Cheating in Contest

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Abstract

We analyze a game between three players: two athletes and the Inspector. Athletes contest one with another, while each athlete can cheat in order to increase his/her chances to win. The Inspector is interested to prevent cheating. S/he performs test of athletes, in order to explore cheating. The test is costly for the Inspector. We compare to approaches for tests: ex ante approach, where the Inspector decides which athlete to test before s/he knows results of the competition; and ex post approach, where the Inspector decides which athlete to test when results of the competition are known.

1   Introduction

Suppose two athletes are competing in a contest. Each one of them can cheat, for example, to use an illegal drug (doping). Cheating significantly increases chances of the athlete to win the contest. However, authorities, which are interested in a fair play, perform doping tests, and if an athlete is caught on cheating s/he will be severely punished. The doping tests are costly for authorities, so they are interested to optimize a use of them. On the one hand,

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authorities are interested to minimize the undetected cheating, on the other hand, they are interested to minimize test's cost.

Following seminal paper by Tullock [1980] this contest model may be applied not only to sports, but to variety of areas, like political competition, rent-seeking competition, etc.. [1]

An inspector (representative of authorities) has two basic approaches to doping tests. First, s/he may choose a probability with which each athlete will be tested for doping disregarding the contest's results (*the ex ante approach*). Second, s/he may decide which one of athlete to test taking into account the contest's results (*the ex post approach*). S/he can also use some mix of these two approaches.

For example, the World Anti-Doping Agency (WADA) selects athletes for testing both randomly and according to competition results. The policy of the German anti-doping agency (NADA) is that "selection of athletes ... can either be decided by placing, by name (target control) as well as by drawing lots" (www.nada.de). Similar policy is adopted by agencies in other countries. Actually 'target control' is the ex ante approach, while selecting by placing of athletes is the ex post approach.

The formal model will be described below. The main prediction is that the ex post approach is more efficient than the ex ante approach, namely, the expected number of doping tests is lower with the ex post approach and the probability of cheating is lower. However, a selection of athletes for testing according to a mix of the ex post approach and a random selection can decrease the probability of doping and increase the expected utility of the anti-doping agency.

This research belongs to the inspection games literature, where one of players decides whether to violate some rule or not, and the second player (the inspector) chooses an optimal strategy to detect the violation. See Avenhaus et al. [2002] for survey of this literature.

There are few game-theoretic papers on contests with possible cheating. In Kirstein [2014] the game is between one athlete and an enforcer (the inspector). The athlete can

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[1]See Konrad [2009] for survey of contest literature.

dope or not dope, an imperfect signal about athlete's decision is sent to the enforcer, which chooses whether to punish the athlete or not. In Berentsen [2002] two players participate in a contest and can cheat, but the probability of detection of cheating by inspector is exogenous, and not a strategic decision of the inspector.

2 Model

Let the Favorite (F) and the Underdog (U) be two athletes. Each one of them chooses one of two actions: to dope (D), or not to dope (N). Let a_i, $i \in \{D, N\}$ be the action chosen by $i \in \{F, U\}$. Let $p(a_F, a_U)$ be the probability that F wins the contest if actions a_F and a_U are chosen by F and U, respectively. It is assumed that U wins with probability $1 - p(a_F, a_U)$. Denote

$$p(a_F, a_U) = \begin{cases} p & , a_F = a_U \\ p^* & , a_F = D, a_U = N \\ p^{'} & , a_F = N, a_U = D \end{cases}$$

The Favorite is stronger than the Underdog, in the sense that $\frac{1}{2} \leq p < 1$. The doping is effective, namely, $p < p^* < 1$ and $0 < p^{'} < p$.

The Inspector (I) chooses one out of three actions: to test F (TF), to test U (TU) or not to test anyone of them (NT). It is assumed that budged constraint does not allow I to test both F and U. It is also assumed that to test is a costly action, namely, if I tests F or U, s/he pays cost c, $0 < c < 1$. The test is reliable, namely, if an athlete is either guilty in doping or innocent, the test, once performed, will correctly detect this.

Next to preferences of players. Both F and U prefer to win and not to be tested for doping (or to be tested and to be found innocent). The utility of F and U in this case is normalized to 1. The worst outcome for F and U is to be tested and to be found guilty (regardless contest result). The utility of F and U in this case is M, $M < 0$. The utility of F, or U, if s/he loses the contest and is not found guilty is 0.

F \ U	D	N
D	$M, 1-p, 1-c$	$M, 1-p^*, 1-c$
N	$p', 1-p', -c$	$p, 1-p, 1-c$

I chooses TF

F \ U	D	N
D	$p^*, M, 1-c$	$p^*, 1-p^*, -c$
N	$p', M, 1-c$	$p, 1-p, 1-c$

I chooses TU

F \ U	D	N
D	$p, 1-p, 0$	$p^*, 1-p^*, 0$
N	$p', 1-p', 0$	$p, 1-p, 1$

I chooses NT

Table 1: Utilities of players

The Inspector is interested to prevent doping or to catch an athlete who dopes. It is a bad outcome for I to let a guilty athlete to escape the test, while another athlete is guilty and not caught. Also cost of the test is taken into account. If neither F nor U dopes, I receives utility 1 if s/he performed no test, and $1 - c$ if the test was performed. Suppose next a player $i \in \{F, U\}$ used the doping. If the Inspector tests i, I receives utility $1 - c$, and 0 if no test is performed. If I tests player $i \in \{F, U\}$, and it is found that i is innocent, while the second player $j \neq i$, $j \in \{F, U\}$, was guilty, I obtains $-c$.

Utilities of players are summarized in Table 1.

Three games are considered. The first, G_1, models the ex ante approach. The Inspector moves first. S/he fixes probabilities z_F and z_U, with which TF and TU, respectively, are chosen. NT is chosen with probability $1 - z_F - z_U$. z_F and z_U are common knowledge. Next, F and U choose D or N. The contest takes place, after the contest F and U are tested with probabilities z_F and z_U, respectively, and payoffs, as defined in Table 1, are realized. Denote by x_i, $i \in \{F, U\}$, the probability that i chooses D.

Proposition 2.1. There exists M^* such that for $M < M^*$, in the unique equilibrium of G_1, $x_F = x_U = c$. The Inspector chooses to test F with a probability which increases in p^*, and to test U with probability which decreases in p'. The probability that I tests either F or U increases in M.

The probability that F and U choose to dope increases in c. As test becomes more expensive for the Inspector, athletes dope with a higher probability. The more effective is the doping for F (higher p^*), the higher is the probability that the Inspector will test F. The same logic applies for U. The more effective is the doping for U (lower p'), the higher is the probability that U will be tested. The more severe is the punishment for an athlete caught doping (lower M), the less probability of test is required.

The second game, G_2, models the ex post approach. First, F and U simultaneously choose N or D. Their choice is a private knowledge. Then the contest takes place. The Inspector observes who is the winner, and then decides to test F, U or none of them. Payoffs of players are defined in Table 1. Denote by y_i, $i \in \{F, U\}$, the probability that i chooses D. Let $P_T(i|j)$ be the probability that I tests i given that j is the winner, $i, j \in \{F, U\}$.

Proposition 2.2. There is $\tilde{M}$ such that for $\tilde{M} < M$, in a Nash equilibrium of G_2, $0 < y_F < 1$, $0 < y_U < 1$, $0 < P_T(i|i) < 1$ and $P_T(i|j) = 0$ for every $i, j \in \{F, U\}$, $j \neq i$.

Proposition 2.2 predicts that both F and U choose to dope with a positive probability. However, only winners are tested by the Inspector. A reason is that since the Inspector has a constraint on the number of tests s/he can perform, s/he prefers not to waste it on testing the loser. In equilibrium, the probability that the winner used doping is higher than the probability that the loser used doping.

A full characterization of an equilibrium of G_2 is cumbersome. A numerical solution for $p = 0.6$, $p' = 0.4$, $c = 0.3$, $M = -5$, based on (13), (14) (17) and (18) is given in Figure 2.1. One can learn that the probability that the Favorite dopes decreases with effectiveness of doping p^*. An intuition is, that as more effective is the doping, the higher is probability that the Inspector tests F if s/he wins, and then F is more reluctant to dope. The probability that F or U use doping (y_F and y_U, respectively) is lower than the probability (c) that they use doping in G_1.

The next result holds only for symmetric case, where F and U has equal chances to win,
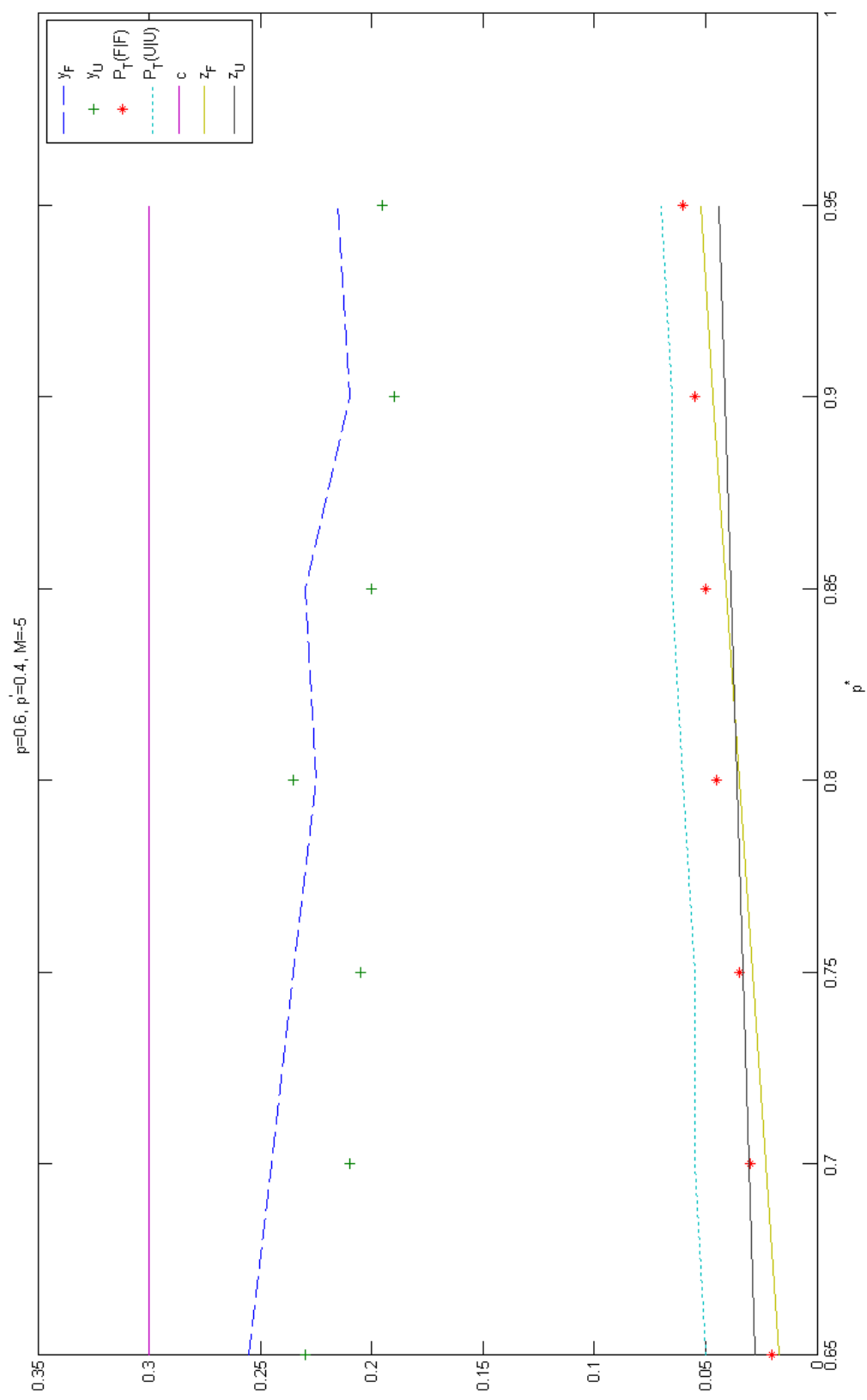
Figure 2.1: Equilibrium of G_2 in a non-symmetric case

namely, $p = \frac{1}{2}$, $p^* = 1 - p'$.

Proposition 2.3. Let $p = \frac{1}{2}$, $p^* = 1 - p'$. Denote by x the probability that $i \in \{F, U\}$ chooses D in the equilibrium of G_1, by y the probability that i chooses D in the symmetric equilibrium of G_2, by z the probability that I tests i in the equilibrium of G_1 and by $P_{T,W}$ the probability that I tests the winner in the symmetric equilibrium of G_2. There exists M' such that for $M < M'$, the following holds.

(i) $y < x$, and both y and x are increasing in c.

(ii) $P_{T,W} < z$, and z and $P_{T,W}$ are increasing in c.

Proposition 2.3 states that the ex post approach is more efficient than the ex ante approach. In the symmetric G_2 the Inspector performs the test with lower probability than in G_1. Namely, considering test's cost, the ex post approach is cheaper. Moreover, $y < x$ means that athletes take doping with a lower probability if the ex post approach is adopted.

In the symmetric case, in both G_1 and G_2, the probability that an athlete dopes is increasing in c. This is intuitive. As higher is the cost of doping test, the higher is incentive of an athlete to use doping. More surprising is that the Inspector performs test with a higher probability, the more expensive the test is. It is less surprising if we take into account strategic considerations. As c increases, the Inspector expects that athletes use doping with higher probability, and then tests them with higher probability.

Let $0 < \epsilon < 1$. The third game, $G(\epsilon)$, is defined as G_2, with one difference. If the Inspector decides not to test either F or U, the Nature with probability ϵ, forces the Inspector to test F (TF), with the same probability to test U (TU), and with probability $1 - 2\epsilon$ to test none of them (NT). Still, maximum one of athletes F or U can be tested. Utilities of players are given in Table 1.

The following proposition holds for symmetric case of $G(\epsilon)$.

Proposition 2.4. Consider symmetric game $G(\epsilon)$. Assume $p = 0.5$, $p' = 1 - p*$. Let each of athletes dope with probability $y(\epsilon)$.
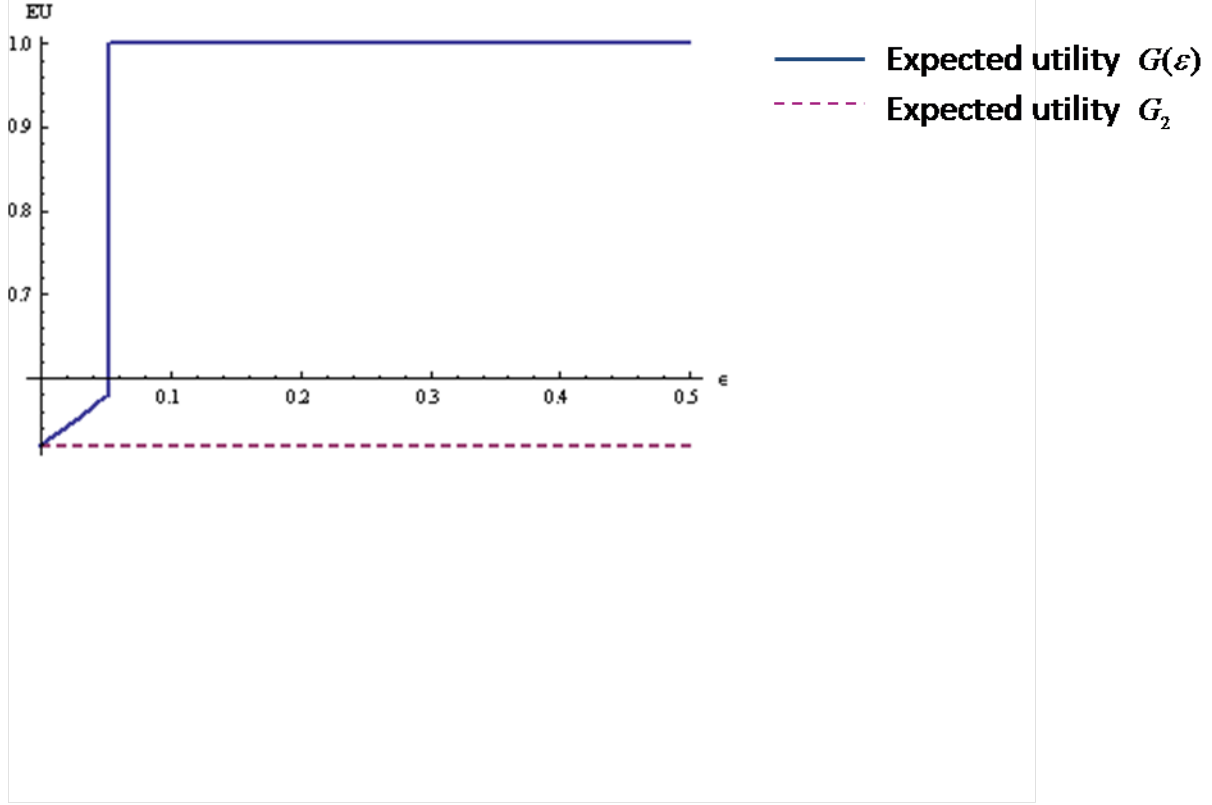
Figure 2.2: Expected utility of the Inspector in $G(\epsilon)$ vs G_2

(i) Let $\epsilon > \frac{p^*-0.5}{p^*-M}$. There is an equilibrium, where both athletes do not dope with certainty.

(ii) Let $\epsilon < \frac{p^*-0.5}{p^*-M}$. There is $\tilde{M} < 0$ such that for $M < \tilde{M}$, the expected utility of I is

$$EU(\epsilon) = (1 - y(\epsilon))^2 + 2\epsilon y(\epsilon),$$

where $y(\epsilon)$ is a solution of (26).

All proofs appear in the Appendix.

Figure 2.2 illustrates that for some parameters ($p^* = 0.8$ and $c = 0.4$) the expected utility of the Inspector is higher in $G(\epsilon)$) than in G_2.

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Appendix

Proof of Proposition 2.1 Let z_F and z_U be probabilities TF and TU, respectively, are chosen with. Let x_i, $i \in \{F, U\}$ be the probability that i chooses D. F prefers D iff

$$(1 - z_F)[p^*(1 - x_U) + p x_U] + z_F M \geq p(1 - x_U) + p' x_U, \tag{1}$$

and U prefers D iff

$$(1 - z_U)[(1 - p')(1 - x_F) + (1 - p)x_F] + z_U M \geq (1 - p)(1 - x_F) + (1 - p^*)x_F. \tag{2}$$

Player I prefers TF to TU iff

$$1 - (1 - x_F)x_U \geq 1 - (1 - x_U)x_F, \tag{3}$$

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and I prefers TF to NT iff

$$-c + 1 - (1 - x_F)x_U \geq (1 - x_U)(1 - x_F), \tag{4}$$

Lemma 1. There is $\bar{M}$ such that for $M < \bar{M}$ there is no equilibrium such that $x_F = x_U = 1$.

Proof Suppose by contrary that both F and U choose D with certainty. Then NT is an inferior action of I, and it will be chosen with probability 0. Therefore, $z_U = 1 - z_F$. By substitution of $x_F = x_U = 1$ and $z_U = 1 - z_F$ into (1) and (2),

$$\frac{1 - p^* - M}{1 - p - M} \leq z_F \leq \frac{p - p'}{p - M}. \tag{5}$$

Since $\frac{1 - p^* - M}{1 - p - M} \to 1$ and $\frac{p - p'}{p - M} \to 0$ as $M \to -\infty$, there exists $\bar{M}$ such that (5) does not hold for any z_F if $M < \bar{M}$. $\square$

Lemma 2. Suppose $M < \bar{M}$. In every Nash equilibrium $0 < z_F < 1$ and $0 < z_U < 1$.

Proof If I chooses TF with probability 1, the best reply of F is N, but then I is better off by deviating to NT, contradiction. If I chooses TF with probability 0, the best reply of F is D. If F chooses D with probability 1, by Lemma 1 U chooses D with probability less than 1, therefore, I is better off by deviating to TF, contradiction. Similarly, TU cannot be a pure strategy of I in an equilibrium. $\square$

By Lemma 2, equality holds in (3), thus, for $M < \bar{M}$

$$x_F = x_U. \tag{6}$$

Lemma 3. Suppose $M < \bar{M}$. In every Nash equilibrium $0 < x_F < 1$ and $0 < x_U < 1$.

Proof Suppose $x_F = 0$. Then the best reply of I is $z_F = 0$, contradiction to Lemma 2. Next, suppose $x_F = 1$. By Lemma 1, in equilibrium U chooses D with probability less than

1, therefore, I's best reply is $z_F = 1$, contradiction to Lemma 2. A proof of $0 < x_U < 1$ is similar. $\square$

The following lemma states that for sufficient low M, in equilibrium I chooses NT with a positive probability.

Lemma 4. There exists M^* such that for $M < M^*$, $z_U + z_F < 1$ in equilibrium.

Proof Let $M < \bar{M}$. Therefore, (6) holds. By contrary, suppose $z_U + z_F = 1$. By Lemma 3, equality holds in (1) and (2). By substitution of (6) and $z_F = 1 - z_U$ into (1) and (2), it easy to verify that there is $M^* < \bar{M}$ such that for $M < M^*$ equality cannot hold in both (1) and (2), contradiction. $\square$

By Lemmas 2 and 4, equality holds in (3) and (4), thus

$$x_F = x_U = c. \tag{7}$$

By (1),

$$z_F = \frac{(p^* - p)(1 - c) + (p - p')c}{p^*(1 - c) + pc - M} \tag{8}$$

and

$$z_U = \frac{(p - p')(1 - c) + (p^* - p)c}{(1 - p')(1 - c) + (1 - p)c - M} \tag{9}$$

The rest of the proof follows directly from (7), (8) and (9). $\square$

Proof of Proposition 2.2 Recall, y_i, $i \in \{F, U\}$, is the probability that i chooses D, $P_T(i|j)$ is the probability that I tests i given that j is the winner, $i, j \in \{F, U\}$.

Lemma 5. There is $\tilde{M}$ such that for $M < \tilde{M}$ there is no equilibrium such that $y_F = y_U = 1$.

Proof Suppose $y_F = y_U = 1$. F prefers D if

$$(1 - P_T(F|F))p + M[pP_T(F|F) + (1 - p)P_T(F|U)] \geq p', \tag{10}$$

and U prefers D iff

$$P_T(F|U)(1-p) + M[(1 - P_T(F|U))(1-p) + p(1 - P_T(F|F))] \geq 1 - p^*. \qquad (11)$$

By summarizing (10) and (11),

$$(1 - P_T(F|F))p + P_T(F|U)(1-p) + M \geq 1 - p^* + p', \qquad (12)$$

and there is $\tilde{M}$ such that for $M < \tilde{M}$ (12) does not hold. $\square$

Lemma 6. Suppose $M < \tilde{M}$. In every Nash equilibrium $0 < y_F < 1$ and $0 < y_U < 1$. There is no equilibrium where $P_T(F|F) = P_T(F|U) = 1$ or $P_T(F|F) = P_T(F|U) = 0$ or $P_T(U|F) = P_T(U|U) = 1$ or $P_T(U|F) = P_T(U|U) = 0$.

Proof Note, $P_T(i|F) = P_T(i|U) = 1$, $i \in \{F, U\}$ means that I tests i with probability 1 irrespective of result of the contest, and $P_T(i|F) = P_T(i|U) = 0$ means that I tests i with probability 0 irrespective of result of the contest. The proof is similar to Lemmas 2 and 3. $\square$ The next lemma states that there is no equilibrium where the winner is tested with certainty.

Lemma 7. Let $i \in \{F, U\}$. There is no Nash equilibrium where $P_T(i|i) = 1$.

Proof Suppose by contrary that player i is tested with probability 1 if s/he wins. If i dopes, his/her expected payoff is $MP(i \text{ wins } |i \text{ dopes})$. This value is negative, therefore, i is better off by deviating to N, contradiction to Lemma 6. $\square$

For $i, j \in \{F, U\}, i \neq j$, the Inspector prefers to test i to NT, given j wins, iff

$$-c+1-P(j \text{ chose D}|j \text{ wins})[1-P(i \text{ chose D}|j \text{ wins})] \geq [1-P(j \text{ chose D}|j \text{ wins})][1-P(i \text{ chose D}|j \text{ wins})]$$

which is equivalent to

$$P(i \text{ chose D}|j \text{ wins}) \geq c, \qquad (13)$$

and prefers to test j to NT, given j wins, iff

$$-c+1-P(i \text{ chose D}|j \text{ wins})[1-P(j \text{ chose D}|j \text{ wins})] \geq [1-P(j \text{ chose D}|j \text{ wins})][1-P(i \text{ chose D}|j \text{ wins})]$$

which is equivalent to

$$P(j \text{ chose D}|j \text{ wins}) \geq c. \tag{14}$$

Given j wins, I prefers to test j to test i iff

$$-c+1-P(j \text{ chose D}|j \text{ wins})[1-P(i \text{ chose D}|j \text{ wins})] \leq$$
$$\leq -c+1-P(i \text{ chose D}|j \text{ wins})[1-P(j \text{ chose D}|j \text{ wins}),$$

which is equivalent to

$$P(i \text{ chose D}|j \text{ wins}) \leq P(j \text{ chose D}|j \text{ wins}). \tag{15}$$

The next lemma states that an athlete which lose in the contest is tested with probability 0.

Lemma 8. Let $i, j \in \{F, U\}, i \neq j$. In equilibrium, $P_T(i|j) = 0$.

    Proof Suppose F wins. Suppose by contrary that I tests U with a positive probability. By (15), $P(U \text{ chose D}|F \text{ wins}) \geq P(F \text{ chose D}|F \text{ wins})$, which is equivalent to

$$P(F \text{ chose D and } F \text{ wins}) \leq P(U \text{ chose D and } F \text{ wins})$$

and it could be verified that this is equivalent to

$$y_F(1-y_U)p^* \leq y_U(1-y_F)p'. \tag{16}$$

Since $p' < p^*$, by (16)

$$y_F(1-y_U) < y_U(1-y_F)$$

and

$$y_F(1 - y_U)(1 - p^*) < y_U(1 - y_F)(1 - p'),$$

which is equivalent to

$$P(F \text{ chose D and } U \text{ wins}) \leq P(U \text{ chose D and } U \text{ wins})$$

and by (15) I tests with certainty U if U wins, contradiction to Lemma 7. A proof for the case where U wins is similar. □

To complete the proof observe that $P_T(j|j) > 0$ in an equilibrium. By Lemma 8, $P_T(i|j) = 0$, then $P_T(j|j) > 0$ follows from Lemma 6. □

F prefers D to N iff

$$p(1 - y_U) + p' y_U \leq [p^*(1 - y_U) + p y_U][1 - P_T(F|F) + P_T(F|F)M] \tag{17}$$

U prefers D to N iff

$$(1 - p)(1 - y_F) + (1 - p^*) y_F \leq [(1 - p')(1 - y_F) + (1 - p) y_F][1 - P_T(U|U) + P_T(U|U)M] \tag{18}$$

Proof of Proposition 2.3 By Proposition 2.1, $x = c$. By Proposition 2.2, $0 < P_{T,W} < 1$. Namely, if $i \in \{F, U\}$ wins, I is indifferent between testing i and NT. Therefore, equality holds in (14). After substitution of $p = 0.5$ and $p' = 1 - p^*$, we obtain

$$c = \frac{y[0.5y + (1 - y)p^*]}{y[0.5y + (1 - y)p^*] + (1 - y)[y(1 - p^*) + 0.5(1 - y)]},$$

and the root of this equation between 0 and 1 is

$$y = \frac{p^* - \sqrt{(p^*)^2 - 2cp^* + c}}{2p^* - 1}. \tag{19}$$

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It is straightforward to verify that $y < c = x$. y is increasing in c,

By substitution of $p = 0.5$ and $p' = 1 - p^*$ in (8),

$$z = \frac{p^* - 0.5}{p^*(1 - c) + 0.5c - M}. \tag{20}$$

By Lemma 6, in G_2 an athlete i is indifferent between doping and no doping. In the symmetric case it implies the following equation:

$$y(1 - p^*) + 0.5(1 - y) = [0.5y + (1 - y)p^*](1 - P_{T,W} + P_{T,W}M),$$

equivalently,

$$P_{T,W} = \frac{p^* - 0.5}{[0.5y + (1 - y)p^*](1 - M)}. \tag{21}$$

Recall, $y < c$ and $p^* > 0.5$. Then by (20) and (21),

$$P_{T,W} < z.$$

$P_{T,W}$ is increasing in y. Since y is increasing in c, $P_{T,W}$ is also increasing in c. $\square$

Proof of Proposition 2.4 Assume $p = 0.5$, $p' = 1 - p*$. Let each of athletes dope with probability $y(\epsilon)$. Let $i, j \in \{F, U\}$, $i \neq j$. Let $P_{T,\epsilon}(i, i)$ be the probability that I tests i if i wins, and $P_{T,\epsilon}(j, i)$ be the probability that I tests j if i wins. Observe that for $\epsilon > \frac{p^* - 0.5}{p^* - M}$ there is an equilibrium where $y(\epsilon) = 0$.

Suppose hereafter $\epsilon < \frac{p^* - 0.5}{p^* - M}$. Similarly to Lemmas 5, 6, 7 and 8, it can be shown that there is $\tilde{M}$ such that for $M < \tilde{M}$, in equilibrium $0 < y(\epsilon) < 1$, $0 < P_{T,\epsilon}(i, i) < 1$ and $P_{T,\epsilon}(j, i) = 0$.

Without loss of generality, suppose F wins the contest. The inspector is indifferent

between TF and NT iff

$$(1 - 2\epsilon)[1 - P(F \text{ chose D}|F \text{ wins})][1 - P(U \text{ chose D}|F \text{ wins})]+ \tag{22}$$

$$+ \epsilon[2 - P(U \text{ chose D}|F \text{ wins})(1 - P(F \text{ chose D}|F \text{ wins}))- \tag{23}$$

$$- P(F \text{ chose D}|F \text{ wins})(1 - P(U \text{ chose D}|F \text{ wins})) - 2c] = \tag{24}$$

$$= 1 - c - P(U \text{ chose D}|F \text{ wins})(1 - P(F \text{ chose D}|F \text{ wins})). \tag{25}$$

After substitution of

$$P(F \text{ chose D}|F \text{ wins}) = \frac{y(\epsilon)[0.5y(\epsilon) + p^*(1 - y(\epsilon))]}{y(\epsilon)[0.5y(\epsilon) + p^*(1 - y(\epsilon))] + (1 - y(\epsilon))[0.5(1 - y(\epsilon)) + (1 - p^*)y(\epsilon)]}$$

and

$$P(U \text{ chose D}|F \text{ wins}) = \frac{y(\epsilon)[0.5y(\epsilon) + (1 - p^*)(1 - y(\epsilon))]}{y(\epsilon)[0.5y(\epsilon) + (1 - p^*)(1 - y(\epsilon))] + (1 - y(\epsilon))[0.5(1 - y(\epsilon)) + p^*y(\epsilon)]}$$

into (22), I is indifferent between TF and NT iff

$$y^2(\epsilon)[(p^* - 0.5)(\frac{\epsilon}{1 - \epsilon} - 1)] + y(\epsilon)[p^* + \frac{\epsilon}{1 - \epsilon}(1 - p^*)] - \frac{c(1 - 2\epsilon)}{2(1 - \epsilon)} = 0. \tag{26}$$

It can be verified that the expected utility of the Inspector in $G(\epsilon)$, $EU(\epsilon)$ is

$$EU(\epsilon) = (1 - y(\epsilon))^2(1 - 2\epsilon) + 2\epsilon(1 - y(\epsilon))(1 - y(\epsilon)) =$$

$$= (1 - y(\epsilon))^2 + 2\epsilon y(\epsilon). \ \square$$