{ND}}\left(\theta_y\right)=\lim_{y\to\infty}P_f^{\mathrm{ND}}\left(\theta_y\right)=E[X].$$

This is sufficient to show that if $\ell=1$ there is at least one solution y for the equilibrium condition (13) in which $P_f^{\mathrm{ND}}\left(\theta_y\right)\leq E_z\left[P^{\mathrm{ND}}\left(z,\theta_y\right)\right]$.

□

A.5 Proof of Proposition 4

Proof. A manager of type x expects to obtain, in case she withholds information, a payoff of

$$\begin{aligned}\hat{U}^{\mathrm{ND}}(x)&\equiv r\left\{(1-p)E_z\left[P^{\mathrm{ND}}\left(z\right)\right]+p\cdot P^{\mathrm{ND}}\left(x\right)\right\}+(1-r)E\left[X\mid\mathrm{ND};\alpha\right]\\&=rU^{\mathrm{ND}}(x)+(1-r)E\left[X\mid\mathrm{ND};\alpha\right]\end{aligned}$$

where $U^{\mathrm{ND}}(x)$ is the expected payoff in the base model, as defined in (9).

The manager's optimal strategy depends on the relation between x and $\hat{U}^{\mathrm{ND}}(x)$. A certain value x satisfies $x \lesseqqgtr \hat{U}^{\mathrm{ND}}(x)$ if and only if

$$x \lesseqqgtr \frac{r - rp \cdot \hat{\rho}(x)}{1 - rp\hat{\rho}(x)} \left[\frac{1-p}{1 - p \cdot \hat{\rho}(x)} E_z \left[P^{\mathrm{ND}}(z)\right] + \frac{p - p\hat{\rho}(x)}{1 - p \cdot \hat{\rho}(x)} P_f^{\mathrm{ND}}\right] + \frac{1-r}{1 - rp\hat{\rho}(x)} E\left[X \mid \mathrm{ND}; \alpha\right]$$

Thus, an equilibrium strategy for a given parameter r, $\alpha_r^*(x)$, must satisfy

$$\alpha_r^*(x) = 1 \iff x \leq \frac{r - rp}{1 - rp\overline{\rho_{\alpha_r^*}}} E_z \left[P^{\mathrm{ND}}(z, \alpha_r^*)\right] + \frac{rp - rp\overline{\rho_{\alpha_r^*}}}{1 - rp\overline{\rho_{\alpha_r^*}}} P_f^{\mathrm{ND}}(\alpha_r^*) + \frac{1-r}{1 - rp\overline{\rho_{\alpha_r^*}}} E\left[X \mid \mathrm{ND}; \alpha_r^*\right]$$
$$\alpha_r^*(x) = 0 \iff x \geq r(1-p) E_z \left[P^{\mathrm{ND}}(z, \alpha_r^*)\right] + rp P_f^{\mathrm{ND}}(\alpha_r^*) + (1-r) E\left[X \mid \mathrm{ND}; \alpha_r^*\right]$$

where $\overline{\rho_{\alpha_r^*}}$ is defined as in the base model (Equation (14)). Thus, Lemma 1 also holds in this model – an equilibrium strategy must contain two thresholds, one under which the firm never discloses, and one above which it always discloses, . $\square$

We are left to show that the same two-threshold equilibrium exists for any r, as in Proposition 2. For a given equilibrium strategy α_r^*, define $\hat{x}_l(\alpha_r^*)$ as the threshold that below it all types withhold ($\alpha_r^* = 1$) and $\hat{x}_h(\alpha_r^*)$ as the threshold above it all types disclose ($\alpha_r^* = 0$). Using the conditions above we obtain

$$\begin{aligned}
\hat{x}_l &= \frac{r - rp\overline{\rho_{\alpha_r^*}}}{1 - rp\overline{\rho_{\alpha_r^*}}} \left[\frac{1-p}{1 - p\overline{\rho_{\alpha_r^*}}} E_z \left[P^{\mathrm{ND}}(z)\right] + \frac{p - p\overline{\rho_{\alpha_r^*}}}{1 - p\overline{\rho_{\alpha_r^*}}} P_f^{\mathrm{ND}}\right] + \frac{1-r}{1 - rp\overline{\rho_{\alpha_r^*}}} E\left[X \mid \mathrm{ND}; \alpha_r^*\right] \\
&= r \frac{1 - p\overline{\rho_{\alpha_r^*}}}{1 - rp\overline{\rho_{\alpha_r^*}}} x_l(\alpha_r^*) + \frac{1-r}{1 - rp\overline{\rho_{\alpha_r^*}}} E\left[X \mid \mathrm{ND}; \alpha\right] \\
\hat{x}_h &= r\left[(1-p) E_z \left[P^{\mathrm{ND}}(z)\right] + p P_f^{\mathrm{ND}}\right] + (1-r) E\left[X \mid \mathrm{ND}; \alpha\right] \\
&= r \cdot x_h(\alpha_r^*) + (1-r) E\left[X \mid \mathrm{ND}; \alpha_r^*\right]
\end{aligned}$$

where x_l and x_h are defined as in the base model (Equation (16)). From the proof of Proposition 2 it is clear that the two threshold equilibrium with mixing region exists if and only if $\hat{x}_l < \hat{x}_h$: if $\hat{x}_l > \hat{x}_h$ then any type $x \in [\hat{x}_h, \hat{x}_l]$ is a threshold equilibrium, while if $\hat{x}_l < \hat{x}_h$ then any type $x \in [\hat{x}_l, \hat{x}_h]$ plays a mixed strategy.

In the proof of Proposition 2 we show that if Lemma 1 then $E_z \left[P^{\mathrm{ND}}(z, \alpha_r^*)\right] < P_f^{\mathrm{ND}}(\alpha_r^*)$ and therefore $x_l < x_h$. This also holds here. Moreover, in the proof of Lemma 2 we show that

$$E\left[X \mid \mathrm{ND}; \alpha_r^*\right] = E_x\left[P^{\mathrm{ND}}\left(x, \alpha_r^*\right) \mid \mathrm{ND}\right] < E_x\left[P^{\mathrm{ND}}\left(x, \alpha_r^*\right)\right] = E_z\left[P^{\mathrm{ND}}\left(z, \alpha_r^*\right)\right].$$

Thus

$$E\left[X \mid \mathrm{ND}; \alpha_r^*\right] < x_l(\alpha_r^*) < x_h(\alpha_r^*).$$

We now show that $\hat{x}_l(\alpha_r^*) < \hat{x}_h(\alpha_r^*)$ for any equilibrium α_r^*. We abuse notation and define the following functions

$$\begin{aligned} \hat{x}_l(r, \alpha) &= r\frac{1 - p\overline{\rho_\alpha}}{1 - rp\overline{\rho_\alpha}} x_h(\alpha) + \frac{1 - r}{1 - rp\overline{\rho_\alpha}} E\left[X \mid \mathrm{ND}; \alpha\right] \\ \hat{x}_h(r, \alpha) &= r \cdot x_l(\alpha) + (1 - r) E\left[X \mid \mathrm{ND}; \alpha\right]. \end{aligned}$$

Observe that

$$\hat{x}_l(0, \alpha) = \hat{x}_h(0, \alpha) = E\left[X \mid \mathrm{ND}; \alpha\right] \tag{25}$$

and

$$\hat{x}_l(1, \alpha) = x_l(\alpha) < x_h(\alpha) = \hat{x}_h(1, \alpha). \tag{26}$$

Differentiating with respect to r we obtain

$$\begin{aligned} \frac{\partial \hat{x}_h(\cdot, \alpha)}{\partial r} &= x_h(\alpha) - E\left[X \mid \mathrm{ND}; \alpha\right] \\ \frac{\partial \hat{x}_l(\cdot, \alpha)}{\partial r} &= \frac{1 - p\overline{\rho_\alpha}}{\left(1 - rp\overline{\rho_\alpha}\right)^2}\left[x_l(\alpha) - E\left[X \mid \mathrm{ND}; \alpha\right]\right]; \end{aligned}$$

observe that (1) $\hat{x}_h(r, \alpha)$ and $\hat{x}_l(r, \alpha)$ are increasing in r, and (2) $\hat{x}_h(r, \alpha)$ is linear w.r.t. r and $\hat{x}_l(r, \alpha)$ is convex w.r.t. r. Equations (25), (26) and the two properties above imply that $\hat{x}_l(r, \alpha) < \hat{x}_h(r, \alpha)$ for any $r \in (0, 1]$ and an equilibrium strategy α, and specifically,

$$\hat{x}_l(\alpha_r^*) = \hat{x}_l(r, \alpha_r^*) < \hat{x}_h(r, \alpha_r^*) = \hat{x}_h(\alpha_r^*).$$

This proves the desired result.

A.6 Proof of Proposition 5

Proof. Observe first that because $P_f^{\mathrm{ND}}(\alpha) = E[X \mid \mathrm{ND}, S = Z; \alpha]$ (Equation (5)) and the minimum principle (see explanation in Section 3.3.1), Equation (21) also implies that $\underline{x} = P_f^{\mathrm{ND}}(\theta_{\underline{x}})$.

Substituting (11) in the equilibrium condition $x^*(s) = P^{\mathrm{ND}}\left(s, \theta_{x^*(s)}\right)$, we obtain two fixed point operators, depending whether we use the upper or lower leg of $P^{\mathrm{ND}}(s, \theta_{x^*})$. First assume that $s \geq x^*(s)$ to obtain the condition

$$x^*(s) = P_f^{\mathrm{ND}}\left(\theta_{x^*(s)}\right),$$

whose unique fixed-point solution is $\underline{x}$. This is a valid equilibrium solution for signals $s \geq \underline{x}$, and proves part 2 of the proposition.

Now assume that $s < x^*(s)$ to obtain the condition

$$x^*(s) = \overline{\rho_{x^*(s)}} \cdot s + \left(1 - \overline{\rho_{x^*(s)}}\right) P_f^{\mathrm{ND}}\left(\theta_{x^*(s)}\right). \tag{27}$$

We consider possible equilibrium $x^*(s)$ for two cases:

1. First, consider signals $s > \underline{x}$. In this case the RHS of (27) is always greater than $\underline{x}$ and thus $x^*(s) > \underline{x}$. By the minimum principle, this implies $P_f^{\mathrm{ND}}\left(\theta_{x^*(s)}\right) < x^*(s)$. But under Condition (27), $x^*(s)$ is between s and $P_f^{\mathrm{ND}}\left(\theta_{x^*(s)}\right)$, implying $x^*(s) < s$ – a contradiction.

2. Now consider signals $s < \underline{x}$. In this case type $\underline{x}$ strictly prefers to disclose and thus $x^*(s) < \underline{x}$. By the minimum principle, this implies $P_f^{\mathrm{ND}}\left(\theta_{x^*(s)}\right) > x^*(s)$ and therefore $x^*(s) > s$, as assumed above. We now show that a fixed-point solution to condition (27) exists and is unique for any $s < \underline{x}$. Because $x^*(s) = P_f^{\mathrm{ND}}\left(\theta_{x^*(s)}\right)$ has a fixed point solution $x^*(s) = \underline{x}$, and $x^*(s) = s$ has a trivial solution, then, by continuity, a solution to (27) exists and satisfies $x^*(s) \in (s, \underline{x})$. Moreover, To see that this solution is unique, observe that $P_f^{\mathrm{ND}}(\theta_{x^*})$, and therefore also the RHS of (27), is decreasing in x^* over the interval $x^* \in (s, \underline{x})$.

This proves part 1 of the proposition.

Finally observe that the RHS of (27) is increasing in s for any $x^*(s)$. This is sufficient to show that $x^*(s)$ is increasing in s for $s < \underline{x}$, and weakly increasing in general. $\square$