

The Interaction of Minimum Wage and Severance Payments in a Frictional Labor Market: Theory and Estimation

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Abstract

We introduce a minimum wage and severance payments in an equilibrium labor market model with search frictions. We analyse how these policies affect endogenous job creation and destruction decisions and, more generally, the general equilibrium allocation. We structurally estimate the model's parameters using data from Chile, a country where labor market regulations prescribe high severance payments and minimum wage; we build three subsamples corresponding to three different levels of educational attainment. With the resulting sets of estimates we perform a quantitative welfare analysis. We reach three main conclusions. First, when the dispersion in wages found in the subsample is low and the share that workers receive from the surplus their job generates is at an appropriate level, the economy's maximum welfare level is reached in a policy-free environment; on the other hand, if the workers' share is too low, the maximum level of welfare can be attained using any of the following possibilities: severance payments or a minimum wage by themselves or an appropriate combination of these two policies. Second, as dispersion in wages increases, the minimum wage, by itself, can no longer reach the economy's maximum level of welfare. Third, when the dispersion in wages is high enough, no policy in isolation can attain the economy's maximum level of welfare, and a particular combination of labor market policies is required.

Keywords: search model, minimum wage, severance payments, welfare.

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1 Introduction

A country's economic development and the well-being of its population are influenced by its labor market performance. The performance of the labor market and the policies that regulate it are tightly linked. We study the positive and normative equilibrium implications of the interaction of severance payments and minimum wage. These two policies are found together in many countries,² yet the analysis of their interaction has not received much attention. We formulate and structurally estimate an equilibrium search-and-matching model of an economy where these two policies coexist, and use the estimates of the model's parameters to perform counterfactual experiments.

To this purpose, our model must have two basic ingredients. Firstly, labor market policies may have an impact on all individuals in the model, not only on those that are directly affected. For example, a minimum wage may change the outside option for all workers, thus affecting the entire wage distribution. Therefore, it is essential to cast the analysis in general equilibrium. Secondly, we should be able to analyze the impact of policies on all firms' decisions, that is, job creation and destruction. Prat (2010) meets these two requirements, creating a framework particularly suited for estimation and the performance of welfare analysis.

Prat (2010) develops a continuous-time equilibrium model of the labor market with search frictions and Nash bargaining in a stationary environment. Ex-ante homogenous risk-neutral workers and firms make contact according to a standard matching technology. Upon contact, they draw their match-specific productivity, a dimension of initial heterogeneity that generates a job creation decision. After the initial draw, productivity fluctuates stochastically, a source of ex-post heterogeneity that introduces an endogenous destruction decision in the model. We extend this framework in two directions. First, wages cannot be lower than a mandatory, exogenously set, minimum wage. Second, we allow for two possible large shocks leading to exogenous match destruction; only one of the two entitles the worker to receive severance payments. Notice that by assuming risk-neutral workers, we are neglecting the insurance aspect of severance pay; instead, we focus on other, at least equally important, aspects: its effect on the wage distribution and on the job creation and destruction decisions.

Parametric assumptions on the distribution of initial productivity and on its stochastic evolution permit us to explicitly derive the likelihood of the model. We estimate the model using data from Chile, which is an interesting case because a large proportion of the Chilean population earns the minimum wage and severance payments are high by international standards. We find that the model implies a good fit of our data, capturing the general shapes of the wage and employment duration distributions.

We then perform counterfactual experiments that allow us to answer questions about optimal policy combinations. Armed with the estimated model, we perform a quantitative analysis of the steady state welfare effects of severance payments and the minimum wage.

²Refer for example to the Employment Outlook, OECD (2004).

To control for ex-ante heterogeneity, we estimate the model for three subsamples corresponding to different levels of workers' education: low education workers (Education 1), which is composed of workers with eight years of schooling or less; high school education workers (Education 2); and those with a college degree or more (Education 3). We perform a welfare analysis for each subsample as well.

We find that in equilibrium, a binding minimum wage affects the whole wage distribution; directly for low wages, and indirectly, through the value of unemployment for higher wages, but its effect is still the strongest at the bottom of the wage distribution. Additionally, our estimates imply a fair amount of dispersion in the sampling distribution, and thus initial productivity draws will result in productivities mostly in a region away from the cutoff. Therefore, small changes in the minimum wage have a large impact on the job creation and destruction threshold, but their impact on labor demand and market tightness is modest. On the other hand, severance payments affect the whole wage distribution, and all workers are equally eligible to receive an amount that is increasing in the wage. Being proportional to productivity, the distortion that severance payments introduce in the creation and destruction decisions (that is, for low productivities) is relatively weak. In this way, we can increase (decrease) severance payments enough so to reach the desired lower (higher) level of vacancy creation without leading firms to discard (create) too many matches.

Our results imply that there exists a unique value for the share that workers receive from the surplus their job generates such that the economy's maximum welfare level is reached in a policy-free environment in all subsamples. On the other hand, if the workers' share is below such unique value, there is room for policies to improve welfare; for Education 1 and 2, the maximum level of welfare can be attained using any of the following three possibilities: severance payments, minimum wage, or an appropriate combination thereof. However, for Education 3 no policy in isolation can attain the economy's maximum level of welfare, and a particular combination of labor market policies is required.

Authors studying the impact of employment protection policies have focused primarily on its tax dimension. Indeed, when wages are flexible, Lazear (1990) showed that the wage of newly recruited workers is reduced in an amount equal to the expected value of the future severance payments transfer. Therefore, in such settings, severance payments have no effect on the equilibrium allocation. We use what Mortensen and Pissarides (1999) call the insider wage model, that is, outsiders face the same wage setting mechanism as incumbent workers, breaking the neutrality of severance payments. Many empirical papers support this non-neutrality result; for example, Di Tella and MacCulloch (2005) and Heckman and Pages (2000) find a significant impact of severance payments on employment.

One of the exceptions closer to our model is Cahuc and Zylberberg (1999). They also study the interaction of severance payments and minimum wage in a search model; however their characterization of wage profiles is different than ours. As shown by Garibaldi and Violante (2005), the wage setting

mechanism is essential in determining the effects of severance payments. In Cahuc and Zylberberg (1999), the wage of entrants is reduced because of severance payments. In addition, they assume that workers cannot observe the productivity of the match, and thus, wage renegotiations only take place by mutual agreement. This implies that, in equilibrium, renegotiations are started by employers, and only when the idiosyncratic productivity shock is so bad that they have a credible threat to destroy the match. This characterization leads to wages that can only decrease with tenure. Cahuc and Zylberberg conclude that severance payments have a real impact on the labor market (in particular on employment, there is no thorough welfare analysis) when the minimum wage is high. Comparing this to our analysis, where all workers face the same wage negotiation mechanism and wages can increase with tenure, severance payments can have a significant impact on employment and, more importantly, on welfare, even in the absence of a binding minimum wage.

Closer to our analysis (modeling of labor market policy, estimation and welfare analysis) is Flinn (2006). He introduces a minimum wage in a search model with wages determined through Nash bargaining, stochastic job matching and endogenous participation. As in our model, a binding minimum wage can increase welfare by increasing the effective bargaining power of workers. Minimum wages, however, can have a potentially larger impact on welfare in Flinn's setting, because they have the additional benefit of increasing participation. Finally, as only exogenous destruction exists in Flinn's model, it is not the best alternative to study the effect of severance payments.

In Section 2 we present some stylized facts about the Chilean labor market and Chilean legal framework to motivate the specification of policies in our model. The model is presented in Section 3 and a sample of its likelihood derivation is presented in Section 4. For the complete likelihood derivation, please refer to the Appendix. In Section 5 we describe our data, discuss identification, present our estimation results and briefly describe a sensitivity analysis. Our welfare analysis is presented in Section 6 and we draw our conclusions in Section 7.

2 Stylized Facts

In this section we present stylized facts about the Chilean labor market and Chilean legal framework, to motivate the specification of policies in our model.

The minimum wage and severance payments are arguably two of the most important labor market policies in Chile. There is also in place an unemployment insurance policy, which has been reformed in 2001 and 2009 in order to increase its coverage and efficiency. However, its benefit replacement rates and duration are still very low compared to OECD countries (see OECD 2010).

The mandatory minimum wage applies to all private sector workers between the ages of 18 and 65.³ The minimum wage is modified every July by Congress, based on expected inflation and productivity. Severance payments are paid to workers with an indefinite contract who are fired because of

³The mandatory minimum wage for workers outside this age range is 25% lower.

Table 1: Descriptive Statistics

Characteristic	All	Education 1	Education 2	Education 3
Spells	14650	3062	8800	2708
Individuals	8131	1947	4703	1481
Ratio (av. wage/ min. wage)	4.74 (5.84)	2.94 (2.16)	4.07 (3.88)	8.43 (10.08)
Av. employment duration	43.99 (42.85)	45.66 (44.13)	42.94 (41.95)	45.72 (44.51)
Av. unemployment duration	10.55 (15.51)	10.39 (15.21)	10.62 (16.13)	10.63 (13.36)
Receive SP	46.4%	41.0%	48.6%	44.5%
Earn min. wage	17.0%	20.2%	13.0%	5.2%

Standard deviations in parenthesis. Source: Social Protection Survey.

firm's necessities (*necesidades de la empresa*). Layoffs due to changes in demand or in the economy or firm modernization fall in this category. The law starts binding after 12 months of tenure and the worker is eligible for a severance payment (SP) of one month of wages for each year worked at the firm, up to an upper bound of 11 years. The base to compute the monthly wage is the last wage received by the worker.⁴

To assess the impact of severance payments and the minimum wage on the Chilean labor market, we present some descriptive statistics of our data. Note that a more detailed description of our data and the subsamples we use in the estimation is presented in Section 5. We draw our longitudinal data set from the Social Protection Survey. Our panel data set contains individual's labor market histories since 1990, with information on wages, spell durations, and reception of severance payments for completed employment spells. Our sample consists of almost 15000 censored and completed spells belonging to 8131 individuals. Table 1 presents some descriptive statistics for our pooled sample and for each of the three subsamples we use in the estimation process: low education workers (*Education 1*), which is composed of workers with eight years of schooling or less; high school education workers (*Education 2*); and those with a college degree or more (*Education 3*).

Employment spells were found to last, on average, almost 44 months, whereas the average unemployment spell length was 10.6 months, and none of these averages vary significantly across education groups. 46 percent of the completed employment spells ended with the reception of severance payments; this proportion changes across education groups, however the proportion is always over 40

⁴Severance payments rules have been modified through the years. The ones described here apply since 1990.

Table 2: Wages and Incidence of Severance Payments

Normalized Hourly Wage w	% of Spells Ending with SP			
	Full Sample	Education 1	Education 2	Education 3
$w \leq 1.1$	37.3	37.5	37.2	37.7
$1.1 < w < 1.6$	42.0	35.2	44.0	43.5
$1.6 \leq w < 3.3$	39.7	28.6	41.7	42.7
$3.3 \leq w < 5.1$	52.2	41.3	58.2	48.9
$w \geq 5.1$	58.0	46.9	59.9	59.4

Hourly wages are normalized as the ratio of the hourly wage to the minimum wage.

Each wage bracket contains approximately 20% of the full sample. Source: Social Protection Survey.

Table 3: Tenure and Incidence of Severance Payments

Tenure T (in months)	% of Spells Ending with SP			
	Full Sample	Education 1	Education 2	Education 3
$12 \leq T \leq 17$	42.4	39.6	44.8	39.7
$18 \leq T \leq 26$	46.5	51.1	49.0	46.1
$27 \leq T \leq 38$	47.9	35.9	49.1	53.4
$39 \leq T \leq 60$	51.8	43.4	55.2	51.1
$T \geq 61$	52.3	43.0	55.5	54.9

Tenure is the length of completed employment spells. Each tenure bracket contains approximately 20% of the full sample. Source: Social Protection Survey.

percent. A mass of almost 17 percent of employees earns the minimum wage and, consistent with lower education workers earning lower average wages, this mass decreases rapidly when education increases, from 20 percent for workers with the lowest education, to 13 percent for those that finished high-school and five percent for those with graduate studies. Therefore, across all education groups, we find a large mass earning the minimum wage and more than 40 percent of the employees receiving severance payments.

An interesting feature of severance payments found in our data is shown in Table 2: the proportion of completed employment spells that end with severance payments increases with the wage. When dividing the sample by education level, this relationship holds in all groups (although, with the proportion increasing at different rates), except for those in the lowest education bracket (eight years of schooling or less). This relationship will help us specify severance payments in the model presented in Section 3.

We assume that severance payments are proportional to productivity, variable of which wages are an affine function in our model. Since in our model, productivity is also positively correlated with tenure, for the sake of the model's fit to the data, we should expect to find in our data a positive

relationship between the incidence of severance payments and tenure too. We do in fact find such relationship (the results are presented in Table 3), which validates our severance payments specification.

3 Model

The model is set in continuous time and we assume a stationary environment. The labor market is populated by a measure one of workers that are either employed or unemployed and searching for jobs. There is a continuum of firms that can produce or search for workers to fill vacancies. Workers and firms meet according to a CRS matching technology, and produce a homogeneous good in matches formed by one firm and one worker. Their match-specific initial productivity is drawn from a distribution G . After observing the initial draw, they decide whether to start producing. If they resolve not to produce, the worker remains in the unemployment state and the firm maintains its vacancy. Otherwise, the job is created and production starts. In active matches, idiosyncratic productivity stochastically fluctuates according to a geometric Brownian motion with parameters μ and σ . Therefore, the law of motion of productivity x is given by:

$$\frac{dx_t}{x_t} = \mu dt + \sigma dB_t$$

where dB_t is the increment of a Wiener process.⁵

Firms and workers here are assumed to be risk-neutral. Given the value of unemployment, workers maximize the expected present discounted value of wages. Similarly, given the value of a vacancy, firms maximize the expected present discounted value of profit flow. Wages are continuously renegotiated via Nash bargaining, where the worker's net return from the relationship is equal to a fraction β of the total surplus of the match. Firms and workers use the same discount rate r .

We use the stylized facts presented in Section 2 to specify severance payments and the minimum wage. We assume wages cannot be lower than a statutory and exogenously set minimum wage m . To mimic severance payment law, we would have to introduce a trial period where the severance payment is not binding and model severance payments as proportional to the last wage, with the proportionality coefficient equal to the minimum between tenure and 11 years of employment. This immensely complicates the derivation of the model's likelihood function. Therefore, we implement an approximation to the Chilean setting by modeling severance payments as proportional to the productivity the job had at the time of the break. As wages are an affine function of productivity in our model, this specification captures the relationship between severance payments and wages. Specifically, we

⁵Beyond tractability (which will be discussed later), the assumption of a geometric Brownian motion allows us to study the effects of the variance of innovations, as opposed to the long-run variance, and identify it through some aspect of the data. Additionally, the geometric Brownian motion assumption enables us to capture some important labor market stylised facts such as a hump-shaped hazard rate of job separation and an unimodal wage density with a right tail of Pareto functional form (see Prat (2006)).

model severance payments as a fraction τ of the workers' final productivity. In Chile, because severance payments are paid only when the worker is fired due to firm's necessities, at any given final wage we find spells that ended both with and without severance payments; this is confirmed by the numbers in Table 2. Based on this, we introduce two exogenous shocks, one that entitles the worker to receive severance payments, while the other does not. We assume these shocks have Poisson arrival rates δ_1 and δ_2 respectively. Therefore, the shocks with arrival rate δ_1 are shocks to the firm, demand side shocks, which force the firm to lay off workers, thus to pay severance pay. In equilibrium, matches with productivity below a certain threshold will be endogenously destroyed. We also assume that firms do not have to pay severance in such circumstance. Given that endogenous destruction can be thought of as bankruptcy, this assumption is consistent with the Chilean reality; in our sample period, it was not an obligation for firms to make provisions to provide workers with severance payments in the case of bankruptcy, thus many could not pay.⁶ Additionally, this assumption guarantees that our model is consistent with the fact that the proportion of completed employment spells that end with severance payments increases with the wage.⁷

Finally, as we do not differentiate sectors or contract types in our model, the same severance payments rules apply to all workers.⁸

3.1 Bellman Equations

Two equilibrium cut-offs for productivity will naturally arise in this setting. First, firms optimally choose a threshold x_r , such that, if the initial productivity draw is below it, the match is not created. As we are assuming that no severance payments are due when the match is endogenously terminated, destruction is determined by the same threshold as creation. Given a wage schedule, firms will create/maintain a job as long as the present discounted value of its net return is equal to the value of posting a vacancy. Let x_m be the level of productivity for which the wage implied by the Nash bargaining is equal to the minimum wage m . If x_m is larger than the creation/destruction cut-off x_r , then the minimum wage is binding, and the mass of people earning the minimum wage is comprised of workers employed with productivities between x_r and x_m .

Let $w_{NB}(x)$ be the wage defined in the Nash bargaining when productivity is equal to x . Therefore, workers will receive a wage equal to the minimum wage m when productivity is in the interval $[x_r, x_m]$ and a wage given by $w_{NB}(x)$ when productivity exceeds x_m . Thus, the general wage function

⁶Refer to Cowan and Micco (2005).

⁷Indeed, if we let x_r be the productivity cut-off for creation/destruction, then the chance of receiving severance payments conditional on separation in the next instant and given a productivity $x_{T-\Delta t}$ in $T - \Delta t$, is given by:

$$\frac{\delta_1}{\delta_1 + \delta_2 + \mathbb{P}\{x_T = x_r | x_{T-\Delta t}\}}$$

which is increasing in $x_{T-\Delta t}$ and, as we will prove, in our model wages are increasing in productivity.

⁸Therefore, in our estimation we only use information on employment spells in the private sector with indefinite contracts.

$w(x)$ can be defined as:

$$w(x) = \begin{cases} w_m = m & x \in [x_r, x_m) \\ w_{NB}(x) & x \geq x_m \end{cases}$$

Hence, under a binding minimum wage, this relationship between wages and productivity implies that the value function of workers and firms are also defined piecewise. Given the value of unemployment U and the value V of a vacancy, the value function of workers (W) and firms (J) are defined as:

$$K(x) = \begin{cases} K_m(x) & x \in [x_r, x_m) \\ K_{NB}(x) & x \geq x_m \end{cases} \quad \text{for } K \in \{W, J\}$$

where:

$$rW_i(x) = w_i(x) + (\delta_1 + \delta_2)(U - W_i(x)) + \delta_1\tau x + \frac{\mathbb{E}[dW_i(x)]}{dt} \quad i \in \{m, NB\} \quad (1)$$

and:

$$rJ_i(x) = x - w_i(x) + (\delta_1 + \delta_2)(V - J_i(x)) - \delta_1\tau x + \frac{\mathbb{E}[dJ_i(x)]}{dt} \quad i \in \{m, NB\} \quad (2)$$

For a non-binding minimum wage m , that is, when $x_m \leq x_r$, the value function for workers and firms are given by $W_{NB}(x)$ and $J_{NB}(x)$, respectively, for $x \geq x_r$. In what follows, we study the case of a binding minimum wage, the non-binding minimum wage model being a special case.

Equation (1) represents an asset equation in a perfect capital market. $W_m(x)$ is the asset value of a worker matched to a job with productivity $x \in [x_r, x_m)$. Consequently, its capital cost, $rW_m(x)$, must equal its return. The return components include: the flow value of a wage, in this case equal to the minimum wage; the net return of changing state, $U - W_m(x)$, which happens according to a Poisson process of rate $(\delta_1 + \delta_2)$; the return from receiving severance payments τx after a δ_1 shock; and the return from expected changes in the valuation of the asset. $W_{NB}(x)$ has exactly the same interpretation, and the only difference in its formulation is that the relevant wage is the one determined using Nash bargaining, not the minimum wage.

The intuition of the firm's value function J in (2) follows that of the worker's. The firm's flow value is the output less the pay to the worker. If a destruction shock arrives, the firm will have a vacancy of value V . If δ_1 hits, then the firm has to pay severance payments to the worker.

3.2 Wage Determination

Wages are determined using Nash bargaining, and we assume the insider wage model proposed by Mortensen and Pissarides (1999). Here, the same wage setting rules apply to outsiders, newly hired

workers, and to insiders, previously hired workers.⁹ Additionally, we assume that if the worker and the firm cannot reach an agreement in their wage renegotiation, then the firm must provide a severance payment to the worker. Following Millard and Mortensen (1997), we acknowledge the fact that severance payments enhance the workers' bargaining position, and therefore, severance payments are considered in the threat points of workers and firms.¹⁰ The Nash bargained wage is the solution of the following maximization problem:

$$\max(W - U - \tau x)^\beta (J - V + \tau x)^{1-\beta}$$

whose first order condition is:

$$W_{NB}(x) - (U + \tau x) = \beta[W_{NB}(x) + J_{NB}(x) - V - U] \quad (3)$$

Equation (3) states that the worker's net return from the relationship is equal to a fraction β of the total surplus of the match. We will refer to β as the workers' bargaining power or share. We use W_{NB} and J_{NB} because this negotiation is relevant only when productivity is larger than x_m .

The amount of vacancies in the market is determined according to a free-entry condition, that is, vacancies are created until discounted profits equal the cost of entry. Thus, the value of vacancies, V , is equal to zero.

Applying equations (1) through (3), and the free-entry condition, we derive the following expression for wages when $x \geq x_m$:

$$w_{NB}(x) = rU + \beta(x - rU) + \tau x(r + \delta_2 - \mu) \quad (4)$$

The first two terms are common in Nash bargaining. The worker receives his outside option, rU , plus a fraction β of the net surplus that the match creates, $x - rU$. The third term is the positive effect of severance payments on wages. This term can be decomposed in two effects: if we did not include severance payments in the threat points, and therefore severance payments only had an effect through the value functions, the third term would just be $-\delta_1 \tau x$, and thus workers would pre-pay

⁹That is, we rule out the two-tier wage structure; structure upon which is built the severance payments neutrality result (see Lazear (1990)). However, there is evidence of a "hold-up" problem, making the two-tier structure not credible (see, for example, Lindbeck and Snower (1988)).

¹⁰We are assuming that in case of disagreement severance payments must be paid, but Nash bargaining is an axiomatic, cooperative solution. Binmore et al. (1986) show that the Nash bargaining solution can be obtained as the limit of the sequential equilibrium of a non-cooperative game. They analyze an alternating offer game with risk of breakdown in between rounds of offers, in which agents receive exogenously set payoffs in the event of a breakdown. They proved that, as the time interval between rounds goes to zero, the equilibrium implies an immediate agreement and the resulting split is analogous to that of a Nash bargaining solution where the threat points are the exogenous payoffs of the agents. This game is consistent with our setting if the breakdown of real time negotiations is caused by a shock arriving at rate δ_1 , which prevents the firm from returning to the bargaining table (and therefore the firm is obliged to pay severance). In this case, the exogenous payoffs of the agents are $V - \tau x$ and $U + \tau x$ for the firm and the worker, respectively.

the severance payments they may receive at the end of the job. Now, when considering severance payments in the wage negotiation, we also have to add $\tau x(r + \delta_1 + \delta_2 - \mu)$.¹¹ That is, each period the worker receives interest for his future holdings of severance payments, where the relevant rate is the market's interest rate adjusted for the probability of match destruction and the drift in productivity. The sum of these two terms is the last term of the wage in equation (4).

3.3 Solution of Value Functions

The assumption of a geometric Brownian motion for the productivity process, together with Ito's lemma, allows us to compute expected changes in the value functions. We obtain:

$$\frac{\mathbb{E}[dK_i(x)]}{dt} = \frac{\sigma^2}{2}x^2K_i''(x) + \mu xK_i'(x) \quad \text{for } i \in \{m, NB\} \text{ and } K \in \{W, J\}$$

Given this result and the expression for the wage in equation (4), we solve the differential equations (1) and (2) to obtain solutions of the form:

$$W_m(x) = Ax^{R_1} + Bx^{R_2} + \frac{m + \delta U}{r + \delta} + \frac{\delta_1 \tau x}{r + \delta - \mu} \quad (5)$$

$$W_{NB}(x) = Cx^{R_1} + Dx^{R_2} + \frac{\beta x}{r + \delta - \mu} - \frac{r\beta U}{r + \delta} + \tau x + U \quad (6)$$

$$J_m(x) = Ex^{R_1} + Fx^{R_2} + \frac{x(1 - \delta_1 \tau)}{r + \delta - \mu} - \frac{m}{r + \delta} \quad (7)$$

$$J_{NB}(x) = Gx^{R_1} + Hx^{R_2} + \frac{x(1 - \beta)}{r + \delta - \mu} - \frac{(1 - \beta)rU}{r + \delta} - \tau x \quad (8)$$

where A, B, C, D, E, F, G and H are unknown scalars, $\delta = \delta_1 + \delta_2$, and $\{R_1, R_2\}$ are the roots of the characteristic equation:

$$\frac{\sigma^2}{2}z(z - 1) + \mu z - (r + \delta) = 0 \quad \text{with } R_1 < 0 < R_2$$

The last two terms of W_m in (5) represent the expected present value of producing when the level of productivity x is in $[x_r, x_m)$ forever. The worker receives a flow wage m ; and in each instant there is a probability δ of becoming unemployed, and receiving U , and a probability δ_1 of receiving severance payments τx . Notice that the effective discount rate for terms not involving productivity is the one usually found when there are Poisson processes involved; that is, the risk free rate plus the destruction rate. For terms involving productivity, the effective discount rate takes into consideration that the expected value of productivity exponentially grows according to the drift μ , which must then

¹¹Given that the last term in equation (4) is positive, for a fixed positive τ the wage could become larger than the productivity of the match as β converges to one. This could be seen as a problem of the specification of the bargaining problem proposed here; however, as τ is always chosen optimally, if it is positive is because β is low enough, rendering this issue irrelevant in practice.

be subtracted. Finally, given that the last two terms of W_m represent the value of producing forever in the region $[x_r, x_m)$, the first two terms must embody the option value to separate, plus the value of going into the region where the minimum wage stops binding.

The interpretation of W_{NB} is similar. The last four terms represent the expected present discounted value of the revenue stream when the initial productivity is x in $[x_m, \infty)$. At each instant, the worker receives a fraction β of the net return that he/she generates in the match, $x - rU$; Nash bargaining wages also add the flows rU and $\tau x(r + \delta_2 - \mu)$. Finally, we have to include δU from the probability of turning unemployed and $\delta_1 \tau x$ from the probability of getting severance payments. The expected present value of the sum of these five terms, using the appropriate discount rate for each of them, are the last four terms in W_{NB} . As before the first two terms of W_{NB} represent the value of going into the region where $x < x_m$.

These equations imply that we must determine 10 parameters: eight coefficients plus the two productivity cut-offs. As the last four terms of W_{NB} represent the expected present value of producing forever given an initial productivity in the region $[x_m, \infty)$, the first two terms represent the value of the option to separate. As productivity increases, the probability of crossing the cut-off x_r goes to zero, and then so should the value of the option to separate. When productivity goes to infinity, x^{R_1} goes to zero but x^{R_2} diverges. Thus, for this solution to have a valid economic meaning, we need D to be zero. Applying this analysis to the value of the firm, we conclude that H will also need to equal zero.

By definition, x_m is the productivity at which Nash bargaining implies a wage equal to the minimum wage. Thus, from equation (4):

$$x_m = \frac{m - (1 - \beta)rU}{\beta + \tau(r + \delta_2 - \mu)} \quad (9)$$

Therefore, we need six restrictions to determine the remaining coefficients, plus one additional equation for x_r , resulting in seven required restrictions.

At x_r firms are indifferent between creating (destroying) the match and remaining idle (keep producing). Additionally, the firm's optimality requires a smooth pasting condition at x_r . These two conditions together imply:

$$J_m(x_r) = 0 \quad \text{and} \quad J'_m(x_r) = 0 \quad (10)$$

Furthermore, we have three value matching conditions. The firm's and worker's value of producing at x_m must be the same whether we approach x_m from the left or the right; and the value of the

worker at the destruction cut-off must be equal to the unemployment value. Thus, we have that:

$$J_m(x_m) = J_{NB}(x_m) \quad (11)$$

$$W_m(x_m) = W_{NB}(x_m) \quad (12)$$

$$W_m(x_r) = U \quad (13)$$

The two remaining equations come from the requirement of smooth value functions:¹²

$$J'_m(x_m) = J'_{NB}(x_m) \quad (14)$$

$$W'_m(x_m) = W'_{NB}(x_m) \quad (15)$$

Equations (9) to (15) form an implicit system for value function coefficients and two cut-offs, as the function of model parameters and the equilibrium value of unemployment U . We determine formulas for each of the unknowns as functions of the model parameters, x_r and U . In particular, the conditions on the value of a firm imply an implicit equation for the threshold x_r that will be useful when performing counterfactual experiments:

$$0 = x_r^{R_2} x_m^{1-R_2} [\beta + \tau(r + \delta_2 - \mu)](r + \delta - R_1\mu) + x_r(1 - \delta_1\tau)(R_1 - 1)(r + \delta) - R_1m(r + \delta - \mu) \quad (16)$$

3.4 Closing the Model

To complete the model, we describe how workers and firms meet and derive the equilibrium equations for the value of unemployment and the value of a vacancy.

As it is common in this strand of literature, we assume a constant returns to scale matching technology M . This matching function depends upon the unemployment and vacancy rates and determines the number of matches per unit of time in the economy. The CRS assumption implies that we can write:

$$M(u, v) = vM\left(\frac{u}{v}, 1\right) \equiv vq(\theta) \quad \text{with } \theta = \frac{v}{u}$$

Thus, the contact rate per vacancy $M(u, v)/v$ is given by $q(\theta)$; and the contact rate per unemployed worker $M(u, v)/u$ is equal to $\theta q(\theta)$.

The value of unemployment U is described by:

$$rU = -s + \theta q(\theta) \left[\int_{x_r}^{x_m} (W_m(x) - U) dG(x) + \int_{x_m}^{\infty} (W_{NB}(x) - U) dG(x) \right] \quad (17)$$

where s is the cost to the worker of exerting search effort, net of the value of leisure. The standard

¹²Since the wage schedule is continuous and $x_r > 0$, the classic theory for ODE tells us that there is a unique C^2 solution to each of the initial-value problems defining the value functions of workers and of firms. See, for example, Codrington and Levinson (1955) for details.

asset interpretation follows, with the right hand side of equation (17) representing the net return from searching; this is, the expected net return of making contact with a firm less search costs. The equation for the value of a vacancy is formulated analogously, and after applying the free entry condition, it becomes:

$$c = q(\theta) \left[\int_{x_r}^{x_m} J_m(x) dG(x) + \int_{x_m}^{\infty} J_{NB}(x) dG(x) \right] \quad (18)$$

With these elements, we are ready to close the model and formally define the equilibrium.

Definition: Given a minimum wage m and a severance payment coefficient τ , an equilibrium is a collection $\{W, J, U, x_r, x_m, \theta\}$, where the value functions W and J , the value of unemployment U , the productivity thresholds $\{x_r, x_m\}$ and the market tightness θ satisfy the conditions (5) to (15), (17) and (18).

4 Estimation

4.1 Data

To estimate the model parameters we use data from the Chilean Social Protection Survey (*Encuesta de Protección Social*, EPS hereafter). It is a panel household survey implemented by the Micro-data Center of the Department of Economics of Universidad de Chile. It was first conducted in 2002 and continued every two years thereafter. For this study, we use the 2002, 2004 and 2006 rounds. As the EPS was created as a tool to study the behavior of individuals affiliated with the pension system, the 2002 EPS is representative of that particular group of the population.¹³ Since 2004, the scope of the EPS was expanded to make it representative of the entire Chilean population aged 18 and over.

EPS data includes socio-demographic information as well as past and current labor market information. When individuals are interviewed for the first time, they are asked about their labor market activities since 1980, or since they were 15, whichever occurred last. Individuals that were interviewed in previous rounds were asked about their labor market activities since the last time they were surveyed. Each activity must be labeled as employed, unemployed, looking for a job for the first time or inactive; they were also asked to provide the initial and final month and year for every spell.

The information specifically relevant to this study includes the duration of job and unemployment spells, monthly wages, hours worked monthly, reception of severance payments and type of job. Given the structure of the survey, we obtain information on the monthly duration of every activity. The question about wages changed after the first round of the survey; in 2002, currently employed individuals were asked about their wages in the last month, but there was no wage question for past job spells. Starting in 2004, only the within-spell average wage was asked for current and past employment spells. For each completed employment spell, individuals were asked whether they received severance payments at the end of the spell, but not the specific amount. Employment spells are classi-

¹³In 2002 around 80% of the population aged 15 and over was affiliated to the pension system.

fied by sector and contract type. As only private sector workers with an indefinite contract are eligible to receive severance payments, we only use in our sample employment spells corresponding to that type of jobs.¹⁴ We also discarded spells corresponding to workers younger than 18 years old or older than 65 years old, as they are under a different minimum wage regime. We further restricted our sample to spells that started on or after 1990. Until 1989 Chile was under a military dictatorship with very different labor market institutions.¹⁵

Three issues arise with this dataset. First, we decided to use average wages as if they were current wages for censored employment spells and last wage for completed spells. We do this for technical reasons; in particular, the joint density of average wages over an employment spell and spell length is not easy to derive analytically in our setting. To test this assumption, we followed current employment spells in 2002 EPS to 2004 EPS and compared the current wage reported in 2002 with their corresponding average wage given in 2004. Almost all 2004 average real wages were significantly higher than their respective 2002 current wage. This result could be attributed to rising wages; however, given that we are looking only at a two year period and that the increase is generally substantial, this result suggests that people tend to report their last wage when asked about average wages. This result provides support to our assumption.

Second, some of those that declared themselves as currently unemployed were not searching for a job. The EPS survey has two sections where individuals are asked regarding their current labor market activity. In the labor market history section respondents just have to give their labor market status (inactive, looking for a job for the first time, unemployed, employed) since 1980 up to the present, with the respective starting and ending month and year for each activity. In the current labor market status section, they are detailed asked about their ongoing labor market activity; whether they have a job or not, and if not, whether they have been looking for one and for how long. When comparing the answers from these two sections, we conclude that it was common for people to say that they were unemployed (in the labor market history section) but not searching for a new job (in the current labor market status section). A striking implication of this behaviour are the quite different unemployment spell length means we obtain for spells in each of these two sections: for example, for the year 2002, if consider unemployment spells from the labor market history section, the mean unemployment spell lasts for 66 weeks, whereas such mean for spells from the current labor market status is only 15 weeks. As an unemployed worker in our model is defined as someone searching, we decided to only include in the sample those spells in which the individual declared to be searching in some round of the survey. Even though this implies not using all information available, this strategy resulted in enough unemployment spells as to deliver a precise estimate of the contact rate. As we are not choosing the spells related to any individual characteristic, this should not introduce any selection bias.

¹⁴This is a significant proportion of the Chilean workforce. For example, in 2002, 71.23% of workers had an indefinite contract and 62.1% worked in the private sector.

¹⁵See Mizala (1998) for more information on changes in labor market regulation from 1975 to 1995.

As a result, our sample disposition implies the fractions of employed and unemployed workers are not representative of such fractions in the Chilean labor market. We use only selected completed unemployment spells, and employment spells (censored and completed) corresponding to jobs in the private sector with an indefinite contract. Therefore the proportions we find in our data of workers in the employed and unemployed states do not properly represent the real values of such fractions. Since we cannot extract valid information from the observed fraction of workers in each labor market state, we decided to implement a maximum likelihood procedure conditional on such states.

Third, as we do not have direct information on hourly wages, we construct our wage measure dividing total earnings by the number of hours worked. To deal with a potential measurement error problem, we assume that the observed wage is equal to the true wage multiplied by a Lognormal error term, with mean one and variance σ_{ME} . However, we assume that the minimum wage is a “focal” point and therefore easy to report correctly, thus, we treat minimum wage observations as accurate.

This leaves us with a sample that comprises 17 years, each year with a different minimum wage. As ours is a steady state equilibrium, to estimate the model we rescale wages every year, so that the rescaled minimum wages and average wages are the same for all years in the sample. When using this sample to estimate our steady state equilibrium model, we are therefore assuming that changes in the minimum wage were expected and that the economy converged rapidly to its steady state after each change. Figure 1 represents the ratio between the minimum wage and the average wage for the Chilean economy. The ratio is fairly stable prior to 1997 and after 2000. We see an increase in the ratio from 1998 to 2000, caused by the inability of legislators to adjust minimum wages in the face of the Asian crisis.¹⁶ Therefore, we have a fairly stable environment with an exogenous shock in 1998. We further discuss this issue in Section 5.5.

Additionally, around half of the individuals in our sample contribute with more than one spell. As we can expect to find individual effects, using spells belonging to the same individual could violate the independence assumption of ML. We also study this issue further in Section 5.5.

4.2 Subsamples

It is well known that the level of education of individuals greatly affects the labor market outcomes. As we do not model this kind of heterogeneity directly, we divide our sample in three education levels, and we perform the estimation and welfare analysis for each of these subsamples previously described (Education 1, 2 and 3), assuming separate labor markets for each subsample.

4.3 Likelihood

The selection method we use to build our sample implies that the proportions of workers we find in our data in the employed and unemployed states do not properly represent the real values of such

¹⁶The minimum wage is usually reset every July. However, in 1998 Congress decided to set the minimum wage for the following three years, at an average annual rate of 11.9%.

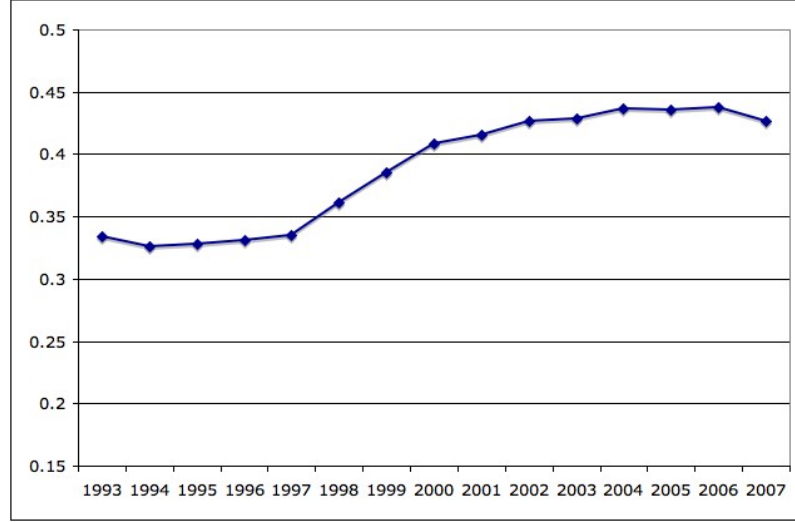


Figure 1: Chilean labor market 1993-2007: ratio of minimum wage to average wage. Source: National Institute of Statistics (INE).

fractions. Since we cannot extract valid information from the observed fraction of workers in each labor market state, we decided to implement a maximum likelihood procedure conditional on such states.

Following Prat (2010), we assume a Lognormal distribution for the distribution G of initial draws, for two reasons. First, Lognormal distributions are recoverable in the sense of Heckman and Flinn (1982), that is, its parameters are identified. Second, the assumption of a Lognormal sampling distribution, together with the assumption of a geometric Brownian motion for the productivity process, allows us to derive explicit expressions for equilibrium unemployment and the ergodic distribution of productivity, as well as the likelihood contribution of each of the spell types. This includes two types of unemployment spells, censored and completed spells. In the data, there are three types of censored employment spells: those with a wage equal to the minimum wage, those with a wage larger than the minimum and those without wage information. Uncensored employment spells can contain information on wages and on the reception of severance payments; there are three possibilities for wages w , $\{w = m, w > m, \text{no information on } w\}$, and three possibilities for severance payments, $\{\text{received SP, did not receive SP, no information about SP}\}$. Thus, we obtain nine different contributions from completed employment spells.

For each type of spell we compute its density conditional on the labor market state, or joint conditional density in the cases where, in addition to spell length, we have information on wages or on the reception of severance payments. We present here the basic steps of the derivation of the conditional likelihood contribution of a completed employment spell with wage information and that ended with the reception of severance payments. More detailed calculations for this type of spell, and for all other types of spells, can be found in the Appendix.

First, we must provide some definitions:

T_i = time of arrival of the first exogenous shock δ_i , $i = 1, 2$. Let f_i be its pdf, and F_i its cdf.

$T_r = \min\{t > 0 | X_t = x_r\}$, time of endogenous separation. Let f_r be its pdf, and F_r its cdf.

$t_e = \min\{T_1, T_2, T_r\}$, that is, the duration of completed employment spells.

By assumption:

$$f_i(t) = \delta_i e^{-\delta_i t} \quad \text{and} \quad F_i(t) = 1 - e^{-\delta_i t} \quad i = 1, 2$$

The following generalization of the reflection principle is going to be useful,¹⁷ for $x_0 \geq x_r$:

$$\mathbb{P}(T_r \leq t, X_t \in dx) = \begin{cases} \frac{1}{x\sigma\sqrt{t}} \left(\frac{x_r}{x_0}\right)^{\frac{2\bar{\mu}}{\sigma^2}} \phi\left(\frac{\ln(x/x_0) - 2\ln(x_r/x_0) - \bar{\mu}t}{\sigma\sqrt{t}}\right) dx & x \geq x_r \\ \frac{1}{x\sigma\sqrt{t}} \phi\left(\frac{\ln(x/x_0) - \bar{\mu}t}{\sigma\sqrt{t}}\right) dx & x < x_r \end{cases}$$

where ϕ is the standard Normal density function and $\bar{\mu} = \mu - \frac{\sigma^2}{2}$. This implies:

$$\begin{aligned} \mathbb{P}(T_r > t, X_t \leq \bar{x}) &= \Phi\left(\frac{\ln(\bar{x}/x_0) - \bar{\mu}t}{\sigma\sqrt{t}}\right) - \Phi\left(\frac{\ln(x_r/x_0) - \bar{\mu}t}{\sigma\sqrt{t}}\right) \\ &\quad + \left(\frac{x_r}{x_0}\right)^{\frac{2\bar{\mu}}{\sigma^2}} \Phi\left(\frac{-\ln(x_r/x_0) - \bar{\mu}t}{\sigma\sqrt{t}}\right) - \left(\frac{x_r}{x_0}\right)^{\frac{2\bar{\mu}}{\sigma^2}} \Phi\left(\frac{\ln(\bar{x}/x_0) - 2\ln(x_r/x_0) - \bar{\mu}t}{\sigma\sqrt{t}}\right) \end{aligned} \quad (19)$$

We first compute the joint density of productivity and the duration t_e of an employment spell, and then use the change of variable formula to derive the formula for wages.

For $\bar{x} \geq x_r$:

$$\mathbb{P}(t_e \leq t, X_{t_e} \leq \bar{x}, \text{ SP paid}) = \int_0^t \mathbb{P}(X_s \leq \bar{x}, s < T_r) [1 - F_2(s)] dF_1(s)$$

Then, we differentiate this probability with respect to \bar{x} and t to derive the density d for $\bar{x} \geq x_r$. Using equation (19) we obtain:

$$\begin{aligned} d(t, \bar{x}, \text{ SP paid}) &= \left[\frac{1}{\bar{x}\sigma\sqrt{t}} \phi\left(\frac{\ln(\bar{x}/x_0) - \bar{\mu}t}{\sigma\sqrt{t}}\right) - \frac{1}{\bar{x}\sigma\sqrt{t}} \left(\frac{x_r}{x_0}\right)^{\frac{2\bar{\mu}}{\sigma^2}} \phi\left(\frac{\ln(\bar{x}/x_0) - 2\ln(x_r/x_0) - \bar{\mu}t}{\sigma\sqrt{t}}\right) \right] \bar{F}_2(t) f_1(t) \end{aligned}$$

¹⁷See Harrison (1985) for details on its derivation.

Let the term in squared brackets be $A(t, \bar{x})$. Using this definition and the change of variable formula, the joint density of a completed spell of length t , with wage w , and that ended up with severance payments is:

$$h(t, w, \text{reception of SP}) = \begin{cases} \frac{A(t, x(w)) \bar{F}_2(t) f_1(t)}{\beta + \tau(r + \delta_2 - \mu)} & w > m \\ \bar{F}_2(t) f_1(t) \int_{x_m}^{x_r} A(t, x) dx & w = m \end{cases}$$

where $x(w) = (w - (1 - \beta)rU)/(\beta + \tau(r + \delta_2 - \mu))$ is the productivity implied by wage w .

The density h is the likelihood contribution of this spell, conditional on the initial draw.¹⁸ To derive the final expression of the likelihood contribution, we take the average over x_0 , which is distributed according to a Lognormal distribution truncated at x_r . We omit this final calculation in this paper due to its length.

4.4 Identification

The parameters that appear directly in the log likelihood formula, and that we are able to consistently estimate given our data, are the job contact rate λ , the rates of the destruction shocks δ_1 and δ_2 , the location (ν) and scale (ξ) parameters of the distribution of initial productivity draws, the productivity process parameters μ and σ , and the variance σ_{ME} of measurement error. Since the average length of unemployment spells in our model is given by $1/(\lambda(1 - G(x_r)))$, the information on the length of unemployment spells in our data is critical in the identification of λ . The proportion found in the data of spells ending with a severance payment helps determine the relative values of δ_1 and δ_2 , whereas the length of employment spells determines their levels. ML uses wage information to identify the parameters from the sampling distribution, the productivity process and measurement error.¹⁹ For example, the correlation between wages and tenure found in the data helps determine whether wage variance should come from the initial sampling distribution or from the productivity process' dispersion.

The search cost s is not identified, because it only enters the likelihood as a part of the equation that determines the value of unemployment U . Thus, as is usual in this literature, we treat the endogenous threshold x_r as a parameter in the estimation process and infer s from the equilibrium equation for U . The workers' bargaining power β also appears in the likelihood function. Even though in theory it is identified, Flinn (2006) showed that, in practice, a very large sample would be needed. Thus, we fix the value for β . As expected, the discount rate r is also unidentified; therefore, we fix it as well.

The severance payments coefficient τ is not identified. The frequency of severance payments obtained from the survey allows us to identify destruction shocks, however, as we do not have in-

¹⁸To be precise, the density h is the *conditional* likelihood contribution of this spell, however, hereafter I will omit the conditional.

¹⁹Recall, one of the reasons to choose a Lognormal for the sampling distribution is that its parameters are identified.

formation on the amount actually paid, we cannot identify τ . Based on Chilean law, if we know the length of a past employment spell that ended with severance payments, then we know how many months of wages the worker must have received as severance payments: the minimum of 11 (ceiling for severance payments) and the number of years he or she worked for the firm. Therefore, using duration data, we can compute the average ratio ρ^{data} between severance payments and final wage for all past employment spells that ended with a severance payment. The theoretical counter part of this moment, $\rho^{model}(\alpha)$, is given by:

$$\rho^{model}(\alpha) \equiv \mathbb{E} \left[\frac{\tau x}{w(x)} \middle| \text{reception of SP} \right]$$

where $\alpha = (\lambda, \delta_1, \delta_2, x_r, \mu, \sigma, \nu, \xi, \sigma_{ME})$ is the collection of the parameters we estimate using ML. We introduce the restriction $\rho^{model}(\alpha) = \rho^{data}$ in the estimation. For any given α , this restriction determines a particular level for the severance payments coefficient τ , allowing us to estimate τ using a concentrated likelihood. Once we determine the collection $\hat{\alpha}$ that maximizes the likelihood, we determine its standard errors with the usual formulas, and then compute the standard errors of the implied $\hat{\tau}$ using the delta method.

If information on vacancies were available, we could estimate one parameter of the matching technology q . Given the lack of such information, we will assume a functional form for q without unknown parameters. With this assumption, together with the free entry condition, we can “back up” the vacancy cost c , which will be necessary for the policy experiments. The resulting value of c depends upon the elasticity of the chosen matching function. Thus, in principle, the elasticity could be chosen so that the expected cost of hiring an employee, measured in units of monthly wages, is consistent with any desired number. We can derive the expected cost of hiring an employee from the free entry condition (18), repeated here for ease of reference:

$$\frac{c}{q(\theta)} = \int_{x_r}^{x_m} J_m(x) dG(x) + \int_{x_m}^{\infty} J_{NB}(x) dG(x) \quad (18)$$

As we directly estimate the job contact rate λ in the ML, changes in the functional form of the matching function do not affect the estimation process. Therefore, as we also estimate the threshold x_r directly, the right hand side of equation (18) is completely determined by the values obtained in the estimation process.²⁰ Accordingly, the left hand side must remain constant as we change the elasticity of q . Therefore, we cannot use this device to identify the matching function elasticity, instead we set it to 0.5, which is the average of what is usually found in the literature. In particular, we use the

²⁰As changes in the elasticity do change the equilibrium value of the market tightness θ , it may seem unintuitive that wages do not change as well. The reason is that wages depend on θ only through the value of unemployment U . As we estimate x_r and λ directly, U is determined once we have the estimates. Therefore, changing the elasticity changes c and θ , but does not affect U , wages or profits.

functional form $q(\theta) = \theta^{-1/2}$ for the matching function.

4.5 Results

By fixing the discount rate at 5% annually and the workers bargaining power β at 0.3, we obtain the following ML estimates for each subsample.²¹ Results are presented in Table 4.

The estimate of the drift μ of the productivity process implies that productivity increases by around 0.7% per month for the lowest educated group and at 0.8% for the most educated, which implies that wages increase rapidly with tenure for all groups and that they do it faster for those with more education.²²

On the other hand, the small estimate for productivity dispersion σ across subsamples, implies that productivity grows almost deterministically, and thus, endogenous destruction is not a common event, with the implied income volatility and number of separations being higher for those with the lowest education. Simulations give us some insight on these low estimates: larger levels of σ 's imply levels of wage dispersion across tenure that we do not observe in our subsamples; in particular, the implied dispersion of wages corresponding to longer employment spells is much larger than what we observe. However, we do observe a fair amount of dispersion in our data, which is captured by the sampling distribution; consistent with our data, the mean and dispersion implied by ν and ξ are increasing in the education level. So, most of the observed wage dispersion is explained by the model as luck in matching when leaving unemployment. Additionally, due to the lack of job-to-job transitions in our model, the high wages at low tenure we find in our data set can only be explained by a high initial productivity.

The estimate for the job contact rate λ implies that, on average, unemployed workers from the two lowest education groups receive a wage offer every 10 and a half months, which is almost half the frequency found by Prat (2010) using United States data. This frequency for those with the highest education is 9.9 months. The implied unemployment duration is 10.4 months for those in Education 1 and 10.6 for the others, which is exactly what we found in the data. These estimates imply that virtually all contacts result in matches for Education 1 and 2, whereas 6.8% of contacts are discarded for those with higher education. The resulting unemployment rate is near 18% for all subsamples.

The estimates for the Poisson rates δ_1 and δ_2 are very stable across subsamples, capturing precisely the probabilities of receiving severance payments we see in our data: 41%, 48% and 46% for Education 1, 2 and 3, respectively. The dispersion of the measurement error implied by the estimates σ_{ME} is much smaller for those more educated, thus, if measurement error derives from incorrect reporting, this entails that the information provided by workers in Education 3 is more accurate. Finally,

²¹A value of 0.3 for β is an upper bound to the estimates found in the literature (see Cahuc et al. (2006) or Yashiv (2003)).

²²Notice that $r + \delta_1 + \delta_2 - \mu > 0$ for all groups, and therefore, the agents' problem is well defined. i.e. their maximization problems do not diverge.

Table 4: Estimates for each Education Subsample

Parameter	Sample		
	Education 1	Education 2	Education 3
λ	0.096 (0.003)	0.094 (0.003)	0.101 (0.006)
δ_1	0.009 (1e-4)	0.010 (1e-5)	0.009 (6e-6)
δ_2	0.012 (1e-4)	0.012 (2e-5)	0.011 (2e-5)
x_r	0.725 (1e-7)	0.752 (1e-4)	0.701 (9e-4)
μ	0.007 (1e-4)	0.007 (9e-6)	0.008 (1e-6)
σ	0.024 (4e-6)	0.001 (8e-7)	0.005 (9e-4)
ν	0.488 (1e-6)	1.023 (3e-4)	1.143 (1e-5)
ξ	0.038 (2e-8)	0.169 (3e-4)	1.005 (0.045)
σ_{ME}	0.512 (2e-4)	0.582 (0.008)	0.115 (3e-6)
τ	1.723 (0.027)	1.099 (0.007)	1.344 (0.013)
ln L	-15891.5	-50243.0	-16479.5

Standard Errors in Parenthesis

the maximized log likelihood is much smaller for the Education 2 sample, but this difference can be explained by the differences in sample size. The samples for groups 1 and 3 are very similar (3062 and 2708, respectively), while Education 2 has almost three times more observations (8800). To assess the fit of the model to the data, Figure 2 presents in its upper panels simulations of the density of wages above the minimum wage for ongoing employment spells (upper left) and of the duration density of such spells (upper right), for Education 2. The bottom panels represent the corresponding densities found in the data. Both simulated densities are a smoother version of the one found in the data, and even though the fatter tail of duration observed in the data is not captured in the model, the general shapes of wages and durations distributions are captured.²³

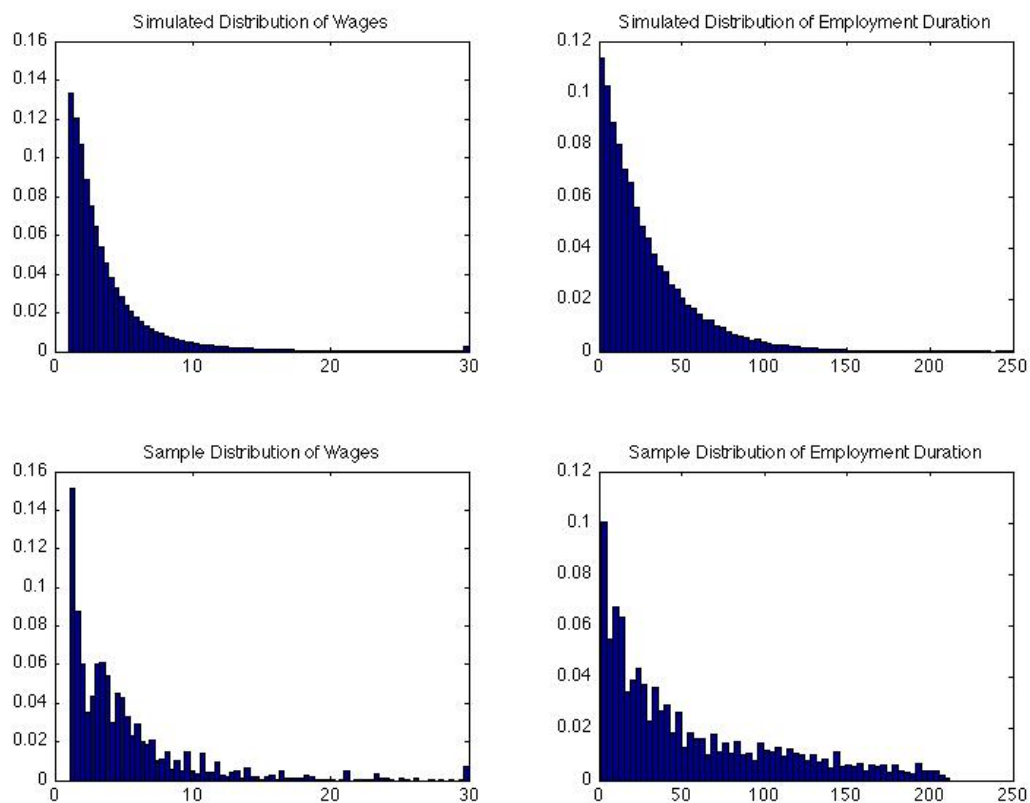


Figure 2: Hourly wages and duration of ongoing employment spells (in months): simulation (top panels) vs. data (bottom panels). The simulations consist of 100,000 random draws, using the Metropolis-Hastings algorithm, from the respective densities implied by the model.

The corresponding equilibrium market tightness and the other parameters that we back up from equilibrium restrictions are presented in Table 5. We compute standard errors of backed-up parameters using the delta method. On average, the flow cost of a vacancy is 40 times the average wage in the

²³The same conclusion is reached for both Education 1 and Education 3; their respective density distribution are not presented.

Table 5: Estimates of Remaining Parameters and Equilibrium Values

Parameter	Sample		
	Education 1	Education 2	Education 3
Vacancy Cost c	457.0 (15.7)	1108.3 (29.4)	1935.9 (156.8)
Search Cost s	2.66 (0.016)	5.19 (0.287)	9.66 (0.715)
Expected Hiring Cost in Wages $\frac{c}{(1-G(x_r))q(\theta)\bar{\omega}}$	40.0	82.6	75.8
Market Tightness θ	0.009	0.009	0.010

Standard Errors in Parenthesis

labor market composed of those with less education, and around 80 for the other two groups. In relative terms, these results are intuitive in the sense that it is more expensive to hire more skilled workers. On the other hand, in terms of magnitude, these flow costs are much higher than those found in the literature.²⁴ However, this is not a particularity in our data, but rather, a particularity of search-and-matching models. The vacancy cost c is computed from the free entry condition, which equates it to the ex-ante expected profits of firms. These high expected profits have at least two possible explanations: the high dispersion of wages we observe from the data and the low share of the surplus we assume workers receive. Differences in wages are captured by the model as differences in match quality, thus, the high dispersion we find in the wage data together with the low workers' share, translates into a distribution of firms values with a high dispersion. Since the value of a firm is bound below by zero, high dispersion implies that expected profits, and therefore c , must be high. In fact, when reestimating the model with a higher workers' share, c drops significantly.²⁵

The low estimates of market tightness are consistent with the high unemployment rate implied by all estimate sets (around 18% in each sample). Our results indicate that the high estimated unemployment rate is the product of long unemployment spells (which mimics exactly what we observe in the data), and therefore due to few vacancies, rather than of a high incidence of unemployment. These

²⁴The literature imply that a reasonable range for the expected recruiting cost per vacancy is from 9% of the average monthly wages up to 42% (see Toledo and Silva (2009), Abowd and Kramarz (2003)).

²⁵Examples found in the search-and-matching literature of high expected vacancy costs estimates can be found in Prat (2010) and Flinn (2006), who obtain values of 3.6 and 20.8 in terms of average wages, respectively (neither Flinn (2006) nor Prat (2010) explicitly compute the expected vacancy cost, however, it can be directly computed from the information given in the respective papers). Their samples present similar levels of dispersion (they have almost the same coefficient of variation), however, Flinn (2006) uses a smaller level for the workers' share (0.42 versus 0.5).

few vacancies and high unemployment rate explain the low market tightness.

4.6 Sensitivity Analysis

As we mentioned previously, there are two important issues regarding our sample. First, around half of the individuals in our sample contributed with more than one spell. As we can expect to find individual specific effects, using spells belonging to the same individual could violate the independence assumption of ML. Second, our sample encompasses 17 years, each with a different minimum wage. This may defy our stationarity assumption. To test how our estimates are affected by these two facts, we re-estimate the model with two different restrictions on our subsamples. To control for the unobserved individual effects, we construct a new sample (Sample B_i) by randomly selecting one spell from each individual in Education i (Sample A_i) for $i \in \{1, 2, 3\}$. As discussed previously, the ratio of the minimum wage to the average wage remained fairly stable over the 2002-2007 period (Figure 1). Therefore, to control for both issues, for each subsample, we build a third sample (Sample C_i) by randomly choosing one spell per individual from the set of spells composed of spells active in 2002 and those that began after 2002.

The estimates from the sensitivity analysis are precisely estimated for all education groups. Table 6 shows the results for Education 3, the smallest subsample. For all education levels, when going from Sample A_i to Sample B_i , and from Sample B_i to Sample C_i , the estimates imply that: the unemployment rate decreases, employment spells are longer, and expected productivity is higher. There are two possible explanations. Firstly, as more skilled people within each education group tend to have more and better (in terms of wages and duration) employment spells, whereas low skilled workers transit more through unemployment, in Samples B_i and C_i employment spells with higher wages and that last longer, as well as long unemployment spells, are over represented. Secondly, at least part of the changes found when going from Sample B_i to Sample C_i , are due to the fact that the period 2002-2007 is characterized by higher average real wages (real wages had steadily increased since 1990).

Since we do not observe dramatic changes in the estimates, and general trends can be explained, these results suggest that individual effects and non-stationary changes in the minimum wage do not have significant impacts on estimates and support our decision of not discarding data.

5 Welfare Analysis

In perfectly competitive models, labor market policies, such as minimum wages and severance payments, distort the agents' behavior, which leads to inefficient outcomes. On the other hand, in models with frictions that prevent the economy from reaching the socially efficient second-best outcome, policies can be used as third-best tools for reaching the constrained efficient allocation and be welfare-improving. In particular, in models with search frictions, if rents are not being distributed "appropriately" between the firm and the worker, then labor market policies can have a positive role. When job

Table 6: Sensitivity Analysis Results for Education 3

Parameter	Full Sample (A_3)	One Spell (B_3)	One Spell Since 2002 (C_3)
λ	0.101 (0.006)	0.099 (0.008)	0.100 (0.006)
δ_1	0.009 (6e-6)	0.008 (8e-5)	0.007 (7e-6)
δ_2	0.011 (2e-5)	0.010 (8e-5)	0.007 (1e-5)
x_r	0.701 (9e-4)	0.628 (9e-4)	0.591 (2e-6)
μ	0.008 (1e-6)	0.008 (1e-4)	0.008 (4e-7)
σ	0.005 (9e-4)	0.013 (0.001)	0.003 (0.001)
ξ	1.143 (1e-5)	1.098 (0.043)	1.213 (0.040)
ν	1.005 (0.045)	0.977 (0.040)	0.947 (0.032)
σ_{ME}	0.115 (3e-6)	0.942 (0.012)	0.087 (0.014)
τ	1.344 (0.013)	1.342 (0.019)	1.301 (0.013)
ln L	-16479.5	-9358.5	- 8127.6
Number Obs.	2708	1481	1201

Standard Errors in Parenthesis

contact rates depend on market tightness, and market tightness depends on wages, congestion externalities arise when negotiating agents do not take into account that their decisions will affect searching workers and firms. The appropriate wage internalizes such externalities. Hosios (1990) showed that in a search model where homogenous firms and workers meet according to a CRS matching function, and where wages are negotiated according to Nash bargaining, the equilibrium allocation is efficient if the share of the surplus that workers receive is equal to the elasticity of the matching function with respect to the unemployed. In such settings, if the share of workers is too low, then labor market policies can increase the “effective” share of workers and improve aggregate welfare.

Our setting has match-specific heterogeneity that leads to endogenous creation and destruction. Pissarides (2000) extends Hosios’ result to models with endogenous creation and to models with endogenous destruction where productivity jumps after being hit by a shock. We introduce the destruction margin in our setting by letting productivity continuously fluctuate, leading to a non-degenerate equilibrium distribution of productivity. Thus, to find the economy’s second-best allocation, that is, the allocation (x_r, θ) chosen by a planner that maximizes aggregate welfare subject to frictions, we must keep the resulting multi-dimensional object as a state variable. Therefore, we are not able to analytically find the second-best allocation for our setting, nor to determine analytically whether Hosios’ result holds or not. The same infinite-dimensional state variable appears in the dynamic optimization problem faced when solving for the economy’s third-best allocation, that is, when looking for the policy levels that maximize welfare subject to frictions and equilibrium conditions. Given that the described technical difficulty in solving the planner’s dynamic optimization problem also applies to its numerical solution (because of the dimensionality curse), they will be subject of future research, and here we present an analysis of steady state welfare.²⁶

Our analysis is focused on the equilibrium impact of severance payments and minimum wage, rather than on their optimal levels. The model is designed to capture some of the essential trade-offs inherent in the use of these policies, therefore it should provide a reliable guidance on the direction in which these policies should change, relative to existing values. However the model is not rich enough to recommend precisely a practical recommendation for the actual optimal levels of these policies. Thus, it can be best thought as a guide to local policy improvement analysis.

We use the three sets of estimates obtained previously to determine the impact on the labor market of counterfactual changes in policies. As before, we treat the three labor markets as completely separate. We assume that the parameters of the model are invariant to changes in policies, and we study the impact of this change on equilibrium and welfare. Our utilitarian welfare measure is, as in Hosios (1990), the sum of the average values of the agents in the labor market, weighted by their measure in the economy. As discussed, we maximize steady state welfare with respect to policy menus, subject

²⁶Note that if there were no discounting in the model, comparisons of steady state welfare would allow us to assess whether Hosios’ result holds in our setting or not. However, with a positive discount rate, to evaluate Hosios’ result, we should also consider welfare during the transitional path between the corresponding steady states.

to frictional unemployment and equilibrium conditions: the free entry condition, the formula for the value of unemployment, the equation for the cut-off x_r derived from the firm's boundary and optimality conditions, and the equation for the cut-off x_m .²⁷ Thus, we solve the following steady state problem:

$$\begin{aligned} & \max_{m, \tau} (1 - u) \mathbb{E}[W(x) + J(x) | x \geq x_r] + uU \\ & \text{s.t.} \\ & u = \frac{\delta}{\delta + \lambda \left[\bar{G}(x_r) - x_r^\kappa \exp\left(\frac{\kappa^2 \xi^2}{2} - \kappa \nu\right) \Phi\left(\frac{-\log(x_r) - \kappa \xi^2 + \nu}{\xi}\right) \right]} \\ & \text{and equations (9), (16), (17), (18)} \end{aligned}$$

where $\bar{G} = 1 - G$, $\kappa = \frac{\bar{\mu} + \gamma}{\sigma^2}$ with $\gamma = \sqrt{\bar{\mu}^2 + 2\delta\sigma^2}$ and $\bar{\mu} = \mu - \sigma^2/2$.

Intuitively, the introduction of a continuously evolving productivity should not affect, at least directly, the channel by which the decisions of meeting firms and workers have an impact on searching agents. Therefore, we would expect Hosios' result to hold in our setting. If that were the case, then we should have that the level of dynamic welfare (that is, the one considering transitional dynamics from one steady state to the other) reached when setting β equal to the matching function elasticity (0.5) and without policies, is the maximum level of dynamic welfare that can be attained in the economy (with or without policies). Now, as we are considering only steady state welfare in a setting with positive discounting, and thus neglecting welfare during the transitional path, we cannot expect Hosios' result to hold. To assess the deviation from Hosios' result when using steady state welfare, as part of our experiments we maximize steady state welfare on β in a policy-free environment (last row of each panel in Table 7). We find that for the three education levels, in the absence of policies, the maximum steady state welfare is reached under $\beta = 0.47$, instead of 0.5. Therefore, there is not a significant deviation from the result we would expect if we were considering dynamic welfare. We refer to the welfare on the last row of each panel in Table 7 as β -welfare.

The results of the counterfactual experiments depend critically upon the difference between the workers' bargaining power β and the matching function elasticity. In line with the literature, we fix the elasticity at 0.5. We performed our welfare analysis for different values of $\beta \leq 0.47$ and, consistent with Hosios, the larger the difference between β and the elasticity, the larger the welfare improvement policies could implement. However, the equilibrium effects of the minimum wage and severance payments resulted qualitatively analogous under different β 's. As we are interested in equilibrium effects rather than on optimal levels, using different values of β for each subsample will not contribute to our analysis, and we present the results for $\beta = 0.3$ for every subsample.²⁸ This

²⁷The steps for the derivation of the unemployment rate are given in the Appendix.

²⁸With adequate data, our estimation procedure can easily be extended to estimate β and η . In particular, if employer-

Table 7: Welfare Analysis Results

Education 1							
Case	β	m	τ	welfare	x_r	θ	$G(x_r)$
Estimated	0.3	1.0	1.7	172.3	0.725	0.009	0.0000
Optimum Only SP	0.3	-	7.7	174.3	0.307	0.007	0.0000
Optimum Only m	0.3	1.18	-	174.3	0.845	0.007	0.0000
Optimum Combined	0.3	multiple		174.3	multiple		
$m = \tau = 0$, max on β	0.47	-	-	174.3	0.244	0.007	0.0000

Education 2							
Case	β	m	τ	welfare	x_r	θ	$G(x_r)$
Estimated	0.3	1.0	1.1	127.1	0.752	0.009	0.0000
Optimum Only SP	0.3	-	8.8	145.3	0	0.005	0.0000
Optimum Only m	0.3	1.7	-	145.3	1.266	0.005	0.0000
Optimum Combined	0.3	multiple		145.3	multiple		
$m = \tau = 0$, max on β	0.47	-	-	145.3	0	0.005	0.0000

Education 3							
Case	β	m	τ	welfare	x_r	θ	$G(x_r)$
Estimated	0.3	1.0	1.3	469.9	0.701	0.010	0.0681
Optimum Only SP	0.3	-	9.5	504.9	0.499	0.006	0.0338
Optimum Only m	0.3	2.6	-	501.8	1.790	0.008	0.2886
Optimum Combined	0.3	1.9	7.6	528.3	1.387	0.006	0.2086
$m = \tau = 0$, max on β	0.47	-	-	500.9	0.381	0.006	0.0180

means we give room to policies to improve welfare.

Table 7 presents the results of our counterfactual experiments, with one panel for each level of education. Columns two to five represent the workers' share, policy levels, and the implied welfare level; the last three columns present equilibrium variables. We compute all these variables for five scenarios (first column): under the current levels of policies; the optimum when only one policy is available: severance payments (second row) or minimum wage (third row); the optimum when both policies can be used; and finally, we set the policy levels to zero and look for the optimal workers' share β . Note that for the first four cases β is fixed at 0.3, which is the value we used for our estimations.

We find that both policies can improve welfare in each of the three cases, and that the maximum increase is 1.2%, 12.6% and 11.1% for Education 1, 2 and 3, respectively. For Education 1 and 2, β -welfare is the highest welfare that can be reached. Therefore, similarly to Hosios' result, no policy can improve steady state welfare when the workers' share is at an appropriate value, however, that value is no longer the elasticity but a slightly smaller one. Additionally, for these two education groups, the maximum welfare can also be implemented using either policy by itself: by incrementing the minimum wage 18% or the severance payments coefficient by 4.5 times for the least educated ones, and by increasing the minimum wage 70% or multiplying τ by eight for those in the middle group.²⁹ The last three columns of Table 7 present the implied equilibrium for each menu of policies. We concentrate our analysis on the values of the productivity cut-off x_r and the market tightness θ , because once these are given, equilibrium equations pin down the values for the rest of the equilibrium variables. When the workers' share is too low, labor is cheap and firms create too many vacancies, implying relatively large market tightness. The introduction of policies can increase the workers' "effective" share and improve welfare. The seventh column of Table 7 shows the equilibrium market tightness for the cases under study. In the first two panels, we see that market tightness has the same value in all of the cases where maximum welfare is reached, 0.007 for the least educated and 0.005 for the middle education group; whereas the cut-off x_r differs greatly, however, these differences are not significant in terms of their impact on the equilibrium turnover. In fact, in terms of the creation decision, the estimates for the parameters of the initial productivity distribution (ν and ξ) imply that, under these cut-offs, almost all matches are created (the accumulation of these thresholds is shown in column 8 of Table 7). Second, as we discussed previously, given the small variance of the productivity process, once a match is created, changes in x_r have almost no impact on job destruction. Therefore, even though these x_r have significant magnitude differences, their impact on equilibrium outcomes is

employee data were available, the workers' share could be estimated using a method similar to that of Cahuc et al. (2006). With data on vacancy rates, together with estimates for the contact rate λ and the unemployment rate, we would be able to estimate one parameter for the matching function. None of this additional data is publicly available for the Chilean labor market.

²⁹For clarity, hereafter, I will refer to the severance payments coefficient just as severance payments or τ .

very similar.

Additionally, for these two sets of parameters, severance payments and the minimum wage are perfect substitutes. Let m_i^* and τ_i^* , for $i \in \{1, 2\}$, be the levels of the minimum wage and severance payments that reach the maximum welfare for Education 1 and 2, respectively. In terms of the combined optimum, in both Education 1 and 2, for any $\bar{m} < m_i^*$ there exists $\bar{\tau} < \tau_i^*$ such that the maximum level of welfare is attained under the policy menu $(\bar{m}, \bar{\tau})$, and vice versa.

The results for Education 3 no longer mimic Hosios' result. In fact, β -welfare is 6.2% larger than current welfare, yet steady state welfare could increase more than 10% if the appropriate policies were implemented. Even more, now the maximum level of welfare cannot be reached using only one policy, both must be implemented; and the level of welfare that can be attained using only severance payments is higher than that obtained imposing only a minimum wage.³⁰

Analyzing the equilibrium impacts of severance payments and minimum wage helps foster our understanding of why severance payments are a better tool in the case of Education 3. Given our estimates, the minimum wage and severance payments have the same qualitative impact on equilibrium. An increase in any of the two policies implies a decrease in market tightness and an increase in the productivity cutoff. The relative effectiveness of severance payments can be explained by the rate at which such trade-off occurs. First, note that a big difference between the results for Education 3 and those for Education 1 and 2, is that, for the latter, almost all matches are created, whereas for Education 3, in each case, there is a significant amount of matches that do not lead to production, implying highly educated workers are “picky”. Therefore, getting the cut-off x_r correctly is now important. Since minimum wages have a relatively stronger effect at the bottom of the wage distribution and a weak effect on higher wages, they affect the creation/destruction cut-off in a particularly strong way, as compared to its effect on vacancy creation. On the other hand, severance payments have a proportional effect across the wage distribution, which enables them to change market tightness without leading to an extremely large threshold for productivity. From the fourth row of the last panel in Table 7, we can see that the pair (x_r, θ) that leads to maximum welfare is $(1.387, 0.006)$; the optimal severance payment of 9.5 (second row) implements the optimal tightness, however it cannot reach the maximum welfare because the implied creation cut-off is around a third the optimal one. On the other hand, the optimal minimum wage equal to 2.6 (third row) does not implement the optimal tightness nor the optimal cut-off: lowering the current tightness of 0.010 to 0.008 already implies a cut-off 30% larger than the optimal one. However, these two policies can complement each other: smaller levels of both policies, $m = 1.9$ and $\tau = 7.6$, can be combined to raise the productivity level enough and, at the same time, affect the rest of the wage distribution significantly.

Just like an increase in the severance payments coefficient τ , an increase in the worker bargaining

³⁰As was discussed before, deviations from Hosios' result are expected when comparing steady state welfare in an environment with a positive discounting rate. Thus, this result does not imply that Hosios does not hold in our setting.

power β impacts wages linearly in productivity, therefore, it is not surprising that there is no level of β that implements the maximum welfare in the absence of policies: there exists a level of β that implements the optimal rate of vacancy creation, however too many matches would lead to production at such level.

An important question is what makes Education 3 different from the other two subsamples. An answer, from a simple inspection of the data, is wage dispersion; wage dispersion in Education 3 is twice the one found in Education 2 and around 4 times the one from Education 1. That wage dispersion influences the behaviour of firms and workers is a very intuitive result. In our model, dispersion in wages can only come from dispersion in productivity; therefore, the wage dispersion found in the sample is interpreted by the model as dispersion in match quality. If match quality dispersion is low, the probability of getting a substantially better initial productivity draw is small, and almost all matches lead to production. On the other hand, as dispersion increases, the option value of waiting for a better match increases and, if the initial productivity draw is sufficiently small, the match will not lead to production.

To test how wage dispersion affects welfare results, we compute optimal welfare (for each of the four last cases in Table 7) for the set of estimates from Education 3, but fixing the shape parameter ξ of the sampling distribution at different levels. As a reference, the estimate for ξ equals 0.038, 0.169 and 1.005 for Education 1, 2 and 3, respectively. We find that, for $\xi \in \{0.1, 0.2\}$, the welfare results are qualitative analogous to those for Education 1 and 2: the maximum welfare reached in each case is the same. On the other hand, for $\xi \in \{0.7, 0.9\}$, the use of policies can lead to welfare higher than that reached under no policy and $\beta = 0.47$, similarly to what we found for Education 3. Finally, we find an additional situation for intermediate wage dispersion; for $\xi \in \{0.3, 0.5\}$, severance payments are able to attain the level of welfare reached under no policy and $\beta = 0.47$, however, the minimum wage by itself is already not able to reach the maximum level of welfare. As dispersion increases, it is optimal to discard some matches, but not too many. Therefore, the impact of the minimum wage at the bottom of the wage distribution is already too strong, whereas the weak effect of severance payments at low wages is enough to reach the small productivity cut-off.

As a final remark, note that the introduction of on-the-job search, while desirable in terms of bringing our model closer to reality, would also bring serious identification issues. However, based on the literature results, there is no reason to believe our welfare results would be modified by the introduction of on-the-job search. The general conclusion of models including on-the-job search, where the workers' bargaining power is estimated, is that lower levels of such share are consistent with the observed wage distribution (see, for example, Cahuc et al. (2006)). The basic idea behind this result is that the bargaining process between the worker and the two interested firms (the incumbent and poaching employers) will transfer some of the match surplus to the worker. Therefore, the introduction of on-the-job search gives more room for policies to improve welfare; for example, Flinn and

Mabli (2008) show that larger levels of minimum wage are required to maximize welfare when on-the-job search is considered. Additionally, given that the bargaining process between the worker and the incumbent and poaching employers do not alter the specification of policies, there does not seem to exist a channel, at least a direct one, through which on-the-job search could affect the equilibrium effects of the minimum wage and severance payments. This remains to be studied formally.

6 Conclusions

We introduce severance payments and a minimum wage in a general equilibrium model of the labor market with endogenous creation and destruction decisions. We also implement a device to structurally estimate the model's parameters.

To estimate the model we use data on employment histories from Chile, a country where labor market regulations prescribe high severance payments and minimum wage. We use the Social Protection Survey, from where we draw up to 17 years of longitudinal information in relation to labor market histories for each individual. With these estimates, we perform counterfactual experiments that allow us to answer questions about optimal policy combinations. Since we do not control for ex-ante heterogeneity in our model, we estimate the model for three subsamples corresponding to different levels of workers' education, assuming that each group belongs to a completely separate labor market.

We conclude that when the share that workers receive from the surplus their job generates is at an appropriate value, the economy's maximum welfare level is reached in a policy-free environment. However, when the workers' bargaining power is below such appropriate value, the level of wage dispersion of the sample starts playing a critical role in determining the optimal policy menu; when the dispersion in wages is relatively low, as the one found in Education 1 and 2, the maximum level of welfare can be attained using any of the following three possibilities: severance payments, or a minimum wage by themselves, or with an appropriate combination of these two policies. That is, for Education 1 and 2, severance payments and a minimum wage are perfect substitutes. In all these cases, it is optimal to create almost all matches, and any creation threshold that accumulates almost no initial draws will suffice.

However, when dispersion in wages is high, as observed in the subsample with the higher education level, the firms' option value to wait increases and, therefore, it is now optimal to discard a significant fraction of matches. In this scenario, no policy in isolation can attain the economy's maximum level of welfare, and severance payments and the minimum wage become complements. Since minimum wages have a relatively stronger effect at the bottom of the wage distribution and a weak effect on higher wages, they affect the creation/destruction cut-off in a particularly strong way, as compared to its effect on vacancy creation. On the other hand, severance payments have a proportional effect across the wage distribution, which enables them to change market tightness with-

out leading to an extremely large threshold for productivity. In this way, a binding minimum wage is required to obtain the relatively large equilibrium productivity cut-off, and severance payments are needed to significantly reduce market tightness to its equilibrium level. In this way, a particular combination of labor market policies is required.

Appendix

A Derivation of Conditional Likelihood Contributions

We use the same notation given in Section 4, repeated here for easier access.

T_i = time of arrival of the first exogenous shock δ_i , $i = 1, 2$. Let f_i be its pdf, and F_i its cdf.

$T_r = \min\{t > 0 | X_t = x_r\}$. Let f_r be its pdf, and F_r its cdf.

$t_e = \min\{T_1, T_2, T_r\}$, that is, the duration of completed employment spells.

Therefore, we have that:

$$\begin{aligned} f_i(t) &= \delta_i e^{-\delta_i t} & i = 1, 2 \\ F_i(t) &= 1 - e^{-\delta_i t} & i = 1, 2 \\ f_r(t) &= \frac{|\ln \frac{x_r}{x_0}|}{\sigma \sqrt{2\pi t^{\frac{3}{2}}}} \exp\left(-\frac{[\ln \frac{x_r}{x_0} - \bar{\mu}t]^2}{2t\sigma^2}\right) \\ F_r(t) &= \begin{cases} \Phi\left(\frac{-\ln \frac{x_r}{x_0} + \bar{\mu}t}{\sigma\sqrt{t}}\right) + \left(\frac{x_r}{x_0}\right)^{\frac{2\bar{\mu}}{\sigma^2}} \Phi\left(\frac{-\ln \frac{x_r}{x_0} - \bar{\mu}t}{\sigma\sqrt{t}}\right) & x_r > x_0 \\ 1 - \Phi\left(\frac{-\ln \frac{x_r}{x_0} + \bar{\mu}t}{\sigma\sqrt{t}}\right) + \left(\frac{x_r}{x_0}\right)^{\frac{2\bar{\mu}}{\sigma^2}} \Phi\left(\frac{\ln \frac{x_r}{x_0} + \bar{\mu}t}{\sigma\sqrt{t}}\right) & x_r < x_0 \end{cases} \end{aligned}$$

where Φ is the standard Normal cumulative distribution function and x_0 is the initial productivity draw. Finally, from the generalized reflection principle we obtain that for $\bar{x} \geq x_r$:

$$\begin{aligned} \mathbb{P}(T_r > t, X_t \leq \bar{x}) &= \Phi\left(\frac{\ln(\bar{x}/x_0) - \bar{\mu}t}{\sigma\sqrt{t}}\right) - \Phi\left(\frac{\ln(x_r/x_0) - \bar{\mu}t}{\sigma\sqrt{t}}\right) \\ &+ \left(\frac{x_r}{x_0}\right)^{\frac{2\bar{\mu}}{\sigma^2}} \Phi\left(\frac{-\ln(x_r/x_0) - \bar{\mu}t}{\sigma\sqrt{t}}\right) - \left(\frac{x_r}{x_0}\right)^{\frac{2\bar{\mu}}{\sigma^2}} \Phi\left(\frac{\ln(\bar{x}/x_0) - 2\ln(x_r/x_0) - \bar{\mu}t}{\sigma\sqrt{t}}\right) \end{aligned} \quad (20)$$

The initial draw x_0 is distributed according to a Lognormal(ν, ξ) density:

$$dG(x_0) = \frac{e^{-\frac{1}{2}\left(\frac{\ln(x_0)-\nu}{\xi}\right)^2}}{x_0\xi\sqrt{2\pi}}dx_0 = \frac{e^{-\frac{1}{2}\left(\frac{\ln(x_0)-\nu}{\xi}\right)^2}}{\xi\sqrt{2\pi}}d\ln(x_0)$$

With the exception of the unemployment rate, all formulas in this appendix are for a given x_0 and to derive the general expression we must integrate with respect to x_0 . Given the assumption that

the initial productivity x_0 is drawn from a Lognormal distribution, such integrations can be explicitly computed in most cases, however, for ease of notation we omit that integral from what follows.

Let us first characterize the equilibrium joint distribution of tenure T and productivity \bar{x} .

For $\bar{x} \geq x_r$

$$\begin{aligned}\mathbb{P}(t_e > T, X_T \leq \bar{x}) &= \mathbb{P}(T_r > T, T_1 > T, T_2 > T, X_T \leq \bar{x}) \\ &= \bar{F}_1(T) \bar{F}_2(T) \mathbb{P}(T_r > T, X_T \leq \bar{x})\end{aligned}$$

where $\bar{F} = 1 - F$. We then derive this expression (using equation (20)) to obtain the density:

$$f(T, \bar{x}) = \frac{1}{\bar{x}\sigma\sqrt{T}} \left[\phi\left(\frac{\ln(\bar{x}/x_0) - \bar{\mu}T}{\sigma\sqrt{T}}\right) - \left(\frac{x_r}{x_0}\right)^{\frac{2\bar{\mu}}{\sigma^2}} \phi\left(\frac{\ln(\bar{x}/x_0) - 2\ln(x_r/x_0) - \bar{\mu}T}{\sigma\sqrt{T}}\right) \right] \bar{F}_1(T) \bar{F}_2(T)$$

Defining $A(T, \bar{x})$ as:

$$A(T, \bar{x}) = \frac{1}{\bar{x}\sigma\sqrt{T}} \left[\phi\left(\frac{\ln(\bar{x}/x_0) - \bar{\mu}T}{\sigma\sqrt{T}}\right) - \left(\frac{x_r}{x_0}\right)^{\frac{2\bar{\mu}}{\sigma^2}} \phi\left(\frac{\ln(\bar{x}/x_0) - 2\ln(x_r/x_0) - \bar{\mu}T}{\sigma\sqrt{T}}\right) \right]$$

we have:

$$f(T, \bar{x}) = A(T, \bar{x}) \bar{F}_1(T) \bar{F}_2(T)$$

To obtain the population joint density of tenure and productivity, we must multiply the density f by the job creation rate, which in steady state is given by $u\lambda\bar{G}(x_r)$. To derive the ergodic population density of productivity, $v(x)$, we integrate the joint density with respect to tenure:

$$v(x) = \int_0^\infty u\lambda\bar{G}(x_r) f(t, x) dt$$

Then, unemployment can be derived from the steady state flow equation:

$$0 = (1 - u)\delta + \frac{1}{2}\sigma^2 x_r^2 v'(x_r) - u\lambda\bar{G}(x_r)$$

which states that the flows in and out of unemployment are equal. The inflow from employment has two sources, exogenous destruction at rate δ , and endogenous destruction caused by productivity crossing the threshold x_r (term captured by $\frac{1}{2}\sigma^2 x_r^2 v'(x_r)$). Outflow from unemployment occurs at the rate $\lambda\bar{G}(x_r)$. Some algebra yields:

$$u = \frac{\delta}{\delta + \lambda \left[\bar{G}(x_r) - x_r^\kappa \exp\left(\frac{\kappa^2 \xi^2}{2} - \kappa\nu\right) \Phi\left(\frac{-\log(x_r) - \kappa\xi^2 + \nu}{\xi}\right) \right]} \quad (21)$$

where $\kappa = \frac{\bar{\mu} + \gamma}{\sigma^2}$ with $\gamma = \sqrt{\bar{\mu}^2 + 2\delta\sigma^2}$ and $\bar{\mu} = \mu - \sigma^2/2$.

A.1 Contribution of Ongoing Employment Spells

The population density of productivity and tenure for an ongoing spell is given by the density f multiplied by the job creation rate. To obtain the probability density, we must normalize this population density by $1 - u$. Finally, using the change of variable formula, the resulting likelihood contribution of wages and tenure for an ongoing spell is:

$$h(t, w) = \begin{cases} \frac{u\lambda\bar{G}(x_r)}{1-u} \frac{A(t, x(w))\bar{F}_1(t)\bar{F}_2(t)}{\beta + \tau(r + \delta_2 - \mu)} & w > m \\ \frac{u\lambda\bar{G}(x_r)}{1-u} \bar{F}_1(t)\bar{F}_2(t) \int_{x_m}^{x_r} A(t, x)dx & w = m \end{cases}$$

For spells with only tenure information, we integrate the joint density with respect to wages to obtain their likelihood contribution.

A.2 Contribution of Unemployment Spells

Since the population unemployment spell duration distribution is an exponential, the density of completed unemployment spells of length t as well as that of ongoing unemployment spells of length t in the steady state are given by $\lambda\bar{G}(x_r) \exp^{-\lambda\bar{G}(x_r)}$.

A.3 Contribution of Completed Employment Spells without Wage Information

Let us first compute the likelihood contribution of a complete employment spell only on tenure information:

$$\begin{aligned} \mathbb{P}\{t_e \leq t\} &= 1 - \mathbb{P}\{t_e \geq t\} \\ &= 1 - \mathbb{P}\{T_1 \geq t, T_2 \geq t, T_r \geq t\} \\ &= 1 - [1 - F_1(t)][1 - F_2(t)][1 - F_r(t)] \end{aligned}$$

Thus, letting $\bar{F} = 1 - F$, we get that the density is given by:

$$f_{t_e}(t) = f_1(t)\bar{F}_2(t)\bar{F}_r(t) + f_2(t)\bar{F}_1(t)\bar{F}_r(t) + f_r(t)\bar{F}_1(t)\bar{F}_2(t)$$

The steps to compute the likelihood contribution of a completed employment spell that ended with the reception of severance payments are explained in the following. Receiving severance payments is equivalent to being destroyed by a δ_1 shock, thus:

$$\begin{aligned}
\mathbb{P}\{t_e \leq t, \text{reception of SP}\} &= \mathbb{P}\{t_e \leq t, T_1 < T_2, T_1 < T_r\} \\
&= \mathbb{P}\{T_1 \leq t, T_1 < T_2, T_1 < T_r\} \\
&= \int_0^t \mathbb{P}\{T_1 \leq s, T_1 < T_2, T_1 < T_r | T_1 = s\} dF_1(s) \\
&= \int_0^t \mathbb{P}\{s < T_2, s < T_r\} dF_1(s) \\
&= \int_0^t [1 - F_2(s)][1 - F_r(s)] dF_1(s)
\end{aligned}$$

Therefore the density is given by:

$$g_{t_e}(t, \text{reception of SP}) = \bar{F}_2(t) \bar{F}_r(t) f_1(t)$$

Analogously, for a spell that ended without severance payments we obtain the following density:

$$\begin{aligned}
g_{t_e}(t, \text{no reception of SP}) &= g_{t_e}(t, T_2 < T_1 \vee T_r < T_1) \\
&= f_r(t) \bar{F}_1(t) \bar{F}_2(t) + f_2(t) \bar{F}_1(t) \bar{F}_r(t)
\end{aligned}$$

where $\bar{F} = 1 - F$.

A.4 Contribution of Completed Employment Spells with Wage Information

For spells that ended with the reception of severance payments, we first compute the joint density of t_e and productivity, and then we use the change of variable formula to derive the formula for wages.

For $\bar{x} \geq x_r$:

$$\begin{aligned}
\mathbb{P}(t_e \leq t, X_{t_e} \leq \bar{x}, \delta_1 \text{ arrived first}) &= \mathbb{P}(T_1 \leq t, X_{T_1} \leq \bar{x}, T_1 < T_2, T_1 < T_r) \\
&= \int_0^t \mathbb{P}(X_{T_1} \leq \bar{x}, T_1 < T_2, T_1 < T_r | T_1 = s) dF_1(s) \\
&= \int_0^t \mathbb{P}(X_s \leq \bar{x}, s < T_r) [1 - F_2(s)] dF_1(s)
\end{aligned}$$

Then, we get the density for $\bar{x} \geq x_r$ from:

$$\begin{aligned}
& g(t, \bar{x}, \delta_1 \text{ arrived first}) \\
&= \frac{\partial^2}{\partial \bar{x} \partial t} \mathbb{P}(t_e \leq t, X_{t_e} \leq \bar{x}, \delta_1 \text{ arrived first}) \\
&= \frac{\partial^2}{\partial \bar{x} \partial t} \int_0^t \mathbb{P}(X_s \leq \bar{x}, s < T_r) \bar{F}_2(s) dF_1(s) \\
&= \frac{\partial}{\partial \bar{x}} \mathbb{P}(X_t \leq \bar{x}, t < T_r) \bar{F}_2(t) f_1(t) \\
&= \frac{1}{\bar{x} \sigma \sqrt{t}} \left[\phi \left(\frac{\ln(\bar{x}/x_0) - \bar{\mu}t}{\sigma \sqrt{t}} \right) - \left(\frac{x_r}{x_0} \right)^{\frac{2\bar{\mu}}{\sigma^2}} \phi \left(\frac{\ln(\bar{x}/x_0) - 2 \ln(x_r/x_0) - \bar{\mu}t}{\sigma \sqrt{t}} \right) \right] \bar{F}_2(t) f_1(t)
\end{aligned}$$

where the last equality comes from (20).

Using the definition for $A(t, x)$ and the change of variable formula, we get that the density of wages and completed spells is:

$$h(t, w, \text{reception of SP}) = \begin{cases} \frac{A(t, x(w)) \bar{F}_2(t) f_1(t)}{\beta + \tau(r + \delta_2 - \mu)} & w > m \\ \bar{F}_2(t) f_1(t) \int_{x_m}^{x_r} A(t, x) dx & w = m \end{cases}$$

Similarly, for spells that did not finish with the reception of severance payments:

$$\begin{aligned}
& \mathbb{P}(t_e \leq t, X_{t_e} \leq \bar{x}, \delta_1 \text{ did not arrive first}) \\
&= \mathbb{P}(t_e \leq t, X_{t_e} \leq \bar{x}, \delta_2 \text{ arrived first} \vee x_r \text{ was reached first}) \\
&= \mathbb{P}(t_e \leq t, X_{t_e} \leq \bar{x} | \delta_2 \text{ arrived first}) \mathbb{P}(\delta_2 \text{ is first}) \\
&\quad + \mathbb{P}(t_e \leq t, X_{t_e} \leq \bar{x} | x_r \text{ is reached first}) \mathbb{P}(x_r \text{ is reached first})
\end{aligned}$$

Using the results from the previous section:

$$\mathbb{P}(t_e \leq t, X_{t_e} \leq \bar{x}, \delta_2 \text{ arrived first}) = \int_0^t \mathbb{P}(X_s \leq \bar{x}, s < T_r) \bar{F}_1(s) dF_2(s)$$

and noting that $X_{T_r} = x_r \leq \bar{x}$, also from previous results we get that:

$$\mathbb{P}(t_e \leq t, X_{t_e} \leq \bar{x}, x_r \text{ is reached first}) = \int_0^t \bar{F}_1(s) \bar{F}_2(s) dF_r(s)$$

Therefore:

$$\begin{aligned}
& g(t, \bar{x}, \delta_1 \text{ did not arrive first}) \\
&= \frac{\partial^2}{\partial \bar{x} \partial t} \int_0^t \mathbb{P}(X_s \leq \bar{x}, s < T_r) \bar{F}_1(s) dF_2(s) + \int_0^t \bar{F}_1(s) \bar{F}_2(s) dF_r(s) \\
&= \frac{\partial}{\partial \bar{x}} \mathbb{P}(X_t \leq \bar{x}, t < T_r) \bar{F}_1(t) f_2(t) + \bar{F}_1(t) \bar{F}_2(t) f_r(t) \\
&= \begin{cases} \frac{\partial}{\partial \bar{x}} \mathbb{P}(X_t \leq \bar{x}, t < T_r) \bar{F}_1(t) f_2(t) & \bar{x} > x_r \\ \bar{F}_1(t) \bar{F}_2(t) f_r(t) & \bar{x} = x_r \end{cases} \\
&= \begin{cases} \left[\frac{1}{\bar{x} \sigma \sqrt{t}} \phi \left(\frac{\ln(\bar{x}/x_0) - \bar{\mu} t}{\sigma \sqrt{t}} \right) - \left(\frac{x_r}{x_0} \right)^{\frac{2\bar{\mu}}{\sigma^2}} \frac{1}{\bar{x} \sigma \sqrt{t}} \phi \left(\frac{\ln(\bar{x}/x_0) - 2 \ln(x_r/x_0) - \bar{\mu} t}{\sigma \sqrt{t}} \right) \right] \bar{F}_1(t) f_2(t) & \bar{x} > x_r \\ \bar{F}_1(t) \bar{F}_2(t) f_r(t) & \bar{x} = x_r \end{cases}
\end{aligned}$$

Then the respective joint density of length of completed employment spells and final wages is given by:

$$h(t, w, \text{ no reception of SP}) = \begin{cases} \frac{A(t, x(w)) \bar{F}_1(t) f_2(t)}{\beta + \tau(r + \delta_2 - \mu)} & w > m \\ \bar{F}_1(t) \bar{F}_2(t) f_r(t) + \bar{F}_1(t) f_2(t) \int_{x_m}^{x_r} A(t, x) dx & w = m \end{cases}$$

Given our previous results we conclude that for spells without severance payment information:

$$g(t, x) = \begin{cases} A(t, x) [f_1(t) \bar{F}_2(t) + f_2(t) \bar{F}_1(t)] & x > x_r \\ \bar{F}_1(t) \bar{F}_2(t) f_r(t) & x = x_r \end{cases}$$

and:

$$h(t, w) = \begin{cases} \frac{A(t, x(w)) [f_1(t) \bar{F}_2(t) + f_2(t) \bar{F}_1(t)]}{\beta + \tau(r + \delta_2 - \mu_0)} & w > m \\ \bar{F}_1(t) \bar{F}_2(t) f_r(t) + [f_1(t) \bar{F}_2(t) + f_2(t) \bar{F}_1(t)] \int_{x_r}^{x_m} A(t, x) dx & w = m \end{cases}$$

A.5 Introduction of Measurement Error

We introduce measurement error in observed wages above the minimum wage, such that:

$$\omega^{obs} = \omega^{real} \epsilon \quad \text{where } \epsilon \sim \text{Lognormal}(1, \sigma_{ME})$$

Therefore, we have to apply the change of variable formula once more to every piece of likelihood evaluated at $x(w)$. If a function f is evaluated at $x(w)$, and possible other variables in the vector y , then we have to replace $f(x(w), y)$ by:

$$\int_0^\infty f\left(\frac{\omega^{obs}/\epsilon - (1 - \beta)rU}{\beta + \tau(r + \delta_2 - \mu)}, y\right) \frac{g(\epsilon)}{\epsilon} d\epsilon$$

where g is the Lognormal density function.

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