Medical Technology Ageing and Growth

Gilad Sorek[†‡•]

Tel Aviv University, Eitan Berglas School of Economics

Ramat Aviv, 69978 Tel Aviv

Israel

January 2008

[†] The study was performed under the most supportive supervision of Prof. Daniel Tsiddon.

[‡] Correspondence may be addressed to Gilad Sorek Berglas School of Economics, Tel Aviv University, Ramat-Aviv, Tel –Aviv, 69978, Israel, or to Sorekgil@post.tau.ac.il, Tel : 972-3-640-7983, Fax: 972-3-640-9908

[•] The study was financially supported by the Israeli National Institution of Health Policy—"N.H.I.P."

Abstract

We model endogenous ageing process of two phases that is driven by progress in medical technology and labor productivity. At first, fertility declines as per-capita income increases and relative price of health services decreases. Then, as (if) real income -in terms of the price of healthcare services- reaches a certain threshold level, the ageing process combines further decline in fertility along with increase in adults' longevity. We analyze the affects of public provision of healthcare and intergenerational trade in the market for healthcare services on these dynamics.

Keywords: Growth; Longevity; Fertility; Human Capital; Health Technology; Health-Policy.

JEL Classification: O11, J11, I .

1. Introduction

During the last two centuries Life expectancy at birth in Europe and North America has more than doubled - from thirty-five to seventy-eight years (Livi-Bacci, 2000). The rise in life expectancy has coincided with phenomenal technological progress (Mokyr, 1990) and continuous growth of per-capita product, and since the last quarter of the nineteenth century it was also accompanied by downward trend in fertility. The transition from high to low rates of fertility and mortality along with remarkable growth in per-capita income is well known in the economic literature as the "Demographic Transition".

However, the decrease in mortality rates was uneven in terms of its causes and measures over time. Up to the beginning of the 20^{th} century the ten-year increase in life expectancy was gained thank to improved nutrition that followed the increase in per capita income due to the industrial revolution (Fogel, 1993). The next twenty-year gain in life expectancy was achieved in the first half of the twentieth century as preventive medicines for bacterial and other diseases (cholera, typhus, malaria etc.) were found. The near-eradication of main infectious diseases marked a great contribution by medical science and technology to mortality reduction. These preventive medicines did not incur a significant cost at the household and public level. This, however, would change in the following fifty years.

In the second half of the twentieth century, life expectancy was extended by another ten years—this time due to significant progress in medical technology as a direct derivative of overall continuing technological development. This revolutionary progress in medical technology offered new methods of diagnosis—MRI, CT, etc.—as well as curative techniques such as heart surgeries, organ transplants and endoscope interventions, not to mention a vast number of pharmaceutical innovations.

These technological breakthroughs and improvements powered much of the life-expectancy extension, which originated mainly in the reduction of mortality rates among adults and the elderly (Cutler and Meara, 2001). To implement these high-tech novelties, a well schooled and trained labor force is needed and the utilization of these technologies incurs significant costs. During these last decades of increase in adult and elderly longevity, fertility rates have continued to fall. We will name this simultaneous increase in adult longevity and decrease in fertility a “two-tail aging process", meaning that the average person in the population is getting old due to both the increase in the number of old and the decrease in the number of young.

A vast existing literature has studied the demographic transition with a focus on the decrease in child's and infant's mortality and its effect on the incentive to invest in children's human capital. The common argument in this line of studies is based on classic quality-quantity trade-off hypothesis in the fertility choice literature: as child's mortality rates decline in becomes more beneficial to invest in the quality of each child on the expense of number of children that are given birth[1].

The focus of the current study is on the interaction among adults' longevity, investment in human capital and fertility choice, in the light of progress in medical technologies. In fact, the present study integrates the earlier literature on fertility choice with the recent growing literature on endogenous longevity and growth (see for example: Finlay, 2005; Pestieau, Ponthiere, and Sato, 2006, Van Zon and Muysken, 2001, Sanso and A’isa, 2006, Cerda 2004).

The two following studies are closely related to the present one: first, Blackburn and Cipriani (2002) model a positive effect of adults’ increased longevity on their

[1] For a comprehensive and critical summary of this literature in the light of the related empirical statistics, see Galor (2004).

investment in their own human capital, while higher investment in education comes at the expense of the number of children they choose to have. However, in this model increase in life expectancy is exogenous to the agents were in the present study the change in longevity is chosen by agents in equilibrium. Second, a study by Cerda (2002), examines endogenous switch from high to low fertility and mortality rates in an infinite horizon economy. The demographic transition in Cerda's analysis is dependant upon the accumulation of physical capital, where the decrease in mortality rate is uniform for all living agents - across all ages (e.i. there is no distinguish between infants and adults mortality), while the present study focuses on human capital accumulation and adults longevity.

The present study is carried within the Overlapping Generations framework. It improves over existing models of endogenous growth and adult's longevity by (1) incorporating agent's simultaneous optimization with respect to the followings: longevity fertility and educational attainment (2) introducing endogenous technological progress in the medical sector and analyzing its role on the dynamics (3) allowing for the very realistic intergenerational trade in the market for healthcare services – i.e. the old bye healthcare services from the young (4) suggesting health tax policy implications.

Our dynamic analysis yield the following result: As long as real income in terms of health services stays below a certain threshold level adults devote increasing share of their time to education and labor activities while they bear fewer but more qualified children (i.e. children's basic human capital increases). The increase in average human capital drives technological progress in the medical sector which is reflected by a decrease in the relative price of health services. As real income surpasses the threshold level the economy switches into a two tail ageing process as adults start to utilize healthcare services that prolong their life expectancy during retirement. The affordable increase in life expectancy stimulates adults' investment in human capital furthermore, and therefore accelerates the decline in fertility.

The rest of the paper is organized as follows: Section 2 presents the full setup of the model; Section 3 constructs the static optimization conditions; Section 4 introduces dynamic analysis including the case of intergenerational exchange; Section 5 explores health-tax policy implications, and Section 6 concludes.

2. The Model

In an overlapping-generations economy that has two production sectors—consumption good and health services—homogeneous agents derive utility from consumption and from the number and quality of their children. The agents live two active periods – adulthood and elderly. Adulthood period is constant and normalized to one. Elderly period is shorter then adulthood period but it may be prolonged by utilizing health services. Adults give birth to children, work in markets (producing consumption good and health services) and generate human capital—their own and that of their children.

2.1 Agents

Agents enter their first active period of life ('adulthood') with initial basic human capital that their parents created (for them) while raising them. During their first period of life, agents allocate their time among (a) investment in education (occupational training), (b) bearing and rearing their offspring, and (c) working.

Education raises adults' human capital and, thereby, the productivity of their working time. The time devoted to rearing each child positively affects the child's basic human capital, i.e., the child's quality. Working income is allocated between health services expenditures during adulthood and saving for consumption at elderly. The amount of health-services utilization positively affects the length of elderly period resembling longevity. This assumption that health services utilization takes place during adulthood greatly simplifies the dynamic analysis of the model by avoiding intergenerational trade in the market for health services. Obviously it is unrealistic, since most healthcare expenditure accrues late in life, where the elderly buy health services from the young workers. However, this is the only way the current related literature analyzes healthcare-service utilization with in the OLG frame work. In section 4.4 we allow for such intergenerational exchange. The second period of life is devoted to consumption[2]. Agents' preferences are represented by the following utility function[3]:

1) $$\begin{aligned} &U(c,n,h) = \rho \cdot \pi(z) \cdot u(c) + \varphi \cdot u(n \cdot h) \\ &where: u' > 0, u'' < 0 \end{aligned}$$

The variables are defined as follows: φ - parameter for the relative utility from children versus consumption, ρ - time discount factor, $\pi(z)$- longevity, which is a function of healthcare-service utilization $-z$, c-consumption, n—number of children, h— basic human capital of each child.

2.2 Production

Household Production and Labor Supply

Workers are endowed with one unit of time in their working period of life (i.e. during adulthood), which they allocate among educational attainment, denoted by e, and

[2] Allowing for consumption in two periods should not affect our qualitative results.

[3] For the sake of convenience, we omit the time index wherever it is unnecessary.

raising their children. Following the conventional fertility choice literature, we assume that giving birth to each child incurs fixed time cost F; and that investing in forming basic human capital of each child incurs a variable cost of v, to be chosen by the parent.

Thus, the time that a worker who has n children can devote to labor activity (i.e., the labor time supply) is: $1-e-n\cdot(F+v)$. The effective labor supply of a worker is the product of the time devoted to work and her total human capital, which is denoted by g. Thus, a worker's supply of effective labor units is:

2) $$l = g\cdot(1-e-n\cdot(F+v))$$

The worker's total human capital, g, is a function of her basic human capital - acquired from her parent as a child- denoted by h, and her chosen educational attainment, denoted by e. We specify total human capital as follows:

3) $$g(h,e) = h\cdot e$$

The production of each child's basic human capital consumes quantity v of parent time and complementary parent education. Hence the basic human capital with which agents reach adulthood in period t may be written as: $h_t = (e_{t-1}, v_{t-1})$. We further assume the Cobb-Douglas functional form:

4) $$h_t(v_{t-1},e_{t-1}) = e_{t-1}^{1-\gamma}\cdot v_{t-1}^{\gamma} \quad (0<\gamma<1)$$

Market Production

The consumption good, denoted by C, and healthcare services, denoted by z, are produced in a two-sector economy. Each sector uses labor and capital as production factors. Both sectors are perfectly competitive. Labor input is measured in terms of effective labor units, which are the product of labor time supply and workers' human capital. Each sector has a different productivity factor, determined by the average level of education in the economy. Thus, the productivity factors vary according to the evolution of the educational level in the economy. When the educational level rises, technological progress takes place. We specify the production function of each sector in the Cobb-Douglas form[4]:

5a) $$C = a(\bar{e}) \cdot L_c^{\alpha} \cdot K_c^{1-\alpha}$$

5b) $$Z = b(\bar{e}) \cdot L_z^{\beta} \cdot K_z^{1-\beta}$$

Where: L - aggregate effective unit of labor input, K -capital , $a(\bar{e}), b(\bar{e})$-productivity factors, which are increasing functions of the average level of education in the economy - $\bar{e}$.

We assume that technological progress in the healthcare sector is faster than in the consumption sector, so that if education follows a rising path, the relative price of healthcare services will decline. This assumption may be interpreted as if the amount of resources required to maintain a *given longevity* do not increase due to a technological progress in the medical sector. Empirical support for this approach is found in Cutler at. all (1998, 2001). An alternative way to model the progress of medical technology is to induce an explicit R&D activity in the healthcare sector (as in Sanso and A'isa, 2006, for example). However, most countries are not involved in

[4] It is commonly assumed the health sector is relatively labor-intensive, i.e.,- $\beta > \alpha$.

medical R&D. They usually implement medical technologies that were developed abroad, and the quality of their healthcare sector may be seen as a derivative of general quality of the labor force. Nevertheless, note that the declining price of healthcare services in the model does not mean that the marginal price of increasing longevity is declining (this point will be elaborated in the analysis of the longevity function). Moreover, the qualitative results of the paper remain under the weaker assumption that income grows faster then the price of healthcare services - we will verify this later on. We further assume that healthcare services are not tradable internationally. This assumption is supported by the observed fact that international trade in healthcare services is negligible (tough it is growing), and by the composition of healthcare expenditure. Indeed, the cost of drugs (and other tradable healthcare goods) is not the main component of healthcare expenditure - most healthcare expenditure accrues to labor inputs.[5] By also assuming that the economy is small and open, we obtain the income - w, and the relative price of healthcare services - p, as a function of the average level of education[6]:

6) $$w = \tilde{A} \cdot a\left(\bar{e}\right)^{\frac{1-\alpha}{\alpha}} \cdot l$$

7) $$p = H \cdot \frac{a(\bar{e})^{\frac{\beta}{\alpha}}}{b(\bar{e})}$$

Where the constants H and $\tilde{A}$ are determined by the values of α, β, r. Recall that we assumed faster productivity growth in the healthcare sector. Assuming, specifically, that productivity in the consumption good is constant, we may obtain the following expressions for working income and for the price of healthcare services:

[5] Only 10% of total healthcare expenditure in the U.S in 2000 went for drugs (Cerda, 2007).

[6] See full elaboration in Appendix 1.

6a) $\tilde{A}\cdot a\left(\overline{e}\right)^{\frac{1-\alpha}{\alpha}}=A\Rightarrow w=A\cdot l$

Laborer's income is a linear function of his effective labor supply.

7a)
$$\begin{aligned}&p_t=p(m,\overline{e}_t)\\&p'_e<0,\ \ p''_e>0,\ \ p''_{e,m}<0,\end{aligned}$$

The price of healthcare services decreases with the economy's average level of education - $\overline{e}$, with a decreasing marginal effect, while the parameter m stands for the sensitivity of the price to the educational level. Thus, for higher values of m the price will fall faster with the increase in education. One may interpret the parameter m as the scientific or technological knowledge that is available to the economy from abroad and that complements the investment in education in determining the efficiency of labor in the healthcare sector.

The production of health itself -measured in longevity - is subject to the folloeing function: $\pi(z)\in[\mu,1]$, which increases with the utilization of healthcare services but with a declining marginal return:

(8)
$$\begin{aligned}&for\quad z_t=0;\quad \pi(0)=\mu,\ where:0<\mu<1\\&for\quad z_t>0;\quad \mu<\pi(z_t)<1\\&and:\ \ 0<\pi'(z_t)<\infty\quad \pi''(z_t)<0\quad \lim_{z\to\infty}:\pi(z_t)\le 1\end{aligned}$$

The parameter μ is the base survival probability that the agent faces if she uses no healthcare services at all. Changes in the base survival probability may account for changes in life expectancy that are exogenous to the use of healthcare services, e.g., improvements in lifestyle, environmental conditions, or any other health-related parameters that do not incur direct costs.

3. Agent Optimization

Once we put together all the characteristics of the economy we have defined, the representative agent should solve the following constrained optimization:

$$9)\quad \max_{c,n,v,z} : U\left(c,n,v\right)=\rho\cdot\pi\left(z\right)\cdot u\left(c\right)+\varphi\cdot u\left(n\cdot h\right)$$

$$\begin{aligned}
&s.t:\\
&c=\left(w-p\cdot z\right)\cdot\left(1+r\right)\\
&w=A\cdot\left(1-e-n\cdot\left(F+v\right)\right)\cdot h_{t-1}\cdot e\\
&e+n\left(F+v\right)\leq 1\\
&n,v\geq 0.
\end{aligned}$$

Using the aforementioned constraints, we may reformulate the maximization problem in the following term:

$$9a)\quad \max_{e,v,n,z} : U=\rho\cdot\pi\left(z\right)\cdot u\left\{\left(1+r\right)\cdot\left[A\cdot\left(1-e-n\cdot\left(F+v\right)\right)\cdot h_{t-1}\cdot e-p\cdot z\right]\right\}+\varphi\cdot u\left(n\cdot e^{1-\gamma}\cdot v^{\gamma}\right).$$

Differentiating for the four chosen variables, we obtain the four first-order conditions that yield the following Lemmas:

Lemma 1: The optimal number of children is a decreasing function of parent's educational attainment and the parameters: v, F, γ.

Proof: in Appendix 2

Lemma 1 resembles the competitive uses of time over the fertile and productive period of adult life, which is expressed in the tradeoff that a parent faces between increasing her own income/consumption and increasing their utility from children. It also resembles the quality–quantity tradeoff within the fertility choice as the number of children is negatively related to (a) the total time invested in each one of them and equals - v, F - and (b) the productivity of this time which is determined by parent education - γ.

Lemma 2: The time invested in each child is fixed by the parameters F and γ.

Proof: in Appendix 2

Lemma 2 specifies the quality–quantity tradeoff that was introduced in Lemma 1 as an "indirect tradeoff" only: the number of children declines with the parent's education level, which, in turn, increases the productivity of the fixed time investment in forming each child's human capital (quality). As parent's education rises, the fixed time invested in each child incurs a higher opportunity cost in terms of forgone income. Two parameters affect positively the optimal amount of time invested: (a) the productivity of time in producing child's human capital - γ and (b) the fixed cost of bearing a child makes quantity costly relative to quality. Before moving on, we use the results we explicitly derive in the Appendix 2 to obtain utility $u(v,n,e)$ as a function of the parent's educational attainment only:

$$(10 \qquad u(n,e,v) = u(e) = u\left[\left(e^{1-\gamma} - 2\cdot e^{2-\gamma}\right)\cdot G\right], \qquad where: G = \left(\frac{1-\gamma}{\gamma\cdot F}\right)^{1-\gamma} .$$

Lemma 3: Demand for healthcare services and life expectancy is zero or rises with real income, depending on whether real income is below or above certain thresh-hold level, respectively.

Proof: In Appendix 2

Putting Lemmas #1 and #3 together we obtain proposition 1:

Proposition 1: As long as real income is below the threshold level, higher education means lower fertility. As real income crosses the threshold level, more education is linked to both lower fertility and higher adult life expectancy.

Proof:

Recall that (according to Lemmas #1 and #2) a higher level of education is associated with higher income per-worker, lower price of healthcare services, and lower fertility rate. Therefore if education level increases over time (along generations), the economy may switch from a path of declining fertility to a two-tail demographic aging. This will accure only if real income will surpass the threshold level (depending on the parameters A, m, μ as will be shown later on, in section 4.2). In the next section we examine the conditions for the existence of an increasing path of education level.

At the switch point from zero to positive demand for healthcare services, the increased education and the decreased price of healthcare services reinforce each other. Thus the level of optimal education around the switch point may not be continuous, inducing a kind of threshold effect. In this neighborhood, the total expected utility function W_e is not single-peak with respect to the optimal level of education; it has two peaks, for a low and a high education level. As the productivity of education surpasses the threshold level, the high education peak becomes higher than the low education peak and the switch takes place. For illustration see simulation results in Appendix 3.

4. Dynamics of Growth and Ageing:

Using the results obtained in the first-order optimization conditions, we rewrite agent utility as a function of a single-choice variable - the level of education, e. The optimal education level should maximize the following utility function:

11) $$\underset{e}{Max}: E\{U\} = \rho \cdot \pi(z(e)) \cdot u((1+r) \cdot [w(e) - p(\bar{e}) \cdot z(e)]) + \varphi \cdot u(n(e) \cdot h(e))$$

Plugging the explicit functions $n(e), h(e), w(e)$ into Equation (11) and differentiating with respect to e, we obtain a single first-order condition that defines the solution for the maximization problem of the agents:

$$\begin{aligned} &\rho \cdot \left[\pi'(z) \cdot z' \cdot u(c) + \pi(z) \cdot u'(c) \cdot c'\right] + \\ &+ G \cdot \left[(1-\gamma) \cdot e^{-\gamma} - (2-\gamma) \cdot 2 \cdot e^{1-\gamma}\right] \cdot \varphi \cdot u'\left[\left(e^{1-\gamma} - 2 \cdot e^{2-\gamma}\right) \cdot G\right] = 0 \quad . \end{aligned} \tag{12}$$

By using the FOC with respect to z and setting ρ to be equal $(1+r)^{-1}$, we may simplify the foregoing expression to:

$$\underbrace{\pi(z) \cdot u'(c) \cdot w'}_{\sigma} + \underbrace{G \cdot \left((1-\gamma) \cdot e^{-\gamma} - (2-\gamma) \cdot 2 \cdot e^{1-\gamma}\right)}_{\psi} \cdot \underbrace{\varphi \cdot u'\left(\left(e^{1-\gamma} - 2 \cdot e^{2-\gamma}\right) \cdot G\right)}_{\zeta} = 0 \tag{12a}$$

By assuming rational-expectations equilibrium in the economy, we impose that the average level of education in the economy - $\bar{e}$ - is equal to the optimal level of education chosen by the representative agent- e. Thus, each agent takes into account the expected decrease in the price of healthcare services if the level of education rises [7] Hence, each level of education e_t^* that solves Equation (12a) sustains equilibrium.

4.1 Converging paths of increasing educational level

Proposition 2: Converging dynamic paths of increasing educational level do exist.

Proof:

Existence of equilibrium: We focus on interior solutions for the agents' optimization that define the equilibrium level of education. We express Equation (12a) as: $\sigma + \psi \cdot \xi = 0$, which is the sum of the marginal effect of educational attainment on the

[7] This consideration is not explicit in Equation (12a) but is implicit in the computed optimal amount of z (and c) out of any possible chosen income.

utility of consumption and on the utility of children. Since σ and ξ are always positive, a necessary condition for an interior solution is - $\psi < 0$, which means:

$$e > \frac{(1-\gamma)}{(2-\gamma)\cdot 2}$$

This condition means that the effect of a marginal increase in education on the total utility of children (i.e., on the product of optimal number and quality) is negative. However, it is already contained in the non-negative income constraint, defined in Appendix 1. By imposing another necessary condition for an interior solution—a positive number of children (defined in Appendix 1) — we obtain lower and upper bounds for the optimal level of education, denoted respectively as e_l and e_u:

$$e_l = \frac{(1-\gamma)}{(2-\gamma)} < e^* < \frac{1}{2} = e_u \ .$$

Within this range, the right term in Equation (12a) is negative (i.e. ;$\psi \cdot \xi < 0$) and decreases with the level of education, while its left term is positive (i.e.$\sigma > 0$). In addition, as education approaches its lower bound ($e \to e_l$), the right-hand side of Equation (12a) and the entire expression in Equation (12a) approach infinity ($\sigma \to \infty$). Accordingly, as education approaches its upper bound ($e \to e_u$), the left-hand side of Equation (12a) and the entire expression in Equation (12a) approach negative infinity $(\psi \cdot \xi \to -\infty)$. Hence there is at least one interior solution within the define range. We will assume that it is unique.

Existence of steady states: For the existence of the steady state, let us examine the explicit term for w'_{e_t} (the partial derivative of income with respect to the level of education), that appears on the left side of Equation (12a):

$$w'_{et} = e_{t-1}{}^{1-\gamma} \cdot v^{\gamma} \cdot A \cdot 2 \cdot e_t \cdot \left(\frac{2}{\gamma} - 1 \right) > 0$$

Note that changes in the productivity parameter - A can perfectly compensate for changes in the level of parents' education - e_{t-1} in determining the value of w'_{e_t} and the level of total income, which determines uniquely the value of the other terms in the expression: $\sigma = \pi(z) \cdot u'(c) \cdot w'$. Hence, since this is the only term where e_{t-1} appears in the equation, each interior solution that is not a steady-state equilibrium $e_t^* \neq e_{t-1}$ for a given value of A is a steady-state equilibrium for another specific value of A, denoted by A', which maintains : $A' \cdot e_t^* = A \cdot e_{t-1}$

Increase and convergence of optimal educational level: For a given equilibrium, an increase/decrease in the level of parents' education alone will increase/decrease the value of w'_{e_t} and will increase the overall marginal productivity of a chosen education level on the utility of consumption. Since the (whole) left side of equation (12a) decreases around the equilibrium level, the return to optimization is achieved by raising the level of education. Since there is an upper bound for the level of education, the increase in educational level must converge to zero. In Appendix 3 we illustrate by simulation the existence of the discussed dynamics.

4.2 Comparative Statics:

The effect of the productivity parameter A on the optimization condition -and thereby on the optimal level of education- is equivalent to the effect of parent's level of education. Thus, a higher value of A increases the optimal level of education along the convergence process and in the steady state.

The effect of base survival rate μ depends on whether the agents utilize healthcare services. Recall that (according to Proposition 3) the demand for healthcare services is zero or it rises with real income $\frac{w}{p}$ depending whether it is below or above a critical minimum level, which is positively dependent on μ. As long as the demand for healthcare services is zero, a higher μ increases the marginal productivity of education through an increased expected marginal utility of consumption. Thus, in this case a higher value of μ will be a followed by higher optimal level of education and lower fertility, as in other models of exogenous life expectancy. In a case where demand for healthcare services is positive a higher μ decreases the marginal productivity of education and, in turn, the optimal education level. This is a kind of crowding-out effect of the free base survival rate on the costly investment in health, due to a decrease in the marginal productivity of healthcare utilization.

The effect of parameter m is ambiguous even along a path of positive demand for healthcare services. Recall that parameter m resembles the sensitivity of price to level of education. According to the functional assumptions, for low/high levels of healthcare utilization an increase in m may increase/decrease the marginal productivity of education on life expectancy and consumption due to the decreasing productivity of healthcare services in the survival function. See simulation results that illustrate our analysis in Appendix 3.

4.3 Intergenerational Exchange —"IGE" – of Healthcare Services:

In the real world elderly people buy healthcare services from young-generation workers. Therefore, it seems very natural to expand the analysis of the model to include this intergenerational exchange that is abstracted from existing OLG model of endogenous longevity.

Proposition 3: The steady-state equilibrium of an intergenerational-exchange economy coincides with the one introduced in the basic model.

Proof:

In an IGE economy, the agents save for their healthcare expenditures in old age. We assume that these savings do not yield interest – otherwise the possibility to postpone health expenditure with postponing the utilization of healthcare services induces an income effect. The utilization of healthcare—i.e., the IGE—occurs just as the periods turn from $t-1$ to t. Each agent who was born in period $t-1$ buys z_t amount of healthcare services and then only $\pi(z_t)$ of such agents survive. Hence, agents should optimize their level of education and consumption allocation according to the expected price of healthcare services in their old age (which will be determined by the educational attainment of their successors). At steady state, the expected price level and the current price level coincide. Thus, the same level of education will obtain in steady state with and without IGE.

Although the possible steady states of the system are identical to those that would prevail in the absence of intergenerational exchange, the analysis of the dynamics and convergence for intergenerational exchange is more complex because it relies on perfect-foresight rational (fulfilled) expectations. The expectations in our model pertain to the education level of future generations but are analogous to the future interest-rate expectations that rest at the core of standard OLG models based on the classic work of Diamond (1965) with intergenerational exchange in capital markets.

Hence, it can be shown that if future decrease in the price of healthcare services - which is equivalent to increase in the parameter m - has a negative effect on education a unique equilibrium exists in an IGE economy, with lower levels of education for each generation - relative to an economy with no IGE. Thus, convergence will be slower in this IGE economy. In the case where a future decrease in the price of healthcare services prompts the current generation to invest more in education, a sufficient condition for the existence of a unique inter-temporal equilibrium is: $e_t'\left(E\{e_{t+1}\}\right)<1$. The term $e_t'\left(E\{e_{t+1}\}\right)$ is the derivative of optimal education level with respect to expected level of education in next period (in response to its effect on future price of healthcare). At this equilibrium, the education level of the each generation is higher than in an economy with no IGE; thus, the convergence is faster.

5. Health-Tax Policy

5.1 Suboptimal Health-Tax:

In many developed economies (e.g., Canada, U.K., Sweden, and Israel), the healthcare sector is largely public and financed by an income tax. Although usually there is room for private provision of healthcare services in these countries, the private sector is severely restricted in size and in the types of activities allowed to it, and therefore at least the main part of medical services provision is supplied solely by the public sector. In this section we assume that the healthcare sector is purely public.

To abstract from the negative (externality) effect of marginal income tax rate on labor supply, let us assume a lump sum health-tax signed by τ, so the provision of per-capita healthcare services is: $z=\dfrac{\tau}{p\left(\overline{e}\right)}$.

The first three optimization conditions that yield lemmas #1 and #2 - regarding the optimal number of children and the optimal investment in each child - do not change, and the fourth condition - regarding the optimal demand for healthcare services - is no longer relevant. The optimal tax, chosen by a social planner to maximize the utility of agents in each generation, will coincide with the healthcare expenditure that is optimally chosen by the agents in the decentralized economy. However, it is of interest to investigate the effects of (small) deviations form this optimal tax level.

Proposition 4: Along an increasing path of health expenditures' share, health tax higher (lower) than the optimum will raise (lower) steady state's level of education[8].

Proof:

Recall that the optimal level of education increases with the productivity parameter - A, and that A and e_{t-1} are compensating elements in the optimization condition. Hence if the economy is on a rising path of optimal health expenditures - which also means that the optimal health expenditures increases with A - and the health tax is higher then optimal, the steady state may be achieved only in higher level of education – where higher e_{t-1} effect the optimality condition as higher level of A.

In Appendix 3 we present simulation results that illustrate the non-monotonic effect of health tax size on the level of education in steady state.

5.2 Pay-As-You-Go Finance of Public Healthcare Services:

Countries that imply health tax usually apply a "Pay As You go" (PAYG) regime, in which the young (workers) pay for the utilization of healthcare services by the elderly (retirees).

[8] The determinants of health expenditures share are defined in the proof for Lemma 3.

Proposition 5: For a steady positive population growth, a Pay As You Go health tax regime is associated with higher educational level – i.e. higher product per capita and life expectancy, and lower fertility.

Proof:

Assume that the tax level is set to be equivalent to the decentralized optimizer, i.e. it resembles the Fully Funded (FF) regime. In this case, under PAYG regime the amount of healthcare services each agent utilizes depends on the (average) fertility of her generation. Hence the amount of healthcare services each agent receives becomes: $z = \frac{n \cdot \tau}{p(\bar{e})}$. For positive population growth (i.e. $n > 1$), the switch from FF to PAYG system increases the amount of healthcare services each agent receives for a given level of education, and therefore it increases the expected marginal utility from consumption for each level of education. Thus a switch to PAYG regime increases the optimal level of education, and by that it increases both per-capita income and population's age in the steady state, and decreases the price of healthcare services.

Note that introducing into the model social security policy which applies risk sharing for foregone savings or PYAG finance regime would imply the opposite effects. Namely, the possibility to annuities saving and the PAYG social security regime will induce income effect results in a lower level of education which means a lower per-capita income and younger population.

6. *Concluding Remarks*

We have developed a model that reproduces the observed dynamics of two-tail ageing process and growth. The model proposed that evolvement of these dynamics is dependent on the progress of medical technology which enables to prolong adults and elderly life expectancy during retirement. We expand the existing OLG models of growth and demography by incorporating a simultaneous optimization of fertility (i.e., quality and quantity of children), longevity and educational choice. We have validated our results to the very realistic structure of intergenerational exchange in the healthcare sector, which is abstract from existing related literature. The analysis of health-tax policy implies that the effect of suboptimal health taxation is dependant on the trend of optimal health-tax along an optimized path. Moreover we find that the effect of PAYG finance of health-tax on education and demographic age in steady-state is opposite to the one of PAYG finance of social security. Relatively to the vast literature on social security policy, little work has been done concerning health policy in a macroeconomic framework and the interaction between the two. Our results encourage further research within these lines.

Appendix 1: Elaborating the Price and Wage Equations

Using Equations (5a) and (5b) and assuming a small open economy, we obtain the following labor-to-capital ratio for each sector:

$$C = a(\bar{e}) \cdot L_c^{\alpha} \cdot K_c^{1-\alpha} = a(\bar{e}) \cdot (N_C \cdot l)^{\alpha} \cdot K_c^{1-\alpha} \Rightarrow MPk_c = (1-\alpha) \cdot a(\bar{e}) \cdot \left(\frac{l}{k_c} \cdot\right)^{\alpha} = r$$

$$Z = b(\bar{e}) \cdot L_z^{\beta} \cdot K_z^{1-\beta} = (N_z \cdot l)^{\beta} \cdot K_z^{1-\beta} \Rightarrow MPk_z = p \cdot (1-\beta) \cdot b(\bar{e}) \cdot \left(\frac{l}{k_z} \cdot\right)^{\beta} = r$$

where k_i is capital per worker in the sector, $i = (c, z)$, and the marginal productivity in the healthcare sector is expressed in terms of a consumption good (i.e., multiplying by the price of healthcare services p). Since both sectors share the same interest rate, we obtain:

$$p = \frac{(1-\alpha)\cdot a(\bar{e})}{(1-\beta)\cdot b(\bar{e})}\cdot l^{\alpha-\beta}\cdot\frac{k_z^{\beta}}{k_c^{\alpha}}$$

Wages in each sector are equal to the marginal productivity of the respective sectors' labor:

$$\omega_c = \alpha\cdot a(\bar{e})\cdot\left(\frac{k_c}{l}\right)^{1-\alpha} \Rightarrow w_c = \alpha\cdot a(\bar{e})\cdot l^{\alpha-1}\cdot k_c^{\,1-\alpha}$$

$$\omega_z = p\cdot\beta\cdot b(\bar{e})\cdot\left(\frac{k_z}{l}\right)^{1-\beta} \Rightarrow w_z = p\cdot\beta\cdot b(\bar{e})\cdot l^{\beta-1}\cdot k_z^{1-\beta}$$

The general equilibrium requires wage equity across the two sectors, hence:

$$p = \frac{\alpha\cdot a(\bar{e})}{\beta\cdot b(\bar{e})}\cdot l^{\alpha-\beta}\cdot\frac{k_c^{1-\alpha}}{k_z^{1-\beta}}$$

Equalizing the two price equations, we find that capital per worker in the healthcare sector is a constant proportion of the capital-per-worker ratio in the consumption sector:

$$\frac{(1-\alpha)\cdot\beta}{(1-\beta)\cdot\alpha} = \frac{k_c}{k_z} \Rightarrow k_z = D\cdot k_c \qquad where:\; D = \frac{(1-\beta)\cdot\alpha}{(1-\alpha)\cdot\beta}$$

Plugging the last equality into the first price equation, we obtain:

$$p = \frac{(1-\alpha)\cdot a(\bar{e})\cdot D^{\beta}}{(1-\beta)\cdot b(\bar{e})}\cdot\left(\frac{l}{k_c}\right)^{\alpha-\beta}$$

Using the first interest rate—MPk_c—condition to express $\frac{l}{k_c}$ in terms of the parameters of the model and the interest rate, we obtain:

$$MPk_c = (1-\alpha)\cdot a(\bar{e})\cdot\left(\frac{l}{k_c}\cdot\right)^{\alpha} = r \Rightarrow \frac{l}{k_c} = \left[\frac{r}{(1-\alpha)\cdot a(\bar{e})}\right]^{\frac{1}{\alpha}}$$

Plugging the above term of $\dfrac{l}{k_c}$ into the last price equation, we obtain:

$$p=\frac{(1-\alpha)\cdot a(\bar{e})\cdot D^{\beta}\cdot r^{\frac{\alpha-\beta}{\alpha}}}{(1-\beta)\cdot b(\bar{e})\cdot(1-\alpha)^{\frac{\alpha-\beta}{\alpha}}}\cdot a(\bar{e})^{\frac{\beta}{\alpha}-1}=H\cdot\frac{a(\bar{e})^{\frac{\beta}{\alpha}}}{b(\bar{e})},\quad where:\ H=\frac{(1-\alpha)\cdot D^{\beta}\cdot r^{\frac{\alpha-\beta}{\alpha}}}{(1-\beta)\cdot(1-\alpha)^{\frac{\alpha-\beta}{\alpha}}}\cdot$$

By plugging the same term for $\dfrac{l}{k_c}$ into the first wage equation, we may define income from labor as:

$$\omega=\alpha\left(\frac{k_c}{l}\right)^{1-\alpha}=\alpha\cdot\left[\frac{a(\bar{e})\cdot(1-\alpha)}{r}\right]^{\frac{(1-\alpha)}{\alpha}}\Rightarrow w=\alpha\cdot\left[\frac{a(\bar{e})(1-\alpha)}{r}\right]^{\frac{(1-\alpha)}{\alpha}}\cdot l=A\cdot a(\bar{e},)^{\frac{1-\alpha}{\alpha}}\cdot l$$

Appendix 2: Proofs for Lemmas 1-3:

The proofs are derived along with the elaboration of the 4 first order conditions.

1. Differentiating for e, we obtain:

$$\rho\cdot\pi(z)\cdot u'(c)\cdot(1+r)\cdot\left[h_{t-1}\cdot A\cdot e\cdot\left(-\frac{1}{e}+2+\frac{1}{e}n\cdot(v+F)\right)\right]=u\left(n\cdot e^{1-\gamma}v^{\gamma}\right)$$

$$e\cdot\left(-\frac{1}{e}+2+\frac{1}{e}n\cdot(v+F)\right)$$

$$e=\frac{1-\left(\dfrac{n^2F}{(n-\gamma)}\right)}{2}$$

$$\frac{2n^2F(n-\gamma)-n^2F}{(n-\gamma)}=\frac{F(n-\gamma)}{(n-\gamma)^2}=\frac{F}{(n-\gamma)}$$

$$v=\frac{F}{\left(\dfrac{n}{\gamma}-1\right)}$$

Optimum level of education e should equalize the marginal effect of education on the utility of consumption occasioned by increasing income to its effect on the marginal utility of children due to the positive effect on children's human capital.

2. Differentiating for n, we get:

$$\rho\cdot\pi(z)\cdot u'(c)\cdot(1+r)\cdot A\cdot h_{t-1}\cdot(F+v)\cdot e=v^{\gamma}\cdot\varphi\cdot u'\left(n\cdot e^{1-\gamma}v^{\gamma}\right)$$

$$\rho\cdot\pi(z)\cdot u'(c)\cdot(1+r)\cdot A\cdot h_{t-1}\cdot(F+v)\cdot e=v^{\gamma}\cdot\varphi\cdot u'\left(n\cdot e^{1-\gamma}v^{\gamma}\right)$$
$$\rho\cdot\pi(z)\cdot u'(c)\cdot(1+r)\cdot A\cdot h_{t-1}\cdot n\cdot e=\gamma\cdot n\cdot v^{\gamma-1}\cdot\varphi\cdot u'\left(n\cdot v^{\gamma}\right)$$
$$v=\frac{\gamma\cdot F}{(n-\gamma)}$$

The optimum number of children equalizes the marginal increase in utility from the marginal child to the marginal loss of utility from consumption due to the time devoted to each child, which has an opportunity cost in working time, i.e., income. Dividing Condition 1 by Condition 2, we obtain Lemma 1:

$$n=\frac{1-2\cdot e}{\gamma\cdot(v+F)}$$

A positive number of children requires: $e<\frac{1}{2}$.

Computing agents' income after plugging in optimal number of children, we obtain:

$$w=A\cdot h_{t-1}\cdot e\cdot\left(\frac{(\gamma-1)+(2-\gamma)\cdot e}{\gamma}\right)$$

Hence, for the wage to be positive, e should satisfy: $e>\frac{(1-\gamma)}{(2-\gamma)}$.

3. Differentiating for v, we get:

$$\rho\cdot\pi\left(z\right)\cdot u'\left(c\right)\cdot\left(1+r\right)\cdot A\cdot h_{t-1}\cdot n\cdot e=\gamma\cdot n\cdot e^{1-\gamma}\cdot v^{\gamma-1}\cdot\varphi\cdot u'\left(n\cdot e^{1-\gamma}\cdot v^{\gamma}\right)$$

$$\rho\cdot\pi\left(z\right)\cdot u'\left(c\right)\cdot\left(1+r\right)\cdot A\cdot h_{t-1}\cdot n\cdot e=\gamma\cdot n\cdot v^{\gamma-1}\cdot\varphi\cdot u'\left(n\cdot v^{\gamma}\right)$$

Optimizing the time invested in each child equalizes the marginal utility of the increase in the child's human capital to the marginal loss of utility due to forgone income and consumption.

Combining Conditions 2 and 3, we obtain Lemma 2:

$$\left(F+v\right)=\frac{v}{\gamma}\Rightarrow v=\frac{F}{\frac{1}{\gamma}-1}$$

4. Differentiating for z, we obtain:

$$\pi'\left(z\right)\cdot u\left(c\right)=\left(1+r\right)\cdot p\cdot\pi\left(z\right)\cdot u'\left(c\right)$$

This first-order condition requires agents to equalize the benefit from utilizing a marginal unit of healthcare service (on the left) to its opportunity cost (the expected marginal utility of consumption), and it allows us to characterize the demand for healthcare services (and life expectancy) as follows: For the allocation of time uses to be optimal, marginal expected utility of consumption must be equal to the marginal utility of children. Due to the convexity of the utility function, if an internal solution exists, it must true be that more education leads to higher income and greater expected utility (and lower expected marginal utility) of consumption, at the expense of lower utility (and higher marginal utility) of children. This means that at least one of the goods—the consumption good and/or healthcare services—is a normal good.

Recalling the survival probability function in (8), As long as $\mu > 0$ and $\pi'(z) < \infty$ (i.e., the base survival probability is positive and the marginal productivity of healthcare services in the survival function is finite), if real income in terms of the price of healthcare services ($\frac{w}{p}$) is low enough a corner solution of income allocation exists, in which demand for healthcare services is zero. The above first order condition may be translated into terms of elasticity:

$$\frac{\pi'(z)}{\pi(z)} = (1+r)\cdot p \cdot \frac{u'(c)}{u(c)} \Rightarrow \frac{\eta_{\pi,z}}{(1+r)\cdot p \cdot z} = \frac{\varepsilon_{u,c}}{c} \Rightarrow \frac{\eta_{\pi,z}}{(1+r)\cdot \varepsilon_{u,c}} = \frac{p \cdot z}{c}$$

where $\eta_{\pi,z}$ is the elasticity of life expectancy with respect to the utilization of healthcare services and $\varepsilon_{u,c}$ is the elasticity of utility with respect to consumption. This equation defines the optimal share of health expenditures in income. As long as $\varepsilon_{u,c}$ decreases faster then $\eta_{\pi,z}$, the share of expenditures will rise as income increases. For a detailed discussion on this issue see Hall and Jones (2007).

One can see that for low real income the left side of the Equation may be greater than the right side when the utilization of healthcare services is zero. This means that as long as real income is below some threshold level, it is not worthwhile to increase life expectancy (and prolong consumption's horizon); instead, it is worthwhile to utilize the base survival rate by consuming all income. This threshold level of real income is positively dependent on the base survival rate.

However, as real income surpasses this threshold level the demand for consumption and for healthcare services should change commensurably at a given price, meaning that higher (lower) income will increase (decrease) the demand for both goods, i.e., both goods are normal.

Appendix 3: Simulations

To demonstrate the existence of the increasing and converging path of the education level, we perform a simulation, assuming the following functional forms:

Relative price of health services: $p_t(\bar{e}_t) = 0.5^{m \cdot \bar{e}_t} \qquad (m > 0)$

Survival function: $\pi(z) = \mu + \dfrac{(1-\mu)\cdot z}{z+1} \qquad (0 < \mu < 1)$

Utility function: $u(c) = \dfrac{c^{1-\phi}}{\phi} \quad \phi = \dfrac{1}{2} \; u(c) = 2\sqrt{c}$

This functional design allows us to formulate an explicit demand for healthcare services:

$$D_Z = \begin{cases} \dfrac{w}{p} > \dfrac{\mu}{2\cdot(1-\mu)} & \sqrt{\dfrac{w\cdot 2\cdot(1-\mu)}{p} + \dfrac{(3-\mu)^2}{4} - \mu} - \dfrac{(3-\mu)}{2} \\ \dfrac{w}{p} \le \dfrac{\mu}{2\cdot(1-\mu)} & 0 \end{cases}$$

We use the following parameter values: $\mu = 0.6, r = 0.05, \gamma = 0.8, F = 0.05, A = 11, m = 11$. The qualitative results of this simulation are stable for a large range of values around this baseline set. The results of the simulation, presented in Figure 1, illustrate the existence of an increasing and converging level of education that defines the optimal paths of all other variables. Life expectancy, consumption, and demand for healthcare services increase while fertility decreases. The simulation yields a fast convergence; it takes about five periods for the variables to reach the steady-state level.

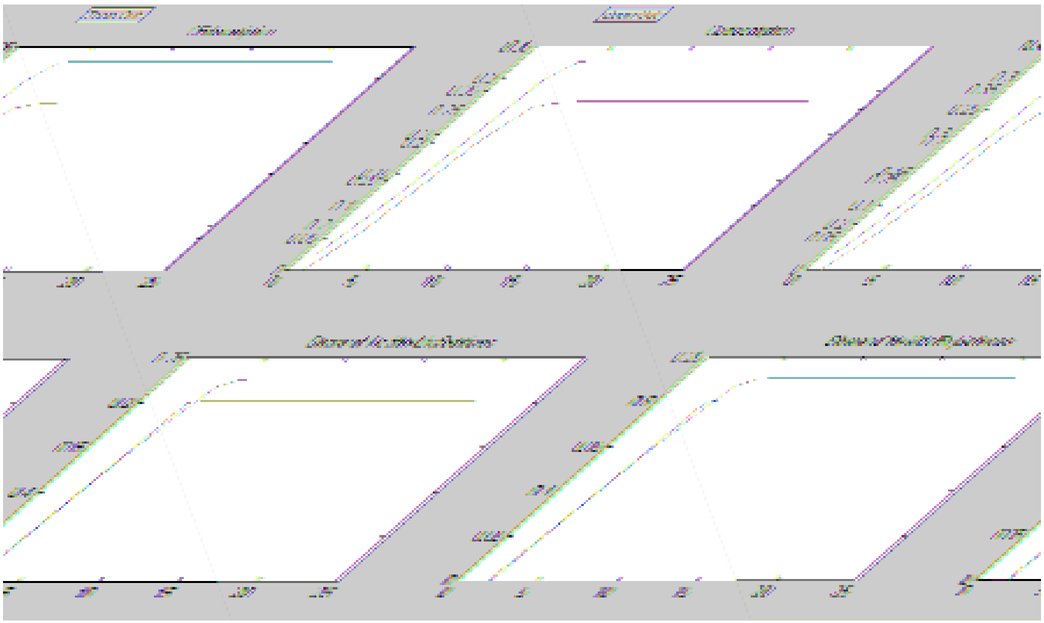

Figure 1

For low (enough) values of the education productivity parameters A and m, demand for healthcare is zero and education sustains a low-level steady state (due to weak incentive to invest in education). From some threshold of these parameters, optimal education and demand for healthcare services jump to a high-level steady state. (The higher the base survival rate, the higher the threshold level of parameters A and m.) For the chosen set of parameterization, this occurs where (both) m and A approximate the value of 10. In the neighborhood of this critical value, the utility function U does not have a single peak, as Figure 2 illustrates. Recall that the parameter A and the level of parents education e_{t-1} has the same effect on the utility thus the increase in A illustrate increase in e_{t-1} along possible growth path. The graphs below show the jump in the optimal level of education around the critical values of A and m—the threshold effect—and the non-concavity of U at this range. On the horizontal axis is the level of education and on the vertical axis is the utility level, with the level of education imposed to be equal for parents and children—i.e., steady state.

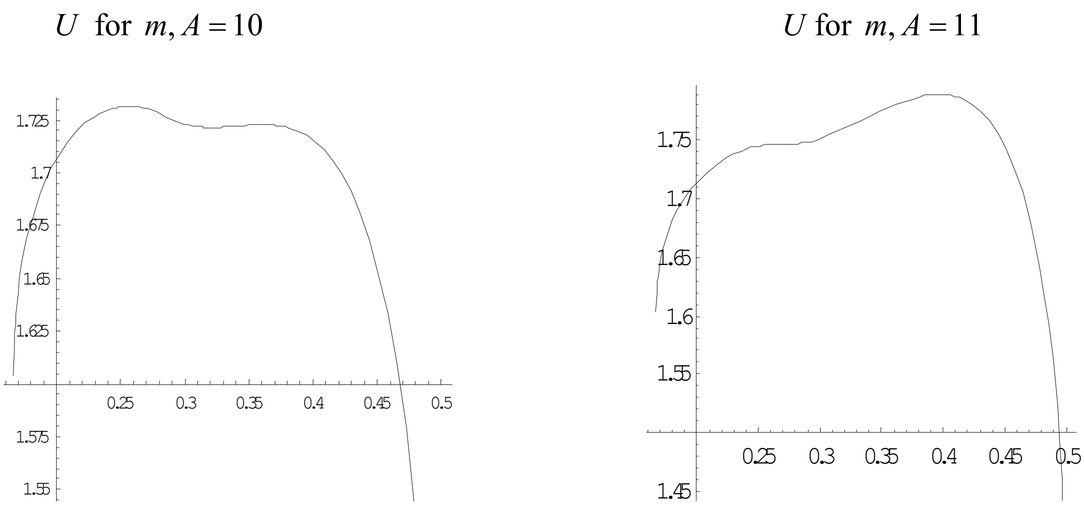

Figure 2

Figures 3, 3a, and 3b demonstrate the effects of parameters - m , A and μ (on the horizontal axes) - on the education level in steady state (on the vertical axes), as analyzed in Section 4.2. The threshold levels of the parameters (around the value of 10) are easy to identify in these figures.

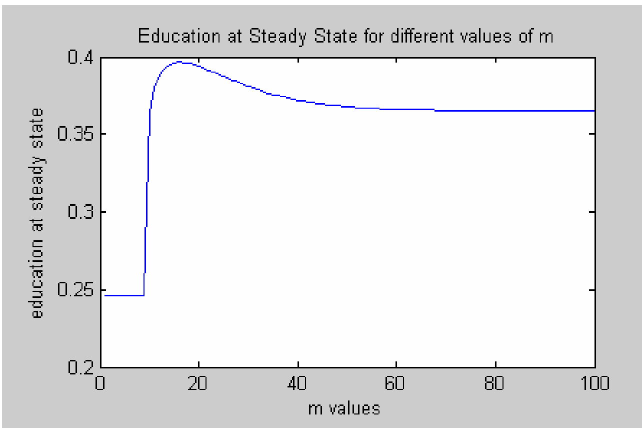

Figure 3

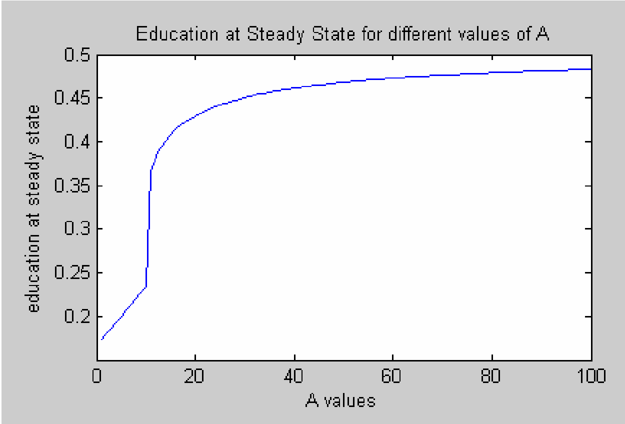

Figure 3a

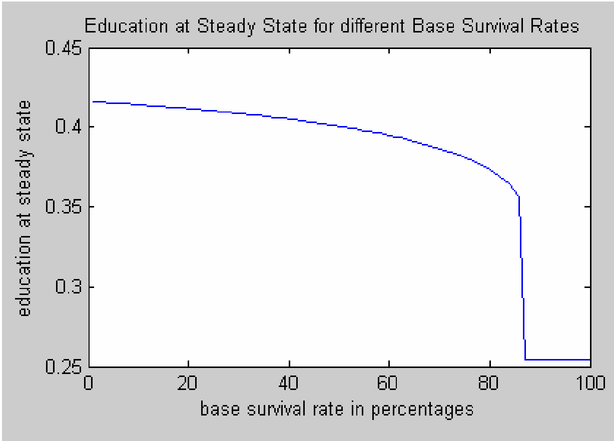

Figure 3b

Figure 4 shows simulation results of the education level in steady state (vertical axis) at different levels of health tax (horizontal axis).

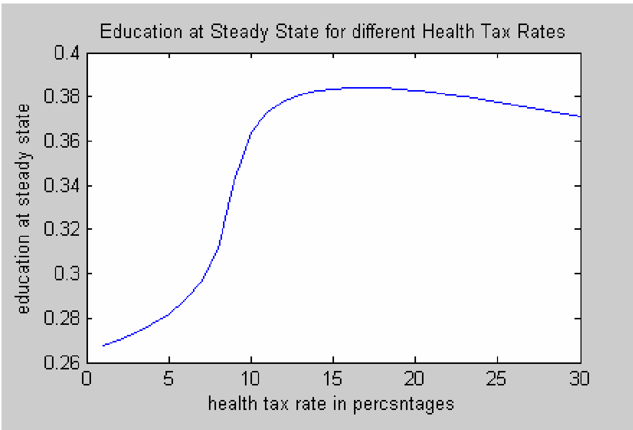

Figure 4

References:

Blackburn, K., and Cipriani, G. P., 2002. “A Model of Longevity, Fertility and Growth”. *Journal of Economic Dynamics and Control* 26, 187–204.

Cerda, Rodrigo A., 2002. "The Economic Foundations of Demographic Transition". PUC Economics Institute Working Paper No. 208.

_______________, 2007. “Endogenous Innovation in the Pharmaceutical Industrial”. *Journal of Evolutionary Economics* 17, 473-515.

Chakraborty, Shankha, 2004. “Endogenous Lifetime and Economic Growth”. *Journal of Economic Theory* 116, 119–137.

Cutler, David M., McClellan Mark, Newhouse Joseph P., Remler Dahlia, 1998. "Are Medical Prices Declining? Evidence from Heart Attack Treatments". *Quarterly Journal of Economics* 113, 991-1024.

Cutler, David M., and Meara, Ellen, 2001. “Changes in the Age Distribution of Mortality over the 19th Century”. NBER Working Paper 8556.

Cutler, David M., McClellan Mark, 2001. "Productivity Change in Health Care". *American Economic Review* 91, 281-286.

Diamond, Peter A., 1965. “National Debt in a Neoclassical Growth Model”. *American Economic Review* 55, 1126–1150.

Finlay, Jocelyn, 2005. “Endogenous Longevity and Economic Growth”. Unpublished working paper, School of Economics, Australian National University.

Fogel, Robert W., 1994. “Economic Growth, Population Theory, and Physiology: The Bearing of Long-Term Processes on the Making of Economic Policy”. *American Economic Review* 84, 369–395.

Galor Oded, 2004. "The Demographic Transition and the Emergence of Sustained Economic Growth." Brown University, Department of Economics Working paper 2004-13.

Hall Robert E., and Jones Charles I., 2007. "The Value of Life and the Rise in Health Spending". *Quarterly Journal of Economics* 122 , 39-72

Livi-Bacci, Massimo, 2000. *The Population of Europe,* Blackwell.

Mokyr, Joel, 1990. *The Lever of Riches,* Oxford University Press.

Pestieau Piere, Ponthiere Georgy, and Sato Motohiro, 2006 "Longevity and Pay As You Go Pensions". Unpublished working paper.

Sanso Marcos, and A'isa Rosa M., 2006, "Endogenous Longevity, Biological Deterioration and Economic Growth". *Journal of Health Economics* 25, 555-578.

Van Zon, Adriaan, and Muysken Joan, 2001. "Health and Endogenous Growth**".** *Journal of Health Economics* 20, 169–185.