

# Information Aggregation Through Stock Prices and the Cost of Capital\*

Olga Gorelkina, Wolfgang Kuhle<sup>†</sup>

March 2, 2015

## Abstract

This paper studies a firm's optimal capital structure in an environment, where the firm's stock price serves as a public signal for its credit worthiness. In equilibrium, equity investors choose whether to acquire information and to trade the firm's equity. This induces a positive relation between the amount of equity issued and the stock price signal's precision. Thus, through its capital structure, the firm can internalize the informational externality that stock prices exert on bond yields. Firms with a strong fundamental therefore issue more equity and less debt than they would if the informational spill-over did not exist.

*Keywords:* Information Aggregation, Capital Structure, Sequential Markets, Market Depth.

*JEL:* C73 D83 G10 G32

## 1 Introduction

Lehman's 2008 bankruptcy may have come as a surprise to those bondholders who believed in its A-ratings. It was less of a surprise to the bondholder who observed that Lehman's stock

---

\*We thank Martin Hellwig for his suggestion to think about information spill-overs between stock prices and bond yields. Julio Davila, Jacques Dreze, Alia Gizatulina, Dominik Grafenhofer, Kristoffel Grechenig, Christian Hellwig, Paul Klemperer, Philipp Koenig, George Lukyanov, Martin Szydlowski and seminar participants in Bonn, CORE Louvain-la-Neuve, MPI Munich, DIW Berlin, and the EEA-ESEM provided helpful comments.

<sup>†</sup>Both authors: Max Planck Institute for Research on Collective Goods, Kurt-Schumacher-Str. 10 53113 Bonn, Germany.

price had fallen from 62.19 at the beginning of 2008 to its 3.65 low on September 12, 2008, the day before Lehman announced its bankruptcy. Similarly, AIG's A-ratings were not more informative during the days before its bail-out. The stock price, which had fallen by more than 90 percent in that year, provided a more informative signal. In both cases, the stock price served as a timely, costless, and arguably unbiased monitoring device for bondholders.

The current paper complements the literature that studies a firm's optimal capital structure in models with default and asymmetric information. It suggests the role of competitive markets as information aggregation devices as another aspect, which shapes the firm's financing decisions. In the present model, firms choose their capital structure to internalize the informational externality that stock prices have on equilibrium bond yields. The main finding indicates that the informational spill-over from stock price signals to equilibrium bond yields makes it optimal for firms which are financially strong to issue more equity and less debt than they would in a world without the information spill-over. This finding relies on a positive relation between the informativeness of the firm's stock price and the amount of equity issued: as the firm issues more equity, it incentivises more equity investors to research and trade the firm's stock. In turn, the firm's stock price becomes more informative and communicates the true financial health of the firm more clearly to bond investors, who use the information contained in the stock price to calculate the firm's default risk, and the corresponding equilibrium bond yield. Compared to capital structure models<sup>1</sup> where the management's choice of an optimal capital structure communicates insider knowledge to outside investors, the present paper analyzes how different capital structures facilitate/optimize the information exchange *between* outside investors.

More precisely, we study a framework where the firm issues bonds  $B$  and sells a number of shares  $K$  to raise an exogenously given revenue  $I$ . The firm's objective is to minimize the capital cost  $C = K(\theta - p) + RB$  subject to the revenue requirement. The dividend  $\theta$  paid to equity investors represents the firm's financial strength. The model is sequential and at the beginning of time, the firm announces a capital structure  $(K, B)$ . Subsequently, the stock market opens and the shares  $K$  are sold at a market-clearing price  $p$  to risk-averse investors who possess private information on the firm's health  $\theta$ . This stock price aggregates the stock investor's dispersed private knowledge and partially reveals the firm's strength  $\theta$ . In turn, the

---

<sup>1</sup>See Harris and Raviv (1991), pp. 306-315, for a survey of models where various aspects of asymmetric information shape optimal capital structures.

bond market opens and risk averse investors, who observe the price signal  $p$ , buy the firm's bonds at a market-clearing net interest rate  $R(p)$ . Bondholders and investors receive their respective pay-outs in the final period when the true strength  $\theta$  of the firm is revealed. Bonds pay a net return  $R$  if  $\theta > 0$ . Otherwise, if  $\theta < 0$ , the firm declares bankruptcy and bond investors take a loss  $L$  on each bond.

The predictions of our model rely on two main components. First, we assume that the stock market is the main source of public information. In our two-period model, this is captured by the assumption that the stock market opens earlier than the bond market. The second main element of our analysis concerns the decision of stock investors to collect information and to trade the firm's stocks. As the firm issues more equity  $K$ , the rents for equity investors increase, which makes it profitable for more investors to trade the firm's stock. As a consequence, the equity price  $p$  becomes more informative. That is, as the firm issues more equity  $K$ , which reduces its indebtedness  $B$ , the market price  $p$  communicates the firm's fundamental  $\theta$  more clearly. In turn, the expected equilibrium bond yield will be lower (higher) if the firm's fundamental is strong (weak) and firms will have an incentive to issue more (less) equity than they would without the informational externality.

Regarding the stock market, we employ the standard noise-trader models of Grossman and Stiglitz (1976, 1980), Green (1975), Hellwig (1980), and more recently Angeletos and Werning (2006), Vives (2008), Albuquerque and Miao (2014). It follows from this framework that the mass of equity investors, who take positions in the firm's stock, increases with the amount of equity issued. In turn, the stock price signal's informativeness increases with the mass of equity investors. Hence, the stock price's informativeness increases with the amount of equity issued. Put differently, we find that larger stock markets allow investors to earn larger rents. In turn, these rents incentivise additional investors to participate in the trade of the firm's equity. Indeed, this theoretical prediction is in line with the empirical findings of Collins et al. (1987), who show that stock prices of firms with a large market capitalization predict future earnings and dividends better than stock prices of firms with a small stock market capitalization.

In turn, the stock market's price signal influences the equilibrium rate of return on the firm's debt. This spill-over influences the firm's optimal capital structure and induces healthy firms to issue more equity than they would if the informational spill-over did not exist. The characterization of this spill-over provides a new aspect to the literature on optimal capital

structures in models with default. That is, we add informational spill-overs from stock prices to bond yields to aspects such as agency costs, corporate control considerations, or the tax-shield-default tradeoff. In terms of the survey on capital structures by Harris and Raviv (1991), the present analysis is closest to models of asymmetric information, where the management's choice of a capital structure transmits insider information to outside investors. However, instead of a transfer of information from insiders to outsiders, the present analysis focuses on the transfer of information *between* outsiders, i.e., bond and stock investors. Similar to our model, Harris and Raviv (1990) study the firm's capital choice in an environment where the firm's indebtedness influences information revelation. In the model of Harris and Raviv (1990), firms produce output using an unobservable technology. In turn, outside investors observe whether the firm can service its debt. Hence, if the firm can meet its debt obligations, outside investors infer a lower bound for the firm's productivity. That is, if a deeply indebted firm meets its obligations it must be very productive; and outside observers can calculate a lower bound for the firm's unobservable productivity.<sup>2</sup>

The rest of the paper is structured as follows. In Section 2, we lay out the model. In Section 3, we derive the main results. In a separate Section 4, we comment on our assumptions and replace some of them to demonstrate the robustness of our findings. First, we discuss the timing of trades, which implies that it is the stock market, rather than the bond market, which aggregates information. Second, we show that the firm's strength  $\theta$  can be derived from a consistent budget constraint. Finally, we present a more general specification for the bond market. Section 5 offers concluding remarks.

## 2 The Model

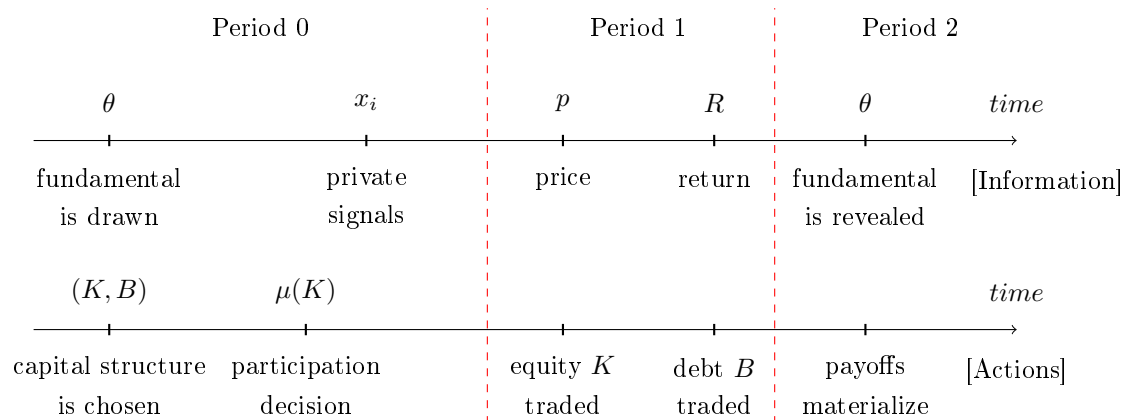
Our model consists of the firm's management, a large mass of potential stock investors, and a unit measure of bond investors. Their interaction is characterized by the following timeline:

- *Period 0*: The firm's management holds a prior  $f$  over the unknown fundamental  $\theta$ .

Based on these expectations, the firm decides on the optimal capital structure  $(K, B)$

---

<sup>2</sup>See Calomiris and Kahn (1991) for a related model. Admati et al. (2010), pp. 28-31, discuss the governance and informational role of a firm's debt. Albagli et al. (2011) develop a model where a firm's management interacts with stock prices that aggregate investors' dispersed private information. In their model, however, firms issue no debt.



which minimizes the expected cost of capital  $\mathbb{E}^f[C] = \mathbb{E}^f[K(\theta - p) + BR]$  subject to the revenue constraint  $I = \mathbb{E}^f[p]K + B$ . After the firm announced a particular plan,  $(K, B)$ , equity investors decide whether to participate or to abstain from the equity market. In equilibrium, the mass of participating agents  $\mu$ , depends on the size  $K$  of the equity market. Finally, equity investors receive private signals  $x_i$  on the firm's strength and submit demand schedules.

- *Period 1*: The equity market opens and an equilibrium stock price  $p$  is observed. The debt market opens after the equity market and bonds are traded at the equilibrium yield  $R(p)$ .
- *Period 2*: The firm's unknown fundamental  $\theta$  is revealed and all payoffs are realized.

Due to the correspondence between stock and bond market, we solve the model recursively. We begin with the equity market, subsequently, we introduce the bond market. Finally, we analyze the ex-ante decision  $(K, B)$  of the firm's management.

**Fundamental and Returns** The returns earned by equity and bond investors depend on the unknown strength of the firm  $\theta$ . In our baseline specification, this fundamental  $\theta$  is exogenous and independent of the capital structure.<sup>3</sup> The left-hand side of Figure 1 indicates that the firm pays bond investors a return  $R$  if the firm is strong enough, i.e.,  $\theta > 0$ . If the firm defaults,  $\theta < 0$ , investors incur a loss  $L$ . Regarding the returns to equity investment, which are depicted

<sup>3</sup>In our baseline model we abstract from a resource constraint for simplicity. In Section 4.2, we return to this omission and show that the incorporation of a resource constraint, where the dividend  $\theta$  depends on the capital structure, strengthens the effects that we derive for the simplified setting.

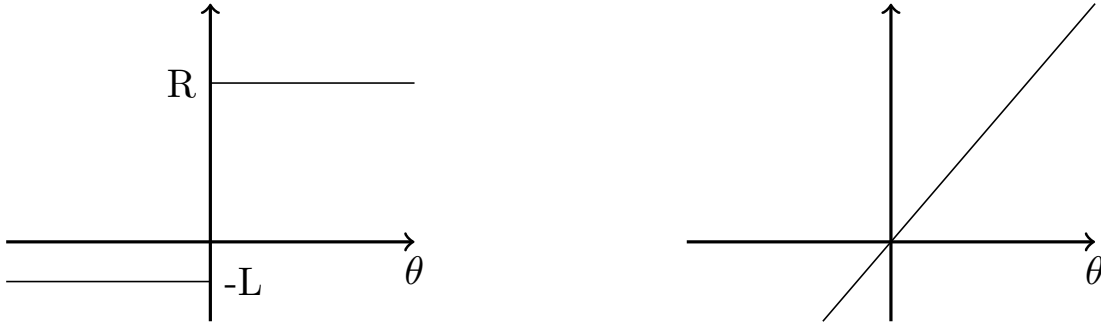


Figure 1: *Net return on debt and equity as functions of  $\theta$ .*

on the right-hand side of Figure 1, we study a CARA-normal model, where  $(\theta - p)k_i$  is the *absolute* amount of consumption goods available to an agent who invested  $k_i$ ;  $p$  and  $\theta$  are the asset's price and pay-out, respectively.<sup>4</sup>

## 2.1 Equity Market

Equity is traded in the standard Grossman and Stiglitz (1976, 1980), Hellwig (1980), Angeletos and Werning (2006), and Vives (2008) linear CARA-normal noise trader market. Equity investors hold an uninformative prior and receive noisy private signals  $x_i = \theta + \sigma_x \xi_i$ , where idiosyncratic noise  $\xi_i$  is i.i.d. across the population and  $\xi_i \sim \mathcal{N}(0, 1)$ . Noise trader activity, which ensures that prices are only partially revealing, is modelled as a demand shock  $K_n^d = \sigma_\varepsilon \varepsilon$ ,  $\varepsilon \sim \mathcal{N}(0, 1)$ . To characterize the market price signal, we proceed in three steps. First, we guess that there exists a linear price function,  $p = \eta_1 \theta + \eta_2 \varepsilon + c$ . Regarding  $\theta$ , this function is informationally equivalent to a signal  $Z \equiv \frac{p-c}{\eta_1} = \theta + \frac{\eta_2}{\eta_1} \varepsilon$ , which reveals the true fundamental with precision  $\alpha_z = \frac{\eta_1^2}{\eta_2^2}$ . Second, given this price function, we characterize individual demands based on the information  $x_i, Z$  and calculate the market equilibrium. Finally, we determine the ratio  $\frac{\eta_1^2}{\eta_2^2}$  as  $\frac{\alpha_x \mu^2}{\gamma^2 \sigma_\varepsilon^2}$ . That is, price signal  $Z$  indeed partially reveals the true fundamental  $\theta$  with precision  $\alpha_z = \frac{\alpha_x \mu^2}{\gamma^2 \sigma_\varepsilon^2}$ .

---

<sup>4</sup>In a different interpretation, which avoids negative prices, we discuss a CRRA-lognormal model. In this interpretation, agent  $i$  derives utility from the *return*,  $\ln(\frac{V}{P})k_i = (\theta - p)k_i$ , that she earns on her investment  $k_i$ ;  $p = \ln P$  and  $\theta = \ln V$  are the natural logarithms of a primitive price  $P > 0$  and a lognormal fundamental  $V > 0$ , respectively.

**Demand** Agents choose their demands  $k_i$  for the asset to maximize expected CARA utility:<sup>5</sup>

$$\begin{aligned}
k_i &= \arg \max_{k_i} \{ \mathbb{E}[-e^{-\gamma(\theta-p)k_i} | x_i, Z] \} \\
&= \arg \max_{k_i} \{ e^{\gamma \mathbb{E}[(\theta-p)k_i | x_i, Z] - \frac{\gamma^2}{2} \text{Var}[(\theta-p)k_i | x_i, Z]} \} \\
&= \arg \max_{k_i} \{ \gamma \left( \frac{\alpha_x}{\alpha} x_i + \frac{\alpha_z}{\alpha} Z - p \right) k_i - \frac{\gamma^2}{2} k_i^2 \frac{1}{\alpha} \}, \quad \alpha = \alpha_x + \alpha_z
\end{aligned}$$

and the individual demand function writes:

$$k_i^d = \frac{\alpha}{\gamma} \left( \frac{\alpha_x}{\alpha} x_i + \frac{\alpha_z}{\alpha} Z - p \right). \quad (1)$$

**Equilibrium** Aggregate supply is given by the firm's equity  $K^S = K$ . Demand is given by  $K^D = K^d + \sigma_\varepsilon \varepsilon$ , where  $K^d$  is the aggregate demand of rational equity investors and  $\sigma_\varepsilon \varepsilon$  is unobservable noise-trader activity  $\varepsilon \sim \mathcal{N}(0, 1)$ . From (1), we find that aggregate demand  $K^d$  is:

$$K^d = \int_{[0, \mu]} k_i di = \frac{\mu \alpha}{\gamma} \left( \frac{\alpha_x}{\alpha} \theta + \frac{\alpha_z}{\alpha} Z - p \right). \quad (2)$$

Equilibrium requires that:

$$K^D = K^S \quad \Leftrightarrow \quad \frac{\mu \alpha}{\gamma} \left( \frac{\alpha_x}{\alpha} \theta + \frac{\alpha_z}{\alpha} Z - p \right) = K - \sigma_\varepsilon \varepsilon. \quad (3)$$

To close the argument, we now resubstitute  $Z = \frac{p-c}{\eta_1}$  and calculate  $\eta_1$  and  $\eta_2$ . First, we solve (3) for  $p$  to obtain:

$$p = \frac{\alpha_x}{\alpha - \frac{\alpha_z}{\eta_1}} \theta + \frac{\gamma \sigma_\varepsilon}{\mu \left( \alpha - \frac{\alpha_z}{\eta_1} \right)} \varepsilon + \frac{1}{1 - \frac{\alpha \eta_1}{\alpha_z}} \left( c + \frac{K \gamma \eta_1}{\mu \alpha_z} \right). \quad (4)$$

Comparison of (4) with our initial guess,  $p = \eta_1 \theta + \eta_2 \varepsilon + c$ , indicates that  $\eta_1, \eta_2$  must satisfy:

$$\eta_1 = \frac{\alpha_x}{\alpha - \frac{\alpha_z}{\eta_1}}, \quad \eta_2 = \frac{\gamma \sigma_\varepsilon}{\mu \left( \alpha - \frac{\alpha_z}{\eta_1} \right)}, \quad c = \frac{1}{1 - \frac{\alpha \eta_1}{\alpha_z}} \left( c + \frac{K \gamma \eta_1}{\mu \alpha_z} \right); \quad \alpha = \alpha_x + \alpha_z. \quad (5)$$

The solution to the first equality is  $\eta_1 = 1$ , thus  $\eta_2 = \frac{\gamma \sigma_\varepsilon}{\alpha_x \mu}$  and  $c = -\frac{K \gamma}{\mu \alpha}$ . Accordingly,  $p$  and  $Z = \frac{p-c}{\eta_1} = \theta + \frac{\eta_2}{\eta_1} \varepsilon$  are given by

$$p = \theta + \alpha_z^{-1/2} \varepsilon - \frac{K \gamma}{\alpha \mu}, \quad Z = \theta + \alpha_x^{-1} \mu^{-1} \gamma \sigma_\varepsilon \varepsilon, \quad \alpha_z = \frac{\alpha_x^2 \mu^2}{\gamma^2 \sigma_\varepsilon^2}. \quad (6)$$

---

<sup>5</sup>See Raiffa and Schlaifer (2000), p. 250, for the standard results on prior and posterior distributions of normally distributed variables which we use throughout the paper.

Hence, the precision  $\alpha_z = \frac{\alpha_x^2 \mu^2}{\gamma^2 \sigma_\varepsilon^2}$  with which  $Z$  reveal the true fundamental  $\theta$  is an increasing function of the equity investor's mass  $\mu$ .

### 2.1.1 Equilibrium Participation

Agents choose whether to trade the firm's equity. Those who decide not to take a position in the firm's equity receive a fixed utility  $U_0$ , which represents an outside option. Agents are just indifferent between both options if:

$$E[E[U|Z, x_i]] = U_0 \tag{7}$$

In Appendix A, we show that, given the firm's stock supply  $K$ , ex-post asset demand (1), and the distribution of equilibrium prices implied by (6), the agent's ex-ante expected utility in (7) can be written as:

$$E[E[U|Z, x_i]] = F(|K|, \mu), \quad \frac{\partial F}{\partial |K|} > 0, \quad \frac{\partial F}{\partial \mu} < 0. \tag{8}$$

Hence, we have the following:

**Proposition 1.** *Increases in the equity market's depth  $|K|$  increases the mass of equity investors  $\mu$  and the price signal's precision  $\alpha_z = \frac{\alpha_x^2 \mu^2}{\gamma^2 \sigma_\varepsilon^2}$ .*

*Proof.* It follows from (7) and (8) that  $U_0 = F(|K|, \mu)$ . Hence, we have  $\frac{\partial \mu}{\partial K} = -\frac{\frac{\partial F}{\partial |K|}}{\frac{\partial F}{\partial \mu}} > 0$ .  $\square$

That is, as the size of the stock market increases, there are larger rents which attract additional equity investors to the market. Hence, the announcement of different capital structures  $(K, B)$  influences the price signal's precision and thus the strength of the informational spillover. As we noted earlier, this prediction is in line with empirical evidence; Collins et al. (1987) show that stock price's informational content indeed increases with the firm's equity.

## 2.2 Debt Market

There is a unit measure of CARA bond investors, and prices are again determined by market clearing. In Section 4.3 we show our results also obtain for more general demand functions. The debt holders' return on the portfolio equals  $b_j \rho(\theta)$ , where  $\rho(\cdot)$  is the net return on debt. If the true fundamental,  $\theta$ , is greater than 0, implying that the firm is solvent, the net return is  $R$ . If the true fundamental is less than 0, debt holders get the fire-sale/liquidation value incurring



a loss  $L$ :

$$\rho(\theta) = \begin{cases} R & \text{if } \theta \geq 0 \\ -L & \text{if } \theta < 0. \end{cases}$$

Demand  $b_j^d(\cdot)$  for bonds solves the utility maximization problem conditional on the available information:

$$\begin{aligned} b_j^d(R) &= \arg \max_{b_j} \mathbb{E}[U(\rho(\theta)b_j) | p] \\ &= \arg \max_{b_j} (-\Pr(\theta \geq 0 | p)e^{-\gamma R b_j} - \Pr(\theta < 0 | p)e^{\gamma L b_j}) \\ &= \arg \max_{b_j} (-\pi_p e^{-\gamma R b_j} - (1 - \pi_p) e^{\gamma L b_j}), \end{aligned}$$

where  $\pi_p = \Pr(\theta \geq 0 | p) = \Pr(\theta \geq 0 | Z) = \Pr(Z - \alpha_z^{-1/2}\varepsilon \geq 0) = \Phi(\alpha_z^{1/2}Z)$  is the firm's survival probability.<sup>6</sup>

That is, agents rely on the stock price signal to calculate the bankruptcy probability  $1 - \pi_p$ , which depends on the unknown value of the fundamental.

Solving the first-order condition yields demand

$$b_j^d(R) = \frac{1}{\gamma(R+L)} \ln \frac{\pi_p R}{(1-\pi_p)L}. \quad (9)$$

Demand for bond decreases in risk aversion  $\gamma$ , the liquidation loss  $L$ , and the conditional bankruptcy probability  $(1 - \pi_p)$ . Increases in net interest  $R$  have an ambiguous effect as they imply higher returns on debt, on the one hand, but higher risks, on the other.

### 2.2.1 Debt Market Equilibrium

For a given supply of bonds,  $B$ , the debt markets' equilibrium condition writes:

$$\int_{j \in [0,1]} b_j^d(R) = B \quad \Leftrightarrow \quad \ln \frac{R}{L} + \ln \frac{\pi_p}{1-\pi_p} = \gamma B (R+L). \quad (10)$$

From (10) we have the implicit expression for the equilibrium return on debt:

$$\ln R = \gamma B (R+L) + \ln L - \ln \frac{\pi_p}{1-\pi_p}. \quad (11)$$

Figure 2 illustrates that the equilibrium condition (11) has two solutions  $R_1$  and  $R_2$ . Where the lower interest rate equilibrium,  $R_1$ , is stable while the high interest rate equilibrium,  $R_2$ , is

<sup>6</sup> $\Phi()$  denotes the standard normal cumulative distribution function.

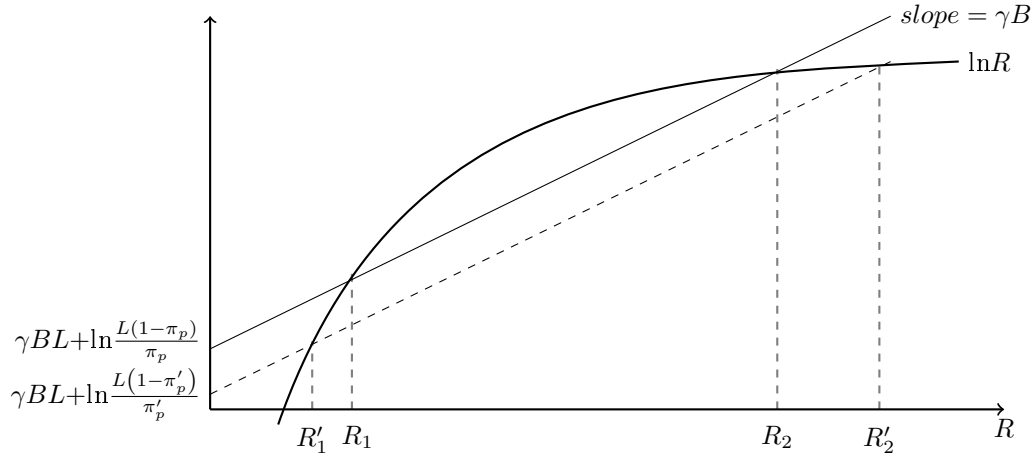


Figure 2: *Debt market equilibria: for a financially stable firm ( $\theta > 0$ ), increases in the stock price signal's precision ( $\alpha_z$ ) increase the inferred survival probability ( $\pi'_p > \pi_p$ ) and reduce the interest rate ( $R'_1 < R_1$ ) in the stable equilibrium.*

unstable.<sup>7</sup> In what follows, we focus on the stable equilibrium. Regarding this equilibrium, we immediately obtain:

**Lemma 1.** *The stable equilibrium rate of return  $R$  decreases in the firm's survival probability  $\pi_p$ , and increases in the loss  $L$ , debt supply  $B$  and risk aversion  $\gamma$ .*

## 2.2.2 Effect of the Stock Price Signal

We recall that increases in the mass of stock investors  $\mu$  increase the price signal's  $Z = \theta + \frac{1}{\sqrt{\alpha_z}}\varepsilon$  informativeness  $\alpha_z = \frac{\mu^2\alpha_x^2}{\gamma^2\sigma_\varepsilon^2}$ . Consequently, we have the following:

**Proposition 2.** *The cost of debt decreases (increases) with the mass of equity investors  $\mu$  when the fundamental  $\theta$  is positive (negative).*

*Proof.*  $\pi_p = \Phi\left(\alpha_z^{1/2}Z\right) = \Phi\left(\frac{\mu\alpha_x}{\gamma\sigma_\varepsilon}\theta + \varepsilon\right)$ , where  $\Phi(\cdot)$  is the c.d.f. of the standard normal distribution. In equilibrium we have  $\frac{\partial R}{\partial \pi} < 0$  and therefore:  $\frac{\partial R}{\partial \pi} \frac{\partial \pi}{\partial \mu} = \frac{\partial R}{\partial \pi} \phi\left(\frac{\mu\alpha_x}{\gamma\sigma_\varepsilon}\theta + \varepsilon\right) \frac{\alpha_x}{\gamma\sigma_\varepsilon}\theta \stackrel{\leq}{\geq} 0$  if  $\theta \stackrel{\geq}{\leq} 0$ .  $\square$

Figure 2 illustrates that, if  $\theta > 0$ , increases in the stock investors' mass reduces borrowing costs.

<sup>7</sup>That is, increases in the rate of return increase (reduce) demand in the low (high) interest equilibrium. Hence, the low (high) interest equilibrium is Walrasian stable (unstable). See Samuelson (1941), pp. 102-106, for a discussion of the Walrasian market mechanism.

### 3 Optimal Capital Structure

In this section, we study how the informational spill-over relates to the choice of an optimal capital structure. We analyze the general equilibrium, where the mass of equity investors and thus the informativeness of the price signal changes with the amount of equity issued, as in Section 2.1.1. In this setting, every capital structure choice implies a distinct informational environment. This endogeneity induces the firm's management to issue more equity and less debt than it would if no informational spill-over existed. That is, an optimizing firm chooses its capital structure to amplify the price-signal's precision, which reduces its expected borrowing costs.

The firm's management is assumed to be risk-neutral. Moreover, it requires external financing  $I > 0$  which it can attract by selling equity/shares  $K$  and debt  $B$ . The firm announces its ex-ante optimal capital structure  $K, B$  at  $t = 0$ , before markets open. That is, the firm chooses its capital structure subject to the anticipated informational interaction between both markets described above. Finally, the management believes that the firm will not default, that is, that  $\theta > 0$ . The expectation operator associated with the management's beliefs is denoted  $\mathbb{E}^f[\cdot]$ , where the superscript  $f$  refers to the firm.

The firm minimizes expected capital costs  $C = (\mathbb{E}^f[\theta] - \mathbb{E}^f[p])K + \mathbb{E}^f[R]B$  subject to the financing constraint  $I$  and the financial market equilibrium. In particular, the firm chooses its capital structure in anticipation of the positive relation between the market depth  $K$  and the informativeness of the price signal described in Proposition 1. Taking into account the equilibria in bond and equity market, the firm's optimization problem reads:

$$\min_{K,B} C(K, \mathbb{E}^f[p], \mathbb{E}^f[R], \mu) = \min_{K,B} \mathbb{E}^f[(\theta - p)K + RB] \quad (12)$$

subject to:

$$\begin{cases} \text{revenue constraint} & \mathbb{E}^f[p]K + B = I \\ \text{equilibrium participation} & \mu = \mu(K) \\ \text{equity market equilibrium} & p = \theta + \frac{1}{\sqrt{\alpha_z}}\varepsilon - \frac{\gamma K}{\alpha\mu}, \quad \alpha = \alpha_x + \alpha_z, \quad \alpha_z = \frac{\alpha_x \mu}{\gamma \sigma_\varepsilon} \\ \text{debt market equilibrium} & \ln R = \gamma B(R + L) + \ln \frac{L(1-\pi_p)}{\pi_p}, \end{cases} \quad (13)$$

where the conditional survival probability for the firm is  $\pi_p = \Phi\left(\frac{\alpha_x \mu}{\gamma \sigma_\varepsilon} Z\right)$ . To study (12) we

reduce it to a problem in  $K$ :

$$\min_K C(K, \mathbb{E}^f[p], \mathbb{E}^f[R], \mu), \quad \text{s.t.} \quad p = p(K, \mu), \quad R = R(I - \mathbb{E}^f[p]K, \mu), \quad \mu = \mu(K), \quad (14)$$

which yields the main result:

**Proposition 3.** *In the endogenous information setting the firm issues more equity and less debt to internalize the informational externality that stock prices exert on bond yields.*

*Proof.* The first order condition:

$$\frac{dC}{dK} = \frac{\partial C}{\partial K} + \frac{\partial C}{\partial \mu} \frac{\partial \mu}{\partial K}, \quad (15)$$

shows that the capital cost minimization problem consists of a direct effect  $\frac{\partial C}{\partial K}$ , and an indirect effect  $\frac{\partial C}{\partial \mu} \frac{\partial \mu}{\partial K}$ , which represents how the informational content of prices changes with the capital structure. In Appendix B, we characterize the properties of the direct effect,  $\frac{\partial C}{\partial K}$ , which describes the capital structure problem for any exogenously given mass of informed traders  $\mu$ . In particular, we show that, if the revenue requirement  $I$  is sufficiently high, there exist values  $K^* > 0$  and  $B^* > 0$ , for which  $\frac{\partial C}{\partial K} = 0$ ; and  $K^* > 0$  and  $B^* > 0$  represent a global cost minimum. To prove that the firm issues more equity and less debt in the endogenous information optimum, such that  $K^{**} > K^*$ , it remains to show that  $\frac{\partial C}{\partial \mu} \frac{\partial \mu}{\partial K} < 0$  at  $K^*$ . That is, if  $\frac{dC}{dK}|_{K^*} = 0 + \frac{\partial C}{\partial \mu} \frac{\partial \mu}{\partial K} < 0$ , then the firm issues more equity and less debt to internalize the informational externality. Regarding this externality, we note that

$$\frac{\partial C}{\partial \mu} \frac{\partial \mu}{\partial K} = \left( \frac{\partial C}{\partial \mathbb{E}^f[p]} - \frac{\partial C}{\partial \mathbb{E}^f[R]} \frac{\partial \mathbb{E}^f[R]}{\partial B} K \right) \frac{\partial \mathbb{E}^f[p]}{\partial \mu} \frac{\partial \mu}{\partial K} + \frac{\partial C}{\partial \mu} \frac{\partial \mu}{\partial K}.$$

Differentiation of the constraints in (13) yields a more explicit expression:

$$\frac{\partial C}{\partial \mu} \frac{\partial \mu}{\partial K} = -\mathbb{E}^f[1 + R + \frac{\partial R}{\partial B} B] K \frac{\gamma}{\alpha \mu^2} \frac{\partial \mu}{\partial K} + B \mathbb{E}^f[\frac{\partial R}{\partial \mu} \frac{\partial \mu}{\partial K}]. \quad (16)$$

From Proposition 2, Lemma 2, and Lemma 1, we know that (i)  $K^* > 0$  such that participation is increasing in  $K$ :  $\frac{\partial \mu}{\partial K} > 0$ , and (ii) the expected equilibrium rate of return on debt decreases in the price signal's precision:  $\frac{\partial R}{\partial \mu} < 0$ . Thus we can estimate the sign of (16), at the exogenous information optimum  $K^*$  where  $\frac{\partial C}{\partial K} = 0$ , as:

$$-\underbrace{(1 + \mathbb{E}^f[R + \frac{\partial R}{\partial B} B^*])}_{+} \underbrace{\frac{\gamma}{\alpha \mu^2} \frac{\partial \mu}{\partial K}}_{+} K^* + \underbrace{\mathbb{E}^f[\frac{\partial R}{\partial \mu} \frac{\partial \mu}{\partial K}]}_{-} B^* < 0, \quad (17)$$

where the negative sign in (17) is ensured if the financing requirement  $I$  is sufficiently high, such that, by Lemma 2,  $B^*$  is positive. Hence, at the exogenous information optimum we have  $\frac{dC^{**}}{dK}(K^*) < 0$ , and an increase in  $K$  from  $K^*$  towards  $K^{**}$  decreases capital costs.  $\square$

That is, (17) reflects that if the firm takes into account that every capital structure is associated with a particular information structure, it issues more equity and less debt since

a marginal increment in equity (i) increases participation and thus allows to sell the equity in place,  $K^*$ , at a higher price and (ii) reduces borrowing costs since debt holders rely more heavily on the price signal, which is on average positive since the expected fundamental is positive  $\mathbb{E}^f[\theta] > 0$ . More precisely, the first term in (17),  $(1 + \mathbb{E}^f[R + \frac{\partial R}{\partial B} B^*]) \frac{\gamma}{\alpha \mu^2} \frac{\partial \mu}{\partial K} K^*$ , reflects that increases in capital supply raise participation, which increases investor demand such that a given stock supply  $K^*$  can be sold at higher prices. The term  $\mathbb{E}^f[R + \frac{\partial R}{\partial B} B^*]$  reflects that this increase in prices allows to reduce debt, which, in turn, reduces borrowing costs on the remaining debt  $B^*$ . The second term,  $\mathbb{E}^f[\frac{\partial R}{\partial \mu} \frac{\partial \mu}{\partial K}] B^*$  is the information externality from stock price signal to bond yield which increases in the stock market's size  $K$ .

## 4 Discussion

A number of our previous assumptions were made to give a parsimonious exposition. In this section, we reflect on our assumptions and replace some of them to demonstrate the robustness of our findings. First, we discuss the assumption that the stock market rather than the bond market is the main source of public information. Second, we show that the firm's dividend  $\theta$ , and thus the survival probability  $\pi(\theta > 0|p)$ , can be derived from a consistent budget constraint. Finally, we present a more general specification for the bond market.

### 4.1 Direction of the Spill-over

Currently, the stock price is the main source of public information. In principle, it is possible to construct an alternative model where bond investors research the firm's financial health and stock investors use the equilibrium yield to infer the firm's fundamental. Alternatively, we could also study the intermediate case where both  $P$  and  $R$  carry information. While such analysis is possible, we believe that the current specification, where the stock price signal influences bond yields, is likely the most relevant one: (i) empirically we find that, even though many firms operate with a capital structure, where the value of the debt far exceeds the value of equity, it is the stock price and not the bond yield (of some reference maturity) which is published most prominently in the media: The financial data provided by Google Finance, Reuters, or Bloomberg make it straightforward to observe the latest stock prices, at the same time, it is difficult to inquire bond yields.

## 4.2 The Firm's Resource Constraint

To simplify the exposition in the main text, we treated the firm's dividend  $\theta$  as independent of the capital structure. In this section we show that the results are indeed strengthened once we add a consistent budget constraint. To do so, we assume that the firm's aggregate resources are given by  $Y$ . In turn, these resources are used to service the debt  $RB$  and to pay a dividend of  $\theta$  per share:

$$Y = K\theta + RB \quad \Leftrightarrow \quad \theta = \frac{Y}{K} - \frac{RB}{K}. \quad (18)$$

In Appendix D, we show that the informational externality, which induces firms to issue more equity and less debt is strengthened once dividends  $\theta$ , and thus the firm's survival probability  $\pi(\theta > 0|p)$ , depend on the interest rate as in (18). In particular, we find that the reduction of expected interest rates, which is associated with more precise stock prices, now also reduces the default probability  $1 - \pi(\theta(R) > 0|p)$ .

## 4.3 Alternative Bond Market

In our baseline model, bond demand is derived from a CARA utility function. However, for our result to obtain we only require that bond demand is given by a continuously differentiable function:

$$B^D = B^D(\pi(\theta > 0|p), R), \quad \frac{\partial B^D}{\partial \pi} > 0, \quad \frac{dB^D}{dR} = \frac{\partial B^D}{\partial \pi} \frac{\partial \pi}{\partial \theta} \frac{\partial \theta}{\partial R} + \frac{\partial B^D}{\partial R} > 0, \quad (19)$$

where  $\theta$  is now defined as in (18). The assumptions regarding the demand function's derivatives indicate that for every given interest rate, demand increases in the firm's survival probability. Moreover, increases in the rate of return increase bond demand. This second assumption is equivalent to the assumption that the bond market equilibrium is Walrasian stable. In a different interpretation, we assume that the increased return  $R$  outweighs the decrease ( $\frac{\partial B^D}{\partial \pi} \frac{\partial \pi}{\partial \theta} \frac{\partial \theta}{\partial R} < 0$ ) in the probability with which  $\theta > 0$ . In equilibrium, where  $B^S = B$ , we have

$$B^D(\pi, R) = B \quad \Leftrightarrow \quad R = R(\pi(\theta > 0|p), B) > 0, \quad \frac{\partial R}{\partial B} > 0, \quad \frac{\partial R}{\partial \pi} < 0. \quad (20)$$

Where the signs of  $\frac{\partial R}{\partial B}$ , and  $\frac{\partial R}{\partial \pi}$ , which are required for our main Proposition 3, follow from (19).

## 5 Conclusion

The present paper provides a simple equilibrium model in which the firm's stock price serves as a costless rating device for investors who buy its debt. The main motivation for our analysis lies in the observation that stock investors, as opposed to rating agencies, which rely on fees from the firm's they rate, have no incentive to misreport their information on the firm's financial health. That is, they do not buy stocks at inflated prices to mislead bond investors.

To study the spill-over from stock price signals to bond yields, we have assumed the perspective of a firm that minimizes its capital cost subject to the informational connection between bond and stock market. In a first step, we have shown that, for a given information structure, firms that are financially strong benefit from an informative stock price which communicates the financial strength of the firm more clearly to bond investors who rely on the stock price signal to infer the firms' default probability. In a second step, we endogenize the strength of the informational spill-over. In an economy where equity investors can choose whether to take a position in the firm's stock, increases in the amount of equity issued incentivise more stock investors to trade the firm's stock, which increases the stock price signal's precision. Hence, the strength of the informational spill-over varies with different capital structures. Consequently, the firm can, through its capital structure, internalize part of the informational externality that stock prices exert on bond yields. Firms with a strong fundamental will therefore issue more equity and less debt, than they would if the informational spill-over did not exist, to generate stock price signals which communicate its strong financial position on average more clearly to bond investors.

In one interpretation, the firm's capital cost minimization problem is simply an exposition device, which helps to illustrate how a firm's stock price interacts with its borrowing costs through its informational content. In a different interpretation, the current paper complements the literature that studies a firm's optimal capital structure in models with default and asymmetric information. It suggests the role of competitive markets as information aggregation devices as another aspect, which shapes a firm's financing decisions.

## A Ex Ante Utility and Participation

*Proof.* Agent's ex-ante expected utility is:

$$U_i^e(\mu, K) = \mathbb{E}U((\theta - p)k_i), \quad (21)$$

Using the properties of CARA utility functions and the law of iterated expectations we have:

$$U_i^e(\mu, K) = -\mathbb{E}[\mathbb{E}[\exp(-\gamma(\theta - p)k_i) | x_i, Z]] \quad (22)$$

Since  $k_i, p$  are constant conditional on  $(x_i, Z)$  and  $\theta$  is conditionally normally distributed, we can write:

$$U_i^e(\mu, K) = -\mathbb{E}\left[\exp\left(-\gamma(\mathbb{E}\theta | x_i, Z) - p)k_i + \frac{1}{2}\gamma^2 k_i^2 \text{Var}(\theta | x_i, Z)\right)\right] \quad (23)$$

Recall that  $\mathbb{E}[\theta | x_i, Z] = \theta_i$ ,  $\text{Var}(\theta | x_i, Z) = \alpha^{-1}$ , and  $k_i = k_i(x_i, Z) = \frac{\alpha}{\gamma}(\theta_i - p)$ , thus:

$$U_i^e(\mu, K) = -\mathbb{E}\left[\exp\left(-\gamma(\theta_i - p)\frac{\alpha}{\gamma}(\theta_i - p) + \frac{1}{2}\gamma^2 \frac{\alpha^2}{\gamma^2}(\theta_i - p)^2 \alpha^{-1}\right)\right] \quad (24)$$

$$= -\mathbb{E}[\exp(-\frac{\alpha}{2}(\theta_i - p)^2)] \quad (25)$$

Substituting the demand function  $\theta_i = \frac{\alpha_x}{\alpha}x_i + \frac{\alpha_z}{\alpha}Z$  and  $p = \theta + \alpha_z^{-\frac{1}{2}}\varepsilon - \frac{K\gamma}{\mu\alpha}$ , we obtain:

$$U_i^e(\mu, K) = -\mathbb{E}\exp\left(-\frac{\alpha}{2}\left(\frac{\alpha_x}{\alpha}x_i + \frac{\alpha_z}{\alpha}Z - \theta - \alpha_z^{-\frac{1}{2}}\varepsilon + \frac{K\gamma}{\mu\alpha}\right)^2\right) \quad (26)$$

Substitute  $x_i = \theta + \alpha_x^{-\frac{1}{2}}\xi_i$ ,  $Z = \theta + \alpha_z^{-\frac{1}{2}}\varepsilon$ , and recall that  $\alpha = \alpha_x + \alpha_z$  (thus  $\theta$  cancels out):

$$U_i^e(\mu, K) = -\mathbb{E}\exp\left(-\frac{1}{2\alpha}\left(\alpha_x^{\frac{1}{2}}\xi_i + \alpha_z^{\frac{1}{2}}\varepsilon - \alpha\alpha_z^{-\frac{1}{2}}\varepsilon + \frac{K\gamma}{\mu}\right)^2\right) \quad (27)$$

Note that  $\alpha_x^{\frac{1}{2}}\varepsilon - \alpha\alpha_z^{-\frac{1}{2}}\varepsilon = \alpha_z^{-\frac{1}{2}}(\alpha_x - \alpha)\varepsilon = -\alpha_z^{-\frac{1}{2}}\alpha_x\varepsilon$ :

$$U_i^e(\mu, K) = -\mathbb{E}\exp\left(-\frac{1}{2\alpha}\left(\alpha_x^{\frac{1}{2}}\xi_i - \alpha_z^{-\frac{1}{2}}\alpha_x\varepsilon + \frac{K\gamma}{\mu}\right)^2\right) \quad (28)$$

Since  $\xi_i$  and  $\varepsilon$  are independent and identically distributed as  $N(0, 1)$ , the random component in (28) is normally distributed around zero with variance  $(\alpha_x^{\frac{1}{2}})^2 + (\alpha_z^{-\frac{1}{2}}\alpha_x)^2 = \alpha_x + \alpha_x^2/\alpha_z = \frac{\alpha_x\alpha}{\alpha_z}$ .

We normalize the expression to obtain:

$$U_i^e(\mu, K) = -\mathbb{E}\exp\left(-\frac{\alpha_x}{2\alpha_z}\left(\varsigma + \frac{K\gamma}{\mu}\sqrt{\frac{\alpha_z}{\alpha_x\alpha}}\right)^2\right) \quad (29)$$

where the new random variable  $\varsigma \sim N(0, 1)$ . Denoting  $\lambda = \left(\frac{K\gamma}{\mu}\sqrt{\frac{\alpha_z}{\alpha_x\alpha}}\right)^2 = \frac{K^2\gamma^2}{\mu^2}/\left(\alpha_x + \frac{\gamma^2\sigma_\varepsilon^2}{\mu^2}\right)$  (8) we observe that  $\varsigma^2$  follows a non-central  $\chi^2$ -distribution with one degree of freedom and the non-centrality parameter  $\lambda$ . We can therefore use the moment generating function,  $\mathbb{E}e^{b\varsigma^2} =$

$\frac{e^{\lambda}}{\sqrt{1-\lambda}} = \frac{e^{\lambda}}{\sqrt{1-\lambda}} = \frac{K^2\gamma^2}{\mu^2}/\left(\alpha_x\left(1 + \frac{\alpha_x}{\alpha_z}\right)\right) = \frac{K^2\gamma^2}{\mu^2}/\left(\alpha_x\left(1 + \frac{\gamma^2\sigma_\varepsilon^2}{\alpha_x\mu^2}\right)\right) = \frac{K^2\gamma^2}{\mu^2}/\left(\alpha_x + \frac{\gamma^2\sigma_\varepsilon^2}{\mu^2}\right)$ , where we substitute  $\alpha_z = \frac{\alpha_x^2\mu^2}{\gamma^2\sigma_\varepsilon^2}$



$(1 - 2b)^{-\frac{1}{2}} e^{\frac{b\lambda}{1-2b}}$  for the non-central  $\chi^2$  distribution,<sup>9</sup> with  $b = -\frac{\alpha_x}{2\alpha_z} = -\frac{1}{2} \frac{\gamma^2 \sigma_\varepsilon^2}{\alpha_x \mu^2}$ , to rewrite the utility as:

$$U_i^e(\mu, K) = -(1 - 2b)^{-\frac{1}{2}} e^{\frac{\lambda b}{1-2b}}. \quad (30)$$

The derivative w.r.t.  $\mu$  (denote  $b' = \frac{\partial b}{\partial \mu}$ ,  $\lambda' = \frac{\partial \lambda}{\partial \mu}$ ):

$$\begin{aligned} \frac{\partial U_i^e(\mu, K)}{\partial \mu} &= -(1 - 2b)^{-\frac{3}{2}} b' e^{\frac{\lambda b}{1-2b}} - (1 - 2b)^{-\frac{1}{2}} e^{\frac{\lambda b}{1-2b}} \left( \frac{(\lambda b)'(1 - 2b) + \lambda b \times 2b'}{(1 - 2b)^2} \right) \\ &= -(1 - 2b)^{-\frac{3}{2}} e^{\frac{\lambda b}{1-2b}} \left( b' \left( 1 + \frac{\lambda}{1 - 2b} \right) + \lambda' b \right) < 0 \end{aligned}$$

since  $b' > 0$ ,  $b < 0$ ,  $\lambda > 0$ ,  $\lambda' < 0$ <sup>10</sup>. Thus, ex-ante utility decreases in  $\mu$ .

Regarding the derivative w.r.t.  $|K|$  (now, denote  $b' = \frac{\partial b}{\partial K} = 0$ ,  $\lambda' = \frac{\partial \lambda}{\partial K^2}$ ) we have:

$$\frac{\partial U_i^e(\mu, K)}{\partial K^2} = -(1 - 2b)^{-\frac{3}{2}} e^{\frac{\lambda b}{1-2b}} (\lambda' b) > 0,$$

since  $b < 0$  and  $\lambda' > 0$ . Thus, ex-ante utility increases in  $|K|$ . □

## B Optimal Capital Structure: The “Direct Effect”

In this appendix, we characterize the direct effect  $\frac{\partial C}{\partial K}$ . The firm minimizes expected capital costs

$$\min_{K, B} C(K, \mathbb{E}^f[p], \mathbb{E}^f[R], \mu) = \min_{K, B} (\mathbb{E}^f[\theta] - \mathbb{E}^f[p])K + \mathbb{E}^f[R]B \quad (31)$$

subject to the market system:

$$\left\{ \begin{array}{ll} \text{revenue constraint} & \mathbb{E}^f[p]K + B = I \\ \text{equilibrium participation} & \mu = \mu(K) \\ \text{equity market equilibrium} & p = Z - \frac{\gamma K}{\alpha \mu}, \\ \text{debt market equilibrium} & \ln R = \gamma B(R + L) + \ln \frac{L(1 - \pi_p)}{\pi_p}, \end{array} \right. \quad (32)$$

---

<sup>9</sup>The moment generating function can be derived as follows:  $\mathbb{E} e^{bz^2} = \frac{1}{\sqrt{2\pi}} \int e^{bz^2} e^{-\frac{(z-\sqrt{\lambda})^2}{2}} dz = \frac{1}{\sqrt{2\pi}} \int e^{-\frac{(1-2b)z^2 - 2\sqrt{\lambda}z + \lambda}{2}} dz = \frac{1}{\sqrt{2\pi}} \int e^{-\frac{(\sqrt{1-2b}z - \sqrt{\frac{\lambda}{1-2b}})^2 - \frac{\lambda}{1-2b} + \lambda}{2}} dz = \frac{1}{\sqrt{1-2b}\sqrt{2\pi}} \int e^{\frac{\lambda b}{1-2b}} e^{-\frac{(\sqrt{1-2b}z - \sqrt{\frac{\lambda}{1-2b}})^2}{2}} d(\sqrt{1-2b}z) = \frac{1}{\sqrt{1-2b}} e^{\frac{\lambda b}{1-2b}}$ , where  $\lambda$  is the non-centrality parameter.  
<sup>10</sup> $\frac{\partial \lambda}{\partial \mu} = -2 \frac{K^2 \gamma^2}{\mu^3} \left( \alpha_x + \frac{\gamma^2 \sigma_\varepsilon^2}{\mu^2} \right)^{-1} + 2 \frac{K^2 \gamma^2}{\mu^2} \left( \alpha_x + \frac{\gamma^2 \sigma_\varepsilon^2}{\mu^2} \right)^{-2} \frac{\gamma^2 \sigma_\varepsilon^2}{\mu^3} = 2 \frac{K^2 \gamma^2}{\mu^3} \left( \alpha_x + \frac{\gamma^2 \sigma_\varepsilon^2}{\mu^2} \right)^{-2} \left( -\alpha_x - \frac{\gamma^2 \sigma_\varepsilon^2}{\mu^2} + \frac{\gamma^2 \sigma_\varepsilon^2}{\mu^2} \right) = -2 \frac{\alpha_x K^2 \gamma^2}{\mu^3} \left( \alpha_x + \frac{\gamma^2 \sigma_\varepsilon^2}{\mu^2} \right)^{-2} < 0$ .

where the conditional survival probability for the firm is  $\pi_p = \Phi\left(\frac{\mu\alpha_\varepsilon}{\gamma\sigma_\varepsilon}Z\right)$ . By substituting for the level of debt  $B = I - \mathbb{E}^f[p]K$ , Problem (31) becomes a problem in  $K$  alone:

$$\min_K C(K, \mathbb{E}^f[p], \mathbb{E}^f[R], \mu), \quad s.t. \quad p = p(K, \mu), \quad R = R\left(\underbrace{I - \mathbb{E}^f[p]K}_B, \mu\right). \quad (33)$$

Where  $p(K, \mu)$  is the solution for the equity market equilibrium (6), and  $R(B, \mu)$  is the stable bond market equilibrium (10). The first-order condition for the cost minimizing  $K^*$  is therefore:<sup>11</sup>

$$\frac{\partial C}{\partial K} = 2\frac{\gamma K}{\alpha\mu}\mathbb{E}^f\left[1 + R + \frac{\partial R}{\partial B}B\right] - \mathbb{E}^f[\theta]\mathbb{E}^f\left[R + \frac{\partial R}{\partial B}B\right] = 0. \quad (34)$$

**Lemma 2.** *For any positive revenue  $I$ , the optimal level of equity  $K^*$  is positive. If the financing requirement  $I$  is sufficiently large, the firm always relies on both markets to raise funds and we have  $B^* > 0$  and  $K^* > 0$  at the optimum.*

*Proof.* See Appendix C □

Lemma 2 leaves open whether there are corner solutions. To complete the characterization of the minimum, we note that all possible minima (local and global) are finite:

**Lemma 3.** *The capital cost minimization problem has a finite global minimum  $K^*, B^*$ .*

*Proof.* See Appendix C □

## C Proof of Lemmas 2 and 3

We recall the revenue constraint  $\mathbb{E}^f[p]K + B = I$ , the equity market equilibrium  $p = Z - \frac{\gamma K}{\mu\alpha}$ , and the debt market equilibrium  $\ln R = \gamma B(R + L) + \ln \frac{L(1-\pi_p)}{\pi_p}$  to rewrite

$$\frac{\partial C}{\partial K} = \mathbb{E}^f[\theta] - \mathbb{E}^f[p] + \left(\frac{\partial C}{\partial \mathbb{E}^f[p]} - \frac{\partial C}{\partial \mathbb{E}^f[R]} \frac{\partial \mathbb{E}^f[R]}{\partial B} K\right) \frac{\partial \mathbb{E}^f[p]}{\partial K} - \frac{\partial C}{\partial \mathbb{E}^f[R]} \frac{\partial \mathbb{E}^f[R]}{\partial B} \mathbb{E}^f[p] = 0 \quad (35)$$

as:

$$\begin{aligned} &= \mathbb{E}^f[\theta] - \mathbb{E}^f[p] - \mathbb{E}^f[R]\mathbb{E}^f[p] + \left(-K - \mathbb{E}^f[R]K - B \frac{\partial \mathbb{E}^f[R]}{\partial B} K\right) \left(-\frac{\gamma}{\alpha\mu}\right) - B \frac{\partial \mathbb{E}^f[R]}{\partial B} \mathbb{E}^f[p] \\ &= \mathbb{E}^f[\theta] - \mathbb{E}^f[p](1 + R) + \frac{\gamma}{\alpha\mu} \left(1 + R + \frac{\partial \mathbb{E}^f[R]}{\partial B} B\right) K - B \frac{\partial \mathbb{E}^f[R]}{\partial B} \mathbb{E}^f[p] \end{aligned}$$

---

<sup>11</sup>See (35) in Appendix C for the differentiation of the constraint set.

$$\begin{aligned}
&= \mathbb{E}^f[\theta] + \left( \frac{\gamma}{\alpha\mu} K - \mathbb{E}^f[p] \right) \left( 1 + R + \frac{\partial \mathbb{E}^f[R]}{\partial B} B \right) \\
&= \mathbb{E}^f[\theta] + \left( 2 \frac{\gamma}{\alpha\mu} K - \mathbb{E}^f[\theta] \right) \left( 1 + \mathbb{E}^f[R] + \frac{\partial \mathbb{E}^f[R]}{\partial B} B \right) = 0.
\end{aligned} \tag{36}$$

**Proof of Lemma 2** First, to prove that  $K^*$  is positive if  $I > 0$ , we show that the first order condition (36) is violated for any  $K \leq 0$ . Namely, we show that the derivative is strictly negative at  $K \leq 0$ , that is, issuing more equity reduces capital costs.

We rearrange the terms of (36) as follows:

$$\frac{dC}{dK} = 2 \frac{\gamma}{\alpha\mu} K \mathbb{E}^f \left[ 1 + R + \frac{\partial R}{\partial B} B \right] - \mathbb{E}^f[\theta] \mathbb{E}^f \left[ R + \frac{\partial R}{\partial B} B \right] = 0. \tag{37}$$

The first term  $2 \frac{\gamma}{\alpha\mu} K \mathbb{E}^f \left[ 1 + R + \frac{\partial R}{\partial B} B \right]$  is non-positive for  $K \leq 0$  since we know (i) from Lemma 1 (debt market equilibrium) that  $R > 0$  and  $\frac{\partial R}{\partial B} > 0$ , and, (ii) the revenue constraint implies that  $B \geq I > 0$  for  $K \leq 0$ . The second term of (37),  $\mathbb{E}^f[\theta] \mathbb{E}^f \left[ R + \frac{\partial R}{\partial B} B \right]$ , is positive as long as  $K \leq 0$  since  $\mathbb{E}^f[\theta] > 0$ . Hence,  $\frac{dC}{dK}$  is negative for  $K \leq 0$ , implying that the cost minimizing  $K^*$  is positive.

Next, we show that  $B^* > 0$ , if the financing requirement  $I$  is sufficiently high. To show this, we note that it follows from the revenue constraint  $I = \mathbb{E}^f[p]K + B$ , the price function  $\mathbb{E}^f[p] = \mathbb{E}^f[\theta] - \frac{\gamma}{\alpha\mu} K$ , and the assumption  $\mathbb{E}^f[\theta] > 0$  that if  $B \leq 0$ , then  $\mathbb{E}^f[p] > 0$  and  $K > 0$ .<sup>12</sup> It therefore follows from (37) that we have  $\frac{dC}{dK} |_{B=0, K=\frac{I}{E^f[p]}} = 2 \frac{\gamma}{\alpha\mu} \frac{I}{\mathbb{E}^f[p]} \mathbb{E}^f[1 + R] - \mathbb{E}^f[\theta] \mathbb{E}^f[R]$ . One can now show that for every given expectation  $\mathbb{E}^f[\theta] > 0$ , a sufficiently large financing requirement  $I$  ensures  $\frac{dC}{dK} |_{B=0, K=\frac{I}{E^f[p]}} > 0$ . That is, a reduction in  $K$ , which implies an increase in  $B$ , reduces capital costs such that  $B^* > 0$ .

**Proof of Lemma 3** We use the revenue constraint to eliminate  $B$  from (36):

$$\begin{aligned}
&= \mathbb{E}^f[\theta] + \left( 2 \frac{\gamma}{\alpha\mu} K - \mathbb{E}^f[\theta] \right) \left( 1 + \mathbb{E}^f \left[ R + \frac{\partial R}{\partial B} \right] (I - K \mathbb{E}^f[p]) \right) \\
&= \mathbb{E}^f[\theta] + \left( 2 \frac{\gamma}{\alpha\mu} K - \mathbb{E}^f[\theta] \right) \left( 1 + \mathbb{E}^f \left[ R + \frac{\partial R}{\partial B} \right] (I - K (\mathbb{E}^f[\theta] - \frac{\gamma}{\alpha\mu} K)) \right).
\end{aligned} \tag{38}$$

---

<sup>12</sup>By contradiction, we find that an allocation where  $\mathbb{E}^f[p] < 0$  and  $K < 0$ , such that  $I = \mathbb{E}^f[p]K > 0$ , is impossible since we have assumed that  $\mathbb{E}^f[\theta] > 0$  such that  $\mathbb{E}^f[p] = \mathbb{E}^f[\theta] - \frac{\gamma K}{\alpha} > 0$  for  $K < 0$  which contradicts the initial assumption that  $\mathbb{E}^f[p] > 0$ .

In equation (38),  $K^3$  is the largest exponent, and it has a non-negative coefficient (38) is therefore positive as  $K \rightarrow +\infty$ .<sup>13</sup> Moreover, we know from Lemma 2 that (38) is negative for  $K \leq 0$ . The continuity of (38) then ensures that it must equal 0 at (at least) one positive finite  $K^*$ , which is a solution to the first order condition.

## D Informational Externality with Budget Constraint

In this appendix we derive the informational externality for the modified specifications given in Sections 4.2-4.3. The capital cost problem:

$$\min_{K,B} C = \min_{K,B} \mathbb{E}^f \left[ (\theta - p)K + RB \right] \quad s.t. \quad I = pK + B \quad (39)$$

is equivalent to

$$\min_{K,B} C = \min_{K,B} \mathbb{E}^f \left[ (\theta - \theta - \alpha_z^{-\frac{1}{2}} \varepsilon + \frac{\gamma}{\alpha \mu} K)K + RB \right] \quad s.t. \quad I = pK + B \quad (40)$$

$$\min_{K,B} C = \min_{K,B} \mathbb{E}^f \left[ \frac{\gamma}{\alpha \mu} K^2 + RB \right] \quad s.t. \quad I = pK + B. \quad (41)$$

From (41), we calculate:

$$\frac{dC}{dK} = \mathbb{E}^f \left[ \frac{\partial C}{\partial K} + \frac{\partial C}{\partial \mu} \frac{\partial \mu}{\partial K} \right] \quad (42)$$

where the externality  $\frac{\partial C}{\partial \mu} \frac{\partial \mu}{\partial K}$  induces firms to issue more equity and less debt since:<sup>14</sup>

$$\begin{aligned} \frac{\partial C}{\partial \mu} \frac{\partial \mu}{\partial K} = & \mathbb{E}^f \left[ \left[ -\frac{\gamma}{\alpha \mu^2} K^2 (1 + R + \frac{\partial R}{\partial B} B) - \frac{\partial R}{\partial \mu} B \right] \frac{\partial \mu}{\partial K} \right. \\ & \left. - (R + \frac{\partial R}{\partial B} B) \frac{\partial \theta}{\partial \mu} \frac{\partial \mu}{\partial K} K \right] < 0. \end{aligned} \quad (43)$$

The first term  $\mathbb{E}^f \left[ -\frac{\gamma}{\alpha \mu^2} K^2 (1 + R + \frac{\partial R}{\partial B} B) - \frac{\partial R}{\partial \mu} B \right] \frac{\partial \mu}{\partial K} < 0$  is identical to the effect (17) from the baseline model. The second term,  $\mathbb{E}^f \left[ - (R + \frac{\partial R}{\partial B} B) \frac{\partial \theta}{\partial \mu} \frac{\partial \mu}{\partial K} K \right] < 0$ , originates from the budget constraint (18). It is negative since we have shown earlier that  $R > 0$ ,  $\frac{\partial R}{\partial B} > 0$ ,  $\frac{\partial \mu}{\partial K} > 0$ , and, as discussed in Proposition 2,  $\mathbb{E} \left[ \frac{\partial \theta}{\partial \mu} \right] = \mathbb{E} \left[ -\frac{\frac{\partial R}{\partial \mu} B}{K} \right] > 0$  since  $\mathbb{E}^f[\theta] > 0$ . Put differently, the reduction in borrowing costs allows to increase dividends, which increases the price at which

<sup>13</sup>If  $2 \left( \frac{\gamma}{\alpha} \right)^2 \frac{\partial R}{\partial B} = 0$  then  $K$  has the largest exponent.

<sup>14</sup>Note that  $\mathbb{E}^f[R] = R(\pi, B) = R(\pi(\mu(K)), I - \theta K + cK^2)$ . Participation  $\mu$  is once again increasing in  $K$ . The proof that information participation increases in  $K$  is parallel to that in Appendix A (once we note that  $R = R(p)$  is known once stocks are traded at price  $p$ ).

shares sell  $\frac{\partial \theta}{\partial \mu} \frac{\partial \mu}{\partial K} K > 0$ . In turn, the firm can sell fewer bonds, which reduces interest expenses  $-R$ . Finally, the reduction in borrowing reduces interest costs on the outstanding debt  $-\frac{\partial R}{\partial B} B$ .

Adding a budget constraint, (18), therefore amplifies the conclusion that the firm issues more equity to internalize the informational externality that stock prices have on bond yields.

## References

- Admati, A. R., DeMarzo, P. M., Hellwig, M. F., and Pfleiderer, P. (2010). Fallacies, irrelevant facts, and myths in the discussion of capital regulation: Why bank equity is not expensive. *Preprint Max Planck Institute for Research on Collective Goods*, (42):1–60. 2
- Albagli, E., Hellwig, C., and Tsyvinski, A. (2011). Information aggregation, investment, and managerial incentives. *NBER Working Paper*, (17330):1–36. 2
- Albuquerque, R. and Miao, J. (2014). Advance information and asset prices. *Journal of Economic Theory*, 149:236–275. 1
- Angeletos, G.-M. and Werning, I. (2006). Crises and prices: Information aggregation, multiplicity, and volatility. *American Economic Review*, 96(5):1720–1736. 1, 2.1
- Calomiris, C. W. and Kahn, C. M. (1991). The role of demandable debt in structuring optimal banking arrangements. *American Economic Review*, 81(3):497–513. 2
- Collins, D. W., Kothari, S. P., and Rayburn, J. D. (1987). Firm size and the informational content of prices with respect to earnings. *Journal of Accounting and Economics*, 9:111–138. 1, 2.1.1
- Green, J. R. (1975). Information efficiency and equilibrium. *HIER Working Paper*, (284):1–29. 1
- Grossman, S. and Stiglitz, J. (1976). Information and competitive price systems. *American Economic Review Papers and Proceedings*, 66(2):246–253. 1, 2.1
- Grossman, S. and Stiglitz, J. (1980). On the impossibility of informationally efficient markets. *American Economic Review*, 70(3):393–408. 1, 2.1
- Harris, M. and Raviv, A. (1990). Capital structure and the informational role of debt. *Journal of Finance*, 45(2):321–349. 1
- Harris, M. and Raviv, A. (1991). The theory of capital structure. *Journal of Finance*, 46(1):297–355. 1, 1

- Hellwig, M. (1980). On the aggregation of information in competitive markets. *Journal of Economic Theory*, 22:477–498. 1, 2.1
- Raiffa, H. and Schlaifer, R. (2000). *Applied Statistical Decision Theory*. Wiley Classics Library. 5
- Samuelson, P. A. (1941). The stability of equilibrium: Comparative statics and dynamics. *Econometrica*, 9(2):97–120. 7
- Vives, X. (2008). *Information and Learning in Markets*. Princeton University Press. 1, 2.1