Automation and Unemployment: Help is on the Way

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Abstract

This paper presents a model of technical change that combines two lines of research together. It is a task based model, in which automation turns labor tasks to mechanized ones, and there is also a continuous addition of new labor tasks, as in the expanding variety literature. We impose three simple restrictions on the model. The first is that all new tasks are adopted. The second is that all new automations are adopted and the third is that the share of labor does not converge to zero in the long run. We show that these restrictions imply that unemployment due to automation is expected to converge to zero over time.

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Automation and Unemployment: Help is on the Way

1. Introduction

This paper examines a theoretical model of the process of automation, or of machines that replace workers in various tasks. The paper focuses on the temporary effects of this process of automation on labor, mainly on unemployment. It is obvious that if automation replaces human labor in various tasks, it increases unemployment, as the replaced workers lose their jobs for a period of time, during which they need to search for another job. This is one of the short-run costs of the long-run process of economic growth.

But the paper shows that the relationship between the long-run process and the short-run effect is actually more complex. If we impose certain restrictions on the model, to make it fit the basic stylized facts of long-run economic growth, the model has a surprising result. The rate of unemployment, which is caused by automation, declines over time and converges to zero.

As described above the paper examines automation in a model of production by tasks, where more and more tasks human labor is replaced by machines. In order to make the model more realistic and to fit some basic facts of economic growth, like a positive value to the Solow residual, we also assume that the set of tasks increases continuously. That assumption actually combines the model of automation with a model of expanding variety. We then add to this very general model three plausible working assumptions that impose three additional restrictions on the model.

The first working assumption is that all the new tasks created are indeed adopted by producers. This working assumption leads to the restriction that the elasticity of substitution between all tasks must be greater than 1. The second working assumption is that all the new automats are indeed adopted, namely that wages are sufficiently high to ensure that producers prefer buying the new machine over hiring workers. This working assumption leads to another restriction on the rate of creation of new jobs. These two assumptions are very plausible in a model of automation and expanding variety.

The third and most crucial assumption, which is actually an assumption on the future, is that the share of labor in output is bounded below by some positive number, so that it does not converge to zero. Many economic studies, the most famous is Kaldor

(1961), have found that the share of labor in output is fairly stable, both across countries and over time, around an average of 2/3. In recent decades we observe a decline of the share of labor, but it is far from going down to zero. This restriction is therefore fully in line with our accumulated knowledge of the empirics of modern economic growth, but we discuss this assumption thoroughly later in the paper.

These three additional assumptions lead to the results of the paper and mainly to the result that unemployment due to automation should converge to zero. We can try and explain this result in the following way. Wages depend on how many sectors produce by labor. If there are more such sectors, less workers are employed in each sector, so their marginal productivity is higher and their wage is higher as well. Hence, to ensure adoption of new automation technologies, wages need to be sufficiently high, which requires that creation of new jobs is faster than automation. But that means that automation shifts sectors from production by labor to production by machines at a lower and lower rate. Since this ratio between the number of automated sectors and the total number of labor sectors determines the rate of unemployment due to automation, it converges to zero.

The process of automation, or more broadly replacing human workers by machines, has captivated public debate since the beginning of the industrial revolution. Workers feared that mechanization will drive them out of their jobs, will increase unemployment and will widen poverty and misery to large parts of society. The issue was dormant most of the time, but exploded once in a while, especially during technological breakthroughs.

The first time fear of mechanization appeared was during the early days of the industrial revolution, in the rebellion of the Luddites in 1811-1817. They were mainly artisans, who viewed in awe how the mills of the textile industry threatened their economic existence. They formed a resistance to mechanization and were finally oppressed by a large British army sent to squelch this rebellion. Interestingly, the famous breaking of machines by Luddites was not their main activity.

This issue came up again during the 1920s, a period of rapid technical change as well. On February 26, 1928, Evan Clark, a reporter in the New York Times, published an article titled “March of the Machine Makes Idle Hands.” In this article Clark claimed: “It begins to look as if machines had come into conflict with men – as if the onward march of machines into every corner of our industrial life had driven men out of the factory and into the ranks of the unemployed." At the time the US rate of unemployment was only 4.2 percent, but the Great Recession started within less than two years.

Recently, the issue came up again to public attention following the financial crisis of 2008 and the great recession that followed it. We live in a period of rapid technical change, mainly in computers, information technology (IT), robots and artificial intelligence (AI). Workers find it hard to adjust to this technological revolution, where the main effect is a significant widening of inequality.

The economic literature on automation and its effects has increased significantly lately. The following survey focuses only on two lines of research, to which this paper belongs, one is analysis of automation using task-based models, and the other line studies the effect of automation on unemployment.

Task-based models study economies in which final output is produced by many tasks, where each can be performed either by labor or by a machine, once it is invented. The first such model is Champernowne (1961), which supplies micro-foundations to the aggregate production function. Zeira (1998) embeds such a model in a framework of general equilibrium and economic growth and reaches two main results. The first is that adoption of new technologies requires that wages be sufficiently high, otherwise producers prefer workers over machines. The second is that despite lower demand for labor in mechanized tasks, demand for labor in the remaining tasks increases, since working with more capital raises marginal productivity of such labor. This raises wages and enables adoption of more automation. Zeira (1998) also has two shortcomings, as it implies that the Solow Residual is zero and that the share of labor in output goes to zero in the long-run, which both contrast well-known stylized facts. This paper shows that one way to remove these implications is to assume that there is a continuous addition of new labor tasks, namely to combine the task-based model with the expanding variety model.

This line of research, of task-based production and automation is followed in many ways. Nakamura and Nakamura (2008) and Nakamura (2009) provide a micro-foundation to CES production functions via automation and tasks. Acemoglu (2010) generalizes this line of research to models with 'labor saving technologies.' Perretto and Seater (2013) examine a version of the theory without tasks. Zuletta (2008, 2015) tries to consolidate the result of decreasing share of labor with the empirics. Recently, Aghion, Jones and Jones

(2017) also offer a variation of the basic task-based model, but unlike this paper they do not combine it with the expanding variety model. A paper that combines expanding variety with automation is Hémous and Olsen (2018), but it differ significantly from our model. It also focuses mainly on the effect of skill, while our paper focuses on unemployment. They also assume that automation replaces only unskilled labor, which is different from our assumption in Section 8.

The second line of research examines the short-run effects of automation, mainly unemployment, in addition to the long-run effects. Hrdy (2017) surveys this literature. Cortes, Jaimovich and Siu (2017) relate this issue to the jobless recovery, as experienced after the great recession. Other recent contributions are Autor (2015) and Sachs (2017). Nakamura and Nakamura (2018) examine the effect of automation on frictional unemployment. Most relevant to this paper is a series of papers by Acemoglu and Restrepo (2017a, 2017b, 2018a, 2018b) that examine the labor market effects of automation in a task-based model.[1] Similar to this paper they assume that new labor tasks are continuously added, but they differ by assuming also that old tasks are destroyed as new ones arrive. We show in Section 7, that this destruction does not fit the restrictions on our model, and that is why we reach different results on automation unemployment. The Acemoglu and Restrepo papers also differ from ours in assuming that labor productivity increases with tasks. We show in an appendix that this cannot change our main result, that unemployment due to automation converges to zero.

The paper is structured as follows. Section 2 presents the model, while Section 3 describes the derivation of the equilibrium. Section 4 analyzes economic growth in the model. Section 5 examines the size of unemployment and how it evolves over time. Section 6 examines the case of a balanced growth path. Section 7 examines some potential extensions of the model, like making better machines and eliminating tasks. Section 8 presents another extension to the model that introduces different skill levels and enables us to analyze a version of skill-biased technical change. Section 9 summarizes the paper. The Appendix presents three extensions to the main model that do not affect its main result.

[1] Acemoglu and Restrepo (2018b) is the basic model, Acemoglu and Restrepo (2017b) extend it to demography and Acemoglu and Restrepo (2017a) examines it empirically. Acemoglu and Restrepo (2018a) presents a less technical summary of their work in the area.

2. The Model

Consider an economy that produces a single final good, which is used both for consumption and for investment. In period t the final good is produced by a continuum of intermediate goods $j \in [0, T_t]$ in the following way:

$$(1) \qquad Y_t = \left[\int_0^{T_t} x_t(j)^{\theta} dj \right]^{\frac{1}{\theta}}.$$

The quantity Y_t is final output, while $x_t(j)$ is the quantity of the intermediate good j. The intermediate goods can be thought of as tasks as well. The parameter θ is smaller than 1.

Each intermediate good can be produced by two alternative technologies, by human labor or by a machine, if such a machine has already been invented. Machines can be thought of as robots or automata. If produced by labor, a unit of good j is produced by one unit of labor. It is then also called job j.[2] If intermediate good j is produced by machines, that replace labor for this task, each machine costs $k(j)$ units of capital and it produces one unit of the intermediate good. We assume that the function k is increasing. We also assume that capital has to be invested one period ahead and we assume for the sake of simplicity that it fully depreciates within one period, so that the rate of depreciation of capital is 1.

We assume that automation technologies are available for the following set of intermediate goods $j \in [0, M_t]$, where M_t is the technology frontier and it grows over time: $M_t \geq M_{t-1}$ for all t . We assume that as it grows, T_t increases as well, as there are new tasks created in order to build and take care of the new machines in use. Hence, $T_t \geq T_{t-1}$ for all t. Later, we add further restrictions on θ, on the function k, and on the growth of M and T. These restrictions are required for the equilibrium to satisfy the assumptions we mention in the introduction, that all new tasks are used, that all new automation is adopted, and that the share of labor does not diminish to zero.

Agents in this economy supply one unit of labor in each period and work in specific jobs, or tasks. We assume that the overall population is 1. If a task j becomes automated in period t, all those who worked in this sector lose their jobs. During period t they search for

[2] Appendix B deals with the case of rising productivity of labor over tasks.

a new job and we assume that from period t + 1 on they work in another job. Hence, the number of unemployed in period t due to loss of job j, is the number of people who used to work in that task in the previous period, $l_{t-1}(j)$. In the benchmark simple case we present here, the unemployment due to automation is the only type of unemployment. Appendix C examines the model with an additional type of unemployment, of mismatch between workers and tasks.[3]

This model identifies tasks and jobs, although in the real world, a job might involve a number of tasks. For example, a taxi driver drives a car, makes commitments to passengers, and collects fees. Hence, if self-driving cars become common, taxi drivers might lose their jobs due to automation, although they can still do the other tasks involved with taxi driving. Hence, our identification of tasks and job is a reasonable simplification.

We denote the wage rate in the economy at period t by w_t. We further assume that the economy is small and open. The final good is tradable, while the intermediate goods are assumed to be non-tradable. The economy is open to capital mobility and the world interest rate is constant over time and is equal to r. We denote the sum of the interest rate and the rate of depreciation by $R = 1 + r$. All markets are perfectly competitive and expectations are rational.

3. The Equilibrium

3.1 First Order Conditions of Final Production

We assume that the price of the final good is the numeraire and the price of intermediate good j in period t is $p_t(j)$. The profits of producers of the final good are described by:

$$profits = \left[\int_0^{T_t} x_t(j)^{\theta} dj \right]^{\frac{1}{\theta}} - \int_0^{T_t} p_t(j) x_t(j) dj.$$

Firms decide first how many intermediate goods to use, or how many tasks to apply. If we assume that all the new tasks are applied in production, it means that the following first order condition must hold for all $T < T_t$:

[3] Nakamura and Nakamura (2018) examine the diverse effects of automation on mismatch unemployment.

(2) $$\frac{1}{\theta}Y_t^{1-\theta}x_t(T)^{\theta} - p_t(T)x_t(T) > 0.$$

Hence, if $\theta \le 0$ this FOC cannot hold. This means that our first working assumption, namely that all the new tasks are used in production by the profit maximizing producers, leads to our first additional restriction:

<u>Restriction 1:</u> The coefficient θ is positive: $\theta > 0$.

Due to Restriction 1 the elasticity of substitution between tasks $1/(1-\theta)$ is higher than 1.

The second FOC is with respect to the quantity used of each intermediate good, $x_t(j)$, and it states that:

(3) $$p_t(j) = \left[\frac{Y_t}{x_t(j)}\right]^{1-\theta}.$$

Note, that this FOC together with restriction 1, imply that condition (2) holds for all *T*.

From the FOC (3) we get that the demand for the intermediate good is:

$$x_t(j) = Y_t \cdot p_t(j)^{\frac{-1}{1-\theta}}.$$

Substituting this demand in the production function (1), we get the following equilibrium condition in the market for goods:

(4) $$\int_0^{T_t} p_t(j)^{\frac{-\theta}{1-\theta}} dj = 1.$$

<u>3.2 Prices of Intermediate goods</u>

Consider a firm producing an intermediate *j* with machines. Such a firm produces a quantity *x*, sells it at a price $p_t(j)$ and its profits are:

$$p_t(j)x - Rk(j)x.$$

Due to perfect competition and to free entry, profits must be zero. Hence:

(5) $$p_t(j) = p(j) = Rk(j).$$

Consider next a firm that produces an intermediate good by labor. Let the size of the group of workers in the firm who remain from the previous period be denoted by *l*. The firm receives a number of new workers, who search for a job, which is denoted by z. Hence, the profit of the firm at the present is:

$$p_t(j)\left[l+z(1-q)\right]-w_t\left[l+z(1-q)\right].$$

Due to perfect competition and free entry, the price of such an intermediate good is equal to the cost of labor:

(6) $$p_t(j)=w_t.$$

<u>3.3 Determination of Wages</u>

Equations (5) and (6) describe the supply prices of the goods produced by machines and by labor. We substitute them in the equilibrium condition in the goods market (4) and get:

(7) $$R^{\frac{-\theta}{1-\theta}}\int_0^{M_t} k(j)^{\frac{-\theta}{1-\theta}}dj+\int_{M_t}^{T_t} w_t^{\frac{-\theta}{1-\theta}}dj=1.$$

To analyze how equation (7) determines the wage rate, note that the first addend on the left hand side of (7) is a function of M_t. Denote this function by φ:

$$\varphi(M)=R^{\frac{-\theta}{1-\theta}}\int_0^{M} k(j)^{\frac{-\theta}{1-\theta}}dj.$$

It is easy to see that the function φ is increasing and concave. Equation (7) also implies that in order for wages to be positive, the function φ should satisfy for each M_t:

(8) $$\varphi(M_t)<1.$$

Note, that the process of automation can follow two cases. The first and the most important and interesting one is that M grows unboundedly: $M_t \xrightarrow[t\to\infty]{} \infty$. In that case assumption (8) means that the function φ is bounded from above by 1. It also implies that $k(j)$ is not only increasing in j, but that it grows unboundedly with j. The second case is that M is bounded: $M_t \le M^*$ for all t. In this case of bounded automation, assumption (8) means that $\varphi(M^*)<1$. In the following analysis we refer mainly to the case of unbounded automation, as it is more interesting, but we include the discussion on bounded automation as well, for the sake of completeness.

Given (8), we derive from the equilibrium condition (7) the following equilibrium value of the wage rate:

(9) $$w_t = \left[\frac{T_t - M_t}{1-\varphi(M_t)}\right]^{\frac{1-\theta}{\theta}}.$$

3.4 The Share of Labor

In this sub-section we calculate the share of labor in output in this economy. From the FOC (3) we get for each good that is produced by labor:

$$Y_t = x_t(j) w_t^{\frac{1}{1-\theta}} = \frac{E_t}{T_t - M_t} w_t^{\frac{1}{1-\theta}}.$$

The variable E_t is the amount of employment in period *t*. This equation is based on the result that the equilibrium allocation of labor between the various intermediate goods between *M* and *T* is equal, since all sectors face the same wage and the same price. From this equation we get that the share of labor in output is:

$$\frac{w_t E_t}{Y_t} = \left(T_t - M_t\right) w_t^{\frac{-\theta}{1-\theta}}.$$

Substituting the equilibrium wage from equation (9) we get that the equilibrium share of labor in output is:

(10) $$\frac{w_t E_t}{Y_t} = 1-\varphi(M_t).$$

In a similar way it can be shown that the share of capital in output is $\varphi(M_t)$.

Equation (10) implies that as mechanization *M* increases, the share of labor in output declines, but by less and less due to the concavity of φ. Furthermore, the declining share of labor is bounded below by 0, as assumed above. We next introduce another working assumption, which states explicitly that the share of labor does not converge to zero, but to some positive number. For this assumption to hold, we need to add a second restriction on the function *k*. Since the function φ is increasing and since M_t is an increasing sequence, and due to the boundedness in (8), it has a least upper bound:

$$A = \sup\left\{\varphi(M_t) : t < \infty\right\}.$$

According to (8) this upper bound should be lower than or equal to 1. To make sure that the share of labor would not go to zero over time, we add the following restriction:

Restriction 2: The least upper bound A is strictly lower than 1: $A < 1$.

Note, that due to the concavity of the function φ and to its boundedness, the share of labor might decline over time, but it actually converges to a constant level, $1 - A$, so that we can say, that from some time on it is basically quite stable, as found in many empirical studies. Furthermore, if we add a realistic assumption to the model, that capital is used also in labor tasks, in structures for example, then the decline of the share of labor is even more moderate, as it does not begin at 1, but at a lower level. This is shown explicitly in Appendix A. Hence, this result of a fairly stable share of labor, would look even more realistic in such an extension of the model, which does not affect its main results.

3.5 A Necessary Condition for Technology Adoption

Producers adopt the technology that enables them to produce at a lower cost. Hence, they produce by machines only if:

$$Rk(j) \leq w_t.$$

This condition implies that the goods produced by machines are between 0 and some upper bound. This upper bound is equal to M_t, namely all the invented machines are used in production, if the following condition holds:

$$w_t \geq Rk(M_t). \tag{11}$$

From here on we assume that this condition holds for each period and this is our third working assumption. Note, that if this condition is not satisfied in some period, automation stops at some finite time and does not go beyond it. This third working assumption will lead us to the third restriction.

From inequality (11) we can derive a condition for technical change and automation that must be satisfied in this economy. Substituting the wage rate (9) in inequality (11) implies that:

$$T_t - M_t \geq k(M_t)^{\frac{\theta}{1-\theta}} R^{\frac{\theta}{1-\theta}} \left[1 - \varphi(M_t)\right] = \frac{1 - \varphi(M_t)}{k(M_t)^{\frac{-\theta}{1-\theta}} R^{\frac{-\theta}{1-\theta}}} = \frac{1 - \varphi(M_t)}{\varphi'(M_t)}.$$

An alternative way of writing this inequality is:

(12) $$\frac{1}{T_t - M_t} \le \frac{\varphi'(M_t)}{1-\varphi(M_t)}.$$

This inequality leads to our third restriction:

Restriction 3: The numbers of tasks and of mechanized tasks, T and M respectively, satisfy in each period t the inequality (12).

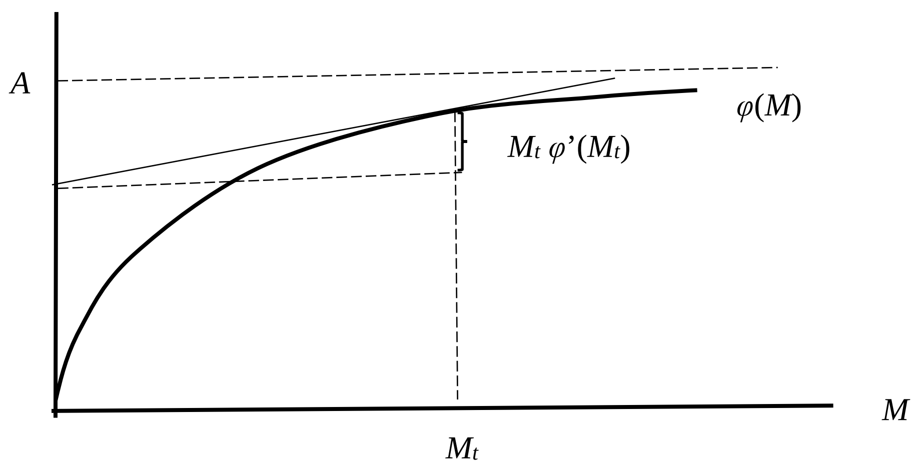

Figure 1: The Curve φ

As mentioned above, there are two possible cases of automation dynamics, one where M is unbounded and one where M is bounded. If M is unbounded then boundedness of the share of capital by A implies that the function φ looks as described in Figure 1. Hence, in this case we can derive from Figure 1 the following result.

Proposition 1: If M_t is unbounded, the number of labor jobs $T_t - M_t$ increases to infinity faster than M_t.

Proof: If M is unbounded, restriction 2 implies that the function φ is an increasing and concave function, which is everywhere bounded above by A, as described in Figure 1. We know that the tangent to the curve intersects the vertical axis at $\varphi(M) - M\varphi'(M)$. It is easy to see that:

$$\varphi(M) - M\varphi'(M) \xrightarrow[M\to\infty]{} A.$$

Since $\varphi(M)$ also converges to A, it follows that:

$$M\varphi'(M) \xrightarrow[M\to\infty]{} 0.$$

Note that inequality (12) implies that:

$$\frac{M}{T-M} \le \frac{M\varphi'(M)}{1-\varphi(M)}.$$

Since the numerator converges to 0 and the denominator converges to $1-A>0$, this proves the Proposition.

4. Economic Growth

In this section we calculate output per worker and from it we derive its rate of growth. We will show that if output grows unboundedly, the amount of labor tasks is unbounded as well. This section also examines the size of the Solow Residual and shows that it is positive in this model, as all empirical studies show.

From equations (9) and (10) we get that output per worker in this model is equal to:

(13) $$y_t = \frac{Y_t}{E_t} = \frac{w_t}{1-\varphi(M_t)} = \frac{\left(T_t - M_t\right)^{\frac{1-\theta}{\theta}}}{\left[1-\varphi(M_t)\right]^{\frac{1}{\theta}}}.$$

From equation (13) we learn that if M is bounded, then output per worker y might be bounded if $T – M$ is bounded as well. If M is unbounded, then according to Proposition 1 $T – M$ is unbounded and so is y. We therefore conclude that output per worker goes to infinity if the level of automation M is unbounded.

Given the level of output per worker in each period we can calculate its rate of growth. From equation (13) we get:

(14) $$\Delta \ln y_t = \frac{1-\theta}{\theta}\Delta \ln\left(T_t - M_t\right) + \frac{1}{\theta}\frac{\varphi'(M_{t-1})\Delta M_t}{1-\varphi(M_{t-1})}.$$

Hence, the rate of growth of output per worker, namely of labor productivity, depends on both the rate of growth of mechanization, M, and on the rate of growth of jobs performed by labor, $T – M$. We next show that in the long-run the rate of growth of output per worker depends only on the rate of growth of $T – M$.

Proposition 2: The dynamics of automation satisfy in the long-run:

$$\frac{\varphi'(M_{t-1})\Delta M_t}{1-\varphi(M_{t-1})} \xrightarrow[t\to\infty]{} 0.$$

Proof: Both in the case that M is bounded in the long-run, and in the case that M is not bounded and is increasing to infinity, $1-\varphi(M_t)$ converges to 1 – A and 1 – A is positive. As a result, its rate of growth converges to zero. But this rate of growth is equal to:

$$\Delta \ln\left[1-\varphi(M_t)\right] = \frac{-\varphi'(M_{t-1})\Delta M_t}{1-\varphi(M_{t-1})}.$$

This proves the Proposition.

From Proposition 2 it follows that in the long-run the only variable that affects the rate of growth of labor productivity is the rate of growth of the number of labor tasks. Equation (14) and Proposition 2 imply that in the long-run, the rate of growth of output per worker is equal to:

$$\Delta \ln y_t \cong \frac{1-\theta}{\theta}\Delta \ln\left(T_t - M_t\right).$$

We can therefore conclude that in this model, that combines the Zeira (1998) tasks model of automation and the Romer (1990) model of expanding variety, the long-run rate of growth is determined by the expanding variety and not by automation. The intuition behind this result is actually Proposition 1, according to which automation progresses more slowly than the expansion of variety. Interestingly, the shares of labor and of capital are determined by the automation part of this combined model, rather than the expanding variety part of it.

We next turn to calculate the Solow Residual in this economy and to examine whether it is indeed positive as all empirical tests show. The amount of capital is calculated by using the first order condition (3) to get that the amount of capital in task j is:

$$K_t(j) = x_t(j)k(j) = Y_t p_t(j)^{\frac{-1}{1-\theta}}k(j) = Y_t R^{\frac{-1}{1-\theta}}k(j)^{\frac{-\theta}{1-\theta}}.$$

Summing over [0, M_t] we get:

$$K_t = \int_0^{M_t} K_t(j)dj = Y_t R^{-1}\int_0^{M_t} R^{\frac{-\theta}{1-\theta}}k(j)^{\frac{-\theta}{1-\theta}}dj = Y_t R^{-1}\varphi(M_t).$$

Hence, the share of capital in output is $\varphi(M)$, and also the capital-labor ratio is:

(15) $$\frac{K_t}{E_t} = \frac{y_t}{R}\varphi(M_t).$$

This enables us to calculate the Solow residual, which is defined by:

$$SR_t = \Delta \ln y_t - \varphi(M_t)\Delta \ln\left(\frac{K_t}{E_t}\right).$$

Using equations (14) and (15) we find that the Solow residual is equal to:

(16) $$SR_t = \left[1-\varphi(M_t)\right]\frac{1-\theta}{\theta}\Delta \ln(T_t - M_t) + \frac{1-\theta}{\theta}\left[\varphi'(M_{t-1})M_{t-1}\right]\frac{\Delta M_t}{M_{t-1}}.$$

Since both mechanization and the number of tasks performed by labor increase over time, the Solow residual is positive, according to equation (16). Furthermore, since according to Proposition 2, $\varphi'(M)M\left[\Delta M / M\right]$ is converging to 0 over time, the Solow residual converges to the rate of growth of output per worker times the share of labor. This is close to what we find in many empirical studies.

5. The Rate of Unemployment

We next turn to describe unemployment in the economy. The group of unemployed are people who worked in the previous period, but their tasks become automated in the present, in period *t*. Once they leave their job due to automation, they search for another job and remain unemployed for one period.

The number of unemployed due to automation is denoted by U_t^A. This is the number of people who were employed in period $t-1$ in the jobs that later become automated in period *t*. The number of workers in period $t-1$ in job *j* is $l_{t-1}(j)$ and it is equal to:

(17) $$l_{t-1}(j) = \frac{E_{t-1}}{T_{t-1} - M_{t-1}}.$$

Hence, the total number of unemployed due to automation in period *t* is equal to:

(18) $$U_t^A = \int_{M_{t-1}}^{M_t} l_{t-1}(j)dj = \frac{E_{t-1}\Delta M_t}{T_{t-1} - M_{t-1}} \le \frac{\Delta M_t}{T_{t-1} - M_{t-1}}.$$

Due to restriction 3, this inequality can be written in the following way:

$$(19) \qquad U_t^A \le \frac{\Delta M_t}{T_{t-1} - M_{t-1}} \le \frac{\Delta M_t \varphi'(M_{t-1})}{1 - \varphi(M_{t-1})}.$$

This upper bound on unemployment due to automation leads to the next Theorem:

<u>Theorem 1:</u> The unemployment due to automation, U_t^A, converges to zero over time: $U_t^A \xrightarrow[t\to\infty]{} 0$.

Proof: There are two cases. The first is that automation stops at some future period and the second is that it does not. If automation stops at some future period, then from that period on $\Delta M_t = 0$. Hence, unemployment due to automation becomes 0 from that period on and the theorem holds. In the second case automation continues infinitely. Of course, *M* could go to infinity or be bounded, as discussed above, but it would keep on increasing. In this case the theorem follows immediately from an application of Proposition 2 to the inequality (19). This proves that if automation continues infinitely, the unemployment it causes converges to zero. QED.

This is a surprising result. The intuition behind it is not trivial, but it is based on the need of wages to be sufficiently high in order for automation to be adopted. According to equation (9), wages can be high in two cases. One is if the share of labor $1-\varphi(M)$ converges to zero. If it does not, the wage can be sufficiently high only if the number of labor jobs, $T - M$, is large enough, since otherwise there will be too many workers crowding each job and their marginal productivity will not rise sufficiently. Once the number of labor jobs is large, it means that the share of jobs being automated each period is declining. This is the intuition behind this surprising result.

6. A Balanced Growth Path

The stylized facts of economic growth are that the rate of growth of the countries of the frontier has been rather stable over a long period of time. GDP per capita in the US has grown at an average annual rate of 1.8 percent at least since 1870 and has been stable over most of the period. This is why many economists examine a balanced growth path, as an important case for many growth models. This section analyzes this specific case as well, by assuming that technology grows at a constant rate. In order to have a full numerical example, we first assume that the function of the cost of machinery, *k*, is described by:

(20) $$k(j) = a(1+j)^{\alpha}.$$

In this case the function φ is equal to:

(21) $$\varphi(M) = \left(aR\right)^{\frac{-\theta}{1-\theta}} \frac{1-\theta}{\theta+\alpha\theta-1}\left[1-\left(1+M\right)^{\frac{1-\theta-\alpha\theta}{1-\theta}}\right].$$

Note that this function is bounded only if $1-\theta-\alpha\theta<0$, or if $\alpha>(1-\theta)/\theta$. We therefore assume that this inequality holds. In this case the function φ satisfies:

$$\varphi(M) \xrightarrow[M\to\infty]{} \frac{1-\theta}{\theta+\alpha\theta-1}\left(aR\right)^{\frac{-\theta}{1-\theta}} = A.$$

Clearly, if *a* is sufficiently large, the limit *A* is smaller than 1.

We assume for the balanced growth case, that the rate of growth of automation is constant over time:

$$\frac{\Delta M}{M} = g_M.$$

We also assume that the rate of growth of new jobs is constant over time as well:

$$\frac{\Delta T}{T} = g_T.$$

Restriction 3, which assumes that inequality (12) holds for each *T* and *M*, enables us to establish a relationship between the two rates of growth.

If we substitute the specific function φ, from (21), and its derivative into equation (11), we get the following inequality:

$$T_t \geq M_t + \left(aR\right)^{\frac{\theta}{1-\theta}}\left[\left(1+M_t\right)^{\frac{\alpha\theta}{1-\theta}}\left(1-A\right)+\left(1+M_t\right)A\right].$$

This inequality implies that in the long-run the two growth rates of *M* and *T* must satisfy:

$$g_T \geq \frac{\alpha\theta}{1-\theta} g_M > g_M.$$

Hence, the rate of growth of new jobs must be higher than the rate of automation, as implied in the general case by Proposition 1. We next assume for the sake of simplicity that:

$$g_T = \frac{\alpha\theta}{1-\theta} g_M.$$

We can now return to equation (14), which describes the rate of growth of output per worker, and substitute in it the rates of growth of *M* and of *T*. We get that in this example the rate of growth of output is equal to:

$$\Delta \ln y = \frac{1-\theta}{\theta}\Delta \ln(T-M) + \frac{1}{\theta}\frac{M(aR)^{\frac{-\theta}{1-\theta}}}{(1-A)(1+M)^{\frac{\alpha\theta}{1-\theta}} + A(1+M)} g_M .$$

Over time, as *M* grows to infinity, the rate of growth of output converges to:

$$\Delta \ln y \xrightarrow[M\to\infty]{} \frac{1-\theta}{\theta} g_T = \alpha g_M . \tag{22}$$

Hence, in the case of a balanced growth path of technology, the rate of growth of output per worker converges to a constant long-run rate as well, which is equal to αg_M . This means that we have a balanced growth path in the long-run, where the rate of growth of output per worker is determined by the rate of growth of the number of jobs, which is reflecting also the rate of growth of automation.

We next examine the bound of the rate of unemployment due to automation in this case of balanced growth path. Combining inequality (19) with the function φ in this example at (21), leads to the following inequality:

$$\begin{aligned} U_t^A &\le \frac{\Delta M_t}{M_{t-1}}\frac{\varphi'(M_t)M_{t-1}}{1-\varphi(M_t)} = g_M \frac{(aR)^{\frac{-\theta}{1-\theta}}(1+M_t)^{\frac{-\alpha\theta}{1-\theta}}M_{t-1}}{1-\left(aR\right)^{\frac{-\theta}{1-\theta}}\dfrac{1-\theta}{\theta+\alpha\theta-1}\left[1-\left(1+M_t\right)^{\frac{1-\theta-\alpha\theta}{1-\theta}}\right]} \\ &\le g_M \frac{(aR)^{\frac{-\theta}{1-\theta}}(1+M_t)^{\frac{1-\theta-\alpha\theta}{1-\theta}}}{1-\left(aR\right)^{\frac{-\theta}{1-\theta}}\dfrac{1-\theta}{\theta+\alpha\theta-1}\left[1-\left(1+M_t\right)^{\frac{1-\theta-\alpha\theta}{1-\theta}}\right]}. \end{aligned} \tag{23}$$

Note that the denominator is converging to 1 – *A* as time goes on, while the numerator converges to zero, since $\alpha > (1-\theta)/\theta$. Hence, the unemployment due to automation goes to zero in this case of a balance growth path as well.

7. Improved Machines and Disappearing Tasks

In this section we deal briefly with two possible critiques of our model, or more precisely with two potential variations of the model in which the unemployment due to automation might not diminish to zero over time. The first extension is that in addition to automation and creating new jobs, technical change also enables us to reduce the cost of machines continuously, as in Sachs (2017) and as discussed in Acemoglu and Restropo (2018a). In that case, the condition that the wage exceeds the cost of machines (restriction 3) becomes less stringent and it might even affect the validity of Theorem 1. The second extension is that unlike our assumption that all tasks are used in production, we might experience disappearance of some of the older tasks, as in Acemoglu and Restropo (2018b).

We first turn to the first possible extension of the model and assume that in addition to automation and new jobs, technical change can also create new improved machines to perform the automated jobs. To analyze the effect of such an extension, assume that the capital cost of machinery *k* declines. Without loss of generality, consider the case of the balanced growth path, which is discussed in Section 6, and assume that the parameter *a* from equation (20) is changing. Note that from equation (23) we get:

$$U_t^A \leq g_M \frac{(1+M_t)^{\frac{1-\theta-\alpha\theta}{1-\theta}}}{(aR)^{\frac{\theta}{1-\theta}} - \frac{1-\theta}{\theta+\alpha\theta-1}\left[1-(1+M_t)^{\frac{1-\theta-\alpha\theta}{1-\theta}}\right]}.$$

Hence a reduction in the cost of machines, or of automation, increases the bound on unemployment due to automation on the RHS of this inequality, so that such technical changes can potentially weaken our main argument.

We next show that this possibility is rather limited, as *a* cannot decline too much. As equation (21) shows, a reduction of *a* increases the share of capital $\varphi(M)$ and if *a* becomes too low, the share of capital might reach 1 and in this case the share of labor becomes zero. This strongly contradicts our restriction 2. Hence, the cost of machinery *a* cannot fall by too much and that means that the rate of unemployment still converges to zero over time. Hence, the result of Theorem 1 still holds in this case as well.

As mentioned in the introduction, Acemoglu and Restrepo (2018a, 2018b) assume that in addition to automation and creation of new jobs, some of the older jobs are disappearing. They assume that the disappearing jobs are exactly the set $[0, T_t - 1]$, so that the set of total tasks remains of size 1. But this assumption cannot be applied to our model with its three restrictions. To understand it consider a more general specification, where the active tasks in each period are $[N_t, T_t]$, where $N_t < M_t < T_t$.

There are two main reasons why we think that this extension cannot be applied to our model. The first reason is that due to our first restriction, $\theta > 0$. That means that the derivative of profits with respect to *N* is negative. Hence, producers have an incentive to reduce *N* to 0, namely to use all tasks that were developed in the past. Hence, a positive value to *N* contradicts profit maximization when the first restriction holds.

The second reason to doubt whether tasks can disappear in our model is more descriptive and it rests on our understanding of what tasks are. We think that they list all the required needs of the human society at a given state of development. Clearly, these needs grow, but it is less likely that they disappear. We are still growing wheat and barley and olives and figs, as in ancient times. We are still producing transportation, though not by animals, but by machines. We still have poetry and art and theater as in ancient times. Tasks don't seem to disappear over time, but rather change.

There are two more differences between our model and the Acemoglu and Restrepo papers. The first is that they derive the rate of growth of *M* and *T* from profit maximization of an R&D sector, while we impose less structure on their dynamics. The second difference is that they assume that labor has increasing productivity over tasks. We examine this possibility in Appendix B and find that even in this extension the main result of the model, that unemployment due to automation converges to zero, still holds.

8. Skill, Wages and Technical Change

Finally, we add to our model of automation the issue of skill. The relationship between skill and automation has received much attention in the public debate.[4] Many have claimed that the recent automation, mainly by Artificial Intelligence, is biased toward workers of high skill and thus hurts workers with low skill quite significantly. Another claim has been that this skill bias has been one of the main reasons for the decline of the share of labor in recent decades. We discuss these two claims below.

Assume that there are two skill levels, high and low. Assume that the number of high skilled in the population is *H* and the number of low skilled is *L*, and $H + L = 1$. For simplicity assume that these shares are constant over time. We also abstract in this section

[4] Acemoglu and Restropo (2018) discuss the issue of skill and its mismatch with automation as well. A recent discussion of skill in a model of automation by tasks appears also in Alesina, Battisti and Zeira (2018).

from the issue of unemployment, by assuming that q is equal to 0. Instead of unemployment we examine the effect on the wages of high and low skilled. Studying the extension with unemployment would lead to similar effects to those of the benchmark model in aggregate, but this section focuses on labor income instead.

Labor tasks can be high or low skilled. There is an indicator function of tasks S, that is equal to 1 if the task requires high skill and to 0 if it requires low skill. We denote:

(24) $$d(x,y) = \frac{1}{y-x}\int_x^y S(j)dj.$$

This means that d is the density of skilled tasks between x and y. The rest of the economy is as in the benchmark model and hence the prices of the intermediate goods satisfy:

$$\int_0^{T_t} p_t(j)^{\frac{-\theta}{1-\theta}} dj = 1.$$

The supply prices of the intermediate goods, or tasks, are:

$$p_t(j) = \begin{cases} Rk(j), \text{ if } 0 \le j \le M_t \\ w_t^H, \text{ if } M_t \le j \le T_t \text{ and } S(j) = 1. \\ w_t^L, \text{ if } M_t \le j \le T_t \text{ and } S(j) = 0 \end{cases}$$

We further assume that all automations are adopted. This means that:

(25) $$w_t^L \ge Rk(M_t).$$

Clearly this condition holds also for the wage of high skilled, as it is even higher.

We next turn to calculate the wages of high and low skilled. Substituting the supply prices in the equilibrium condition we get:

$$1 = \varphi(M_t) + \int_{M_t}^{T_t} (w_t^H)^{\frac{-\theta}{1-\theta}} S(j)dj + \int_{M_t}^{T_t} (w_t^L)^{\frac{-\theta}{1-\theta}} [1 - S(j)]dj.$$

Some calculation leads to:

(26) $$\frac{1-\varphi(M_t)}{T_t - M_t} = \left(w_t^H\right)^{\frac{-\theta}{1-\theta}} d(M_t, T_t) + \left(w_t^L\right)^{\frac{-\theta}{1-\theta}} \left[1 - d(M_t, T_t)\right].$$

From the first order conditions of tasks we get that skilled non-automated tasks satisfy:

(27) $$Y_t = x_t(j)\left(w_t^H\right)^{\frac{1}{1-\theta}} = \frac{H}{(T_t - M_t)\, d\left(M_t, T_t\right)} \left(w_t^H\right)^{\frac{1}{1-\theta}}.$$

Similarly, each low skilled task that is not automated satisfies:

(28) $$Y_t = x_t(j)\left(w_t^L\right)^{\frac{1}{1-\theta}} = \frac{L}{\left(T_t - M_t\right)\left[1-d\left(M_t,T_t\right)\right]}\left(w_t^L\right)^{\frac{1}{1-\theta}}.$$

Dividing (27) by (28) we get:

(29) $$\left(\frac{w_t^H}{w_t^L}\right)^{\frac{1}{1-\theta}} = \frac{d\left(M_t,T_t\right)}{1-d(M_t,T_t)}\frac{L}{H}.$$

From this equation we see that the skill premium, w_t^H / w_t^L, increases with *d*. It means that if the density of high skilled jobs among the new jobs increases, the skill premium increases as well. If the number of high skilled workers increases, the skill premium declines. Hence the rise in the skill premium in the last three decades, despite the parallel rise in education, can be interpreted as a rise in the share of skilled tasks among new jobs. It is important not to interpret it strictly a strict skill biased technical change (SBTC). There is of course an implicit connection, as the new tasks are usually created to deal with the new automation. But it is possible that the determination whether a new task is high or low skilled might not always reflect technological considerations. It could reflect also social considerations. For example, if the workers in a certain job prefer to interact with highly educated people, it might affect the definition of the job.

To simplify notation we use from here on, $d = d(M_t, T_t)$. From equations (26) and (29) we can solve the two wage levels, of high and low skilled. The wage of low skilled is:

(30) $$w_t^L = \left[\frac{T_t - M_t}{1-\varphi(M_t)}\right]^{\frac{1-\theta}{\theta}}\left[\left(\frac{1-d}{d}\frac{H}{L}\right)^{\theta} d + 1 - d\right]^{\frac{1-\theta}{\theta}}.$$

The wage of high skilled is:

(31) $$w_t^H = \left[\frac{T_t - M_t}{1-\varphi(M_t)}\right]^{\frac{1-\theta}{\theta}}\left[d + (1-d)\left(\frac{d}{1-d}\frac{L}{H}\right)^{\theta}\right]^{\frac{1-\theta}{\theta}}.$$

We can now rewrite condition (25) for technology adoption in the following way and get:

$$\frac{1}{T_t - M_t} \leq \frac{\varphi'(M_t)}{1-\varphi(M_t)}\left[\left(\frac{1-d}{d}\frac{H}{L}\right)^{\theta} d + 1 - d\right].$$

Hence, the dynamic condition imposed by the model on the number of labor jobs remains similar to that in the benchmark case.

Equations (27) and (28) enable us to find the share of labor in output:

$$\text{(share of labor)}_t = \frac{w_t^H H + w_t^L L}{Y_t} = \left(T_t - M_t\right)\left[d(w_t^H)^{\frac{-\theta}{1-\theta}} + (1-d)(w_t^L)^{\frac{-\theta}{1-\theta}} \right].$$

Applying (30) and (31) we get:

(32) $$\text{(share of labor)}_t = 1 - \varphi(M_t).$$

This result means that the share of labor in output does not depend at all on *d*, namely on the density of high skilled tasks in the economy.

We can now relate this section to the extensive literature on skill biased technical change. If we interpret a rise in high skilled jobs in newly created jobs as SBTC, it fits the standard view by having the same effect on the skill premium. But in this model the skill premium is not affected at all by automation, only by the type of new tasks created. Also, according to our model, the recently observed decline in the share of labor cannot be caused by skill biased technical change. Hence, SBTC alone cannot explain all the recent developments in the labor markets. It can account for some of the widening skill premium, but to explain the decline in the share of labor we need other explanations, like the decline in the power of unions, or a rise in monopoly power in the economy, as suggested by Barkai (2017) and by Autor, Dorn, Katz, Patterson and Van Reenen (2017), or a rise in monopsony power in labor markets, as suggested by Benmelech, Bergman and Kim (2018).

9. Summary

This paper presents a simple model that enables us to analyze the relationship between the long-run process of automation and the short-run phenomenon of unemployment. Our model implies that the part of unemployment that should be attributed to automation is actually declining over time and converges to zero in the long-run. This is a surprising result and its intuition depends on the need of wages to be sufficiently high in order to enable continuous adoption of technologies. The wage depends on having sufficient number of new jobs created. As the number of new jobs rises sufficiently fast, sooner the share of jobs that are automated each period will be relatively small and as a result the rate of unemployment as well.

The model supplies a strong result and that is a good reason to treat it with some caution. In general, economic models should help us to explain processes and mechanisms and dwell less on predicting future developments. We think that this model should be treated in a similar way. Its main message is that although automation causes unemployment, by turning tasks that used to be performed by labor into tasks that are performed by machines, it might also ignite a mechanism that reduces this unemployment as well. One reason is that automation also contributes to creation of new labor jobs, to produce and to maintain the new machines. If the number of the new jobs grows sufficiently fast, the labor jobs will absorb automation better and better, since automation will affect a smaller and smaller share of this growing set of tasks. This is a point to bear in mind when we consider the effect of automation on unemployment and on the labor market in general.

The prediction that the unemployment due to automation is converging to zero should be dealt with caution for an additional reason. It relies on the assumption that the share of labor in output does not converge to zero over time. This sounds like a reasonable assumption, as shares of labor tend to be quite stable and are around two thirds across countries and over long periods of time. But no one knows for sure how the future will look like. If indeed the share of labor will continue to decline and even go all the way to zero, then it might as well happen that the unemployment due to automation will remain at a stable size and will not converge to zero. We shall wait and see…

Appendix

A. Adding Capital Costs to Production by Labor

Assume the following addition to the benchmark model, that production by labor of intermediate good *j* requires not only one worker, but also a structure of size *s*. Assuming that this capital also fully deteriorates within one period, it can be shown that the price of such a good becomes:

(A.1) $$p_t(j) = w_t + Rs.$$

Substituting this price in the equilibrium condition (4) we get that the wage rate is equal:

(A.2) $$w_t = \left[\frac{T_t - M_t}{1-\varphi(M_t)}\right]^{\frac{1-\theta}{\theta}} - Rs.$$

We next turn to calculate the share of labor in output in this economy and in a similar way to the calculation of equation (10) we get:

(A.3) $$\frac{w_t E_t}{Y_t} = 1-\varphi(M_t) - Rs\frac{\left[1-\varphi(M_t)\right]^{\frac{1}{\theta}}}{(T_t - M_t)^{\frac{1}{\theta}-1}}.$$

This equation implies that even at the start of automation, when $M_t = 0$ and $\varphi(M_t) = 0$ as well, the share of labor is lower than 1. This is of course the result of using capital, mainly structures, even before automation, or even before the industrial revolution. We next show that as automation proceeds and is unbounded, the share of labor in output converges to 1 – *A*, as in the benchmark model.

To see this consider the condition for adopting automation:

(A.4) $$w_t + Rs \geq Rk(M_t).$$

From this condition we get, using (A.2) that:

(A.5) $$(T_t - M_t)^{\frac{1}{\theta}-1} \geq Rk(M_t)\left[1-\varphi(M_t)\right]^{\frac{1}{\theta}-1}.$$

Inequality (A.5) enables us to find a bound to the discrepancy between the share of labor in (A.3) and $1-\varphi(M_t)$:

(A.6) $$Rs\frac{\left[1-\varphi(M_t)\right]^{\frac{1}{\theta}}}{(T_t - M_t)^{\frac{1}{\theta}-1}} \leq Rs\frac{\left[1-\varphi(M_t)\right]^{\frac{1}{\theta}}}{Rk(M_t)\left[1-\varphi(M_t)\right]^{\frac{1}{\theta}-1}} = s\frac{1-\varphi(M_t)}{k(M_t)}.$$

The RHS of (A.6) is monotonically decreasing in any case, so this discrepancy is reduced over time. The RHS converges to zero if automation is unbounded. In the case of bounded automation the share of labor converges to some other constant than 1 – *A*.

B. Growing Productivity of Labor

In this appendix we assume that the productivity of labor is not constant, but it increases with tasks, namely the amount of workers required to produce one unit of intermediate good *j* is equal to $n(j)$, where *n* is a decreasing function. As a result the price of a task produced by labor is:

(A.7) $$p_t(j) = w_t n(j).$$

Substituting these prices into the equilibrium condition (4) in the benchmark model, which holds here as well, leads to the following equilibrium wage rate:

(A.8) $$w_t = \left[\frac{\psi(M_t, T_t)}{1-\varphi(M_t)}\right]^{\frac{1-\theta}{\theta}},$$

where we use the following notation:

$$\psi(M_t, T_t) = \int_{M_t}^{T_t} n(j)^{\frac{-\theta}{1-\theta}} dj.$$

We next turn to calculate the share of labor in this extension of the model. Note that from the first order condition of the final good (3) we get in this case:

$$x_t(j) = Y_t w_t^{\frac{-1}{1-\theta}} n(j)^{\frac{-1}{1-\theta}}.$$

Hence, the amount of labor producing good j is equal to:

$$l_t(j) = x_t(j) n(j) = Y_t w_t^{\frac{-1}{1-\theta}} n(j)^{\frac{-\theta}{1-\theta}}.$$

Summing up the amount of labor for all labor produced tasks we get that aggregate employment is described by:

(A.9) $$E_t = \int_{M_t}^{T_t} l_t(j) dj = Y_t w_t^{\frac{-1}{1-\theta}} \psi(M_t, T_t).$$

We therefore get, using also equation (A.8) that the share of labor in output is:

(A.10) $$\frac{w_t E_t}{Y_t} = w_t^{\frac{-\theta}{1-\theta}} \psi(M_t, T_t) = 1 - \varphi(M_t).$$

Hence, the share of labor in this extension is the same as in the benchmark model.

We finally turn to the condition for adoption of automation, which is equal in this extension to:

(A.11) $$w_t n(M_t) \geq Rk(M_t).$$

Raising by the power of $\theta/(1-\theta)$ we get from (A.11):

(A.12) $$\psi(M_t, T_t) n(M_t)^{\frac{\theta}{1-\theta}} \geq \frac{1-\varphi(M_t)}{\varphi'(M_t)}.$$

We next turn to the upper bound of unemployment due to automation:

$$U_t^A = \int_{M_{t-1}}^{M_t} l_{t-1}(j)dj \leq \Delta M_t n(M_t)^{\frac{-\theta}{1-\theta}} Y_{t-1} w_{t-1}^{\frac{-1}{1-\theta}}.$$

Using (A.9) we get:

$$U_t^A \leq \Delta M_t n(M_t)^{\frac{-\theta}{1-\theta}} \frac{E_{t-1}}{\psi(M_{t-1}, T_{t-1})} \leq \Delta M_t \frac{n(M_t)^{\frac{-\theta}{1-\theta}}}{\psi(M_{t-1}, T_{t-1})}.$$

Using inequality (A.12) we get:

(A.13) $$U_t^A \leq \frac{\Delta M_t \varphi'(M_{t-1})}{1-\varphi(M_{t-1})} \left[\frac{n(M_t)}{n(M_{t-1})}\right]^{\frac{-\theta}{1-\theta}}.$$

Since the RHS of (A.13) converges to zero, so does the equilibrium unemployment due to automation.

C. Unemployment Due to Mismatch and Automation

In this appendix we extend the model to include unemployment due to mismatch. Assume that agents belong to overlapping generations. Each one lives L periods, supplying one unit of labor in each period. Each generation is a continuum of size $1/L$, so the overall population is 1. In the analysis below we assume for simplicity that the number of cohorts is $L = 2$. Workers try to work in each period but might find themselves unemployed during that period for two reasons, one is mismatch between the worker and the job and the other is due to losing their job to automation.

We assume that workers can fit a specific task or not. If they fit, it becomes their job, while if they don't, they lose this job, remain unemployed for one period and then search in the next period for another job. This is mismatch unemployment. We assume that the probability of not fitting a job is q, and it is equal across workers and across tasks. Workers can also be unemployed if their task becomes automated in the second period. Assume that the information that a task is automated arrives at the beginning of the period, so that workers, who search for a new job, do not try this job. Only those who worked in the job in the past need to leave it and search for a new job, so that q of them become unemployed. Hence, unemployment due to automation hits older workers only.

There are three groups of unemployed in the population. The first group are young, who enter the job market for the first time and some of them find themselves mismatched to the job they try. Their number is: $q/2$. The young do not suffer from unemployment due to automation. The second group of unemployed are older workers, who have not found a job yet, and are still looking for it. They sample new jobs in their second period of life and q of them suffer from mismatch. Their total number is $q^2/2$.

The third group of unemployed are people who were matched well and worked in the previous period, but the tasks they found become automated in the present, in period t. Once they leave their job due to automation, they search for another job and q of them do not find one. The number of unemployed of this third group, of older workers, is denoted U_t^A. This is the rate of unemployment due to automation.

The number of the third group of unemployed due to automation, is the number of young people who were employed in period $t-1$ in the jobs that later become automated in period t, when the workers are already old. Notice that young workers are not distributed equally across jobs, since the set of jobs is increasing and every period new jobs are added. The new jobs don't have older workers from the past, so they employ more young workers. The number of old people in each job affects the number of young in the job and that correlation has a result that the distribution of old and young in a job is not equal across jobs. This makes the calculation of the size of unemployment due to automation quite complicated. As an alternative, we can try to find an upper bound to this unemployment.

The unemployed due to automation in period t are people who worked as young in these jobs in period $t-1$. The number of young workers in period $t-1$ in job j, which we denote by $l_{t-1}^y(j)$, is bounded from above by the total number of workers in the job. This number is equal across jobs and hence we get:

(A.14) $$l_{t-1}^{y}(j) \leq \frac{E_{t-1}}{T_{t-1} - M_{t-1}}.$$

Hence, the total number of unemployed due to automation in period *t* satisfies:

(A.15) $$U_t^A = q \int_{M_{t-1}}^{M_t} l_{t-1}^{y}(j)dj \leq q \frac{E_{t-1}\Delta M_t}{T_{t-1} - M_{t-1}} \leq q \frac{\Delta M_t}{T_{t-1} - M_{t-1}}.$$

Note that due to restriction 3, this inequality can be written in the following way:

(A.16) $$U_t^A \leq q \frac{\Delta M_t}{T_{t-1} - M_{t-1}} \leq q \frac{\Delta M_t \varphi'(M_{t-1})}{1 - \varphi(M_{t-1})}.$$

This upper bound on unemployment caused by automation shows that the main result of the paper, in Theorem 1, holds in this extension as well.

Note that in this case the total rate of unemployment, U_t, satisfies:

$$U_t \xrightarrow[t \to \infty]{} \frac{q}{2} + \frac{q^2}{2}.$$

This means that the long-run rate of unemployment is due to mismatch only and not to automation.

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