A Financial Accelerator Based on Payout as a Signal of Firm Value*

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Abstract

The paper examines whether signaling frictions associated with firms' cash distributions to shareholders can account for aggregate labor volatility. A financial accelerator model is presented in which the firm's dividend and share buybacks are rationally used by financial intermediaries to estimate the firm's value. Because the estimated value of the firm determines its ability to borrow, the management of the borrowing constraint becomes a driver of the firm's payout policy. In the case of negative productivity shocks, firms subdue the transmission of the shock to payout in order to preserve lenders' image of them, at the cost of an excessive and inefficient contraction of production. In contrast, positive productivity shocks are paid out immediately and do not generate irregular production expansions. In the first stage, a firm-value estimation function that is consistent with rational expectations is derived and its implications for the firm's labor demand are studied. It is then shown that a financial accelerator model which allows the firm to borrow less than the borrowing limit is not only able to reproduce the volatility of US work hours but also their asymmetry around the mean.

Keywords: Signaling; Financial Accelerator; Credit Constraints; Labor Demand; Labor Volatility.

JEL Codes: D82, E32, G35, J23.

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1 Introduction

It is widely believed among economists that the operation of the credit markets is central to the workings of the real economy. A quantitatively appealing way to express this view is through a financial accelerator Real Business Cycle (RBC) model along the lines of **?** or **?**. In such models, producers are credit-constrained, i.e. their borrowing depends on their ability to post collateral. In turn, the credit they are able to obtain determines the economy's level of investment and production. This framework stands in contrast to the frictionless world of **?**, in which all profitable investment and production opportunities are exploited and the form of financing is irrelevant.

Figure 1 presents the credit flow into the US non-financial corporate sector.[1] The sharp slowdowns during recessions suggest that reductions in credit availability may play a role in exacerbating economic contractions. This motivates the study of credit as a key macroeconomic variable.

Details vary from one financial accelerator model to the next, but a few common themes can be identified. First, producers are given an incentive to borrow. For example, in **?**, which is the closest model to the one presented here, a tax incentive makes credit financing cheaper than equity financing. Second, asymmetric information problems are put in place so that producers are unable to borrow enough to finance every profitable investment opportunity. Finally, any alternative financing route, most notably equity, is either assumed away or is rendered inflexible in an ad-hoc manner.

Naturally, financial accelerator models are prime candidates for studying the impact of shocks that originate in the financial system. Recently, several working papers have indicated that the empirical estimation of financial accelerator models may assign considerable weight to financial shocks (**?**; **?**; **?**). In addition, the inclusion of a propagation mechanism for financial shocks should enable an investigation of the appropriate policy response.

All financial accelerators work by transforming small disturbances into large fluctuations. One measure for the level of amplification needed is to require that the sizable volatility of work hours would be reproduced in a model in which the sole input is productivity disturbances, which are relatively minute. This requirement is in the spirit of **?**'s requirement that a search and matching model should be able to generate the volatility of unemployment. Like search and matching, RBC models typically produce too little work hours volatility. The usual method by which this volatility is driven to plausible levels is by assuming an intertemporal elasticity of labor supply that is higher than that indicated by micro-studies (**?**).

[1] Source: US Flow of Funds, F101, line 28.

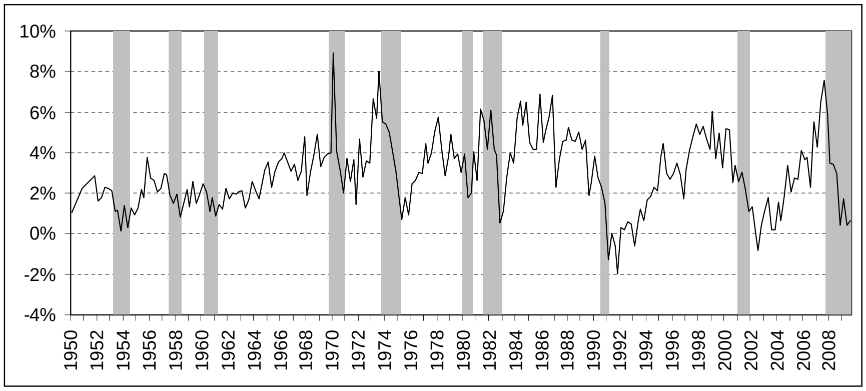

Figure 1: US non-financial corporate credit-liability quarterly increase as percent of 3-year trailing average GDP (annualized rate, recessions are highlighted in gray).

By including a financial accelerator we can allow labor supply remain relatively inelastic and shift our attention to labor demand. If labor demand is governed by available credit and if available credit is sufficiently volatile because of credit-related amplification mechanisms, it may provide the model with a labor volatility that is consistent with the empirical evidence. The financial accelerator model to be presented is able not only to reproduce the volatility of hours, but also their substantial asymmetry around the mean.

Furthermore, this paper seeks to assert if the needed amplification could be generated using relatively plausible micro-foundations. Specifically, the model attempts to relax the strong assumptions about equity financing adopted in the literature. The common practice is to assume equity financing is not available or is associated with ad-hoc costs that render it unable to respond to shocks. As a frictionless pro-cyclical payout would have the effect of eliminating all financial distress, these assumptions, though they are implausible, are key to generating the model's amplification. **?**, for example, assume that when payout moves away from its steady-state level, the firm incurs a real cost that rises quadratically with the deviation of payout from its steady-state. The level of amplification is thus determined by the parameter that states the rate by which the real costs rise.

The rationale **?** cite for the quadratic cost assumption relies on stock issuance costs and asymmetric information costs. For stock issuance costs it can be argued that the representative firm of a well calibrated model will always pay a positive payout since this is the case in the aggregate economy (**?**). Thus, the representative firm never needs to issue stock. As for asymmetric information problems, many microeconomic models have been suggested in order to describe why a firm may seek to smooth payout (for a review see **?**); however, it is unclear why these would produce the assumed cost structure. Indeed, one of the original goals of this study was to see if this structure is reasonable in a general equilibrium setting.

In the model presented, the ad-hoc rigidity in payout is replaced by a signaling-associated friction in equity financing. The collateral backing the firm's borrowing is the value of the firm, which will be received by lenders, net of bankruptcy costs, in the event of default. Credit markets are operated by financial intermediaries whose role is to ensure that firms do not borrow more than a certain fraction of their value. The model abstracts from the rise in borrowing costs that could sometimes compensate lenders for higher expected default risk. Instead, we assume a collateral constraint that mimics the effect of credit rationing. However. The firm's true condition is observable to financial intermediaries only with a one period delay. Therefore they must estimate the firm's current value from the payout, which is the only firm-specific variable available to them on a current basis.

Naturally, financial intermediaries perceive movements in the firm's current payout policy as a signal of changes in the firm's value. This generates a friction in equity financing whereby the reduction of a firm's payout leaves more money for operations, but reduces the perceived value of the firm. As the firm's perceived value drops, financial intermediaries call back outstanding loans. Management knows that only a certain fraction of every dollar cut from payout becomes available for operations and thus it is more restrained in lowering its payout.

The model rests on the assumption that financial intermediaries use a particular and publicly-known function to estimate the firm's value. The function takes as inputs the firm's current payout and last-period true value. The form and coefficients of the value estimation function are known to management, which selects its payout in order to respond optimally. In practice, this is achieved by including the derivatives of the value estimation function in the firm's first-order conditions. The firm's objective and first-order conditions are presented in Section 2, along with the rest of the model.

To have any merit, a value estimation function must generate estimates that are close to the true value of the firm. If a function predicts the firm's value correctly, it is considered to be a rational value estimation function. Searching for such a function and studying its implications is the focus of Section 3.

To facilitate the search, it is assumed that the value estimation function linearly links percentage deviations from market-wide averages. This implies a log-log regression-like functional form. Its inputs are the deviations of the firm's payout and last-period true value from market averages. Its output is the deviation of current firm value from the market average. The function's parameters represent the elasticity of estimated value with respect to payout and the last-period true value.

According to the assumed information structure, it is difficult to estimate the value of a specific firm, but the current true value of the entire stock market is always known to financial intermediaries. The rationale is that if obtaining the true value of each specific firm is associated with some fixed cost and the firm is small, (in the model all firms have zero mass), financial intermediaries would not find it worth while to uncover the firm's true value. However, even if the cost of obtaining the true value of the stock market is large, which is unlikely because firm specific factors should generally average out, financial intermediaries can profit greatly from having this information and would be willing to invest heavily in obtaining it. As it turns out, in the model, the cost of obtaining the true value of the stock market amounts to making a trivial computation.

One concern that could be raised about the soundness of the information structure is whether it is likely that financial intermediaries would go in to the trouble of estimating firms' values in a world where share prices provide continuous price revelation services. This concern can be addressed by changing the name of the agents that estimate the firm's value. Instead of calling them financial intermediaries we can call them stock market traders. Because each firm is small, they too, would find uncovering firms' true value overly costly. To the degree that share prices respond to payout changes, as has been repeatedly documented in the literature (**?**; **?**; **?**; **?**; **?**), and to the degree that available credit responds to share prices, as is implied by the empirical inverse association of stock returns and credit spreads (**?**), firms would be prudent to consider the affect of lowering their payout on their credit lines, which is the driving force of the presented model. However, in practice, credit flows are too stable to comply with the notion that they are imposed by stock market fluctuations. Therefore, to align the model with empirical evidence, one must consider the value of the firm for collateral purposes to be somewhat removed from it's empirical stock market counterpart.

The search for parameters that will make the value estimation function consistent with rational expectations is conducted in a partial equilibrium setting. The procedure involves negatively shocking the productivity of a single firm and attempting to predict its value over an impulse response. In this setup, market prices are fixed at their steady-state levels and any change to the firm's value originates only in changes in its future payout stream.

Once parameters consistent with rational expectations have been found, the resulting system is examined in a general equilibrium setting (Section 4). In the RBC tradition, all firms are identically and simultaneously shocked and market prices are allowed to adjust. Since all firms are symmetric and financial intermediaries know the true value of the stock market, firms' estimated value no longer plays a role. However, the parameters of the value estimation function still affect the model because they remain part of the firms first-order conditions.

It is then shown that while there are many convergence paths that are consistent with the models equations, one of them reproduces the distribution of US working hours surprisingly well. Interestingly, it is the one at which the sensitivity of the stock market to changes in aggregate payout equals the sensitivity at which each rationally estimated firm value responds to its own payout. Put simply, it is the one at which aggregation does not affect payout's signaling of underlying strength.

2 The Model

There are five types of agents in the model: households, final good producers, intermediate good producers, financial intermediaries and a government. Households follow the standard assumptions. The final good producers are of the monopolistic competition variety (**?**). The government provides a subsidy to corporate borrowing and issues money.

The intermediate good producers, which encounter frictions in both equity and debt financing, are at the heart of the model. The frictions originate from the financial intermediaries, which are assigned the role of correctly estimating, under temporally asymmetric information conditions, the value of owning an intermediate good producing firm. Financial intermediaries use the firm's estimated value to guarantee that intermediate good producers would not be allowed to borrow more than what is safe. Lowercase letters denote real values while uppercase letters denote nominal values.

2.1 Households

The representative household solves the following maximization problem

$$\max_{c_t, l_t, b_t, s_{it}} E_0 \sum\nolimits_{t=0}^{\infty} \beta^t \left[\ln c_t + \alpha \ln\left(1 - l_t\right)\right]$$

s.t.

$$\begin{aligned} w_t l_t \quad + \quad & b_{t-1}\left(1 + r_{t-1}\right) + \int_0^1 \left[s_{i,t-1}\left(\delta_{it} + q_{it}\right)\right] di = \\ & \int_0^1 \left[s_{it} q_{it}\right] di + b_t + c_t + \chi_t \end{aligned} \tag{1}$$

where β is the discount rate, α is the weight of leisure in the utility function, c_t is consumption, l_t is labor, w_t is the real wage rate, b_t is total lending to intermediate good producers, r_{t-1} is the real interest rate on loans made in $t-1$, s_{it} is the number of shares held in intermediate good producer i at the end of period t, q_{it} is the market price of one of these shares, δ_{it} is its payout (dividends plus stock buybacks minus stock issuance) and χ_t is a lump-sum tax charged by the government.

The Lagrangian:

$$\mathcal{L}_h \equiv E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{l} \ln c_t + \alpha \ln (1 - l_t) \\ +v_t \cdot \left(\begin{array}{c} w_t l_t + b_{t-1} (1 + r_{t-1}) - c_t - \chi_t - b_t \\ + \int_0^1 [s_{i,t-1} (\delta_{it} + q_{it}) - s_{it} q_{it}] \, di \end{array} \right) \end{array} \right\}$$

First Order Conditions (FOCs):

$$\frac{\partial \mathcal{L}_h}{\partial c_t} : \frac{1}{c_t} = v_t \tag{2}$$

$$\frac{\partial \mathcal{L}_h}{\partial l_t} : \frac{\alpha}{1 - l_t} = v_t w_t \tag{3}$$

$$\frac{\partial \mathcal{L}_h}{\partial b_t} : v_t = \beta (1 + r_t) E_t v_{t+1} \tag{4}$$

$$\frac{\partial \mathcal{L}_h}{\partial s_{it}} : v_t q_{it} = \beta E_t [v_{t+1} (\delta_{i,t+1} + q_{i,t+1})] \tag{5}$$

2.2 Final good producers

The final good producers assemble the final good from a continuum of intermediate goods.

$$y_t = \left(\int_0^1 x_{it}^{\eta} di \right)^{\frac{1}{\eta}} \tag{6}$$

And solve:

$$\max_{x_{it}} \left(\int_0^1 x_{it}^{\eta} di \right)^{\frac{1}{\eta}} - \int_0^1 p_{it} x_{it} di,$$

where p_{it} is the real price of the intermediate good produced by firm i. This yields the standard monopolistic competition demand for intermediate goods:

$$\left(\int_0^1 x_{jt}^{\eta} dj \right)^{\frac{1}{\eta} - 1} x_{it}^{\eta - 1} = p_{it}, \tag{7}$$

$$y_t^{1-\eta} x_{it}^{\eta - 1} = p_{it}. \tag{8}$$

2.3 Government

The government subsidizes interest payments from intermediate good producers to households, buys back old money, issues new money and collects a lump sum tax:

$$\chi_t + \frac{M_t}{P_t} = \frac{M_{t-1}}{P_t} + \tau r_{t-1} b_{t-1}. \tag{9}$$

Its objective is to maintain a fixed price of the final good $P_t = 1$. The amount of money is determined by demand. As a result, the lump-sum tax will be set to:

$$\chi_t = \tau r_{t-1} b_{t-1} - (M_t - M_{t-1}). \tag{10}$$

The inclusion of money in the model serves to provide intermediate good producers with a safe liquid financial asset, both to facilitate daily operations and serve as a financing source of last resort. A discussion of monetary policy would require debt and interest rates to be stated in nominal terms as well, which is avoided in order to keep the model simpler.

2.4 Intermediate good producers

There is a continuum of length 1 of intermediate good producers, each one facing an idiosyncratic $AR(1)$ productivity shock. The model abstracts from capital. Firm i's production function is given by:

$$x_{it} = z_{it} (l_{it})^{1-\theta} \tag{11}$$

and its gross revenue by:

$$p_{it} x_{it} = y_t^{1-\eta} x_{it}^{\eta-1} x_{it} = y_t^{1-\eta} z_{it}^{\eta} (l_{it})^{\eta(1-\theta)} \tag{12}$$

A production constraint imposes the condition that a real operating balance of M_{it}/P_t must be maintained in order to facilitate the firm's daily operations. This assumption will play a key role in the transmission of shocks from the credit flow to the real economy. As the firm finds that it is short on operating balance, it will have to reduce its demand for labor. Alternatively and more realistically, we could assume that a delay in production necessitates financing a stock of working capital, the amount of which determines maximum labor demand. However, a concurrent production constraint was preferred so as to better focus on the effects of the asymmetric information setup suggested.

In addition to the production constraint, a credit constraint imposes the that the firm's debt will remain below a proportion ϕ of the firm's estimated value. By fixing maximum leverage in this way the model abstracts from the rise in cost of borrowing that may compensate lenders for the risks of leverage. This abstraction is necessary since in this model there is no actual bankruptcy, so there are no real costs to leverage.

Financial intermediaries face a particular temporal information asymmetry in estimating the value of the intermediate good producing firm. They can observe only the current payout of the firm but must wait one period in order to learn its true value. Let $\bar{q}(d_{it}, q_{i,t-1})$ denote the value intermediaries infer from currently observable variables. The key feature of the intermediate good producing firm maximization problem is that the function $\bar{q}$ is known. Thus, the firm must take into account the affect of changing its payout on its perceived value and its implied access to credit.

The firm solves the following maximization problem:

$$\max_{l_{it},\delta_{it},b_{it},M_{it}} E_0 \sum_{t=0}^{\infty} \beta^t \frac{v_t}{v_0} \delta_{it}$$

s.t.

$$\begin{aligned} b_{it} \quad + \quad & y_t^{1-\eta} z_{it}^{\eta} \left(l_{it}\right)^{\eta(1-\theta)} + \frac{M_{i,t-1}}{P_t} = \\ & \left(1 + (1-\tau)\, r_{t-1}\right) b_{i,t-1} + w_t l_{it} + \delta_{it} + \frac{M_{it}}{P_t} \end{aligned} \tag{13}$$

$$b_{it} \leq \phi \bar{q}\left(\delta_{it}, q_{i,t-1}\right) \tag{14}$$

$$w_t l_{it} \leq \omega \frac{M_{it}}{P_t} \tag{15}$$

$$\ln z_{it} = \rho \ln z_{i,t-1} + \sigma \varepsilon_{it} \tag{16}$$

where $\phi, \omega, \rho, \sigma \geq 0$ are parameters, z_{it} is the firm's productivity and ε_{it} is IID standard normal.

The Lagrangian is as follows:

$$\mathcal{L}_i = E_0 \sum_{t=0}^{\infty} \beta^t \frac{v_t}{v_0} \left\{ \begin{array}{c} \delta_{it} \\ +\lambda_{it} \left[\begin{array}{c} b_{it} + y_t^{1-\eta} z_{it}^{\eta} \left(l_{it}\right)^{\eta(1-\theta)} + M_{i,t-1}/P_t \\ -\left(1 + (1-\tau)\, r_{t-1}\right) b_{i,t-1} - w_t l_{it} - \delta_{it} - M_{it}/P_t \end{array} \right] \\ +\lambda_{it}\mu_{it} \left[\phi \bar{q}\left(\delta_{it}, q_{i,t-1}\right) - b_{it}\right] \\ +\lambda_{it}\kappa_{it} \left[\omega M_{it}/P_t - w_t l_{it}\right] \end{array} \right\}$$

The FOC for optimal payout is given by:[2]

$$\begin{aligned} \frac{\partial \mathcal{L}_i}{\partial \delta_{it}} \quad : \quad & v_t \left(1 - \lambda_{it} + \lambda_{it}\mu_{it}\phi \frac{\partial \bar{q}\left(\delta_{it}, q_{i,t-1}\right)}{\partial \delta_{it}}\right) \\ + \quad & \beta E_t \left[v_{t+1}\lambda_{i,t+1}\mu_{i,t+1}\phi \frac{\partial \bar{q}\left(\delta_{i,t+1}, q_{it}\right)}{\partial q_{it}} \frac{\partial q_{it}}{\partial \delta_{it}}\right] = 0 \end{aligned} \tag{17}$$

Let $V_{it} = \delta_{it} + q_{it}$ stand for the object of optimization. This implies that:

$$\frac{\partial q_{it}}{\partial \delta_{it}} = \frac{\partial V_{it}}{\partial \delta_{it}} - 1 \tag{18}$$

[2]A large part of the complexity of the model's first-order conditions is due to the presence of $q_{i,t-1}$ in $\bar{q}\left(\delta_{it}, q_{i,t-1}\right)$. However, the resulting complexity is a by-product and does not constitute an object of interest in itself.

However, since δ_{it} is set to maximize V_{it}, the derivative $\partial V_{it}/\partial \delta_{it}$ is equal to zero and therefore:

$$v_t \left(1 - \lambda_{it} + \lambda_{it}\mu_{it}\phi \frac{\partial \bar{q}\left(\delta_{it}, q_{i,t-1}\right)}{\partial \delta_{it}}\right) = \beta E_t \left[v_{t+1}\lambda_{i,t+1}\mu_{i,t+1}\phi \frac{\partial \bar{q}\left(\delta_{i,t+1}, q_{it}\right)}{\partial q_{it}}\right] \quad (19)$$

The FOC for labor demand is given by:

$$\frac{\partial \mathcal{L}_i}{\partial l_{it}} : \quad v_t\lambda_{it}\left[\eta\left(1-\theta\right) y_t^{1-\eta} z_{it}^{\eta}\left(l_{it}\right)^{\eta(1-\theta)-1} - w_t\left(1+\kappa_{it}\right)\right] + E_t\left[\beta v_{t+1}\lambda_{i,t+1}\mu_{i,t+1}\phi \frac{\partial \bar{q}\left(\delta_{i,t+1}, q_{it}\right)}{\partial q_{it}} \frac{\partial q_{it}}{\partial l_{it}}\right] = 0. \quad (20)$$

By envelope conditions:

$$\frac{\partial q_{it}}{\partial l_{it}} = \frac{\partial V_{it}}{\partial l_{it}} = 0 \quad (21)$$

and therefore:

$$\eta\left(1-\theta\right) y_t^{1-\eta} z_{it}^{\eta}\left(l_{it}\right)^{\eta(1-\theta)-1} = w_t\left(1+\kappa_{it}\right). \quad (22)$$

The FOC for debt demand is given by:

$$\frac{\partial \mathcal{L}_i}{\partial b_{it}} : \quad v_t\lambda_{it}\left(1-\mu_{it}\right) + \beta E_t\left[v_{t+1}\lambda_{i,t+1}\left(\mu_{i,t+1}\phi \frac{\partial \bar{q}\left(\delta_{i,t+1}, q_{it}\right)}{\partial q_{it}} \frac{\partial q_{it}}{\partial b_{it}} - \left(1+\left(1-\tau\right) r_t\right)\right)\right] + \beta^2 E_t\left[v_{t+2}\lambda_{i,t+2}\mu_{i,t+2}\phi \frac{\partial \bar{q}\left(\delta_{i,t+2}, q_{i,t+1}\right)}{\partial q_{i,t+1}} \frac{\partial q_{t+1}}{\partial b_{it}}\right] = 0. \quad (23)$$

By envelope conditions:

$$\frac{\partial q_{it}}{\partial b_{it}} = \frac{\partial V_{it}}{\partial b_{it}} = 0 \quad (24)$$

$$\frac{\partial q_{i,t+1}}{\partial b_{it}} = \frac{\partial V_{i,t+1}}{\partial b_{it}} = -\lambda_{i,t+1}\left(1+\left(1-\tau\right) r_t\right) \quad (25)$$

and therefore:

$$v_t\lambda_t\left(1-\mu_t\right) = \beta\left(1+\left(1-\tau\right) r_t\right) \cdot E_t\left[\lambda_{t+1}\left(v_{t+1} + \beta v_{t+2}\lambda_{t+2}\mu_{t+2}\phi \frac{\partial \bar{q}\left(\delta_{i,t+2}, q_{i,t+1}\right)}{\partial q_{i,t+1}}\right)\right]. \quad (26)$$

The FOC for the operating balance is given by:

$$\frac{\partial \mathcal{L}_i}{\partial M_{it}} : \quad v_t\lambda_{it}\left(\omega\frac{\kappa_{it}}{P_t} - \frac{1}{P_t}\right) + E_t\left[\beta v_{t+1}\lambda_{i,t+1}\left(\frac{1}{P_{t+1}} + \mu_{i,t+1}\frac{\partial \bar{q}\left(\delta_{i,t+1}, q_{it}\right)}{\partial q_{it}} \frac{\partial q_{it}}{\partial M_{it}}\right)\right] + E_t\left[\beta^2 v_{t+2}\lambda_{i,t+2}\mu_{i,t+2}\frac{\partial \bar{q}\left(\delta_{i,t+2}, q_{i,t+1}\right)}{\partial q_{i,t+1}} \frac{\partial q_{i,t+1}}{\partial M_{it}}\right] = 0. \quad (27)$$

By envelope conditions:

$$\frac{\partial q_{it}}{\partial M_{it}} = \frac{\partial V_{it}}{\partial M_{it}} = 0 \tag{28}$$

$$\frac{\partial q_{i,t+1}}{\partial M_{it}} = \frac{\partial V_{i,t+1}}{\partial M_{it}} = \frac{\lambda_{i,t+1}}{P_{t+1}} \tag{29}$$

and therefore:

$$\begin{aligned} & v_t \frac{\lambda_{it}}{P_t} \left(1 - \omega\kappa_{it}\right) = \\ & \beta E_t \left[\frac{\lambda_{i,t+1}}{P_{t+1}} \left(v_{t+1} + \beta v_{t+2} \lambda_{i,t+2} \mu_{i,t+2} \frac{\partial \bar{q}\left(\delta_{i,t+2}, q_{i,t+1}\right)}{\partial q_{i,t+1}} \right) \right] \end{aligned} \tag{30}$$

Combining the FOC for the operating balances (30) with that for debt demand (26) and price stability, we obtain that:

$$\kappa_{it} = \frac{1}{\omega} \frac{\left(1-\tau\right) r_t + \mu_{it}}{\left(1 + \left(1-\tau\right) r_t\right)}, \tag{31}$$

implying that while the credit constraint may not be binding, the operating balance constraint always is.

2.5 Closing the model

Since it is possible for the firm to raise funds through a negative payout, we have no need for stock issuance and can safely fix the number of shares at:

$$s_t = 1. \tag{32}$$

The clearing of labor, debt and money markets is expressed as follows:

$$l_t = \int_0^1 l_{it} di, \tag{33}$$

$$b_t = \int_0^1 b_{it} di, \tag{34}$$

$$M_t = \int_0^1 M_{it} di. \tag{35}$$

In order to find the expression for clearance in the final good market, we first aggregate the budget constraints of all intermediate good producers:

$$b_t + y_t + M_{t-1} = \left(1 + \left(1-\tau\right) r_{t-1}\right) b_{t-1} + w_t l_t + \delta_t + M_t, \tag{36}$$

We then add the household and government budget constraints and combine with $(32-35)$ to obtain:

$$y_t = c_t \tag{37}$$

2.6 Functional form of the value estimation function

Let $\bar{q}$ take the form of a log-log regression in the deviations from market-wide averages:

$$\bar{q}\left(\delta_{it}, q_{i,t-1}\right) = \exp\left[\begin{array}{c} \ln q_t + a_\delta \cdot \left(\ln \delta_{it} - \ln \delta_t\right) \\ +a_q \cdot \left(\ln q_{i,t-1} - \ln q_{t-1}\right) \end{array}\right]. \tag{38}$$

where a_δ, a_q are parameters and:

$$\begin{aligned} q_t &\equiv \int_0^1 q_{it} di, \\ d_t &\equiv \int_0^1 d_{it} di. \end{aligned}$$

Naturally, if all firms make the same decisions this rule automatically estimates their true value, such that:

$$\bar{q}\left(\delta_t, q_{t-1}\right) = q_t. \tag{39}$$

The derivatives of the functional form are:

$$\frac{\partial \bar{q}\left(\delta_{it}, q_{i,t-1}\right)}{\partial \delta_{it}} = \bar{q}_{it} \frac{a_\delta}{\delta_{i,t}} \tag{40}$$

$$\frac{\partial \bar{q}\left(\delta_{it}, q_{i,t-1}\right)}{\partial q_{i,t-1}} = \bar{q}_{it} \frac{a_q}{q_{i,t-1}} \tag{41}$$

where $\bar{q}_{it} \equiv \bar{q}\left(\delta_{i,t}, q_{i,t-1}\right)$.

3 Finding Parameters that are Consistent with Rational Expectations

Appendix A summarizes the general equilibrium equation system and Appendix B presents the equations of the deterministic steady-state.

To find rational values for a_δ and a_q that provide a good estimate of the firm's value, we can consider a partial equilibrium in which only one firm is shocked. In this partial equilibrium, every firm but one is in its deterministic steady-state.

3.1 Partial equilibrium

Let the index i denote the firm that is not placed in its steady-state. The objective, at this point, is to find the set of (a_δ, a_q)-pairs for which the estimated value of firm i approximates its true value. This is done by measuring the difference between the true value of the firm and its estimated value over an impulse response.

The disturbance path for the impulse response is described by $\varepsilon_{i,t=0} = -0.02$, $\varepsilon_{i,t>0} = 0$. The reason for using a negative shock is that it generates a convergence path on which the credit constraint is always binding. The method of computing the impulse response utilizes certainty equivalence, which involves substituting actual future values into the first-order conditions and ignoring the expectation operator, as if agents, except financial intermediaries, had perfect foresight. This means abstracting from households' precautionary saving and firms' precautionary hoarding of money.

The equations from all periods of the impulse response are then numerically and simultaneously solved [3]. Following is the equation set to be solved, in addition to the steady-state equations $(68-80)$ (note the mixture of steady-state values that represent market-wide conditions, denoted by an ss subscript, and firm-specific variables, denoted by an i subscript):

$$\frac{\partial \mathcal{L}_i}{\partial \delta_t} : 1-\lambda_{it}+\lambda_{it}\mu_{it}\phi\bar{q}_{it}\frac{a_\delta}{\delta_{it}}-\beta\lambda_{i,t+1}\mu_{i,t+1}\phi\bar{q}_{i,t+1}\frac{a_q}{q_{it}}=0, \tag{42}$$

$$\frac{\partial \mathcal{L}_i}{\partial l_t} : \eta\left(1-\theta\right)y_{ss}^{1-\eta}z_{it}^{\eta}\left(l_{it}\right)^{\eta(1-\theta)-1}-w_{ss}\left(1+\kappa_{it}\right)=0, \tag{43}$$

$$\begin{aligned}\frac{\partial \mathcal{L}_i}{\partial b_t} &: \lambda_{it}\left(1-\mu_{it}\right)-\beta\left(1+\left(1-\tau\right)r_{ss}\right)\cdot \\ &\quad \lambda_{i,t+1}\left(1+\beta\lambda_{i,t+2}\mu_{i,t+2}\phi\bar{q}_{i,t+2}\frac{a_q}{q_{i,t+1}}\right)=0,\end{aligned} \tag{44}$$

$$\begin{aligned}\frac{\partial \mathcal{L}_i}{\partial M_t} &: \lambda_{it}\left(1-\omega\kappa_{it}\right) \\ &- \beta\lambda_{t+1}\left(1+\beta\lambda_{t+2}\mu_{t+2}\bar{q}_{t+2}\frac{a_q}{q_{t+1}}\right)=0\end{aligned} \tag{45}$$

$$\begin{aligned}\text{budget} &: b_{it}+y_{ss}^{1-\eta}z_{it}^{\eta}\left(l_{it}\right)^{\eta(1-\theta)}+M_{i,t-1} \\ &\quad -\left(1+\left(1-\tau\right)r_{ss}\right)b_{i,t-1}-w_{ss}l_{it}-\delta_{it}-M_{it}=0,\end{aligned} \tag{46}$$

$$\text{credit} : b_{it}-\phi\bar{q}_{it}=0, \tag{47}$$

$$\text{capacity} : w_{ss}l_{it}-\omega M_{it}=0, \tag{48}$$

$$\text{productivity} : z_{it}=\left(z_{i,t-1}\right)^{\rho}e^{\sigma\varepsilon_{it}}, \tag{49}$$

$$\text{estimated value} : \bar{q}_{it}=\exp\left[\begin{array}{l}\ln q_{ss}+a_\delta\cdot\left(\ln\delta_{it}-\ln\delta_{ss}\right)\\ +a_q\cdot\left(\ln q_{i,t-1}-\ln q_{ss}\right)\end{array}\right] \tag{50}$$

$$\frac{\partial \mathcal{L}_h}{\partial s_{it}} : q_{it}=\beta\left(\delta_{i,t+1}+q_{i,t+1}\right). \tag{51}$$

A survey of the value estimation function goodness-of-fit to the true firm value for different values of a_δ and a_q, and their respective im-

[3]The numerical solution procedure makes use of a sparse-matrix version of **?**. Note that the pair (a_δ, a_q) affects the steady-state and therefore we need to re-solve the steady state for every new pair that is tried.

pulse responses will be presented in Subsection 3.3. However, since the solution will be numeric, the model must first be calibrated

3.2 Calibration

The model focuses on the dynamics of the non-financial corporate sector, which accounts for about 77% of the non-financial business gross value added. Calibrating the model requires comparing balance sheet stocks with production flows. The primary data source for the calibration is Table S.5.: Non-financial Corporate Business in the Integrated Macroeconomic Accounts for the United States (IMAUS). It combines data from the Flow of Funds Accounts and the National Income and Product Accounts in a data-consistent manner (**?**).

As Figure 2 shows, the empirical ratio of the non-financial corporate sector debt to its market value[4] has ranged from 35% to 100% during the last 47 years. However, for long periods the ratio appears to have remained relatively stable around a particular value. Thus, from 1960 to 1972 the ratio remained in the vicinity of 40%; from 1973 to 1990 it remained in the vicinity of 80%; and finally, from 1991 onward, it settled back to around 40%. The question of why the ratio deviated from 40% and then returned to it falls outside the scope of this paper, but the timing suggest it might have something to with inflation. In the discussion to follow, ϕ is calibrated to 0.4, as this was the prevailing regime in most of the years.

Between 1960 and 2007 the ratio of quarterly wages to operating balance[5] ranged from 120% to 240% (Figure 3). ω was calibrated to 1.74, which is the average for the period.

During the same period, the ratio of wages to output[6] ranged from 68% to 78% (Figure 4). θ was calibrated to 0.19 in order to arrive at a steady-state ratio of 0.73 which is in line with the average for this period.

The weight of leisure in the utility function, α, was calibrated to 1.7 in order to arrive at steady-state aggregate labor of 0.3. In order to obtain an average price markup of 10%, η was set to 0.91. The period of the model is a quarter. The time discount, β, was set to 0.9825, in order to produce an annual return on equity of 7.3%. The subsidy on taxes, τ, is set to 0.35 in order to reproduce the effect of tax deductible interest payments with a 35% marginal tax rate on corporate profits. Productivity persistence, ρ, and the shock's standard deviation, σ, were set to 0.87 and 0.008, respectively. These were calibrated to generate,

[4]Debt is defined as Securities other than Shares, and Loans (IMAUS, S.5, lines 125 and 129, respectively). Market value is defined as Corporate Equity (IMAUS, S.5, line 135), which is a market-based measure of equity (as opposed to a book-based measure).

[5]Wages are defined as Compensation of Employees (IMAUS, S.5, line 4, divided by four). Operating balances are defined as Currency and Deposits (IMAUS, S.5, line 99).

[6]Output is define as non-financial corporate Net Value Added (IMAUS, S.5, line 3).

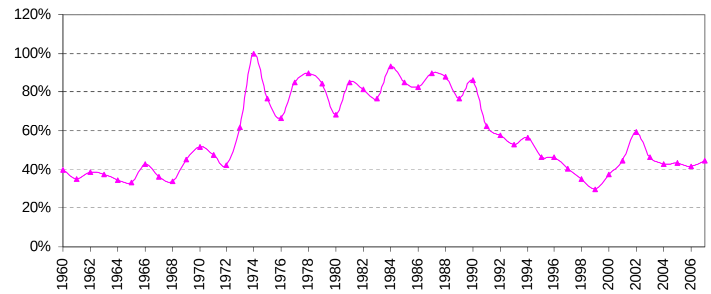

Figure 2: Ratio of US Non-financial Corporate Sector Credit Liabilities to its Equity at Market Value.

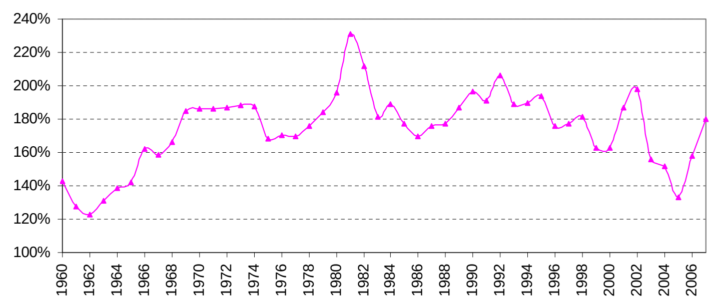

Figure 3: Ratio of US Non-financial Corporate Sector Quarterly Compenstation of Employees to Currency and Deposits.

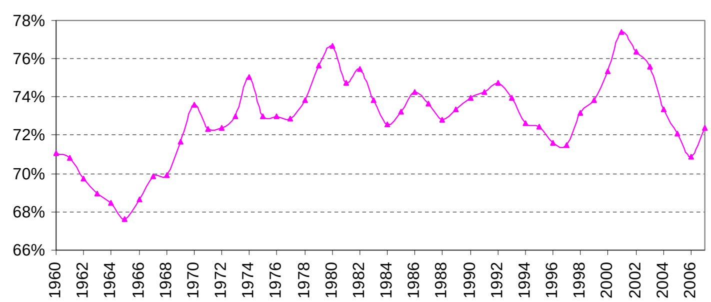

Figure 4: Ratio of US Non-financial Corporate Sector Compenstation of Employees to Net Value Added.

after HP-filtering ($\lambda = 1600$), an auto-correlation of 0.67 and a standard deviation of 0.01, as in the case of the similarly filtered log US business sector output per hour.

Having finished the calibration, we can return to the task of numerically finding rational parameters for the firm's value estimation function.

Parameter	Model context	Calibrated value	Calibration method
ϕ	$\phi = b_{ss}/\bar{q}_{ss}$	0.4	Equating to the mean ratio of debt to market value over the prevailing regime
ω	$\omega = (w_{ss}l_{ss})\,/M_{ss}$	1.74	Equating to the mean ratio of quarterly wages to operating balance
θ	$x_{ss}= z_{ss}\,(l_{ss})^{1-\theta}$	0.19	Adjusting until the ratio of wages to output ($w_{ss}l_{ss}/y_{ss}$) would equal its empirical counterpart (0.73)
α	$\alpha = w_{ss}\,(1 - l_{ss})\,/c_{ss}$	1.7	Adjusting until aggregate labor (l_{ss}) would equal its empirical counterpart (0.3)
β	$\beta = 1/\,(1 + r_{ss})$	0.9825	Conforming to previous literature
τ	Interest subsidy	0.35	Conforming to previous literature
ρ, σ	$\ln z_t= \rho \ln z_{t-1}+\sigma\varepsilon_t$	$0.87, 0.008$	Adjusting until an HP-filtering would produce an auto-correlation of 0.67 and a standard deviation of 0.01, as in output per hour

Table 1: Calibration Summary

3.3 A survey of the payout sensitivity parameter space

Figure 5 presents a survey of the (a_δ, a_q)-space around the best pair that was found. Each data point represents two 70-period impulse responses to -2% and -1% productivity shocks that were computed using the corresponding (a_δ, a_q)-pair. The first shock, -2%, hits at $t = 1$ and the second shock hits at $t = 71$. Each two such impulse responses are rated by the goodness-of-fit between the estimated value of the firm, $\bar{q}_t$, and the true value of the firm, q_t.

Goodness-of-fit is measured by a weighted version of Root Mean Square Deviations (RMSD), which assigns half the weight to the periods

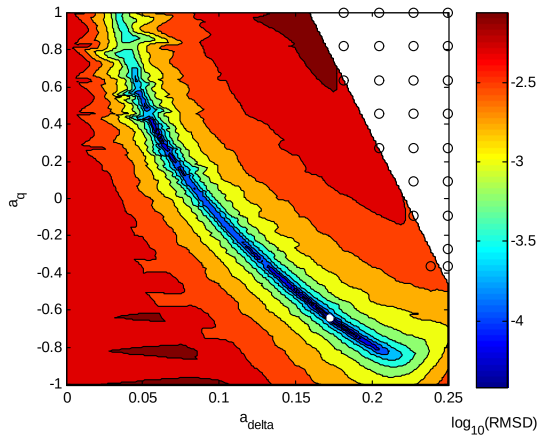

Figure 5: Goodness-of-fit survey for the firm's estimated value following a -2% and -1% productivity shocks.

in which shocks occur and half to the entire path. The measure is defined by:

$$goodness\ of\ fit = \log_{10}\left(\sqrt{\begin{array}{c}\frac{1}{4}\left(\ln \bar{q}_{i1} - \ln q_{i1}\right)^2 + \frac{1}{4}\left(\ln \bar{q}_{i71} - \ln q_{i71}\right)^2 \\ +\frac{1}{2}\left(\frac{1}{140}\sum_{t=1}^{140}\left(\ln \bar{q}_{it} - \ln q_{it}\right)^2\right)\end{array}}\right). \tag{52}$$

Lower values of this measure indicate a better fit: -2 indicates that 1% deviations are the norm, -3 indicates that 0.1% deviations are the norm; etc.

The assignment of so much weight to the periods in which shocks occur is due to the view that the environment is a volatile one, firms are constantly subject to shocks. In such an environment it is more important for lenders to correctly estimate the value of a firm that has just been affected by an unseen shock than it is to be accurate about firms that are going through serene times.

The survey is conducted such that the density of sampling rises with the goodness-of-fit provided by near by samples. 3000 samples were made. White dots mark the best 50 samples found - they are all meshed into one small white stain around $(0.17, -0.64)$. Empty circles mark (a_δ, a_q)-pairs for which the set of equations could not be solved numerically. Isolines are generated by interpolation and are used only to illustrate the "geography" revealed by the survey.

The best (a_δ, a_q)-pair found in the survey is $(0.1720, -0.6383)$ with a goodness-of-fit of -4.42. Denote the model in using this pair as the signaling model. Figure 6 presents the impulse response for the signaling

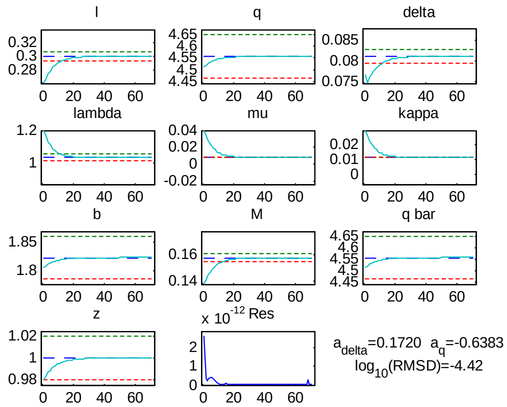

Figure 6: Response to a -2% shock to productivity for the signaling model.

model after a -2% shock. A dashed blue line marks the steady-state value; a solid light blue line marks the path of the variable; and the densely-dashed red and green lines mark -2% and $+2\%$ deviations from the steady-state value, respectively. The box labelled Res displays the numerical error in the solution of the equation system (which in this case is below 10^{-11} for all t).

A natural reference for comparison is the impulse response provided by a firm-value estimation function that always assigns the firm its true value. Denote the model in which this is the firm value estimation function as the full information reference model. Appendix C presents this model and derives its FOCs; Figure 7 presents its impulse response.

The differences between the two models are striking. Most significantly, the decrease in labor demand in the first period of the rational-pair impulse response is almost double that of the reference model (see Figure 8). The driving force for the decline in labor demand in both models is the credit contraction that follows the fall in firm value. However, while in the reference model this effect is mitigated by a one time drop in payout, in the signaling model, the signaling mechanism and the way it links payout to credit availability make any irregular declines in payout sub-optimal (see Figure 9). Rather, along the optimal path the necessary funds are drawn from the operating balance, thus forcing an outsized contraction of labor demand.

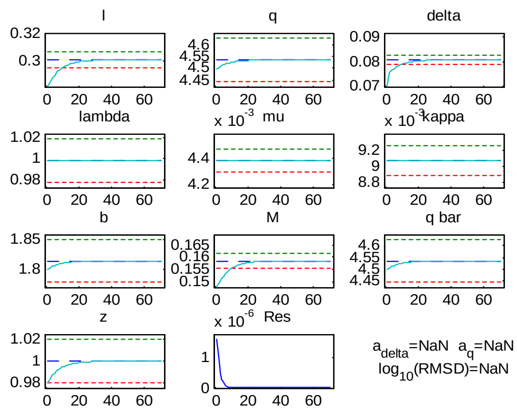

Figure 7: Response to a -2% negative shock to productivity for the full information reference model.

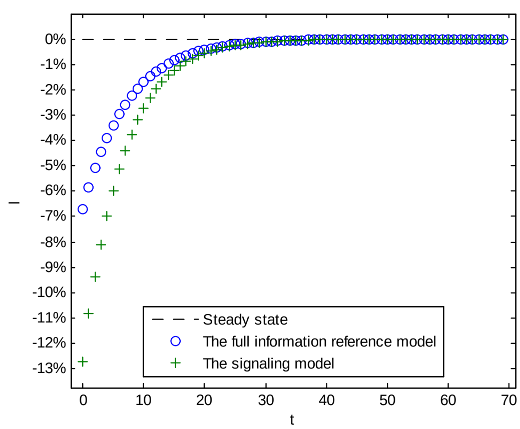

Figure 8: Labor demand response to a -2% productivity shock for the reference model and the signaling model.

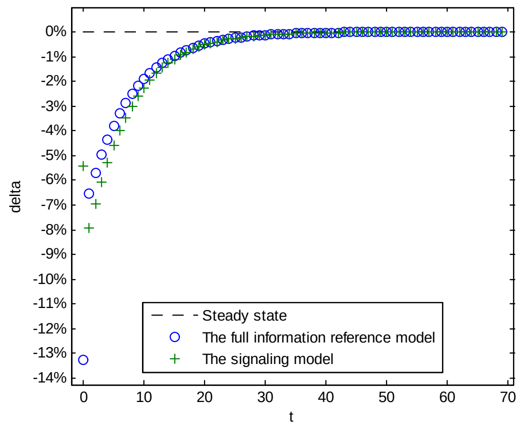

Figure 9: Payout response to a -2% productivity shock for the reference model and the signaling model.

3.4 Asymmetry in the response to positive shocks

Solving the equation system for a positive shock under the assumption that the credit constraint is binding reveals a contradiction, in that the credit constraint multiplier is found to be negative in the first period. In order to solve the system of equations, one must assume that the credit constraint is non-binding in the first period. By making this assumption the equation set can be solved without generating contradictions and we obtain the impulse response presented in Figure 10. As can be seen, the impulse response to a 2% positive shock is quite different from being the mirror image of the response to a 2% negative shock.

The difference originates from the release of the credit constraint in the first period. As a result, the firm experiences a one-period irregular availability of credit. However, it can only profitably expand production to a certain degree. Some of the additional credit that is made available goes out as a one-time outsized payout,[7] most is just left unused.

The release of the credit constraint induces a kink in the model dynamics. This can be seen in Figure 11 where the labor demand in the first three periods is presented for a range of shocks. Accelerator effects are present only as long as the credit constraint is binding.

[7] The irregular payout is somewhat smaller than the one for the payout-insensitive and reference models, though it is still significant (not shown).

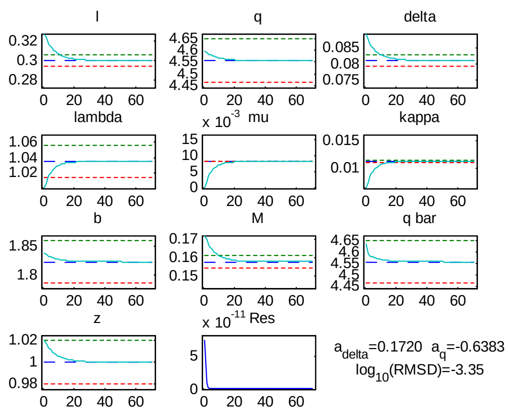

Figure 10: Response to a 2% productivity shock for the signaling model.

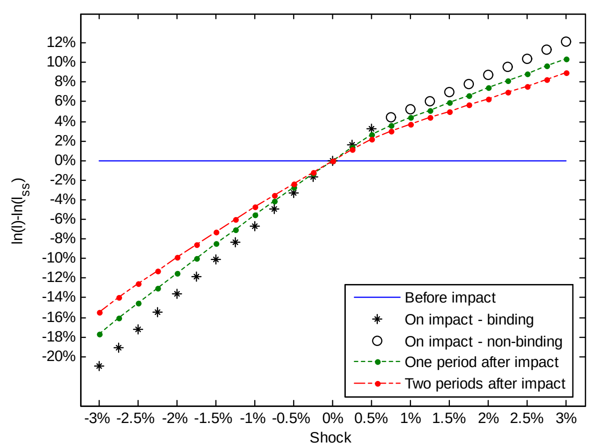

Figure 11: Labor demand response to various sizes of productivity shocks for the signaling model.

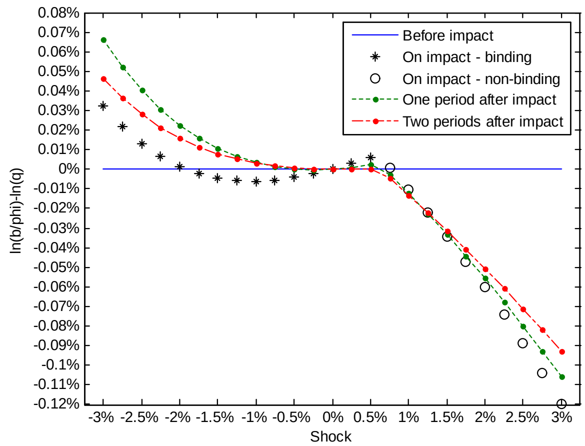

Figure 12: Firm-value estimation error response to various sizes of productivity shocks for the signaling model.

3.5 Value estimation errors

Before continuing, it is worthwhile testing the accuracy of value estimations for productivity shocks other than -2%. Figure 12 presents the value estimation errors for shocks between -3% and 3% for the signaling model. For positive shocks when the credit constraint is non-binding, the value estimate is replaced with the value implied by actual borrowing b_t/ϕ.

Note that since shocks are normally distributed, minor shocks are more common than extreme ones. We see that this value estimation function generally provides estimates that are within $\pm 0.01\%$ of the true value for shocks between -2σ and $+\sigma$ ($\sigma = 0.8\%$). On impact, it exhibits minor under-valuation in response to shocks between 0% and -2% and increasing over-valuations for shocks below -2%.

Estimation errors can be viewed as an indication that the log-log functional form is a poor choice. It is certainly possible that a different functional form would improve accuracy. Yet, one benefit of using a log-log form is that it provides a natural interpretation of the coefficients, which in turn facilitates the interpretation of estimation errors.

Moving left from zero in Figure 12, as small negative shocks hit the firm, initially payout, and thus estimated value, falls at a faster rate than true value. However, starting from around -1% the trend reverses and payout begins falling at a slower rate than true value. This implies that the firm is making efforts to hide the severity of its condition by keeping payout artificially afloat.

This pattern of estimation errors is also the reason that a -2% shock was used to search for rational pairs. In this setup, under-valuation turns into over-valuation around -2% and subsequently grows quite rapidly. Using a more moderate shock shifts the crossover point rightward and implies that relatively common negative shocks could result in significant over-valuation.

4 General Equilibrium

This section will examine the path taken by the economy when all firms receive a shock of $-\sigma$. In such a setting all firms are symmetric and no firm deviates from the market average. As a result, financial intermediaries always know the true value of firms. As will be shown, there are multiple paths that maintain the general equilibrium equation system for the rational pair (see Appendix A for the equation set).

4.1 Indeterminacy of the equilibrium path

To see that there are multiple paths that fulfill the general equilibrium equation system, consider the following restriction on the amount of total lending, connecting it to the deviations of payout and last-period true value from their steady-state levels:

$$b_t \leq \phi \exp \left[\begin{array}{l} \ln q_{ss} \\ +a_{\Delta} \cdot (\ln \delta_t - \ln \delta_{ss}) \\ +a_Q \cdot (\ln q_{t-1} - \ln q_{ss}) \end{array} \right] \tag{53}$$

where a_{Δ} and a_Q are parameters. Clearly, if the exponent in (53) equals the value of the stock market, we would have a solution to the original model.

A survey of the (a_{Δ}, a_Q)-space is conducted in a model in which the aggregate borrowing constraint is stated by (53). Whenever the exponent is close to the true value of the stock market we suspect that the convergence path may be a solution to the original model. This is later confirmed by providing the computed convergence path as an initial guess to the numerical solution of the original equation system and seeing that it generates as output a path that is similar.

Figure 13 presents the goodness-of-fit survey of the (a_{Δ}, a_Q)-space. A straight, deep and even-leveled "valley" can be seen stretching from low to high aggregate payout sensitivity. The best 30 sampled pairs out of the 1200 samples taken are marked by the white dots that are meshed into one white stain around $(a_{\Delta} = 0.1568,\ a_Q = 1.442)$.

As will be shown, each pair along the "bottom of the valley" is associated with a markedly different solution to the original model. The existence of multiple (a_{Δ}, a_Q)-pairs that provide accurate market-wide

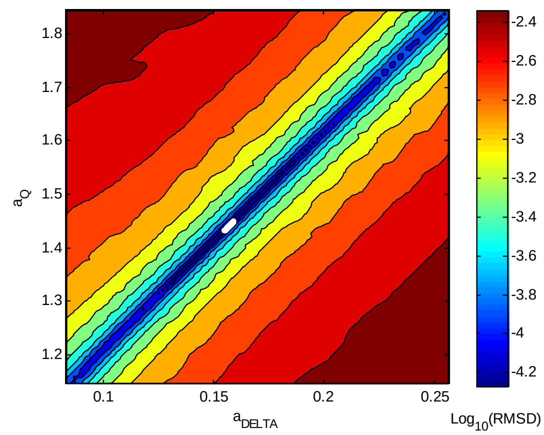

Figure 13: Goodness-of-fit survey for the estimated economy-wide market value following a market-wide -0.8% productivity shock.

value estimations while generating different convergence paths implies that in this system the fact that financial intermediaries know the true value of the stock market does not provide sufficient information to identify the general equilibrium. To do so, one must also specify the sensitivity of the value of the stock market to deviations in aggregate payout.

Figures 14 and 15 present two impulse responses drawn from the "bottom of the valley", with the first selected arbitrarily from the continuum of possible convergence paths and the second selected such that the sensitivity of the value of the stock market to deviations in aggregate payout would mirror the sensitivity of firm-value to deviations in firm payout, namely $a_\Delta = a_\delta$.

The indeterminacy of the equilibrium path can be interpreted as the indeterminacy of the path of the true market value of firms. Shares trace a lower path in the strong accelerator equilibrium (Figure 15) than in the weak accelerator equilibrium (Figure 14). As a result, more debt has to be repaid initially and less is available to fund operations subsequently. Wage expenditures must contract more drastically along the entire path. Labor demand remains lower and holds wages down. Households earn less and as a result they try to work more and save less. The rise in labor supply is weaker than the decline in labor demand, and therefore the quantity of labor input traces a lower path. However, the contraction in the supply of debt outpaces the decline in demand for debt, so the interest rate rises. Finally, the higher path of the interest rate justifies the lower path of the true value of firms.

Furthermore, because the situation is more dire along the strong

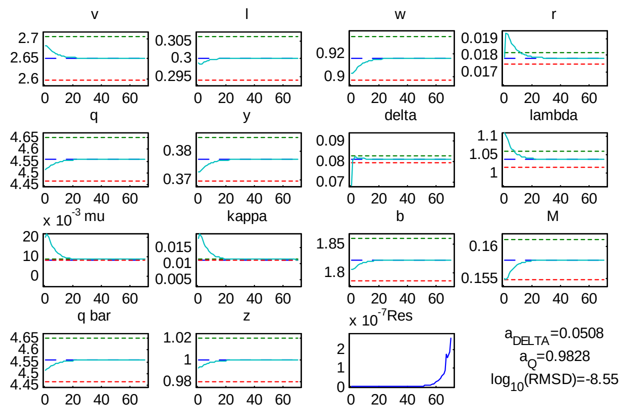

Figure 14: Response to an economy-wide -0.8% productivity shock for a weak accelerator equilibrium (a_Δ selected arbitrarily).

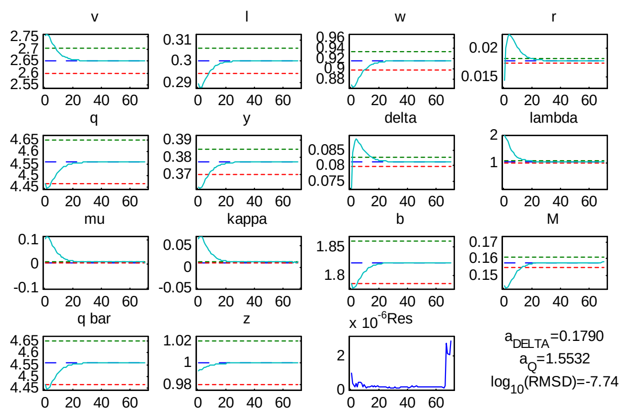

Figure 15: Response to an economy-wide -0.8% productivity shock for a strong accelerator equilibrium ($a_\Delta = a_\delta$).

accelerator convergence path, there is more incentive for each firm to attempt to keep its own estimated value artificially afloat by maintaining payout above what would otherwise be optimal. For this reason, firms on the strong accelerator equilibrium, though facing the same productivity as on the weak accelerator, cannot reduce payout to fund the same level of operations. Moreover, while each firm attempts to keep its own estimated value artificially afloat, firms efforts have no impact on aggregate credit availability since the true aggregate value of firms is known.

In order to ascertain whether the convergence path is determined by share prices at the time of the shock's impact, the convergence path was computed for a model where the first-period true value of the firm is set as a parameter. If the indeterminacy is simply a matter of the first-period stock market value, then whatever parameter value was chosen, would also turn out to be the discounted value of the next-period share price and payout. However, this is not the case and similarly, the equilibrium is not determined by the first-period interest rate or wage rate. It can be concluded that the indeterminacy requires altering the entire convergence path in order to maintain rationality. Therefore, it is expectations that determine which equilibrium path plays out, not observed prices.

A promising way to select one of the convergence paths is to rely on the stableness of the signaling mechanism in aggregation. If we consider a setting in which the number of the firms shocked is $\psi \in [0, 1]$, we have that $\psi = 0$ produces the partial equilibrium while $\psi = 1$ generates the general equilibrium. The path connecting the partial and general equilibriums travels through the three-dimensional space (ψ, a_Δ, a_Q). As we start moving away from $\psi = 0$ along this continuum we have that while firms still send the same signals, financial intermediaries need to adjust to changes in market prices. The argument is that along this route the signals transmitted by firms pass partly through a_δ and partly through a_Δ, but because firms always see only a_δ, it should be optimal for financial intermediaries to maintain $a_\Delta = a_\delta$ at all $\psi \in (0, 1)$ and make the adjustment using only a_Q. However, due to its computational complexity the assertion of this claim is left to a future study.

At any rate, from this point on we focus on the convergence path associated with $a_\Delta = a_\delta$. There are two reasons for this: First, it is natural to assume that aggregate value maintains the same relation with aggregate payout as firm value does with firm payout. Second, $a_\Delta = a_\delta$ is close to $(a_\Delta = 0.1568,\ a_Q = 1.442)$ which provided the best prediction of market value in the goodness-of-fit survey.

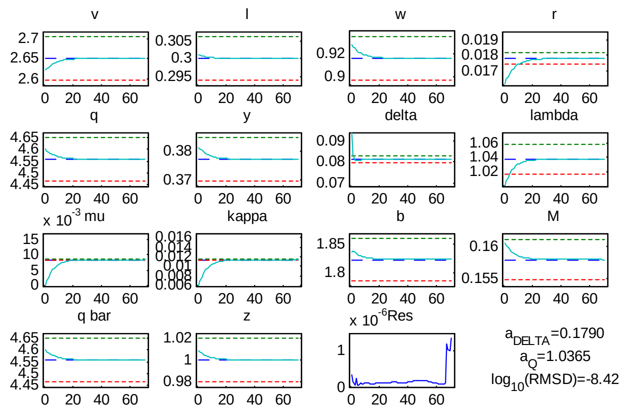

Figure 16: Response to an economy-wide 0.8% productivity shock for a strong accelerator equilibrium ($a_{\Delta} = a_{\delta}$).

4.2 Cyclical asymmetry of the general equilibrium

As with the partial equilibrium, the general equilibrium shows asymmetric responses to negative and positive shocks. As in the case of a negative shock, the convergence path from a positive shock similarly allows for many different routes back to the steady-state. However, they are less distinct from one another and in addition, the relation seen in Figure 13 shifts to somewhat lower a_Q values (not shown).

The convergence path from a positive shock associated with $a_{\Delta} = a_{\delta}$, shown in Figure 16, requires the use of a lower a_Q than the convergence path from a negative shock. As was the case in the partial equilibrium, the liquidity constraint is released on impact and the financial accelerator mechanism ceases to operate.

Unlike in the case of the partial equilibrium, once the financial accelerator mechanism ceases to operate, the link between productivity and labor is severed. Figure 17 demonstrates this by showing that all positive shock levels are associated with almost the same level of labor. To see why this is the case observe that combining the equations for households' labor supply (3), firms' labor demand (22), marginal utility of consumption (2) and clearance of the final good market (37) we find that:

$$l_t = \left(\frac{\alpha \left(1 + \kappa_t\right)}{\eta \left(1 - \theta\right)} + 1 \right)^{-1}, \tag{54}$$

implying that labor rises only if the marginal value of another unit produced, κ_t, declines.

However, the decline in κ_t is limited by the fact that it must always

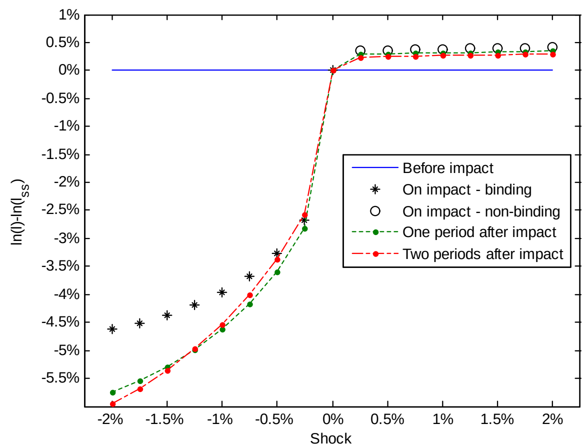

Figure 17: Labor input's response to different shocks for a strong accelerator equilibrium $(a_{\Delta} = a_{\delta})$.

remain above zero. In particular, in the general equilibrium κ_t is given by:

$$\kappa_t = \frac{1}{\omega} \frac{(1-\tau) r_t + \mu_t}{(1 + (1-\tau) r_t)}. \tag{55}$$

In positive shocks the credit constraint does not bind, $\mu_t = 0$, and κ_t which was minute to begin with, declines further as increased supply drives the interest rate downward. While κ_t never reaches zero, positive shocks drive it close enough to zero that it ceases to have any affect on labor, which consequently becomes nearly fixed.

Moreover, κ_t can be interpreted in terms of welfare as having one component that is consistent with efficient production because it expresses a financing cost which is part of the production function, $\omega^{-1} (1 + (1-\tau) r_t)^{-1} (1-\tau) r_t$, and one component that is not consistent with efficient production because it is due only to information problems, $\omega^{-1} (1 + (1-\tau) r_t)^{-1} \mu_t$. In contrast to the case of positive shocks, when the economy is subject to a negative productivity shock, the borrowing constraint binds and tightens along with the capacity constraint. Thus in this case, μ_t, an inefficiency wedge absent from the response to positive shocks, grows into the dominant force driving labor dynamics.

4.3 The distribution of work hours

The labor distribution generated by the signaling model and $a_{\Delta} = a_{\delta}$ closely matches the distribution of US hours. The comparison is conducted as follows: First, the log of Quarterly US Business Sector Hours

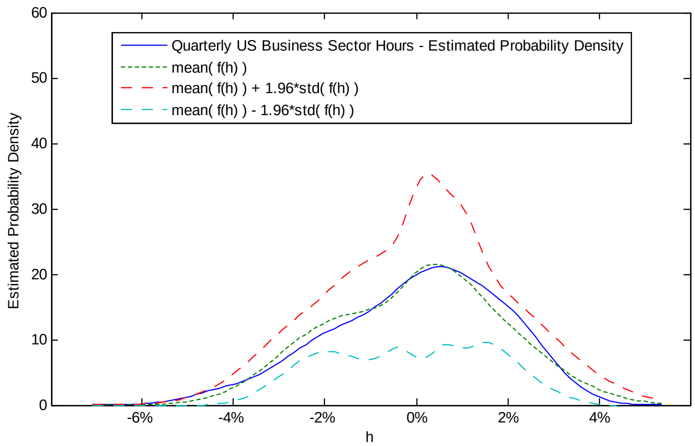

Figure 18: Probability density of logged HP-filtered US hours and a simulated distribution of hours generated by the model, both estimated using kernel smoothing.

from 1947:Q1 to 2009:Q2 is HP-filtered ($\lambda = 1600$) and run through a kernel-smoothing density estimation procedure.[8] Second, 100 series of an $AR(1)$ with $\sigma = 0.008$ and $\rho = 0.87$, as in the productivity process, are simulated. The length of each of the series matches that of the hours sample. Third, using a spline interpolation of the relation observed in Figure 17, each simulated productivity value is matched with its corresponding one-period-after-impact labor input. This generates 100 series of simulated log hours. Finally, each series is HP-filtered ($\lambda = 1600$).

Each series is now passed through the kernel-smoothing density estimation procedure to produce an estimated probability density function; denote each such function as $f(h)$. A population of 100 estimated probability density functions is thus obtained. From this population the mean and standard deviation of $f(h)$ are computed for every h in the relevant range and a 95% confidence band is constructed. As can be seen in Figure 18, the estimated density of US hours fits well within this band; thus, it is generated quite accurately by the model.

5 Conclusion

The question addressed in this paper is whether signaling-induced frictions in equity financing can help reproduce labor volatility. With al-

[8] MATLAB's *ksdensity* is used with its default parameters.

lowance for the assumptions needed to obtain the result - most notably the selection of the $a_\triangle = a_\delta$ convergence path - we conclude that they can. Moreover, not only is the model able to reproduce hours' second moment, i.e. its volatility, it is also able to reproduce its third moment, i.e. its asymmetry, and all following moments. But does the model capture the true dynamics driving labor demand?

While many studies have documented the direct effect of payout announcements on stock prices (**?**; **?**; **?**; **?**; **?**), it has yet to be shown that micro-data can accommodate a response of the form and intensity used here. Furthermore, the partial equilibrium model can produce time series that should be tested against firm-level data. Clearly, some firms are more credit-constrained than others and if we are to accept the partial equilibrium's description of labor demand, it should be able to explain variations both across firms and over time.

The same goes for the general equilibrium. Thus, if the convergence path of $a_\triangle = a_\delta$ is truly the reason for aggregate labor volatility, $a_\triangle$ must not conflict with aggregate level regressions. And again, the general equilibrium model can produce time series that should be tested against aggregate-level data in terms of reproducing co-movements, as is common in the RBC literature.

Finally, the way the model considers financial intermediaries could very well be over-simplified. Certainly, a model where financial intermediaries are profit maximizing is preferable. One option is to model them as debt free, practically lending their own capital. With small fixed profits on the up-side and substantial losses on the down-side it is clear that their optimal value estimation function would be different from the one optimizing on the symmetric goodness-of-fit used in this study. A more realistic approach would model them as almost entirely debt financed, the way they really are. In this case, while any firm-specific losses are paid by equity holders, aggregate negative shocks below some level drive the financial intermediary into government receivership, forcing tax-payers to finance losses. As the equity of the financial intermediary draws near zero, its operations become dominated by option-value considerations. It is hard to say, offhand, whether this implies it is optimal for it to become more payout-sensitive, or less.

The paper offers a somewhat novel approach in incorporating micro-level rational signaling in an RBC setting. Having seen its use, it may be worthwhile to point that the signaling mechanism presented follows **?**'s Handicap Principle of Evolutionary Biology, which states that when a social signal is carried by a continuous variable, it must be costly to produce increases in the level of the signal or else weak players would be able to disguise themselves as strong ones. Because such signals can be trusted by receivers, they are not easily undone. As in the classic example of the peacock's tail, the resulting signal may be enormously costly for the average sender, but may nonetheless prevail, as senders are unable to coordinate a binding agreement to send less of it.

Interestingly, the cost of signaling in the model presented here is the loss of profitable production opportunities. In addition, only a firm that is credit-constrained has any use for sending this signal. What is then the nature of the inflated payout signal? It is essentially saying to financial intermediaries: "Granted, this firm is in a bad shape, but it is still a better borrower than some of the *other* firms. If it wasn't, it could not afford to distribute that much cash to its shareholders".

Appendix

A The General Equilibrium Equation System

After imposing price stability and the equilibrium condition, the model amounts to four equations for households (the government budget constraint is substituted into the household's, which is then dropped due to Walras' law):

$$\begin{aligned}\frac{\partial \mathcal{L}_h}{\partial c_t} &: \quad \frac{1}{y_t} = v_t, \\ \frac{\partial \mathcal{L}_h}{\partial l_t} &: \quad \frac{\alpha}{1-l_t} = v_t w_t, \\ \frac{\partial \mathcal{L}_h}{\partial b_t} &: \quad v_t = \beta\left(1+r_t\right) E_t v_{t+1},\end{aligned} \tag{56}$$

$$\frac{\partial \mathcal{L}_h}{\partial s_{it}} : \quad v_t q_{it} = \beta E_t\left[v_{t+1}\left(\delta_{i,t+1} + q_{i,t+1}\right)\right], \tag{57}$$

one equation for the final goods firm:

$$y_t = \left(\int_0^1 z_{it}^{\eta}\left(l_{it}\right)^{(1-\theta)\eta} di\right)^{\frac{1}{\eta}}, \tag{58}$$

and nine equations for each intermediate good producer:

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial \delta_t} &: \quad v_t\left(1-\lambda_{it}+\lambda_{it}\mu_{it}\phi\bar{q}_{it}\frac{a_\delta}{\delta_{it}}\right) \\ &\quad -\beta E_t\left[v_{t+1}\lambda_{i,t+1}\mu_{i,t+1}\phi\bar{q}_{i,t+1}\frac{a_q}{q_{it}}\right] = 0,\end{aligned} \tag{59}$$

$$\frac{\partial \mathcal{L}}{\partial l_t} : \eta\left(1-\theta\right) y_t^{1-\eta} z_{it}^{\eta}\left(l_{it}\right)^{\eta(1-\theta)-1} - w_t\left(1+\kappa_{it}\right) = 0, \tag{60}$$

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial b_t} &: \quad v_t\lambda_{it}\left(1-\mu_{it}\right) - \beta\left(1+\left(1-\tau\right)r_t\right)\cdot \\ &\quad E_t\left[\lambda_{i,t+1}\left(v_{t+1}+\beta v_{t+2}\lambda_{i,t+2}\mu_{i,t+2}\phi\bar{q}_{i,t+2}\frac{a_q}{q_{i,t+1}}\right)\right] = 0,\end{aligned} \tag{61}$$

$$\frac{\partial \mathcal{L}}{\partial M_t} : v_t \lambda_{it} (1 - \omega \kappa_{it}) - \beta E_t \left[\lambda_{i,t+1} \left(v_{t+1} + \beta v_{t+2} \lambda_{i,t+2} \mu_{i,t+2} \bar{q}_{i,t+2} \frac{a_q}{q_{i,t+1}} \right) \right] = 0, \quad (62)$$

$$\text{budget} : b_{it} + y_t^{1-\eta} z_{it}^{\eta} (l_{it})^{\eta(1-\theta)} + M_{i,t-1} - (1 + (1-\tau) r_{t-1}) b_{i,t-1} - w_t l_{it} - \delta_{it} - M_{it} = 0, \quad (63)$$

$$\text{credit} : b_{it} - \phi \bar{q}_{it} = 0, \quad (64)$$

$$\text{capacity} : w_t l_{it} - \omega M_{it} = 0, \quad (65)$$

$$\text{productivity} : z_{it} = (z_{i,t-1})^{\rho} e^{\sigma \varepsilon_{it}}, \quad (66)$$

$$\text{estimated value:} \quad \bar{q}_{it} = \exp \begin{bmatrix} q_t + a_\delta \cdot (\ln \delta_{it} - \ln \delta_t) \\ + a_q \cdot (\ln q_{i,t-1} - \ln q_t) \end{bmatrix}. \quad (67)$$

If the liquidity constraint is not binding, then $\mu_t = 0$ replaces (64).

B Deterministic Steady-State

In the deterministic steady-state, $z_{it} = 1$ for every i and t. The model reduces to the following four equations for households:

$$\frac{\partial \mathcal{L}_h}{\partial c_t} : \frac{1}{c_{ss}} = v_{ss}, \quad (68)$$

$$\frac{\partial \mathcal{L}_h}{\partial l_t} : \frac{\alpha}{1 - l_{ss}} = v_{ss} w_{ss}, \quad (69)$$

$$\frac{\partial \mathcal{L}_h}{\partial b_t} : r_{ss} = \frac{1}{\beta} - 1, \quad (70)$$

$$\frac{\partial \mathcal{L}_h}{\partial s_{it}} : q_{ss} = \frac{\beta}{1 - \beta} \delta_{ss}, \quad (71)$$

two equations for the final goods firm and equilibrium:

$$y_{ss} = l_{ss}^{1-\theta}, \quad (72)$$

$$y_{ss} = c_{ss}, \quad (73)$$

and seven equations for the representative intermediate good producer:

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial \delta_t} &: \quad 1-\lambda_{ss}+\lambda_{ss}\mu_{ss}\phi\left(q_{ss}\frac{a_\delta}{\delta_{ss}}-\beta a_q\right)=0, && (74)\\
\frac{\partial \mathcal{L}}{\partial l_t} &: \quad \eta\left(1-\theta\right)y_{ss}^{1-\eta}\left(l_{ss}\right)^{\eta(1-\theta)-1}-w_{ss}\left(1+\kappa_{ss}\right)=0, && (75)\\
\frac{\partial \mathcal{L}}{\partial b_t} &: \quad 1-\mu_{ss}-\beta\left(1+\left(1-\tau\right)r_{ss}\right)\left(1+\beta\lambda_{ss}\mu_{ss}\phi a_q\right)=0, && (76)\\
\frac{\partial \mathcal{L}}{\partial M_t} &: \quad 1-\omega\kappa_t-\beta\left(1+\beta\lambda_{ss}\mu_{ss}a_q\right)=0, && (77)\\
\text{budget} &: \quad y_{ss}^{1-\eta}\left(l_{ss}\right)^{\eta(1-\theta)}-\left(1-\tau\right)r_{ss}b_{ss}-w_{ss}l_{ss}-\delta_{ss}=0, && (78)\\
\text{credit} &: \quad b_{ss}-\phi q_{ss}=0, && (79)\\
\text{capacity} &: \quad w_{ss}l_{ss}-\omega M_{ss}=0. && (80)
\end{aligned}$$

C The reference model

Households, final good producers and government remain the same. The difference lies in the intermediate good producers and financial intermediaries, whereby the friction in equity funding is removed. Financial intermediaries observe the true value of the firms and allow them to borrow up to ϕ of their true value.

The firm solves:

$$\max_{l_{it},\delta_{it},b_{it},M_{it}} E_0\sum_{t=0}^{\infty}\beta^t\frac{v_t}{v_0}\delta_{it}$$

s.t.

$$\begin{aligned}
b_{it} + y_t^{1-\eta}z_{it}^{\eta}\left(l_{it}\right)^{\eta(1-\theta)}+\frac{M_{i,t-1}}{P_t} &= \\
\left(1+\left(1-\tau\right)r_{t-1}\right)b_{i,t-1}+w_tl_{it}+\delta_{it}+\frac{M_{it}}{P_t} & \qquad (81)
\end{aligned}$$

$$\begin{aligned}
b_{it} &\leq \phi q_t && (82)\\
w_tl_{it} &\leq \omega\frac{M_{it}}{P_t} && (83)\\
\ln z_{it} &= \rho\ln z_{i,t-1}+\sigma\varepsilon_{it} && (84)
\end{aligned}$$

Lagrangian (dropping the index i and substituting $P_t=1$ for brevity):

$$\mathcal{L}=E_0\sum_{t=0}^{\infty}\beta^t\frac{v_t}{v_0}\left\{\begin{array}{c}\delta_t\\ +\lambda_t\left[\begin{array}{c}b_t+y_t^{1-\eta}z_t^{\eta}\left(l_t\right)^{\eta(1-\theta)}+M_{t-1}\\ -\left(1+\left(1-\tau\right)r_{t-1}\right)b_{t-1}-w_tl_t-\delta_t-M_t\end{array}\right]\\ +\lambda_t\mu_t\left[\phi q_t-b_t\right]\\ +\lambda_t\kappa_t\left[\omega M_t-w_tl_t\right]\end{array}\right\}$$

The FOC for optimal payout is given by:

$$\frac{\partial \mathcal{L}}{\partial \delta_t} : 1 - \lambda_t + \lambda_t \mu_t \phi \frac{\partial q_t}{\partial \delta_t} = 0 \tag{85}$$

but since

$$\frac{\partial q_t}{\partial \delta_t} = -1 \tag{86}$$

we have

$$1 - \lambda_t - \lambda_t \mu_t \phi = 0. \tag{87}$$

The FOC for labor demand is given by:

$$\frac{\partial \mathcal{L}}{\partial l_t} : \eta (1-\theta) y_t^{1-\eta} z_t^{\eta} (l_t)^{\eta(1-\theta)-1} - w_t (1+\kappa_t) = 0 \tag{88}$$

since by envelope conditions:

$$\frac{\partial q_t}{\partial l_t} = \frac{\partial V_t}{\partial l_t} = 0. \tag{89}$$

The FOC for debt demand is given by:

$$\frac{\partial \mathcal{L}}{\partial b_t} : v_t \lambda_t (1-\mu_t) + \beta E_t \left[v_{t+1} \lambda_{t+1} \left(\mu_{t+1} \phi \frac{\partial q_{t+1}}{\partial b_t} - (1 + (1-\tau) r_t) \right) \right] = 0. \tag{90}$$

By envelope conditions:

$$\frac{\partial q_{t+1}}{\partial b_t} = \frac{\partial V_{t+1}}{\partial b_t} = -\lambda_{t+1} (1 + (1-\tau) r_t) \tag{91}$$

and so

$$\frac{\partial \mathcal{L}}{\partial b_t} : v_t \lambda_t (1-\mu_t) - \beta (1 + (1-\tau) r_t) E_t \left[v_{t+1} \lambda_{t+1} \left(1 + \phi \mu_{t+1} \lambda_{t+1}\right) \right] = 0. \tag{92}$$

The FOC for the operating balance is given by:

$$\frac{\partial \mathcal{L}}{\partial M_t} : v_t \lambda_t (\omega \kappa_t - 1) + \beta E_t \left[v_{t+1} \lambda_{t+1} \left(1 + \mu_{t+1} \phi \frac{\partial q_{t+1}}{\partial M_t} \right) \right] \tag{93}$$

By envelope conditions:

$$\frac{\partial q_{t+1}}{\partial M_t} = \frac{\partial V_{t+1}}{\partial M_t} = \lambda_{t+1} \tag{94}$$

and so

$$\frac{\partial \mathcal{L}}{\partial M_t} : v_t \lambda_t (\omega \kappa_t - 1) + \beta E_t \left[v_{t+1} \lambda_{t+1} \left(1 + \mu_{t+1} \phi \lambda_{t+1}\right) \right]. \tag{95}$$

References