Certification Design with Common Values*

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Abstract

This paper studies how certification design is affected by the objective of the designer. Our model features a profit-maximizing certifier offering his services to the seller of a good of unknown quality. We allow for common values as the seller's cost may depend on the quality of the good. We compare certifier-optimal with transparency-maximizing certification design. The certifier-optimal certification design implements the evidence structure of Dye (1985) – a fraction of sellers acquire information while the remaining sellers are uninformed – and results in partial disclosure to the market. A transparency-maximizing regulator prefers a less precise signal which conveys more information to the market through a higher rate of certification and unraveling (Grossman, 1981; Milgrom, 1981) at the disclosure stage.
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1 Introduction

In many markets, buyers cannot readily observe the quality of the offered goods. Sellers often respond by voluntarily disclosing verifiable product information in the form of certificates, labels, or ratings. Sellers obtain such verifiable information from intermediaries – or certifiers – that evaluate the quality of the goods against a given standard.

The extent to which sellers disclose information to the market is of central interest to the analysis of quality certification. Viscusi (1978), Grossman and Hart (1980), Grossman (1981), and Milgrom (1981) provide a powerful *unraveling* argument for complete information disclosure. However, this result relies on the assumption that all sellers are informed about the quality of their goods. If some sellers are uninformed, Dye (1985) shows that disclosure remains partial as sellers of low quality conceal their information to pool with the uninformed sellers.

As the assumptions regarding the information of sellers determine disclosure behavior, we endogenize the information structure by studying certification design and its effects on the extent of certification and information disclosure. We thereby aim to answer the following questions: How does a monopolistic certifier optimally design the standard of certification? How many sellers obtain certification and what do they disclose under the certifier-optimal design? How do these outcomes compare to the case in which the certification standard is devised by a designer who aims to maximize transparency on the market?

These questions are of high relevance in practice where certification standards are designed by different types of institutions. In some markets, the certifiers design the standard. This is the case in financial markets where credit rating agencies evaluate financial products using their own rating scales ranging from triple-A to C in around twenty steps. In other markets, the standard designer neither is the certifier nor shares the certifier's profit-maximizing objective. The International Organization of Standardization (ISO) – a non-governmental organization (NGO) – provides a large variety of different standards such as the management quality standard ISO 9001. In many intermediate good markets, suppliers seek a certificate according to this standard to assure their buyers of the quality of their products. The certification process results in a binary outcome: either the certificate is awarded or not. Standards may also be designed by governmental bodies. The United States Environmental Protection Agency (US EPA) founded the "Energy Star"-label as a standard against which to evaluate the energy efficiency of electronic products. As certified goods may receive the additional distinction "Most Efficient", the certification process generates three possible grades. Independently of the type of the certification designer, sellers typically obtain certificates from a profit-maximizing certifier. For instance, the service of certification according to the quality management standard ISO 9001 is offered by the "Big Four" accounting firms Deloitte, Ernst & Young, KPMG, and PwC.[1]

We consider the following setup. A seller may sell a good on a market composed of homogeneous buyers. Initially, neither the seller nor the buyers know the quality of the good. We allow for common values in that the quality influences both the value of the buyers as well as the opportunity cost of the seller. Before entering the market, the seller may obtain a hard signal about the good's quality from a monopolistic certifier at a fee. The certifier incurs a cost from certifying the seller. Upon paying the fee, the seller observes a realization of the signal and decides whether or not to disclose the signal realization to the buyers. The buyers do not observe whether the seller obtains the signal and can therefore not differentiate between sellers who are uninformed and sellers who are informed but conceal their signal realization. We model certification design as the choice of a signal from a set of technologically feasible signals.

We obtain three main results. First, we provide an elegant and useful characterization of certifier-optimal certification design. In particular, the certifier's optimal signal maximizes the buyer's willingness to pay across all feasible signals. This characterization holds for any arbitrary set of feasible signals and is irrespective of the probability at which the seller obtains the optimal signal in equilibrium. Second, we use the characterization to show that any certifier-optimal signal induces partial certification and, consequently, partial disclosure in equilibrium. Thus, the information structure of Dye (1985) arises endogenously. Third, we show that certifier-optimal certification design is suboptimal from the perspective of a regulator who aims to convey as much information to the market as possible. In particular, we show that by lowering the informativeness of a certifier-optimal signal, the regulator can increase the information on the market through higher equilibrium rates of certification and disclosure. Moreover, we provide conditions under which the regulator would always optimally choose a signal that induces full certification and unraveling as in Grossman (1981) and Milgrom (1981).

Two comments regarding these findings are in order. First, our results require the conditions of common values and costly certification. Common values between the seller and the buyers naturally arise in many markets in which certification is important.[2] In product markets, producers of higher quality often incur higher production costs. In markets for durable goods, owners of high quality goods derive higher utility from

[1] Sellers who wish to obtain US EPA's Energy Star label are required to pay a "Professional Engineer" to review their application.

[2] By common values, we mean that the valuations of seller and buyers are both determined by the same underlying parameter, the quality, but are not identical. This follows the use of the term in Maskin and Tirole (1990, 1992).

keeping the good. In financial markets, sellers of more profitable financial assets typically expect higher prices when selling their asset either in the future or through a different channel. Certification costs seem to be an equally natural condition as a certifier may incur some costs when signing a contract with the seller, reviewing the seller's documents, or providing access to the certificate for potential buyers. Moreover, our results hold even in the case where certification costs are strictly positive but arbitrarily small.[3] Our second remark concerns the regulator's objective to increase the information on the market. The amount of information on the market is a key metric in the literature on information disclosure. However, it does not coincide with social welfare. Indeed, social welfare and market informativeness are perfectly misaligned in our model as full trade prevails independently of market information while certification costs are a waste from a social welfare perspective. However, this stark contrast is an artefact of the simplicity of our model. If information has social value either due to an ex-ante investment of the seller (Ben-Porath, Dekel and Lipman, 2018) or an ex-post investment of the buyer (Shavell, 1994), market informativeness and social welfare become much more closely aligned.

As an important intermediate step of our analysis, we characterize the seller's demand and the certifier's optimal certification fee for any signal. We start by making a case distinction regarding the signal which is central to our analysis. We say that a signal induces *strong common values* if the buyers' expected valuation after the worst signal realization lies strictly below the seller's unconditional expected value from keeping the good. By contrast, a signal induces *weak common values* if the buyers' expected valuation after the worst signal realization lies weakly above the seller's unconditional expected value from keeping the good. The case distinction of weak and strong common values is closely connected to the signals' informativeness in the sense of Blackwell (1951, 1953). A more informative signal induces a wider range of posterior means. Hence, if a signal induces strong common values, so does any more informative signal.

Building on a full characterization of the seller's demand for any given signal, we find that the distinction between weak and strong common values determines whether the certifier's optimal fee implements full or partial disclosure. If a signal induces weak common values, the certifier optimally sells the signal to the seller with probability one. The unraveling argument of Grossman (1981) and Milgrom (1981) applies and leads to full disclosure. If a signal induces strong common values, the certifier optimally sells the signal with some positive probability strictly less than one, resulting in partial disclosure and an active market of uncertified goods as in Dye (1985). To see how this result comes about, suppose the certifier always sells the signal. By unraveling, the equilibrium price

[3] In the main model, we assume that the certifier incurs no costs beyond the aforementioned transaction costs. In particular, any available signal can be generated at no costs. In Section 6, we extend our analysis to signal-dependent costs. We discuss the case of costless certification in Appendix B.

of uncertified assets equals the buyers' valuation after the worst signal. If the signal induces weak common values, an uninformed seller prefers to sell at this price rather than keeping the good, and thus the seller is willing to pay a fee up to difference between the unconditional expected valuation of the buyers and the worst posterior mean. If the signal induces strong common values, an uninformed seller prefers to keep the good. Thus, the willingness to pay for the signal equals the difference between the expectations of the buyers' and seller's valuations for the good, i.e., the expected gains from trade. Can the certifier attain a higher profit by selling the signal only to a fraction of sellers? If the certifier sells the signal with less than probability one, disclosure is partial as in Dye (1985) and the price for uncertified assets increases. This raises the expected payoff for a seller who obtains certification as it now becomes an attractive option to conceal bad signal realizations and to sell without disclosure. If the signal induces weak common values, the expected payoff for an uninformed seller rises more strongly as this seller always sells without disclosure. Thus, the seller's willingness to pay for being informed decreases and the certifier's profit falls. However, if the signal induces strong common values, the certifier can indeed increase her profit. In particular, the certifier optimally sells the signal with a probability such that the price on the market for uncertified assets equals the seller's expected payoff of keeping the good. This increases the expected payoff from being informed – due to the option value from selling without disclosure – and leaves the expected payoff from being uninformed unchanged. Hence, the seller's willingness to pay for the signal increases. If the certifier sets the fee equal to this willingness to pay, neither informed nor uninformed sellers, nor buyers earn a positive rent. Thus, the seller's revenue equals the expected gains from trade. To extract this return, the certifier needs to certify only a fraction of sellers. Thus, certification costs are lower and profits higher than under full unraveling. Importantly, our analysis of certifier-optimal fees implies that any optimal fee equals the seller's maximal willingness to pay for this signal across all equilibria of the disclosure game, independently of the probability at which this signal is sold.

The analysis of optimal signal fees paves the way toward a simple yet powerful characterization of the set of certifier-optimal signals. Given that the certifier-optimal fee maximizes the seller's willingness to pay for the signal, the certifier picks only signals that maximize the seller's maximal willingness to pay across all feasible signals, again independently of the probability with which the signal is sold in equilibrium. The characterization directly implies that any optimal signal induces strong common values. Thus, partial disclosure as in Dye (1985) arises endogenously under certifier-optimal certification design. To see this later point, note that the revenue under a signal inducing weak common values is bounded by the expected gains from trade. Any signal which induces strong common values generates a revenue equal to the expected gains from trade. Moreover, expected certification costs are lower as the seller remains uncertified with a strictly positive probability.

Finally, we analyze regulator-optimal certification design. First and seemingly paradoxical, we show that the regulator may increase the information on the market by lowering the informativeness of certification. Indeed, any signal that induces strong common values can be garbled into a new signal that leads – under a certifier-optimal fee – to more information on the market in the sense of Blackwell (1951, 1953). The increase in information on the market is due to higher probabilities of certification and disclosure. If the certification costs lie below the expected gains from trade, the transparency-increasing garbling takes a simple form: posterior means below some cutoff are pooled into the same signal realization whereas the posterior means above the cutoff are perfectly revealed. The cutoff is chosen such that the signal induces weak common values and is therefore sold to all sellers and fully disclosed. If some feasible signal Blackwell-dominates all other feasible signals, garbling the Blackwell-dominant signal as just described yields a transparency-maximal signal. Thus, this signal would be optimal for the transparency-maximizing regulator.

Our analysis uncovers an important caveat faced in the regulation of certification design. Very precise certification designs induce certifiers to set high fees at which only a fraction of sellers obtains certification. Less precise certificates induce lower fees from certifiers and may result in more transparency through more certification and higher disclosure activity.

The paper is organized as follows. In the remainder of this section, we present the related literature. We present the model in Section 2. In Section 3, we provide a complete characterization of the seller's demand for certification. Section 4 presents the analysis of the certifier's optimal pricing of a given signal. In Section 5, we study optimal certification design from the perspectives of the certifier and the regulator. In Section 6, we extend our analysis to the case where certification costs may depend on the signal. Section 7 concludes. The proofs and a discussion of the case of costless certification appear in the Appendices A and B.

1.1 Related Literature

Our paper relates to the literature on voluntary disclosure of verifiable information. This literature is concerned with the strategic disclosure of evidence by a sender to a competitive market. Viscusi (1978), Grossman and Hart (1980), Grossman (1981), and Milgrom (1981) consider a perfectly informed sender who can disclose any evidence at no cost. In this setup, information unravels as any sender type discloses in equilibrium. Verrecchia

(1983) shows that unraveling breaks down if disclosure is costly as senders with relatively low values choose to conceal their evidence. Dye (1985) and Jung and Kwon (1988) show that the unraveling result also fails if the market is uncertain regarding whether or not the sender possesses evidence. In this setup, senders with unfavorable evidence pool with uninformed senders by not disclosing their information. In all of these seminal contributions as well as in the more recent literature on disclosure games (Glazer and Rubinstein, 2008; Sher and Vohra, 2015; Hart, Kremer and Perry, 2017; Ben-Porath, Dekel and Lipman, 2019; Lichtig and Weksler, 2022), each sender type is exogenously endowed with a set of evidence that the sender may disclose.

Our paper contributes to the growing literature that studies endogenous evidence structures. DeMarzo, Kremer and Skrzypacz (2019) model an endogenous process of evidence gathering without a strategic information intermediary.[4] In their main model, an initially uniformed sender is choosing from a given set of signals. They assume that the signals produce a null result with positive probability, as in the information structure of Dye (1985). For any given set of signals, they characterize the equilibrium signal as the one which induces the minimal price in case the seller does not disclose. In our model, every signal yields a verifiable result with certainty.[5] The evidence structure of Dye (1985) emerges in our analysis as the result of a profit-maximizing pricing decision of a monopolistic information intermediary. Additionally, DeMarzo et al. (2019) show – under a restriction to signals with two possible realizations – that the equilibrium signal choice of the sender is inefficient in terms of information released to the market. In our analysis, the certifier's signal choice is also inefficient in this sense. For any certifier-optimal signal, there is another signal which is less informative in the sense of Blackwell (1951, 1953) but results in more information becoming public due to lower certification fees and higher disclosure rates. Moreover, we study the optimal signal choice of a transparency-maximizing regulator in this environment.

Our paper belongs to the strand of the literature that adds a strategic information intermediary to the basic disclosure setup with a seller (sender) and a competitive market (receiver). We contribute to this literature by studying the implications of common values on the incentive of the intermediary, an aspect which has previously not been studied.

One important strand of the literature analyzes the optimal behavior of a monopolistic information intermediary who offers the seller a signal at a fee. Lizzeri (1999) assumes that the seller is initially fully informed and finds that the intermediary's optimal signal is completely uninformative. Ali, Haghpanah, Lin and Siegel (2022) consider the same question with two major differences. First, the seller is initially uniformed. Second, in

[4]Another example is Ben-Porath et al. (2018). They model an endogenous project choice that plays a similar role to the role of endogenous evidence gathering in DeMarzo et al. (2019).

[5]Thus, a seller who acquired a signal can always distinguish himself from a seller who did not.

case that the strategy of the intermediary induces multiple disclosure equilibria, the worst equilibrium from the intermediary's perspective is chosen. They find that the optimal signal is noisy and features a continuum of possible scores.[6] Our setting includes an initially uninformed seller as in Ali et al. (2022). Thus, our approach is also in the spirit of the literature on information design and Bayesian persuasion (Kamenica and Gentzkow, 2011) which it extends to a framework of verifiable disclosure. We differ from Ali et al. (2022) by assuming a favorable equilibrium selection from the intermediary's perspective as well as by allowing for common values.

Another strand of this literature considers a regulator who wants as much information as possible to be released to the market. Most of this literature deals with the comparison between two regulatory disclosure regimes: mandatory disclosure and voluntary disclosure (Shavell, 1994; Weksler and Zik, 2022; Bar-Gill and Porat, 2020) or the interplay between these two regimes (Friedman, Hughes and Michaeli, 2020; Bertomeu, Vaysman and Xue, 2021; Banerjee, Marinovic and Smith, 2021). An important exception is Harbaugh and Rasmusen (2018). They consider a model where the information intermediary is the regulator himself, the seller is initially fully informed and creating a signal is costly. Harbaugh and Rasmusen (2018) find that, although the information intermediary's objective is to release as much information as possible, the optimal signal is not fully informative. By coarsening the signal, the information intermediary is able to induce more types of sellers to acquire the signal and thus release more information to the public. We also consider a transparency-maximizing regulator whose signal choice we contrast with that of the profit-maximizing certifier. Importantly, in our framework, the regulator is only in charge of the signal choice while the profit-maximizing certifier remains in charge of selling the signal. Moreover, the mechanisms underlying their result and ours are very different. In our model, the information structure of Dye (1985) emerges endogenously whereas the assumptions on the feasible signals in Harbaugh and Rasmusen (2018) imply a disclosure game along the lines of Verrecchia (1983).

[6] Faure-Grimaud, Peyrache and Quesada (2009) consider the intermediate case where the seller is initially partially informed. They allow the intermediary to charge two types of fees, a testing fee and a disclosure fee that may depend of the realization of the signal. On the other hand, they assume that the signal is given and perfect, that is, the intermediary only chooses the selling strategy of the perfect signal. They show that the intermediary is able to extract the entire surplus in this environment.

2 Model

2.1 Market

A risk-neutral seller seeks to sell a good in a competitive market to one of several risk-neutral buyers. The value of the good to the seller and the buyers depends on the state of the world $v \in V \subset \mathbb{R}$. The state is initially unobservable to the seller and the buyers. The set V is compact and its convex hull is denoted by $[\underline{v}, \overline{v}]$. The state is the realization of a random variable ν with the commonly known cumulative distribution function $F(v) \equiv \Pr(\nu \leq v)$. For $v \in V$, each buyer values the good by v while the seller values the good by $\phi(v)$. Depending on the context, the opportunity cost of trade $\phi(v)$ can be understood as a production cost – such as with consumption goods –, the value of keeping the good – as with durable goods –, or the expected payoff from selling the good at a future date or through a different channel – as with financial assets.

2.2 Certification

Before going to the market, the seller can obtain hard information about the state of the world from a certifier. The certifier sets a fee $r \geq 0$ at which the seller can observe the realization of a signal σ. A signal σ is a random variable with generic realization s in the support S_σ that may be correlated with the state ν.[7] For a given signal σ, a signal realization $s \in S_\sigma$ induces the posterior mean $E_\sigma[\nu|s]$ of the buyers' value. The signal σ induces a distribution over the posterior means of the buyers' value of

$$G_\sigma(v) \equiv \Pr(E_\sigma[\nu|\sigma] \leq v).$$

The prior distribution $F(v)$ is a mean-preserving spread of $G_\sigma(v)$ for any signal σ, i.e.,[8]

$$\int_{\underline{v}}^{\overline{v}} v dG_\sigma(v) = E[\nu] \quad \text{and} \quad \int_{\underline{v}}^{v} G_\sigma(x)dx \leq \int_{\underline{v}}^{v} F(x)dx \quad \forall v \in [\underline{v}, \overline{v}].$$

Denote the set of all signals that satisfy these two conditions by Σ_F. Let V_σ be the set of posterior means induced by the signal σ and let $[\underline{v}_\sigma, \overline{v}_\sigma]$ denote the closure of V_σ.

There is common knowledge regarding the distribution $G_\sigma(v)$ and the fee r. If the seller obtains a certificate, the certifier incurs a cost $c > 0$ and the seller observes a signal realization $s \in S_\sigma$. If the seller obtains certification and decides to disclose, the

[7] More formally, a signal structure consists of a Borel-measurable signal space S and a probability measure μ on the σ-algebra of $S \times V$ such that $\int_{S\times\{x\in V: x\leq v\}} d\mu = F(v)$.

[8] Let the random variable ε satisfy $\nu = E[\nu|\sigma] + \varepsilon$. Note that $E[\varepsilon|\sigma] = E[\nu|\sigma] - E[[\nu|\sigma]|\sigma] = 0$. Thus, $F(v)$ is a mean-preserving spread of $G_\sigma(v)$. The next line follows from Proposition 6.D.2 in Mas-Colell, Whinston and Green (1995).

buyers observe the signal realization s. If the seller does not obtain certification or obtains certification and decides not to disclose, the buyers observe the signal realization $N \notin S_\sigma$.

2.3 Payoffs, seller's strategy, and market prices

The parties obtain the following payoffs in some state of the world $v \in V$. A buyer receives $v - p$ when buying the good at price $p \in \mathbb{R}$ and zero otherwise. The payoffs of the seller and the certifier for the price p and the fee r are given by

$$u = \phi(v) + \mathbf{1}(\text{sell})(p - \phi(v)) - \mathbf{1}(\text{cert})r \quad \text{and} \quad \pi = \mathbf{1}(\text{cert})(r - c),$$

where $\mathbf{1}(\text{sell}) = 1$ if the good is traded and $\mathbf{1}(\text{sell}) = 0$ otherwise, and $\mathbf{1}(\text{cert}) = 1$ if the seller obtains certification and $\mathbf{1}(\text{cert}) = 0$ otherwise.

Given a signal σ and a fee r, the seller faces the following sequence of decisions. First, the seller needs to decide whether to obtain certification or not. We denote the probability to buy certification by $a \in [0, 1]$. If the seller does not buy the certificate, she can decide whether to sell the good or not. We denote by $b_U \in [0, 1]$ the probability with which the seller sells the good in this case. If the seller obtains certification and observes $s \in S_\sigma$, she has three options: sell the good and disclose s, sell the good without disclosure, or keep the good. For any $s \in S_\sigma$, let $b_C^D(s) \in [0, 1]$ be the probability to sell with disclosure and $b_C^N(s) \in [0, 1]$ the probability to sell without disclosure. The total probability to sell satisfies $b_C^D(s) + b_C^N(s) \leq 1$. A (behavioral) strategy for the seller is therefore a collection

$$y = (a, b_U, b_C^D(\cdot), b_C^N(\cdot)).$$

If the seller decides to sell, the good is either traded uncertified or with some certificate $s \in S_\sigma$. Given a strategy y, the probability that the good is traded uncertified is

$$\Pr(N) \equiv aE[b_C^N(\sigma)] + (1 - a)b_U.$$

We say that the market for uncertified goods is *active* if $\Pr(N) > 0$ and *inactive* if $\Pr(N) = 0$. We denote by p^N the price on the market for uncertified goods. The function $p^D : S_\sigma \to \mathbb{R}$ assigns a market price to any market for goods that are sold with the certificate $s \in S_\sigma$. Thus, the market prices are given by the collection

$$p = (p^N, p^D(\cdot)).$$

2.4 Equilibrium notion

Next, we define an equilibrium given a signal σ and a fee r. Denote the seller's expected payoff from a strategy y given the prices p, the signal σ and the fee r by

$$U(y,p,\sigma,r) \equiv a\Big(E\big[b_C^D(\sigma)p^D(\sigma) + b_C^N(\sigma)p^N + (1-b_C^D(\sigma)-b_C^U(\sigma))E_\sigma[\phi(\nu)|\sigma]\big] - r\Big) + (1-a)\Big(b_U p^N + (1-b_U)E[\phi(\nu)]\Big).$$

Our equilibrium notion is the following.

Definition 1. *Given a signal σ and a fee r, an equilibrium is a combination of a strategy y for the seller and market prices p such that*

1. *y is optimal for the seller given p, i.e.,*

$$y \in \arg\max_{y'} U(y', p, \sigma, r) \tag{1}$$

2. *p is consistent with y, i.e.,*

$$p^D(s) = E_\sigma[\nu|s],\ \forall s \in S_\sigma, \quad p^N \in \begin{cases} \{E[\nu|N]\} & \textit{if } \Pr(N) > 0, \\ [\underline{v}_\sigma, \bar{v}_\sigma] & \textit{if } \Pr(N) = 0. \end{cases} \tag{2}$$

The set of equilibria is denoted by $\mathcal{E}(\sigma, r)$.

The equilibrium notion consists of two conditions. Condition (1) requires that the seller plays an optimal strategy given the market prices. Condition (2) is a market-clearing condition. As the seller is on the short side of the market, the good is traded at the buyers' expected value. If the market for uncertified goods is inactive, rational expectations do not pin down the buyers' expected value. Given the seller's information, any expectation in the set $[\underline{v}_\sigma, \bar{v}_\sigma]$ remains possible in this case.[9]

One may provide a game-theoretic foundation for this market equilibrium notion. Consider a game in which the seller first decides whether to offer the good and what to disclose, followed by the buyers bidding for the good in a second-price auction. Any (weak) perfect Bayesian perfect equilibrium[10] of this game would then satisfy conditions (1) and (2).

[9]Note that the buyers form beliefs regarding the type of seller they are facing. A feasible belief must be a probability distribution over possible types of the seller. Given a signal σ, a seller could be either uninformed or informed by some realization of the signal σ. It follows that an expected quality outside of $[\underline{v}_\sigma, \bar{v}_\sigma]$ is not possible as it does not correspond to any probability distribution over the seller's types.

[10]As defined in Definition 9.C.3 of Mas-Colell et al. (1995)

2.5 Certifier's pricing problem

We now describe the certifier's problem of pricing a given signal σ. To this purpose, we define the demand $D_\sigma(r)$ for a given signal σ as a function of the fee r. We introduce this equilibrium object as it allows us to cast the certifier's problem in terms of standard monopoly analysis. We define the demand as the highest probability with which the seller acquires a signal σ in any equilibrium for a given fee r.

Definition 2. *The demand function for a signal σ is $D_\sigma(r) \equiv \max_{(y,p)\in\mathcal{E}(\sigma,r)} a$.*

The definition of the demand function implies certifier-preferred equilibrium selection – a standard assumption in contract theory and mechanism design.[11] For any given signal σ and fee $r \geq c$, the certifier's preferred equilibrium satisfies $a = D_\sigma(r)$. For the case $r < c$, the certifier might prefer an equilibrium with $a < D_\sigma(r)$. However, this is not problematic as the certifier can always set r sufficiently high to deter the seller from buying certification.

Thus, the certifier's optimal fee for a given signal σ solves the monopoly problem

$$\max_{r\geq 0} D_\sigma(r)(r-c). \tag{3}$$

2.6 Certification design

We capture certification design as the choice of a signal σ from a set of technologically feasible signals $\Sigma \subseteq \Sigma_F$. We first study the case in which the certifier chooses the signal from the set Σ. We then consider the case in which the signal is chosen by a regulator who seeks to maximize the information conveyed to the market. In both cases, the signal is then priced by the certifier according to the solution of problem (3) and sold to the seller. This assumption is motivated by the structure of certification markets in practice where – as described in the introduction – certification is typically provided by profit-maximizing certifiers while the standard may be designed by a transparency-maximizing institution.

2.7 Assumptions and discussion

We now specify three assumptions that we maintain throughout the analysis and discuss the two key conditions of common values and costly certification. Our first assumption ensures that there are always strictly positive gains from trade.

Assumption 1. *The function $\phi(\cdot)$ satisfies $\phi(v) < v$ for all $v \in V$.*

[11]See Ali et al. (2022) for an analysis of certification design under adversarial equilibrium selection.

We denote the expected gains from trade by

$$\Delta \equiv E[\nu] - E[\phi(\nu)] = \int_{\underline{v}}^{\overline{v}} (v - \phi(v)) dF(v).$$

The second assumption is a technical condition on the set of feasible signals Σ.

Assumption 2. *The set $\Sigma \subseteq \Sigma_F$ satisfies the following conditions:*

i) V_σ is closed for all $\sigma \in \Sigma$,

ii) the set $\{\underline{v}_\sigma\}_{\sigma \in \Sigma}$ is compact,

iii) the set $\{G_\sigma(\cdot)\}_{\sigma \in \Sigma}$ is compact.

The assumption ensures the existence of a certifier-optimal signal in the set of feasible signals Σ. Under condition *i*), conditions *ii*) and *iii*) are satisfied if the set Σ is finite or equal to the set of all signals, i.e., $\Sigma = \Sigma_F$.

Our model features two natural conditions that drive our main results: common values and costly certification. We make the following assumption which specifies a minimal condition for common values to matter in equilibrium.

Assumption 3. *The function $\phi(\cdot)$ satisfies $E[\phi(\nu)] > \underline{v}$.*

The assumption implies that – under some sufficiently informative signal – the buyers' belief regarding the quality of uncertified goods may be so pessimistic – and thus the price for uncertified goods so low – that an uninformed seller prefers to keep the good rather than selling it without disclosure. In this case, the value of keeping the good influences the seller's willingness to pay for the signal and thereby becomes of relevance for the certifier.[12]

As our second key condition, we assume that certification is costly, i.e., $c > 0$. Costs of contracting, reviewing documents, or disseminating credible certificates suggest this to be a mild condition, especially as we allow c to be arbitrarily small.[13] In Section 6, we show how our results can be extended to the case where the certifier faces a signal-dependent cost of generating information in addition to the transaction cost c.

[12]While the condition of common values is naturally satisfied in many markets, our characterization of optimal signals holds also in the case where Assumption 3 is not satisfied. However, our most interesting economic insights arise under Assumption 3.

[13]We discuss the case of costless certification in Appendix B.

3 Demand for a signal

In this section, we take an intermediate step in our analysis by characterizing the demand $D_\sigma(r)$ for a given signal σ as a function of the fee r. We first introduce the following case distinction which is of relevance throughout the analysis.

Definition 3.

1. *A signal σ induces strong common values if $\underline{v}_\sigma < E[\phi(\nu)]$.*
2. *A signal σ induces weak common values if $\underline{v}_\sigma \geq E[\phi(\nu)]$.*

If $\underline{v}_\sigma < E[\phi(\nu)]$, the buyers' belief regarding the value of an uncertified good may be so pessimistic that the price on the uncertified market lies below the uninformed seller's expected value from keeping the good. Hence, the common-value aspect of our model may play an important role as self-consumption is an uninformed seller's best option in this case. The certifier may be able to extract the expected gain from trade Δ if she manages to induce such a sufficiently pessimistic belief regarding the value of uncertified goods.

If $\underline{v}_\sigma \geq E[\phi(\nu)]$, keeping the good is a weakly dominated action for an uninformed seller as the price on the uncertified market always weakly exceeds the expected value from keeping the good. In this case, the common-value aspect of the model plays no role. As the uninformed seller can guarantee a payoff of $\underline{v}_\sigma$, the certifier can extract at most the remainder of the expected gains from trade given by $E[\nu] - \underline{v}_\sigma$.

Given a set of feasible signals Σ, we denote by Σ_s the subset of feasible signals that induce strong common values, and by Σ_w the complementary subset of signals that induce weak common values. Formally, we define

$$\Sigma_s \equiv \{\sigma \in \Sigma \mid \underline{v}_\sigma < E[\phi(\nu)]\} \quad \text{and} \quad \Sigma_w \equiv \{\sigma \in \Sigma \mid \underline{v}_\sigma \geq E[\phi(\nu)]\}.$$

The uninformative signal – if feasible – induces weak common values as its support of posterior means is $\{E[\nu]\}$. If the fully informative signal is feasible, it induces strong common values by Assumption 3 as the range of posterior means is $[\underline{v}, \bar{v}]$.

3.1 Equilibrium analysis

In order to characterize the demand function, we make some important observations regarding equilibrium strategies and prices. Fix some signal σ and suppose the seller buys certification and observes the signal realization $s \in S_\sigma$. By Assumption 1, selling the good with disclosure strictly dominates keeping the good as $p^D(s) = E_\sigma[\nu|s] > E_\sigma[\phi(\nu)|s]$ for all $s \in S_\sigma$. Selling with disclosure is optimal if $p^D(s) = E_\sigma[\nu|s] \geq p^N$ and selling without disclosure is optimal if $E_\sigma[\nu|s] \leq p^N$. Formally, we have in any equilibrium

$$b_C^D(s) = 1 - b_C^N(s) = \begin{cases} \{1\} & \text{if } E_\sigma[\nu|s] > p^N, \\ [0,1] & \text{if } E_\sigma[\nu|s] = p^N, \\ \{0\} & \text{if } E_\sigma[\nu|s] < p^N. \end{cases} \tag{4}$$

Next, suppose the seller has not obtained certification. The seller can then choose between keeping the good at an expected payoff of $E[\phi(\nu)]$ or selling the good without disclosure. Thus, the seller optimally plays

$$b_U \in \begin{cases} \{1\} & \text{if } p^N > E[\phi(\nu)], \\ [0,1] & \text{if } p^N = E[\phi(\nu)], \\ \{0\} & \text{if } p^N < E[\phi(\nu)]. \end{cases} \tag{5}$$

It follows that the seller's expected equilibrium payoff satisfies

$$U(y,p,\sigma,r) = a\Big(\int_{V_\sigma} \max\{v, p^N\} dG_\sigma(v) - r\Big) + (1-a)\max\{p^N, E[\phi(\nu)]\}.$$

The seller's willingness to pay for the signal σ is a function of the price for uncertified goods and given by

$$\Omega_\sigma(p^N) \equiv \int_V \max\{v, p^N\} dG_\sigma(v) - \max\{p^N, E[\phi(\nu)]\}. \tag{6}$$

The willingness to pay is the difference between the expected option value of an informed seller from choosing between selling with or without disclosure, and the option value of an uniformed seller who can choose between selling the uncertified good and keeping it.

Hence, the seller's optimal decision regarding certification satisfies

$$a \in \begin{cases} \{1\} & \text{if } r < \Omega_\sigma(p^N), \\ [0,1] & \text{if } r = \Omega_\sigma(p^N), \\ \{0\} & \text{if } r > \Omega_\sigma(p^N). \end{cases} \tag{7}$$

The conditions (4) to (7) characterize the equilibrium strategy of the seller for a given market price p^N for uncertified goods.

As a next step, we study which form the price p^N takes in equilibrium given an optimal strategy for the seller. Consider an equilibrium with an active market for uncertified goods, i.e., $\Pr(N) > 0$. The market is populated by a mixture of uninformed sellers and informed sellers of relatively low value goods. The expected value of an uncertified good to the buyers can be determined by Bayes' rule, giving us the equilibrium condition

$$\frac{(1-a)b_U E[\nu] + a\int_{\underline{v}}^{p^N} v dG_\sigma(v)}{(1-a)b_U + aG_\sigma(p^N)} = p^N. \tag{8}$$

Consider next an equilibrium with an inactive market for uncertified goods, i.e., $\Pr(N) = 0$. In such an equilibrium, informed sellers do not benefit from concealing their certificate. This requires $p^N \leq \underline{v}_\sigma$. Morever, uninformed sellers do not benefit from selling the good instead of keeping it, hence $p^N \leq E[\phi(\nu)]$.

Thus, we obtain the following equilibrium condition for p^N given an optimal strategy of the seller:

$$p^N \in \begin{cases} \{\underline{v}_\sigma\} & \text{if } \Pr(N) = 0 \text{ and } a > 0, \\ \{v \in [\underline{v}_\sigma, \bar{v}_\sigma] : v \leq E[\phi(\nu)]\} & \text{if } \Pr(N) = 0 \text{ and } a = 0, \\ \left\{\frac{(1-a)b_U E[\nu] + a\int_{\underline{v}}^{p^N} v dG_\sigma(v)}{(1-a)b_U + aG_\sigma(p^N)}\right\} & \text{if } \Pr(N) > 0. \end{cases} \tag{9}$$

We obtain the following equilibrium characterization.

Lemma 1. *For any* $(\sigma, r) \in \Sigma \times \mathbb{R}$, $(y, p) \in \mathcal{E}(\sigma, r) \iff (y, p)$ *satisfies* (4) – (9).

Before proceeding to the characterization of demand, it is helpful to revisit the function $\Omega_\sigma(p^N)$ which captures the seller's willingness to pay for the signal σ as a function of the price p^N on the uncertified market.

Lemma 2. *The function* $\Omega_\sigma(\cdot)$ *is continuous and strictly quasi-concave on* $[\underline{v}_\sigma, \bar{v}_\sigma]$. *Its unique maximum is attained at* $p^N = \max\{E[\phi(\nu)], \underline{v}_\sigma\}$ *and given by*

$$\Omega^*_\sigma \equiv \max_{p^N \in [\underline{v}_\sigma, \bar{v}_\sigma]} \Omega_\sigma(p^N) = \begin{cases} \Delta + \int_{\underline{v}_\sigma}^{E[\phi(\nu)]} G_\sigma(v)dv & \text{if } \sigma \in \Sigma_s, \\ E[\nu] - \underline{v}_\sigma & \text{if } \sigma \in \Sigma_w. \end{cases}$$

If the signal σ induces strong common values, the seller's willingness to pay first increases and then decreases in the price p^N on the uncertified market. An increase in p^N always benefits an informed seller who has the option to conceal the certificate and sell the good at p^N. If $p^N < E[\phi(\nu)]$, an increase in p^N does not affect the payoff of an uninformed seller as self-consumption remains more attractive than selling without disclosure. Thus, the willingness to pay for a signal increases in p^N for $p^N < E[\phi(\nu)]$. If $p^N > E[\phi(\nu)]$, an increase in p^N always benefits an uninformed seller as selling without disclosure is the uninformed seller's best choice. Moreover, the increase of p^N increases the uninformed seller's payoff by more than it increases the informed seller's payoff as the informed seller trades at p^N only with some probability. An increase in p^N therefore reduces the seller's willingness to pay in this case.

If the signal induces weak common values, the seller's willingness to pay decreases in the price p^N as any equilibrium price p^N needs to exceed $E[\phi(\nu)]$. Thus, any increase in p^N increases the payoff of an uninformed seller more than the payoff of an informed seller, resulting in a lower willingness to pay for the signal.

3.2 Equilibrium demand for a signal

We are now in a position to state our characterization of the demand function $D_\sigma(r)$. To this purpose, let $\Omega_\sigma^{-1}(r)$ be the inverse function of the willigness to pay $\Omega_\sigma(\cdot)$ defined on the domain $[E[\phi(\nu)], E[\nu]]$.

Lemma 3.

1. *Suppose* $\sigma \in \Sigma_s$. *If* $\Delta \geq \Omega_\sigma(E[\nu])$, *then*

$$D_\sigma(r) = \begin{cases} 1 & \text{if} \quad r \leq \Delta, \\ \frac{E[\nu]-\Omega_\sigma^{-1}(r)}{r} & \text{if} \quad r \in (\Delta, \Omega_\sigma^*], \\ 0 & \text{if} \quad r > \Omega_\sigma^*, \end{cases}$$

and $D_\sigma(r)$ *is strictly increasing on* $r \in (\Delta, \Omega_\sigma^*)$. *If* $\Delta < \Omega_\sigma(E[\nu])$, *then*

$$D_\sigma(r) = \begin{cases} 1 & \text{if} \quad r \leq \Delta, \\ 0 & \text{if} \quad r \in (\Delta, \Omega_\sigma(E[\nu]), \\ \frac{E[\nu]-\Omega_\sigma^{-1}(r)}{r} & \text{if} \quad r \in (\Omega_\sigma(E[\nu]), \Omega_\sigma^*], \\ 0 & \text{if} \quad r > \Omega_\sigma^*, \end{cases}$$

and $D_\sigma(r)$ *is strictly increasing on* $r \in (\Omega_\sigma(E[\nu]), \Omega_\sigma^*)$.

2. *Suppose* $\sigma \in \Sigma_w$. *Then*

$$D_\sigma(r) = \begin{cases} 1 & \text{if} \quad r \leq E[\nu] - \underline{v}_\sigma, \\ 0 & \text{if} \quad r > E[\nu] - \underline{v}_\sigma. \end{cases}$$

We construct these demand curves in a sequence of Lemmata, going from high to low levels of demand. We start with the segments of the demand curves at which the seller always obtains certification.

Lemma 4.

1. *Suppose $\sigma \in \Sigma_s$. There exists an equilibrium in which the seller always obtains certification if and only if $r \leq \Delta$.*

2. *Suppose $\sigma \in \Sigma_w$. There exists an equilibrium in which the seller always obtains certification if and only if $r \leq E[\nu] - \underline{v}_\sigma$.*

The lemma implies that the demand function takes the value 1 for $r \leq \Delta$ with strong common values and $r \leq E[\nu] - \underline{v}_\sigma$ with weak common values, and takes lower values for higher fees. In any equilibrium in which the seller always obtains certification, disclosure takes the form of full unraveling as studied in Grossman (1981) and Milgrom (1981). Under full unraveling, the price for an uncertified good is $\underline{v}_\sigma$. The certifier may charge any fee up to Δ with strong common values, and up to $E[\nu] - \underline{v}_\sigma$ with weak common values. As long as the fee lies below these thresholds, the seller optimally always obtains certification under the expectation that the market for uncertified good is inactive.

It remains to construct the demand functions for $r > \Delta$ with strong common values, and $r > E[\nu] - \underline{v}_\sigma$ with weak common values. We first consider those segments of the demand curves at which the seller obtains certification with an intermediate probability.

Lemma 5.

1. *Suppose $\sigma \in \Sigma_s$. For $r > \Delta$, there exists an equilibrium in which the seller obtains certification with an interior probability if and only if $r \in (\max\{\Delta, \Omega_\sigma(E[\nu])\}, \Omega^*_\sigma]$. For $r \in (\max\{\Delta, \Omega_\sigma(E[\nu])\}, \Omega^*_\sigma)$, any equilibrium satisfies*

$$a = \frac{E[\nu] - \Omega_\sigma^{-1}(r)}{r}.$$

For $r = \Omega^*_\sigma$, there exists an equilibrium for any $a \in (0, \Delta/\Omega^*_\sigma]$.

2. *Suppose $\sigma \in \Sigma_w$. If $r > E[\nu] - \underline{v}_\sigma$, there is no equilibrium in which the seller obtains certification with an interior probability.*

Consider first the case of strong common values. As in any equilibrium with an interior probability $a \in (0, 1)$, the seller needs to be indifferent between obtaining hard information or not, i.e, $r = \Omega_\sigma(p^N)$. Using this condition, each $r \in (\Omega_\sigma(E[\nu]), \Omega^*_\sigma)$ determines a unique equilibrium price $p^N \in (E[\phi(\nu)], E[\nu])$. As $p^N > E[\phi(\nu)]$, any uninformed seller strictly prefers to sell in this equilibrium,, i.e., $b_U = 1$. The unique equilibrium probability of certification is then pinned down by equation (8). Notice that in the range of fees $(\Omega_\sigma(E[\nu]), \Omega^*_\sigma)$, a higher fee r corresponds to a lower equilibrium price $p^N = \Omega_\sigma^{-1}(r)$, and this equilibrium price, according to equation (8), corresponds to a higher probability of certification $a = \frac{E[\nu]-\Omega_\sigma^{-1}(r)}{r}$. Thus, an increase in the fee goes together with an increase in the unique equilibrium probability of certification.

For $r = \Omega_\sigma^*$, the indifference condition $r = \Omega_\sigma(p^N)$ implies $p^N = E[\phi(\nu)]$. Thus, uninformed sellers are indifferent between selling without disclosure and self-consumption. Varying the probability b_U in $(0, 1]$ allows us to generate equilibrium probabilities of obtaining certification in $a \in (0, \Delta/\Omega_\sigma^*]$.

Next, consider the case of weak common values. For such a signal, an equilibrium with an interior probability of certification may only exist for fees that also admit an equilibrium in which the seller always obtains certification. Lemma 2 implies that the sellers' willingness to pay for the signal is maximal if the market for uncertified goods is inactive, i.e., if $p^N = \underline{v}_\sigma$. In any equilibrium with an interior probability of certification, the seller has to be indifferent between obtaining certification or not. Formally, these two observations imply $r = \Omega_\sigma(p^N) \leq E[\nu] - \underline{v}_\sigma$. Under this condition, there exists by Lemma 4 an equilibrium in which the seller always certifies.

The previous two Lemmata determine those fees for which we can sustain a strictly positive demand for certification. To show that the demand curve is well defined for all other fees – i.e., that equilibria exist for these fees – we provide the following lemma regarding those segments of the demand curves on which the seller never obtains certification.

Lemma 6.

1. *Suppose $\sigma \in \Sigma_s$. There exists an equilibrium in which the seller never obtains certification for $r > \Delta$.*

2. *Suppose $\sigma \in \Sigma_w$. There exists an equilibrium in which the seller never obtains certification for $r > E[\nu] - \underline{v}_\sigma$.*

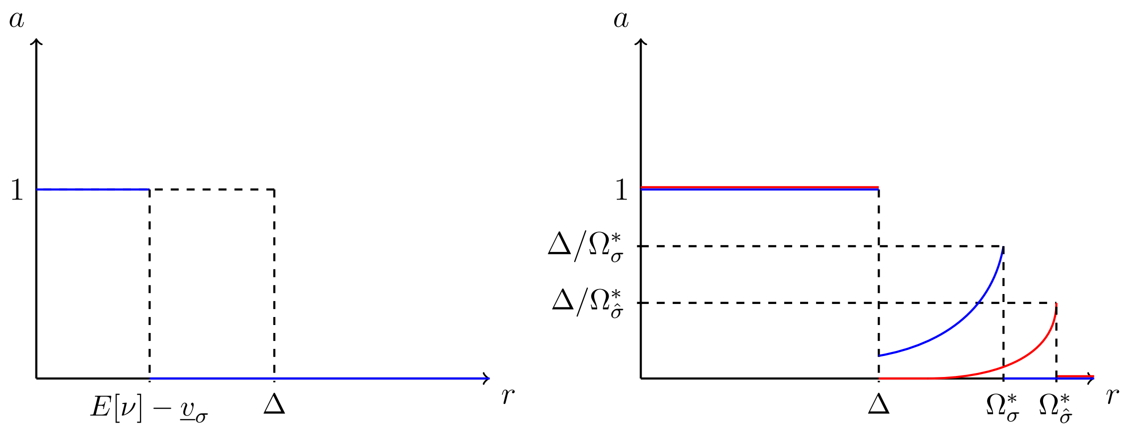
With strong common values and $r > \Delta$, there exists an equilibrium in which the seller is always uninformed and the market for uncertified goods remains inactive. The market for uncertified goods can indeed be inactive for prices in the interval $[\underline{v}_\sigma, E[\phi(\nu)]]$. The seller prefers to remain uninformed if $r \geq \Omega_\sigma(p^N)$. This latter condition is least stringent for $p^N = \underline{v}_\sigma$, implying that an equilibrium without trade and certification exists for $r \geq \Delta = \Omega_\sigma(\underline{v}_\sigma)$.[14]

Next, consider the case of weak common values. For $r > E[\nu] - \underline{v}_\sigma$, there exists an equilibrium in which the seller never obtains certification and the uncertified market is

[14]If $r \geq \Omega_\sigma(E[\nu])$ then there exist also an equilibrium with trade and without certification. In such equilibrium the seller is active in the uncertified market with probability 1. It follows that $p^N = E[\nu]$ and thus the seller would not find the deviation to acquiring σ profitable if and only if $r \geq \Omega_\sigma(E[\nu])$.

active. The latter property implies that the price on this market equals the buyer's expected value $E[\nu]$ for the good. Thus, all sellers indeed refrain from obtaining a certificate if the fee exceeds the value from the signal at this price, i.e., $r > \Omega_\sigma(E[\nu])$. The result then follows from the observation that $\Omega_\sigma(E[\nu]) < E[\nu] - \underline{v}_\sigma$ with weak common values.

Figure 1: Demand for a signal



The left panel depicts the demand functions for a signal σ which induces weak common values. The right panel depicts the demand function for two signals σ (blue) and $\hat{\sigma}$ (red) with both inducing strong common values.

The Lemmata 4 to 6 imply the characterization of demand which we illustrate in Figure 1. The left panel of Figure 1 depicts the demand curve for a signal with weak common values. For such a signal, the demand function is a simple step function. This follows from the observation in Lemma 5 that an equilibrium with strictly positive probability of certification exists only if there exists an equilibrium in which the seller always obtains certification. Thus, the demand for certification takes the form that is typical for settings in which a monopolist sells a good to a group of homogeneous buyers in the absence of externalities.

The right panel of Figure 1 illustrates the demand curves for two signals σ and $\hat{\sigma}$ that induce strong common values. The signal σ with the blue demand curve satisfies $\Delta \geq \Omega_\sigma(E[\nu])$. In this case, the demand function features three different segments. For any low fee $r \leq \Delta$, there exists an equilibrium in which the seller always obtains information. For any intermediate fee $r \in (\Delta, \Omega^*_\sigma)$, we cannot sustain an equilibrium in which the seller always obtains information. However, mixed equilibria exist in which the seller becomes informed with strictly positive probability. Those equilibria build on the fact that buying the signal conveys an option value to the seller if the market for uncertified goods is active. If the fee exceeds the seller's maximal willingness to pay Ω^*_σ, there does not exist an equilibrium in which certification is bought.

If a signal induces strong common values and satisfies $\Delta < \Omega_\sigma(E[\nu])$, the demand function features an additional segment. This is illustrated by the red demand curve for the signal $\hat{\sigma}$ in the right panel of Figure 1. For values of the fee in the interval $(\Delta, \Omega_\sigma(E[\nu]))$, no equilibrium exists in which the seller obtains certification with a positive probability. For these fees, it is not possible to find a price on the uncertified market in $[E[\phi(\nu)], E[\nu]]$ such that the uncertified market is active *and* sellers are indifferent between obtaining certification or not. As $r > \Delta$, it is not possible to sustain an equilibrium in which the seller always buys certification. Moreover, any equilibrium with a strictly positive but interior probability of buying certification requires that the price for uncertified goods makes the seller indifferent between buying certification or not, i.e., $r = \Omega_\sigma(p^N)$. If $r < \Omega_\sigma(E[\nu])$, this can – due to the form of $\Omega_\sigma(p^N)$ – only be achieved by a price $p^N > E[\nu]$. However, such a price cannot be part of equilibrium as it would require the buyers to hold overly optimistic beliefs about the composition of sellers on the uncertified market.

With strong common values, the demand function reflects that the market for uncertified goods induces an externality among the sellers. As a result of this externality, the demand function has an increasing segment. When the certification fee increases, in the intermediate range, the price for uncertified goods needs to decrease in order to keep sellers indifferent between obtaining certification or not. This is because by lowering the price for uncertified goods the payoff of both informed and uninformed sellers decreases. However, it decreases the payoff of uninformed sellers by more as they sell on the market for uncertified goods with a higher probability than informed sellers. Now, in order to sustain a lower price in the uncertified goods market it must be that this market is populated to a greater extent by informed sellers. Thus, the demand for certification increases.

4 Optimal pricing of a signal

In this section, we study how the certifier optimally prices a signal. We find the following solution to the certifier's pricing problem (3) given the demand curves illustrated in Figure 1.

Proposition 1.

1. *Suppose $\sigma \in \Sigma_s$. If $c < \Omega^*_\sigma$, the certifier attains the optimal profit $\Delta - (\Delta/\Omega^*_\sigma)c$ by certifying the seller with probability Δ/Ω^*_σ at the uniquely optimal fee $r = \Omega^*_\sigma$. If $c = \Omega^*_\sigma$, a fee is optimal if and only if $r \geq \Omega^*_\sigma$. If $c > \Omega^*_\sigma$, a fee is optimal if and only if $r > \Omega^*_\sigma$.*

2. *Suppose $\sigma \in \Sigma_w$. If $c < \Omega^*_\sigma$, the certifier attains the optimal profit $E[\nu] - \underline{v}_\sigma - c$ by certifying the seller with probability one at the uniquely optimal fee $r = E[\nu] - \underline{v}_\sigma$. If $c = \Omega^*_\sigma$, a fee is optimal if and only if $r \geq E[\nu] - \underline{v}_\sigma$. If $c > \Omega^*_\sigma$, a fee is optimal if and only if $r > E[\nu] - \underline{v}_\sigma$.*

The key insight generated by the proposition concerns the question whether the certifier prefers unraveling or partial disclosure on the market. We show that the certifier optimally induces an inactive market for uncertified goods through unraveling (Grossman, 1981; Milgrom, 1981) if the signal induces weak common values. By contrast, the certifier induces partial disclosure as in Dye (1985) if the signal induces strong common values.

In both cases of strong and weak common values, the certifier optimally sets the fee equal to the seller's maximal willingness to pay Ω^*_σ for the signal. Importantly, the optimal fee is independent of the certification cost c as long as the optimal fee covers the cost. With strong common values, the optimal fee entails selling only to a fraction of sellers with the remaining fraction of uninformed sellers trading on the market for uncertified goods. With weak common values, all sellers obtain certification and the market for uncertified goods is inactive.

With strong common values, the certifier optimally accomodates an active market for uncertified goods as in Dye (1985) by certifying only a fraction of sellers. In particular, the certifier prefers this outcome over the one induced by certifying all sellers and shutting down the uncertified market. The latter outcome would be optimally induced by a fee $r = \Delta$ which would result in a profit of $\Delta - c$. By setting a fee $r \in (\Delta, \Omega^*_\sigma]$, the certifier can activate the uncertified market. As the demand function is increasing for these fees, the optimal fee with an active uncertified market is $r = \Omega^*_\sigma$. At this fee, the fraction Δ/Ω^*_σ of sellers is certified. Compared to the case of an inactive market for uncertified goods, the certifier obtains the same revenue of Δ for a strictly smaller certification cost of $(\Delta/\Omega^*_\sigma)c$. By maintaining an active market for uncertified goods, the certifier increases an informed seller's option value from concealing the certificate. At the same time, the active uncertified market does not increase the value of being uninformed as the price p^N on the uncertified market equals the expected value of self-consumption $E[\phi(\nu)]$.

It is noteworthy that the certifier may be active even if the costs of certification exceed the expected gains from trade. Indeed, the certifier charges a fee exceeding Δ and realizes a positive margin even if $\Delta < c < \Omega^*_\sigma$.

With weak common values, it is optimal to shut down the market for uncertified goods through unraveling as in Grossman (1981) and Milgrom (1981). To prevent unraveling, the certifier would need to keep a fraction of sellers uninformed and to implement a price for uncertified goods above $\underline{v}_\sigma$. However, the lost revenue could not be recouped from informed sellers. While increasing the price p^N above $\underline{v}_\sigma$ increases the payoff of informed sellers due to higher option value, the payoff of uninformed sellers increases more strongly. Thus, the signal's value to the seller shrinks.

5 Optimal certification design

In this section, we analyze optimal certification design as the choice of a signal σ. We first study the certifier's optimal choice from the set Σ and provide a characterization of certifier-optimal signals. In a second step, we consider a regulator who seeks to maximize transparency on the market and examine regulator-optimal certification design.

We assume that there exists a technologically feasible signal under which the certifier can make a positive payoff. By this assumption, which we maintain throughout the remainder of the analysis, we ensure that certification design is non-trivial.

Assumption 4. *There exists a signal $\sigma \in \Sigma$ with $\Omega^*_\sigma > c$.*

5.1 Certifier-optimal certification design

We start by analyzing the certifier's optimal signal choice from the set Σ. We use the optimal pricing of signals in Proposition 1 to derive the following result.

Proposition 2. *The set of certifier-optimal signals is*

$$\Sigma^* = \{\sigma \in \Sigma : \sigma \in \arg\max_{\Sigma} \Omega^*_\sigma\}.$$

The proposition provides a simple and powerful characterization of the set of certifier-optimal signals: A signal is optimal if and only if the maximal willingness to pay for this signal is the highest across all feasible signals – independently of the probability with which this signal is sold in equilibrium. Therefore, the set of certifier-optimal signals can be identified using the statistic of the maximal willingness to pay Ω^*_σ only. In particular, the set of certifier-optimal signals is independent of the certification cost c. The proof of the proposition builds on the characterization of optimal fees in Proposition 1 and shows that signals with higher maximal willingness to pay Ω^*_σ generate higher revenue and lower certification cost.

We highlight that the characterization of Proposition 2 holds for arbitrary sets of feasible signals satisfying the technical condition of Assumption 2. We do not require further conditions such as all signals to be feasible, i.e., $\Sigma = \Sigma_F$, or the set Σ to be Blackwell-ordered or to contain a Blackwell-dominant signal.

Proposition 2 implies an important corollary: the celebrated evidence structure of Dye (1985) emerges endogenously under certification design.

Corollary 1. *If $\Sigma_s \neq \emptyset$, then $\Sigma^* \subseteq \Sigma_s$. Thus, the seller obtains certification with probability strictly less than 1 and partial disclosure, as in Dye (1985), prevails as a result of certifier-optimal certification design.*

Formally, the corollary can be established as follows. By Lemma 2, the maximal willingness to pay Ω_σ^* under a signal inducing strong common values exceeds the maximal willingness to pay for any signal which induces weak common values. Thus, by Proposition 2, the set of certifier-optimal signals Σ^* contains only signals inducing strong common values whenever at least one such signal is feasible. The optimality of signals with strong common values is also immediate from Proposition 1. If the certifier selects a signal that induces strong common values, the certifier generates a revenue of Δ and bears the cost c with probability strictly less than one. Under a signal with weak common values, the certifier always bears the certification cost c and extracts a revenue of at most Δ.

The corollary further simplifies the procedure of finding the set of certifier-optimal signals if there exist feasible signals which induce strong common values. In this case, one can restrict attention to the set Σ_s and identify all signals $\sigma \in \Sigma_s$ that maximize the statistic

$$\int_{\underline{v}}^{E[\phi(\nu)]} G_\sigma(v)dv.$$

This strengthening of our characterization is a result of the fact that for every $\sigma \in \Sigma_s$ the certifier is able to extract all the gains from trade Δ by charging the fee Ω_σ^*. Thus, it must be that the probability of selling a certificate is equal to $\frac{\Delta}{\Omega_\sigma^*}$. It follows that a signal that maximizes Ω_σ^* also minimizes the certifier's certification cost, and thereby maximizes the certifier's profit.

While the characterization of Proposition 2 applies to arbitrary sets of feasible signals, we obtain the following corollary if the set Σ features a most informative signal.

Corollary 2. *Suppose the set Σ contains a most informative signal $\bar{\sigma}$, i.e.,*

$$\int_{\underline{v}}^{v} G_\sigma(x)dx \leq \int_{\underline{v}}^{v} G_{\bar{\sigma}}(x)dx, \ \forall v \in [\underline{v}, \bar{v}], \ \ \forall \sigma \in \Sigma.$$

Then, $\bar{\sigma} \in \Sigma^*$

The corollary follows directly from Proposition 2 and Lemma 2. As the most informative signal induces the lowest minimal posterior mean, i.e., $\bar{\sigma} \in \arg\min_\Sigma \underline{v}_\sigma$, there exists a signal inducing strong common values if and only if $\bar{\sigma}$ induces strong common values. In both instances, $\bar{\sigma}$ induces the highest maximal willingness to pay. If $\bar{\sigma}$ induces weak common values, then $\Omega^*_{\bar{\sigma}} \geq \Omega^*_{\sigma}$ for all signals σ as $E[\nu] - \underline{v}_{\bar{\sigma}} \geq E[\nu] - \underline{v}_{\sigma}$ for all $\sigma \in \Sigma$. If $\bar{\sigma}$ induces strong common values, then $\Omega^*_{\bar{\sigma}} \geq \Delta$ by Lemma 2 and $\int_{\underline{v}}^{E[\phi(\nu)]} G_{\sigma}(v)dv \leq \int_{\underline{v}}^{E[\phi(\nu)]} G_{\bar{\sigma}}(v)dv$ by the definition of $\bar{\sigma}$. Hence, $\Omega^*_{\bar{\sigma}} \geq \Omega^*_{\sigma}$ for all signals $\sigma \in \Sigma$.

The certifier's optimal choice of signal and fee follows the principle of *maximizing the sellers' willingness to pay*, even if this requires selling only to a fraction of sellers. If the set of possible signals Σ comprises only signals with weak common values, the certifier induces unraveling. Moreover, the certifier's optimal signal minimizes the price on the uncertified market – the same metric as in the private value model of DeMarzo et al. (2019). By contrast, if there are signals with strong common values, minimizing the nondisclosure price p^N is suboptimal. In this case, the certifier cannot increase revenue from pushing the nondisclosure price below $E[\phi(\nu)]$ as this would not affect the uninformed seller's payoff and lower the informed seller's payoff. Instead, the certifier activates the uncertified market to keep revenue constant and to lower the total cost of certification.

Example In order to illustrate the use of our results, we analyze a specific example of our model. Suppose the state ν is uniformly distributed on $V = [0, 1]$ and the seller's value is a fraction $\alpha \in (0, 1)$ of the buyers' value v, i.e., $\phi(v) = \alpha \cdot v$. We consider the set of feasible signals Σ to be the set of all binary threshold signals. In particular, for every $t \in [0, 1]$, denote by σ_t the signal that certifies whether the state is above or below t, and set $\Sigma = \{\sigma_t\}_{t \in [0,1]}$. Finally, suppose $c < \Delta = (1 - \alpha)E[\nu] = \frac{1-\alpha}{2}$.

Note that the set of binary threshold signals is not Blackwell-ordered, nor does it contain a Blackwell-dominant signal. Nevertheless, our results allow us to quickly find the set of certifier-optimal signals. We first identify the sets of signals inducing strong and weak common values. We have $\underline{v}_{\sigma_t} < E[\phi(\nu)] = \frac{\alpha}{2}$ if and only if $t < \alpha$. The sets of signals inducing strong and weak common values are therefore given by $\Sigma_s = \{\sigma_t\}_{t \in [0,\alpha)}$ and $\Sigma_w = \{\sigma_t\}_{t \in [\alpha,1]}$. Corollary 1 implies that all certifier-optimal signals must be in Σ_s. Using Proposition 2, we can now identify the optimal signals as those signals that lead to the largest maximal willingness $\Omega^*_{\sigma_t}$. The signal σ_t induces the posterior means $\frac{t}{2}$ and $\frac{1+t}{2}$ with probabilities t and $1 - t$, respectively. For any signal σ_t with $t < \alpha$, we therefore obtain

$$\Omega^*_{\sigma_t} = \Delta + \int_{\underline{v}_{\sigma}}^{E[\phi(\nu)]} G_{\sigma_t}(v)dv = \frac{1-\alpha}{2} + \int_{\frac{t}{2}}^{\frac{\alpha}{2}} t dv = \frac{1-\alpha}{2} + t\left(\frac{\alpha - t}{2}\right)$$

Clearly, this function is maximized at $t = \frac{\alpha}{2}$. Thus, the set of certifier-optimal signals in our example is the singleton $\Sigma^* = \{\sigma_{\frac{\alpha}{2}}\} = \{\sigma_{E[\phi(\nu)]}\}$.

While the binary threshold signal $\sigma_{E[\phi(\nu)]}$ is uniquely optimal in our example, it remains optimal in any set of feasible signals in which it is contained, beyond the specific setup of the example. To see that this claim is true, note that Corollary 2 implies the fully revealing signal – i.e., the signal inducing the distribution of posterior means equal to the prior $F(v)$ – to be certifier-optimal whenever it is feasible. Moreover, the maximal willingness to pay for the fully revealing signal – given by $\Delta + \int_{\underline{v}}^{E[\phi(\nu)]} F(v)dv$ – is an upper bound on the maximal willingness to pay for any signal as $F(v)$ is a mean-preserving spread of $G_\sigma(v)$ for any $\sigma \in \Sigma_F$. We now want to show that the maximal willingness to pay for the binary threshold signal $\sigma_{E[\phi(\nu)]}$ also attains the upper bound. To this purpose, we only need to demonstrate that the expected payoff of an informed seller is the same under $\sigma_{E[\phi(\nu)]}$ and the fully informative signal as the payoff of the uninformed seller is $E[\phi(\nu)]$ for both signals. If $v < E[\phi(\nu)]$, the informed seller chooses for both signals to sell in the uncertified market and get the price $E[\phi(\nu)]$. If $v \geq E[\phi(\nu)]$, the informed seller discloses and sells under both signals. It follows that conditional on the event $v \geq E[\phi(\nu)]$ the expected selling price is the same under both signals and equal to $E[\nu \mid \nu \geq E[\phi(\nu)]]$. Clearly, the probabilities of the events $v < E[\phi(\nu)]$ and $v \geq E[\phi(\nu)]$ do not depend on the signal and thus the claim follows. This argument implies an even stronger result. Indeed, any signal which perfectly reveals whether the state lies above or below $E[\phi(\nu)]$ generates a maximal willingness to pay equal to that of the fully revealing signal, and is therefore certifier-optimal.

5.2 Regulator-optimal certification design

In this section, we consider a regulator who aims to maximize transparency on the market, i.e., to convey as much information to the market as possible. We do not specify the regulator's preferences any further. Instead, we provide results based on Blackwell comparisons of the information on the market. In our analysis, the certifier retains the right to set the fee. This assumption is motivated by the structure of certification markets in practice as described in the introduction. In addition, this assumption allows us to identify differences in certifier- and regulator-optimal certification design independently from distortions that arise from the certifier's pricing power in the market for certifications.

As our key measure of information on the market, we introduce the distribution over disclosed posterior means $H_\sigma(v)$. Given a signal σ and a certifier-optimal fee – as specified in Proposition 1 –, the equilibrium induces a distribution over disclosed signal realizations in the set $S_\sigma \cup \{N\}$. For any of these signal realizations, the buyers calculate an expectation of the good's quality using Bayes' rule. We therefore obtain a distribution over disclosed posterior means that reflects the precision of the signal, the fraction of informed sellers, as well as the extent of disclosure.

The equilibrium distribution over disclosed posterior means $H_\sigma(v)$ is determined as follows. Consider a signal σ with $c \leq \Omega^*_\sigma$. Assuming that the certifier sets the lowest optimal fee[15], the seller buys certification in equilibrium with the probability $\min\{\Delta/\Omega^*_\sigma, 1\}$. If the signal induces weak common values, the certification is certain and full unraveling occurs. In this case, the distribution over disclosed posterior means equals the distribution $G_\sigma(v)$. If the signal induces strong common values, the probability of certification is strictly interior and informed sellers disclose if the posterior mean weakly exceeds $E[\phi(\nu)]$ while they conceal their information otherwise. Thus, the distribution $H_\sigma(v)$ for an optimally priced signal σ is given by

$$H_\sigma(v) = \begin{cases} 0 & \text{if} \quad v < E[\phi(\nu)], \\ 1 - \min\{\Delta/\Omega^*_\sigma, 1\} + \min\{\Delta/\Omega^*_\sigma, 1\} G_\sigma(v) & \text{if} \quad v \geq E[\phi(\nu)]. \end{cases}$$

When σ induces strong common values and the certifier prices the signal optimally, a seller can never gain from disclosing a signal realization that induces a posterior mean below $E[\phi(\nu)]$. Thus, no posterior means below this value can be induced in equilibrium. Signal realizations above $E[\phi(\nu)]$ are always disclosed in equilibrium. The posterior mean of $E[\phi(\nu)]$ is induced by the signal N which is generated if the seller is either uninformed or informed and refrains from disclosure.

Before we present the first result of this section, we recall the notion of a *garbling* due to Marschak and Miyasawa (1968) as a tool to reduce the informational content of a signal. A signal σ can be garbled through a function $\gamma_\sigma : S_\sigma \to \Delta S$ which maps from the set of signal realizations of σ to the set of probability distributions over some set S of signal realizations. With slight abuse of notation, we denote the garbled signal by $\gamma(\sigma) \equiv \gamma_\sigma \circ \sigma$. Of course, the garbled signal $\gamma(\sigma)$ is less Blackwell informative than σ, i.e.,

$$\int_{\underline{v}}^{v} G_\sigma(x)dx \geq \int_{\underline{v}}^{v} G_{\gamma(\sigma)}(x)dx, \;\; \forall v \in [\underline{v}, \bar{v}].$$

In the following proposition, we show that garbling a signal with strong common values can increase the information on the market.

Proposition 3. *For any $\sigma \in \Sigma_s$ with $\Omega^*_\sigma > c$, there exists a garbled signal $\gamma(\sigma)$ which satisfies $\Omega^*_{\gamma(\sigma)} \geq c$, conveys weakly more information to the market than σ, i.e.,*

$$\int_{\underline{v}}^{v} H_{\gamma(\sigma)}(x)dx \geq \int_{\underline{v}}^{v} H_\sigma(x)dx \;\; \forall v \in [\underline{v}, \bar{v}],$$

and conveys strictly more information to the market than σ if $V_\sigma \cap (E[\phi(\nu)], \bar{v}_\sigma]$ is not a

[15] This assumption is innocuous as the tie can be broken at almost no cost.

singleton. In this case,

$$\int_{\underline{v}}^{v} H_{\gamma(\sigma)}(x)dx \geq \int_{\underline{v}}^{v} H_{\sigma}(x)dx \;\; \forall v \in [\underline{v}, \bar{v}] \;\; \textit{and}$$
$$\int_{\underline{v}}^{v} H_{\gamma(\sigma)}(x)dx > \int_{\underline{v}}^{v} H_{\sigma}(x)dx \;\; \forall v \in (E[\phi(\nu)], \bar{v}_{\sigma}).$$

The proposition shows that the regulator can increase the information on the market by reducing the informativeness of certification. Given some signal which induces strong common values, the regulator can garble this signal and thereby increase the equilibrium rates of certification and disclosure. This results in a strict increase in market informativeness if the original signal features at least two posterior means above the unconditional expected payoff of keeping the good $E[\phi(\nu)]$.

Our proof of Proposition 3 is constructive, i.e., we show for each signal σ inducing strong common values how to construct a garbled signal $\gamma^*(\sigma)$ that satisfies the properties in the proposition. Before defining this garbling, we explain our construction for the case where certification costs lie below the expected gains from trade, i.e., $c \leq \Delta$. To this purpose, consider some signal σ which induces strong common values. Recall that, due to probabilistic certification and partial disclosure, the distributions $G_{\sigma}(v)$ and $H_{\sigma}(v)$ differ under the certifier-optimal fee.

The regulator may be able to strictly increase the information on the market captured by $H_{\sigma}(v)$. Note first that the distribution $H_{\sigma}(v)$ can be obtained from the following procedure: draw posterior means from the distribution $G_{\sigma}(v)$, disclose posterior means above $E[\phi(\nu)]$ with an interior constant probability, pool the remaining probability of posterior means above $E[\phi(\nu)]$ with the posterior means below $E[\phi(\nu)]$. The uniform treatment of posterior means above $E[\phi(\nu)]$ is not optimal from an informational perspective. The regulator can increase the information on the market by disclosing higher posterior means above the threshold $E[\phi(\nu)]$ with a larger probability. In particular, consider a specific garbling of the signal σ – denoted by $\gamma^*(\sigma)$ – which pools all posterior means below a threshold into the same signal realization and perfectly discloses all posterior means above the threshold. The threshold is chosen such that the posterior mean induced by the pooling signal realization equals the expected value from keeping the good $E[\phi(\nu)]$. Thus, the garbled signal $\gamma^*(\sigma)$ induces weak common values and results in full certification and unraveling. Moreover, the distribution of disclosed posterior means $H_{\gamma^*(\sigma)}(v)$ Blackwell-dominates $H_{\sigma}(v)$ as $\gamma^*(\sigma)$ pools only intermediate posterior means while perfectly revealing posterior means at the top whereas $H_{\sigma}(v)$ pools posterior means from the whole range and reveals the extreme posterior means only with some interior probability. The garbled signal $\gamma^*(\sigma)$ strictly improves the market informativeness over σ

whenever it changes the composition of posterior means above $E[\phi(\nu)]$ in the pooling signal realizations relative to $H_\sigma(v)$. This is possible if and only if σ induces at least two posterior means above the threshold, i.e., the set $V_\sigma \cap (E[\phi(\nu)], \bar{v}_\sigma]$ is not a singleton. If $V_\sigma \cap (E[\phi(\nu)], \bar{v}_\sigma]$ is a singleton, the garbled signal has to treat all posterior means above $E[\phi(\nu)]$ identically – a trivial statement as there is just one such posterior mean – and thus it leads to the same market informativeness as the original signal σ.

The proof of Proposition 3 extends our argument to the case of $c > \Delta$. In this case, only signals with strong common values can cover certification costs. Nevertheless, the regulator can still improve upon all signals that generate a positive margin to the certifier, i.e., all signals which satisfy $c < \Omega^*_\sigma$. In particular, the regulator gains market informativeness by a garbling that pools intermediate posterior means into the same signal realization while fully revealing high and low posterior means. In the equilibrium for this garbled signal and the certifier-optimal fee, the pooling signal realization as well as the high posterior means are disclosed. By contrast, the low posterior means are concealed and pooled with a fraction of uninformed sellers.

The garbled signal $\gamma^*(\sigma)$ is constructed as follows. Take a signal $\sigma \in \Sigma_s$ with $\Omega^*_\sigma \geq c$. For this signal, define the garbling function γ^*_σ by

$$\gamma^*_\sigma(s) = \begin{cases} \delta_s & \text{if} \quad E_\sigma[\nu|s] \notin [v'_\sigma, v''_\sigma], \\ \xi'\delta_s + (1-\xi')\delta_{\hat{s}} & \text{if} \quad E_\sigma[\nu|s] = v'_\sigma, \\ \delta_{\hat{s}} & \text{if} \quad E_\sigma[\nu|s] \in (v'_\sigma, v''_\sigma), \\ \xi''\delta_s + (1-\xi'')\delta_{\hat{s}} & \text{if} \quad E_\sigma[\nu|s] = v''_\sigma, \end{cases} \tag{10}$$

where $\hat{s} \notin S_\sigma$, δ_s denotes the Dirac measure on the signal realization s, v'_σ and ξ' solve

$$\int_{\underline{v}_\sigma}^{E[\phi(\nu)]} \min\left\{G_\sigma(x), \xi' G_\sigma(v'_\sigma) + (1-\xi')G_\sigma(v'^-_\sigma)\right\} dx = \max\{c-\Delta, 0\}, ^{16} \tag{11}$$

and v''_σ and ξ'' solve – for given v'_σ and ξ' –

$$E_{\gamma^*(\sigma)}[\nu|\hat{s}] = E[\phi(\nu)]. \tag{12}$$

In essence, the garbled signal $\gamma^*(\sigma)$ pools posterior means that lie in some interval and reveals all posterior means outside of this interval. The probabilities ξ' and ξ'' accommodate the possibility of mass points at the boundaries of the interval, and determine the probability at which the posterior means at the boundary are revealed or pooled with the interior of the interval. If $c \leq \Delta$, the lower end of the interval equals the lowest posterior

[16] We use the notation $G_\sigma(v^-) \equiv \lim_{x \nearrow v} G_\sigma(x)$.

mean and we obtain the structure of lower-censorship described above. If $c \in (\Delta, \Omega^*_\sigma)$, the lower boundary exceeds the lowest posterior mean, resulting in the perfect revelation of posterior means at the extremes and pooling in the middle. If $c = \Omega^*_\sigma$, the boundaries coincide at $E[\phi(\nu)]$. Thus, the garbling returns the original signal in this case.

We return to the case of $c \leq \Delta$ for which we obtain the following corollary of Proposition 3.

Corollary 3. *Suppose $c \leq \Delta$, $V_\sigma \cap (E[\phi(\nu)], \bar{v}_\sigma]$ is not a singleton, and $\gamma^*(\sigma) \in \Sigma$ for all $\sigma \in \Sigma_s$. Any regulator-optimal signal induces weak common values. Thus, full certification and unraveling as in Grossman (1981) and Milgrom (1981) prevails under regulator-optimal certification design.*

Corollaries 1 and 3 demonstrate a stark contrast between certifier- and regulator-optimal certification design. Under relatively mild conditions, any certifier-optimal signal induces the information structure of Dye (1985) and brings about partial certification and disclosure, whereas any regulator-optimal signal induces the information structure of Grossman (1981) and Milgrom (1981) and results in full certification and unraveling.

For the corollary to hold, we require that the garbled signal $\gamma^*(\sigma)$ is feasible for any signal $\sigma \in \Sigma_s$. Thus, the regulator is always able to reduce the informativeness of a signal as described in Proposition 3. Note that this condition is satisfied if the set of feasible signals is closed under garblings, i.e., if any garbling of a feasible signal yields another feasible signal. Such a condition would allow the regulator to reduce the informational content of any feasible signals in an arbitrary way. For the purpose of our result, the feasibility of the garbled signals $\{\gamma^*(\sigma)\}_{\sigma \in \Sigma_s}$ is obviously enough.

While Proposition 3 shows how the regulator can improve the information on the market by reducing the informativeness of a given signal, it does not specify a regulator-optimal signal. As we defined the regulator's preference only with respect to Blackwell's partial order, we have to add structure to the feasible set of signals Σ in order to get such result. In the following corollary, we show that if the set of feasible signals contains a Blackwell-dominant signal, applying the garbling of Proposition 3 yields a regulator-optimal certification design.

Corollary 4. *Suppose the set Σ contains a most informative signal $\bar{\sigma}$, i.e.,*

$$\int_{\underline{v}}^{v} G_\sigma(x)dx \leq \int_{\underline{v}}^{v} G_{\bar{\sigma}}(x)dx, \;\; \forall v \in [\underline{v}, \bar{v}], \;\; \forall \sigma \in \Sigma,$$

and suppose the garbling $\gamma^*(\bar{\sigma})$ is also an element of Σ. Then, $\gamma^*(\bar{\sigma})$ is regulator-optimal, i.e.,

$$\int_{\underline{v}}^{v} H_{\bar{\gamma}(\bar{\sigma})}(x)dx \geq \int_{\underline{v}}^{v} H_\sigma(x)dx, \;\; \forall v \in [\underline{v}, \bar{v}], \;\; \forall \sigma \in \Sigma.$$

Corollaries 2 and 4 study the preferences of the certifier and the regulator regarding a most informative signal $\bar{\sigma}$. By Corollary 2, the most informative signal is always certifier-optimal. By Corollary 4, a garbling of the most informative signal is regulator-optimal. By Corollary 3, this garbling conveys strictly more information to the market than the most informative signal $\bar{\sigma}$ – and is therefore strictly different from $\bar{\sigma}$ – whenever $\bar{\sigma}$ induces strong common values and features at least two posterior means above the threshold $E[\phi(\nu)]$.

The main results of this section, Proposition 3 and its corollaries, build on the observation that the regulator may reduce the informativeness of the certification design to increase the information on the market through higher rates of certification and disclosure. This result yields a stark contrast between certifier- and regulator-optimal certification design under weak assumptions on the set of feasible signals. A reduction in the informativeness of a signal with a single posterior mean above the threshold $E[\phi(\nu)]$ cannot improve the information on the market. However, this does not mean that the contrast between certifier- and regulator-optimal certification design is not relevant for such signals. In the following, we revisit our example of Section 5.1 in which the set of feasible signals consists of binary threshold signals that have a single posterior mean above the threshold $E[\phi(\nu)]$. We show that the conflict between certifier- and regulator-optimal certification design remains present in this class of signals.

Example Recall the setup of the example specified in Section 5.1. We argue that any regulator-optimal signal induces weak common values. This stands in contrast to the certifier-optimal signal $\sigma_{\frac{\alpha}{2}}$ which induces strong common values. In particular, we show that the signal σ_α – which provides evidence as to whether the state lies above or below α and induces weak common values – is strictly preferred by the regulator over any signal that induces strong common values.

Remember that $\Sigma_s = \{\sigma_t\}_{t\in[0,\alpha)}$ and $\sigma_\alpha \in \Sigma_w$ as $\underline{v}_{\sigma_\alpha} = \frac{\alpha}{2} = E[\phi(\nu)]$. We want to show that the regulator prefers the signal σ_α over any signal σ_t with $t < \alpha$. For every signal σ_t with $t \leq \alpha$, the distribution $H_{\sigma_t}(v)$ features two posterior means: $E[\phi(\nu)]$ and some $\bar{\rho}_t > E[\nu]$. As the lower posterior mean is fixed, it is easy to see that the informativeness of $H_{\sigma_t}(v)$ is strictly monotone in $\bar{\rho}_t$. As $\bar{\rho}_t = E[\nu \mid \nu > t] = \frac{1+t}{2}$ is strictly increasing for $t \leq \alpha$, the distribution distribution $H_{\sigma_\alpha}(v)$ is strictly more informative than $H_{\sigma_t}(v)$ for any $t < \alpha$. Thus, any regulator-optimal signal induces weak common values.

The insight of our example again generates beyond the specific setup. Indeed, the binary threshold signal $\sigma_{\hat{t}}$ with $E[\nu|\nu \leq \hat{t}] = E[\phi(\nu)]$ is strictly preferred by the regulator over any binary signal which induces strong common values whenever the prior distribution $F(v)$ is continuous on $[\underline{v}, \bar{v}]$. To see this, recall that any binary signal with posterior means of $\underline{\rho} \leq E[\phi(\nu)]$ and $\bar{\rho} > E[\phi(\nu)]$ generates disclosed posteriors means of $E[\phi(\nu)]$ and $\bar{\rho}$. Thus, the higher $\bar{\rho}$, the more information is conveyed to the market. The binary signal with the highest posterior mean from this class solves the optimization problem

$$\max_{\bar{\rho},\underline{\rho}} \bar{\rho} \quad \text{s.t.} \quad \underline{\rho} \leq E[\phi(\nu)], \quad 0 \leq \int_{\underline{v}}^{v} F(x)dx - \begin{cases} 0 & \text{if} \quad v < \underline{\rho}, \\ (v - \underline{\rho})\frac{\bar{\rho}-E[\nu]}{\bar{\rho}-\underline{\rho}} & \text{if} \quad v \in [\underline{\rho}, \bar{\rho}), \\ v - E[\nu] & \text{if} \quad v \geq \bar{\rho}. \end{cases}$$

The second constraint ensures that the prior distribution $F(v)$ is a mean-preserving spread of the binary distribution over $\underline{\rho}$ and $\bar{\rho}$, where the probability of $\underline{\rho}$ can be computed from $\Pr(\underline{\rho})\underline{\rho} + (1 - \Pr(\underline{\rho}))\bar{\rho} = E[\nu]$. Closer inspection of this second constraint reveals that it is never violated for $v < \underline{\rho}$ and $v > \bar{\rho}$ if it is satisfied for $v \in [\underline{\rho}, \bar{\rho}]$. In this intermediate range, the right-hand side of the constraint attains a unique minimum at $\hat{v}$ defined by the first-order condition $F(\hat{v}) = \frac{\bar{\rho}-E[\nu]}{\bar{\rho}-\underline{\rho}}$. As the choice of $\bar{\rho}$ is only limited by the second constraint, this constraint needs to bind and we obtain the condition $\int_{\underline{v}}^{\hat{v}} F(v)dv = (\hat{v} - \underline{\rho})F(\hat{v})$. Basic manipulations of this condition yield

$$\underline{\rho} = \frac{\int_{\underline{v}}^{\hat{v}} vdF(v)}{F(\hat{v})} = E[\nu|\nu \leq \hat{v}] \quad \text{and} \quad \bar{\rho} = \frac{\int_{\hat{v}}^{\bar{v}} vdF(v)}{1 - F(\hat{v})} = E[\nu|\nu > \hat{v}].$$

Both posterior means are strictly increasing in $\hat{v}$. Hence, the constraint $\underline{\rho} \leq E[\phi(\nu)]$ binds in any optimum, i.e., $\hat{v} = \hat{t}$. Thus, the binary threshold signal $\sigma_{\hat{t}}$ – which induces weak common values – leads to a strictly higher posterior mean $\bar{\rho}$ than any binary signal that induces strong common values. Moreover, the signal $\sigma_{\hat{t}}$ is also preferred by the regulator to any nonbinary signal $\tilde{\sigma}$ which induces strong common values and generates a single posterior mean above $E[\phi(\nu)]$. This follows from the fact that the signal $\tilde{\sigma}$ conveys the same information to the market than the binary signal $\tilde{\sigma}_b$ which is obtained from $\tilde{\sigma}$ by pooling all posterior means below $E[\phi(\nu)]$ and therefore induces strong common values.

6 Signal-dependent certification costs

In this section we extend our analyses of certifier-optimal signals and regulator-optimal signals to the case where the cost of certification depends on the signal. For every feasible signal $\sigma \in \Sigma$ we denote by c_σ the cost of certifying the seller. We maintain the assumption that the certification cost is strictly positive, i.e., $c_\sigma > 0$ for all $\sigma \in \Sigma$. We interpret the certification cost as consisting of a signal-independent, strictly positive transaction cost – which may arise from contracting with the seller, reviewing documents, or providing access to the certificates – and a signal-dependent cost of generating information. Our assumption of signal-independent certification costs from the previous sections may therefore be viewed as the case where information can be costlessly generated.

Our characterization of the certifier-optimal certification fee in Proposition 1 is unaffected by the signal-dependency of certification costs. Hence, we can use the analysis of Section 4 to obtain a characterization of certifier-optimal signals under signal-dependent certification costs. Let the set $\{c_\sigma\}_{\sigma\in\Sigma}$ be compact and assume there is a signal $\sigma \in \Sigma$ such that $\Omega^*_\sigma > c_\sigma$.[17] Defining $\Delta_\sigma \equiv \min\{\Delta, E[\nu] - \underline{v}_\sigma\}$, we obtain the following characterization.

Proposition 4. *The set of certifier-optimal signals under signal-dependent certification cost is*

$$\Sigma^* = \left\{\sigma \in \Sigma : \sigma \arg\max_{\Sigma}\ \Delta_\sigma - \frac{\Delta_\sigma c_\sigma}{\Omega^*_\sigma}\right\}.$$

With signal-dependent costs, a new trade-off arises as a signal with high maximal willingness to pay Ω^*_σ may come at a high cost of certification c_σ. Within the set of signals inducing strong common values, the probability of certification is weighed against the cost of certification. In particular, suppose all feasible signals induce strong common values, i.e., $\Sigma = \Sigma_s$. In this case, the characterization of Proposition 4 takes the simple form:

$$\Sigma^* = \left\{\sigma \in \Sigma : \sigma \arg\max_{\Sigma}\ \frac{\Omega^*_\sigma}{c_\sigma}\right\}.$$

Thus, a signal is optimal if it leads to the highest maximal willingness to pay normalized by the certification cost. Within the set of signals inducing weak common values, the trade-off is between revenue and cost of certification. For $\Sigma = \Sigma_w$, the set of certifier-optimal signals is

$$\Sigma^* = \left\{\sigma \in \Sigma : \arg\max_{\Sigma}\ \Omega^*_\sigma - c_\sigma\right\}.$$

Next, we argue that our analysis of regulator-optimal certification design extends to the case of signal-dependent certification cost. From the regulator's point of view, the cost of a signal $\sigma \in \Sigma$ matters only insofar as it determines whether this signal induces non-negative profits to the certifier, and is thus a viable option. Given a set of feasible signals Σ, the regulator determines the subset $\{\sigma \in \Sigma \mid \Omega^*_\sigma \geq c_\sigma\}$ and chooses from this set without taking the cost of the signals into account any further.

Proposition 3 carries over to the case of signal-dependent costs if we impose the condition of Blackwell monotonicity on the cost of signals.

[17]These two assumptions naturally extend the Assumptions 2 and 4 to the case of signal-dependent certification costs.

Assumption 5. *The cost of certification is Blackwell monotone, i.e.,*

$$\int_{\underline{v}}^{v} G_{\sigma'}(v)dv \leq \int_{\underline{v}}^{v} G_{\sigma}(v)dv, \;\; \forall v \in [\underline{v}, \bar{v}] \implies c_{\sigma'} \leq c_{\sigma}.$$

We then obtain the following extension of Proposition 3.

Proposition 5. *For any $\sigma \in \Sigma_s$ with $\Omega^*_\sigma > c_\sigma$, the garbled signal $\gamma^*(\sigma)$ satisfies $\Omega^*_{\gamma^*(\sigma)} \geq c_{\gamma^*(\sigma)}$, conveys weakly more information to the market than σ, i.e.,*

$$\int_{\underline{v}}^{v} H_{\gamma^*(\sigma)}(x)dx \geq \int_{\underline{v}}^{v} H_{\sigma}(x)dx \;\; \forall v \in [\underline{v}, \bar{v}],$$

and convey strictly more information than σ if $V_\sigma \cap (E[\phi(\nu)], \bar{v}_\sigma]$ is not a singleton. In this case,

$$\int_{\underline{v}}^{v} H_{\gamma^*(\sigma)}(x)dx \geq \int_{\underline{v}}^{v} H_{\sigma}(x)dx \;\; \forall v \in [\underline{v}, \bar{v}] \;\; \textit{and}$$
$$\int_{\underline{v}}^{v} H_{\gamma^*(\sigma)}(x)dx > \int_{\underline{v}}^{v} H_{\sigma}(x)dx \;\; \forall v \in (E[\phi(\nu)], \bar{v}_\sigma).$$

Proposition 3 implies that the garbled signal $\gamma^*(\sigma)$ satisfies the above mentioned informational properties with $\Omega^*_{\gamma^*(\sigma)} \geq c_\sigma$. Using Blackwell monotonicity, we obtain $\Omega^*_{\gamma^*(\sigma)} \geq c_\sigma \geq c_{\gamma^*(\sigma)}$.

The insights of Corollaries 3 and 4 carry over to the case of signal dependent certification costs as well. In particular, both corollaries hold under the condition that $c_\sigma \leq \Delta$ for all $\sigma \in \Sigma$. In this case, the regulator still induces full certification and unraveling as in Grossman (1981) and Milgrom (1981). Moreover, the garbling of the most informative signal remains a regulator-optimal certification design for any set of feasible signals.

As a next step, we revisit the example and show that our previous results extend to a natural case of signal-dependent costs.

Example Recall the example introduced in Section 5.1. Instead of assuming that all binary threshold signals $\{\sigma_t\}_{t\in[0,1]}$ have the same cost, suppose that the signal σ_t has the cost of certification $c(t)$. We assume that $c(t)$ is strictly positive, continuous, single peaked and symmetric around $\frac{1}{2}$, and satisfies $c(\frac{1}{2}) < \Delta$. It is easy to check that $c(t)$ is a positive, decreasing, and continuous transformation of the quadratic loss under a signal σ_t, which is a standard measure of the informativity of a signal.

We want to show that any certifier-optimal signal induces strong common values and any regulator-optimal signal induces weak common values. We first consider certifier-optimal certification design. Write the set of feasible signals as a set of pairs of feasible signals: $\Sigma = \{(\sigma_\tau, \sigma_{1-\tau})\}_{\tau \in [0, \frac{1}{2}]}$. By the symmetry of the cost function, each pair has the same cost of certification. For any pair of signals that contains at least one signal which induces strong common values, the certifier's preferred signal from the pair induces strong common values. This observation follows directly from Corollary 1. If $\alpha > \frac{1}{2}$, any pair contains at least one signal which induces strong common values.[18] It follows that any certifier-optimal signal induces strong common values for $\alpha > \frac{1}{2}$.[19] If $\alpha \leq \frac{1}{2}$, the pairs in $\{(\sigma_\tau, \sigma_{1-\tau})\}_{\tau \in [\alpha, \frac{1}{2}]}$ do not contain a signal which induces strong common values. However, – due to the single-peakedness of the cost function around $\frac{1}{2}$ – the cost of certification of these pairs exceeds the cost of certification of any signal in the set of signals inducing strong common values $\{\sigma_t\}_{t<\alpha}$. As those signals are preferred by the certifier for a constant cost of certification c, these signals certainly remain preferred if they come at a lower cost of certification. Thus, any certifier-optimal signal induces strong common values in the case of $\alpha \leq \frac{1}{2}$ as well.

Next we argue that any regulator-optimal signal induces weak common values. This is an immediate consequence of our analyses of the example in section 5.2, as the regulator cares about the signal cost only to the extent to which it makes some signals inviable. Our assumption $c(\frac{1}{2}) < \Delta$ rules out this option. Thus, the set of regulator-optimal signals is still contained in $\Sigma_w = \{\sigma_t \mid t \geq \alpha\}$. It follows that our main insight regarding the difference between certifier- and regulator-optimal certification design remains valid: the former results in Dye's evidence structure and partial disclosure while the latter results in Grossman's and Milgrom's full certification and unraveling.

7 Conclusion

This paper studies how certification design is affected by the objective of the certification designer. Profit-maximizing certifiers prefer highly precise certification as it allows them to charge higher fees and to extract the gains from trade by certifying only a fraction of sellers. By contrast, transparency-maximizing regulators strike a balance between the precision of certification and the extent of certification and disclosure. By reducing the precision of the certificate, the regulator induces the certifier to charge a lower fee, thereby certifying a higher share of sellers, which results in more information provision to the market.

As a consequence of this logic, profit-maximizing certification design endogenizes the information structure of Dye (1985) and results in partial disclosure whereas transparency-maximizing certification design endogenizes the information structure in Grossman (1981)

[18]Recall that the set of signals inducing strong common values is given by $\{\sigma_t\}_{t<\alpha}$.

[19]Note that we did not use the assumption of single-peakedness to derive this result.

and Milgrom (1981) and results in full disclosure through unraveling.

A Proofs

Proof of Lemma 1

Proof follows from the arguments in the main text. □

Proof of Lemma 2

Using integration by parts,

$$\begin{aligned}\Omega_\sigma(p^N) &= \int_V \max\{v, p^N\} dG_\sigma(v) - \max\{p^N, E[\phi(\nu)]\} \\ &= \int_V (v - \phi(v)) dG_\sigma(v) + \int_{\underline{v}}^{p^N} (p^N - v) dG_\sigma(v) - \max\{p^N - E[\phi(\nu)], 0\} \\ &= \Delta + \int_{\underline{v}}^{p^N} G_\sigma(v) dv - \max\{p^N - E[\phi(\nu)], 0\}.\end{aligned}$$

Clearly, the function $\Omega_\sigma(\cdot)$ is continuous on $[\underline{v}_\sigma, \bar{v}_\sigma]$ for any $\sigma \in \Sigma$. For $p^N = \underline{v}_\sigma$, the function takes the value $\min\{\Delta, E[\nu] - \underline{v}_\sigma\}$. If $E[\phi(\nu)] < \underline{v}_\sigma$, the function is strictly decreasing on $[\underline{v}_\sigma, \overline{v}_\sigma]$ as its derivative equals $-(1 - G_\sigma(p^N)) < 0$. For $E[\phi(\nu)] \geq \underline{v}_\sigma$, the function is strictly increasing on $(\underline{v}_\sigma, E[\phi(\nu)])$ as its derivative equals $G_\sigma(p^N)$. For $p^N \in (E[\phi(\nu)], \bar{v}_\sigma)$, the function is strictly decreasing as its derivative equals $-(1 - G_\sigma(p^N))$. Thus, $\Omega_\sigma(\cdot)$ is strictly quasi-concave and attains its unqiue maximum at $p^N = \max\{E[\phi(\nu), \underline{v}_\sigma\}$. □

Proof of Lemma 3

The result follows directly from the Lemmata 4 to 6 and the following Lemma 7. □

Proof of Lemma 4

If $a = 1$ in equilibrium, condition (9) implies $p^N = \underline{v}_\sigma$. For $\Pr(N) = 0$, this follows directly from (9). For $\Pr(N) > 0$, the condition in (9) simplifies to

$$p^N G_\sigma(p^N) = \int_{\underline{v}_\sigma}^{p^N} v dG_\sigma(v) \iff p^N = \underline{v}_\sigma.$$

If $p^N = \underline{v}_\sigma$ in equilibrium, condition (7) implies that $a = 1$ if and only if $r \leq \Omega_\sigma(\underline{v}_\sigma)$. If $\underline{v}_\sigma < E[\phi(\nu)]$, $\Omega_\sigma(\underline{v}_\sigma) = \Delta$. If $\underline{v}_\sigma \geq E[\phi(\nu)]$, $\Omega_\sigma(\underline{v}_\sigma) = E[\nu] - \underline{v}_\sigma$. □

Proof of Lemma 5

First, consider a signal σ with strong common values. Condition (7) implies that $r = \Omega_\sigma(p^N)$ in any equilibrium with $a \in (0,1)$. If $p^N < E[\phi(\nu)]$ in such an equilibrium, we have $b_U = 0$ by (5). Due to condition (9), this implies $p^N = \underline{v}_\sigma$ and $r = \Omega_\sigma(\underline{v}_\sigma) = \Delta$. This is compatible with condition (9) for any $a \in (0,1)$. If $p^N = E[\phi(\nu)]$ in such an equilibrium, we have $b_U \in [0,1]$ by (5) and condition (9) implies $a = b_U\Delta/(b_U\Delta + \int_{\underline{v}_\sigma}^{E[\phi(\nu)]} G_\sigma(v)dv) \in [0, \Delta/\Omega_\sigma^*]$ and $r = \Omega_\sigma^*$. If $p^N > E[\phi(\nu)]$ in such an equilibrium, we have $b_U = 1$ by (5). Condition (9) implies $a = (E[\nu)] - p^N)/\Omega_\sigma(p^N)$. The result follows from the definition of $\Omega_\sigma^{-1}(r)$.

Second, consider a signal σ with weak common values. Condition (7) implies $r = \Omega_\sigma(p^N)$ in any equilibrium with $a \in (0,1)$. By Lemma 2, $\Omega_\sigma(p^N) \leq \Omega_\sigma(\underline{v}_\sigma) = E[\nu] - \underline{v}_\sigma$. Thus, $r \leq E[\nu] - \underline{v}_\sigma$ in any equilibrium with $a \in (0,1)$. □

Proof of Lemma 6

If $a = 0$ in equilibrium, condition (9) implies that either $\Pr(N) > 0$ and $p^N = E[\nu]$ or $\Pr(N) = 0$ and $p^N \in [\underline{v}_\sigma, E[\phi(\nu)]$. For any equilibrium with $\Pr(N) > 0$ and $p^N = E[\nu]$, condition (7) implies that we may have $a = 0$ if and only if $r \geq \Omega_\sigma(E[\nu])$. For any equilibrium with $\Pr(N) = 0$ and $p^N = \underline{v}_\sigma$, condition (7) implies that we may have $a = 0$ if and only if $r \geq \Omega_\sigma(\underline{v}_\sigma) = \Delta$. If $\underline{v}_\sigma < E[\phi(\nu)]$, the interval $[\underline{v}_\sigma, E[\phi(\nu)]$ is nonempty and thus an equilibrium with $a = 0$ exists if and only if $r \geq \min\{\Delta, \Omega_\sigma(E[\nu])\}$. If $\underline{v}_\sigma \geq E[\phi(\nu)]$, $[\underline{v}_\sigma, E[\phi(\nu)]$ is empty and an equilibrium with $a = 0$ exists if and only if $r \geq \Omega_\sigma(E[\nu])$. □

Statement and Proof of Lemma 7

Lemma 7. *The function* $\frac{E[\nu]-\Omega_\sigma^{-1}(r)}{r}$ *is increasing in* r *on* $(\Omega_\sigma(E[\nu]), \Omega_\sigma^*)$.

Proof. Note first that for $p^N \in (E[\phi(\nu)], E[\nu])$,

$$sign\left(\frac{\partial}{\partial r}\frac{E[\nu] - \Omega_\sigma^{-1}(r)}{r}\right) = sign\left(-\frac{\partial}{\partial p^N}\frac{E[\nu] - p^N}{\Omega_\sigma(p^N)}\right).$$

Moreover, basic manipulations yield for $p^N \in (E[\phi(\nu)], E[\nu])$

$$
\begin{aligned}
\frac{\partial}{\partial p^N}\frac{E[\nu]-p^N}{\Omega_\sigma(p^N)} &= \frac{-\Omega_\sigma(p^N) - \frac{\partial \Omega_\sigma(p^N)}{\partial p^N}(E[\nu]-p^N)}{\Omega_\sigma(p^N)^2} \\
&= \frac{\int \max\{v, p^N\} dG_\sigma(v) + p^N + (1-G_\sigma(p^N))(E[\nu]-p^N)}{\Omega_\sigma(p^N)^2} \\
&= \frac{\int_{\underline{v}}^{p^N} (v - E[\nu]) dG_\sigma(v)}{\Omega_\sigma(p^N)^2}
\end{aligned}
$$

where the last term is strictly negative due to $p^N < E[\nu]$. □

Proof of Proposition 1

With strong common values, the demand function is weakly increasing for $r \in (\Delta, \Omega^*_\sigma)$. Thus, the only candidates for an optimal fee are $r = \Delta$, $r = \Omega^*_\sigma$, and $r > \Omega^*_\sigma$. At $r = \Delta$, the certifier obtains the profit $\Delta - c$. At $r = \Omega^*_\sigma$, the certifier's profit is $\Delta - (\Delta/\Omega^*_\sigma)c > \Delta - c$. At $r > \Omega^*_\sigma$, the certifier makes zero profit. If $c < \Omega^*_\sigma$, $r = \Omega^*_\sigma$ is strictly optimal. If $c = \Omega^*_\sigma$, $r = \Omega^*_\sigma$ and $r > \Omega^*_\sigma$ are optimal. If $c > \Omega^*_\sigma$, only $r > \Omega^*_\sigma$ are optimal. With weak common values, demand is a step function and the only candidates for an optimal fee are $r = E[\nu] - \underline{v}_\sigma$ and $r > E[\nu] - \underline{v}_\sigma$. At $r = E[\nu] - \underline{v}_\sigma$, the certifier makes the profit $E[\nu] - \underline{v}_\sigma - c$. At $r > E[\nu] - \underline{v}_\sigma$, the profit is zero. If $c < E[\nu] - \underline{v}_\sigma$, $r = E[\nu] - \underline{v}_\sigma$ is strictly optimal. If $c = E[\nu] - \underline{v}_\sigma$, $r = E[\nu] - \underline{v}_\sigma$ and $r > E[\nu] - \underline{v}_\sigma$ are optimal. If $c > E[\nu] - \underline{v}_\sigma$, only $r > E[\nu] - \underline{v}_\sigma$ are optimal. □

Proof of Proposition 2

Define $\Delta_\sigma \equiv \min\{\Delta, E[\nu] - \underline{v}_\sigma\}$. By Assumption 4, a signal can only be optimal if it induces strictly positive profit. By Proposition 1, the profit of any such signal σ can be expressed as $\Delta_\sigma - (\Delta_\sigma/\Omega^*_\sigma)c$. By Assumption 2, the maximum of this expression across the Σ exists and the set of certifier-optimal signals is therefore given by

$$
\Sigma^* = \{\sigma \in \Sigma : \sigma \in \arg\max_{\Sigma} \Delta_\sigma - (\Delta_\sigma/\Omega^*_\sigma)c\}.
$$

The expression $\Delta_\sigma - (\Delta_\sigma/\Omega^*_\sigma)c$ is increasing in Δ_σ and Ω^*_σ for all signals that lead to strictly positive profit. Thus, the proposition follows from the observation that – due to Lemma 2 and the definition of Δ_σ – for any two signals $\sigma', \sigma'' \in \Sigma$ that lead to strictly positive profit, $\Omega^*_{\sigma'} > \Omega^*_{\sigma''}$ implies $\Delta_{\sigma'} \geq \Delta_{\sigma''}$. □

Proof of Corollary 1

Proof follows from the arguments in the main text. □

Proof of Corollary 2

Proof follows from the arguments in the main text. □

Proof of Proposition 3

Consider a signal $\sigma \in \Sigma_s$ with $\Omega^*_\sigma > c$. Define the garbled signal $\gamma(\sigma)$ through the function

$$\gamma_\sigma(s) = \begin{cases} \delta_s & \text{if} \quad E_\sigma[\nu|s] \notin [v'_\sigma, v''_\sigma], \\ \xi'\delta_s + (1-\xi')\delta_{\hat{s}} & \text{if} \quad E_\sigma[\nu|s] = v'_\sigma, \\ \delta_{\hat{s}} & \text{if} \quad E_\sigma[\nu|s] \in (v'_\sigma, v''_\sigma), \\ \xi''\delta_s + (1-\xi'')\delta_{\hat{s}} & \text{if} \quad E_\sigma[\nu|s] = v''_\sigma. \end{cases}$$

where $\hat{s} \neq s$ for all signal realizations s in the support of σ, δ_s denotes the Dirac measure on the signal realization s, v'_σ and ξ' solve

$$\int_{\underline{v}_\sigma}^{E[\phi(\nu)]} \min\left\{G_\sigma(x), \xi' G_\sigma(v'_\sigma) + (1-\xi')G_\sigma(v'^{-}_\sigma)\right\} dx = \max\{c - \Delta, 0\},$$

and v''_σ and ξ'' solve – for given v'_σ and ξ' –

$$E_{\gamma(\sigma)}[\nu|\hat{s}] = E[\phi(\nu)].$$

From the definition of the garbled signal $\gamma(\sigma)$, we can determine the distribution $G_{\gamma(\sigma)}$ over posterior means as

$$G_{\gamma(\sigma)}(v) = \begin{cases} G_\sigma(v) & \text{if} \quad v < v'_\sigma, \\ \xi' G_\sigma(v'_\sigma) + (1-\xi')G_\sigma(v'^{-}_\sigma) & \text{if} \quad v \in [v'_\sigma, E[\phi(\nu)]), \\ \xi'' G_\sigma(v''_\sigma) + (1-\xi'')G_\sigma(v''^{-}_\sigma) & \text{if} \quad v \in [E[\phi(\nu)], v''_\sigma), \\ G_\sigma(v) & \text{if} \quad v \geq v''_\sigma. \end{cases}$$

We want to establish that

$$\int_{\underline{v}}^{v} H_{\gamma(\sigma)}(x)dx \geq \int_{\underline{v}}^{v} H_\sigma(x)dx \quad \forall v \tag{13}$$

and the inequality is strict for some v if and only if $V_\sigma \cap (E[\phi(\nu)], \bar{v}_\sigma]$ is not a singleton, or equivalently $G_\sigma(\bar{v}_\sigma^-) > G_\sigma(E[\phi(\nu)])$. Consider a generic signal $\hat{\sigma} \in \Sigma$ and let $a_{\hat{\sigma}} \equiv \min\{1, \Delta/\Omega^*_{\hat{\sigma}}\}$ be the equilibrium probability of certification given $\hat{\sigma}$ and the associated certifier-optimal fee. Using the formulation of $H_{\hat{\sigma}}(v)$ derived in the main text, we obtain

$$\int_{\underline{v}}^{v} H_{\hat{\sigma}}(x)dx = \begin{cases} 0 & \text{if} \quad v < E[\phi(\nu)], \\ (1-a_{\hat{\sigma}})(v - E[\phi(\nu)]) + a_{\hat{\sigma}} \int_{E[\phi(\nu)]}^{v} G_{\hat{\sigma}}(x)dx & \text{if} \quad v \geq E[\phi(\nu)]. \end{cases}$$

If $v < E[\phi(\nu)]$, the weak inequality in condition (13) is trivially satisfied. For $v \geq E[\phi(\nu)]$, we can further reformulate the expression as follows:

$$\begin{aligned} &(1-a_{\hat{\sigma}})(v - E[\phi(\nu)]) + a_{\hat{\sigma}} \int_{E[\phi(\nu)]}^{v} G_{\hat{\sigma}}(x)dx \\ &\qquad = v - E[\phi(\nu)] - a_{\hat{\sigma}} \left(v - E[\nu] + \Delta + \int_{\underline{v}}^{E[\phi(\nu)]} G_{\hat{\sigma}}(x)dx - \int_{\underline{v}}^{v} G_{\hat{\sigma}}(x)dx \right) \\ &\qquad = v - E[\phi(\nu)] - \Delta + \frac{\Delta\left(E[\nu] - v + \int_{\underline{v}}^{v} G_{\hat{\sigma}}(x)dx\right)}{\Delta + \int_{\underline{v}}^{E[\phi(\nu)]} G_{\hat{\sigma}}(x)dx} \qquad (14) \\ &\qquad = v - E[\phi(\nu)] - \Delta + \frac{\Delta \int_{v}^{\bar{v}_{\hat{\sigma}}} (1 - G_{\hat{\sigma}}(x))dx}{\Delta + \int_{\underline{v}}^{E[\phi(\nu)]} G_{\hat{\sigma}}(x)dx}. \qquad (15) \end{aligned}$$

We move from the second line to the third using the equality $a_\sigma = \frac{\Delta}{\Delta + \int_{\underline{v}}^{E[\phi(\nu)]} G_\sigma(x)dx}$ which follows from Proposition 1. The fourth line can be obtained from the third by integration by parts of the numerator in the fraction.

First, we compare the functions $\int_{\underline{v}}^{v} H_\sigma(x)dx$ and $\int_{\underline{v}}^{v} H_{\gamma(\sigma)}(x)dx$ for $v \in [v''_\sigma, \bar{v}_\sigma]$. Note that (i) the denominator in expression (15) for $\hat{\sigma} = \gamma(\sigma)$ equals $\max\{\Delta, c\}$ whereas the denominator for $\hat{\sigma} = \sigma$ strictly exceeds both Δ and c by $\sigma \in \Sigma_s$ and $\Omega^*_\sigma > c$, and (ii) the numerator in expression (15) is identical for $\hat{\sigma} = \sigma$ and $\hat{\sigma} = \gamma(\sigma)$ if $v \in [v''_\sigma, \bar{v}_\sigma]$, strictly positive for $v \in [v''_\sigma, \bar{v}_\sigma)$, and zero for $v = \bar{v}_\sigma$. The observations (i) and (ii) imply that $\int_{\underline{v}}^{v} H_{\gamma(\sigma)}(x)dx$ and $\int_{\underline{v}}^{v} H_\sigma(x)dx$ are identical for $v = \bar{v}_\sigma$ and that $\int_{\underline{v}}^{v} H_{\gamma(\sigma)}(x)dx$ strictly exceeds $\int_{\underline{v}}^{v} H_\sigma(x)dx$ for $v \in (v''_\sigma, \bar{v}_\sigma]$.

Second, we compare the functions $\int_{\underline{v}}^{v} H_\sigma(x)dx$ and $\int_{\underline{v}}^{v} H_{\gamma(\sigma)}(x)dx$ for $v \in [E[\phi(\nu)], v''_\sigma]$. Note that (iii) $\int_{\underline{v}}^{v} H_{\gamma(\sigma)}(x)dx$ is linear for $v \in [E[\phi(\nu)], v''_\sigma]$ while $\int_{\underline{v}}^{v} H_\sigma(x)dx$ is weakly convex, and (iv) both functions are zero at $v = E[\phi(\nu)]$. If $\int_{\underline{v}}^{v} H_{\gamma(\sigma)}(x)dx$ strictly exceeds $\int_{\underline{v}}^{v} H_\sigma(x)dx$ at $v = v''_\sigma$, the observations (iii) and (iv) imply that $\int_{\underline{v}}^{v} H_{\gamma(\sigma)}(x)dx$ strictly exceeds $\int_{\underline{v}}^{v} H_\sigma(x)dx$ for $v \in (E[\phi(\nu), v''_\sigma)$, and the two functions are identical for $v = E[\phi(\nu)]$. If $\int_{\underline{v}}^{v} H_{\gamma(\sigma)}(x)dx = \int_{\underline{v}}^{v} H_\sigma(x)dx$ at $v = v''_\sigma$, the observations (iii) and (iv) imply that $\int_{\underline{v}}^{v} H_{\gamma(\sigma)}(x)dx$ strictly exceeds $\int_{\underline{v}}^{v} H_\sigma(x)dx$ for $v \in (E[\phi(\nu), v''_\sigma)$ if there are posterior means in $(E[\phi(\nu), v''_\sigma)$, i.e., $G_\sigma(v''^{-}_\sigma) > G(E[\phi(\nu)]$. Otherwise, the two functions are identical for $v \in [E[\phi(\nu)], v''_\sigma]$.

We can conclude that the weak equality of conditions (15) always holds, and that the two functions are identical if and only if $v''_\sigma = \bar{v}_\sigma$ and $G_\sigma(v''^{-}_\sigma) = G(E[\phi(\nu)])$, i.e., the signal σ induces only a single posterior mean above $E[\phi(\nu)]$, which is equivalent to saying $G_\sigma(\bar{v}^-_\sigma) > G(E[\phi(\nu)])$. This concludes the proof. □

Proof of Corollary 3

If $c \leq \Delta$ then for every signal $\sigma \in \Sigma_s$ it holds that the garbling $\gamma^*(\sigma) \in \Sigma_w$. From the condition that for every $\sigma \in \Sigma$ there exist more than one posterior above $E[\phi(\nu)]$ we can deduce, according to Proposition 3, that $H_{\gamma^*(\sigma)}$ is a mean preserving spread of H_σ. It follows that for every $\sigma \in \Sigma_s$ the regulator strictly prefers the signal $\gamma^*(\sigma) \in \Sigma_w$ over the signal $\sigma \in \Sigma_s$ and so the regulator would always choose a signal that induces weak common values. Thus, according to the certfier optimal pricing (Proposition 1) full certification and unraveling prevails. □

Proof of Corollary 4

We want to prove that $\gamma^*(\bar{\sigma})$ is regulator-optimal. For any signal $\sigma \in \Sigma$ with $\Delta + \int_{\underline{v}}^{E[\phi(\nu)]} G_\sigma(v)dv < c$, the seller is never certified and $\gamma^*(\bar{\sigma})$ generates more information on the market. For any signal $\sigma \in \Sigma$ with $\Delta + \int_{\underline{v}}^{E[\phi(\nu)]} G_\sigma(v)dv \geq c$, the expression in equation (14) together with $\int_{\underline{v}}^{v} G_\sigma(x)dx \leq \int_{\underline{v}}^{v} G_{\bar{\sigma}}(x)dx$ for all v implies the upper bound

$$\int_{\underline{v}}^{v} H_\sigma(x)dx \leq \begin{cases} 0 & \text{if} \quad v < E[\phi(\nu)], \\ v - E[\phi(\nu)] - \Delta + \frac{\Delta\left(E[\nu] - v + \int_{\underline{v}}^{v} G_{\bar{\sigma}}(x)dx\right)}{c} & \text{if} \quad v \geq E[\phi(\nu)]. \end{cases}$$

Note that $\gamma^*(\bar{\sigma})$ attains the upper bound for $v < E[\phi(\nu)]$ and $v \geq v''_{\bar{\sigma}}$. For $v \in [E[\phi(\nu)], v''_{\bar{\sigma}}]$, $\int_{\underline{v}}^{v} H_{\gamma^*(\bar{\sigma})}(x)dx$ is linear. As $\int_{\underline{v}}^{E[\phi(\nu)]} H_\sigma(x)dx = 0$ and $\int_{\underline{v}}^{v} H_\sigma(x)dx$ weakly convex for all σ, we have $\int_{\underline{v}}^{v} H_{\gamma^*(\bar{\sigma})}(x)dx \geq \int_{\underline{v}}^{v} H_\sigma(x)dx$ for all $v \in [E[\phi(\nu)], v''_{\bar{\sigma}}]$ and $\sigma \in \Sigma$. □

Proof of Proposition 4

Proposition 1 implies that the certifier's profit for a given signal $\sigma \in \Sigma$ can be written as $\Delta_\sigma - \frac{\Delta_\sigma c_\sigma}{\Omega^*_\sigma}$. The maximum of this expression across Σ exists due to Assumption 2 and $\{c_\sigma\}_{\sigma \in \Sigma}$ being compact. □

Proof of Proposition 5

Proof follows from the arguments in the main text. □

B Costless certification

In this Appendix, we consider the case of costless certification, i.e., $c = 0$. At first, we discuss how the absence of certification costs affects the optimal pricing of a given signal σ. The analysis leading to Proposition 1 can be straightforwardly extended to obtain the following results. If the signal induces weak common values, the fee $r = E[\nu] - \underline{v}_\sigma$ remains uniquely optimal due to the demand curve being a simple step function. If the signal induces strong common values, both fees $r = \Omega^*_\sigma$ and $r = \Delta$ are optimal as they lead to the same optimal profit Δ. However, based on our analysis of the case $c > 0$, one may argue that the fee $r = \Omega^*_\sigma$ is robustly optimal as it is remains optimal under small perturbations in the cost parameter c. Thus, probabilistic certification and partial disclosure are robustly optimal outcomes for the certifier.

Assuming that the certifier picks the robustly optimal fee $r = \Omega^*_\sigma$ for any $\sigma \in \Sigma_s$, we consider the problem of certifier-optimal certification design under $c = 0$. The certifier is indifferent between all feasible signals in Σ_s as they lead to an identical profit of Δ. All signals in Σ_w with $\underline{v}_\sigma > E[\phi(\nu)]$ lead to a profit strictly below Δ. Thus, the certifier never picks any of these signals whenever a signal inducing strong common values is feasible. If a signal σ satisfies exactly the condition of $\underline{v}_\sigma = E[\phi(\nu)]$, this signal induces weak common values and leads to the optimal profit. Thus, we can conclude that certifier-optimal certification design also generates the information structure of Dye (1985) for costless certification whenever Σ does not contain a signal satisfying the knife-edge case $\underline{v}_\sigma = E[\phi(\nu)]$.

References

Ali, S Nageeb, Nima Haghpanah, Xiao Lin, and Ron Siegel, "How to Sell Hard Information," *The Quarterly Journal of Economics*, 2022, *137* (1), 619–678.

Banerjee, Snehal, Iván Marinovic, and Kevin Smith, "Disclosing to informed traders," *Available at SSRN 3723747*, 2021.

Bar-Gill, Oren and Ariel Porat, "Disclosure Rules in Contract Law," *The Journal of Legal Studies*, 2020, *49* (1), 103–152.

Ben-Porath, Elchanan, Eddie Dekel, and Barton L Lipman, "Disclosure and choice," *The Review of Economic Studies*, 2018, *85* (3), 1471–1501.

— , — , **and** — , "Mechanisms with evidence: Commitment and robustness," *Econometrica*, 2019, *87* (2), 529–566.

Bertomeu, Jeremy, Igor Vaysman, and Wenjie Xue, "Voluntary versus mandatory disclosure," *Review of Accounting Studies*, 2021, *26* (2), 658–692.

Blackwell, David, "Comparison of Experiments," *Proceedings of the second Berkeley symposium on mathematical statistics and probability*, 1951, *1*, 93–102.

— , "Equivalent Comparisons of Experiments," *The Annals of Mathematical Statistics*, 1953, pp. 265–272.

DeMarzo, Peter M, Ilan Kremer, and Andrzej Skrzypacz, "Test design and Minimum Standards," *American Economic Review*, 2019, *109* (6), 2173–2207.

Dye, Ronald A, "Disclosure of nonproprietary information," *Journal of accounting research*, 1985, pp. 123–145.

Faure-Grimaud, Antoine, Eloic Peyrache, and Lucia Quesada, "The ownership of ratings," *The RAND Journal of economics*, 2009, *40* (2), 234–257.

Friedman, Henry L, John S Hughes, and Beatrice Michaeli, "Optimal reporting when additional information might arrive," *Journal of Accounting and Economics*, 2020, *69* (2-3), 101276.

Glazer, Jacob and Ariel Rubinstein, "A study in the pragmatics of persuasion: a game theoretical approach," *New perspectives on games and interaction*, 2008, pp. 121–140.

Grossman, Sanford J, "The informational role of warranties and private disclosure about product quality," *The Journal of Law and Economics*, 1981, *24* (3), 461–483.

— **and Oliver D Hart**, "Disclosure laws and takeover bids," *The Journal of Finance*, 1980, *35* (2), 323–334.

Harbaugh, Rick and Eric Rasmusen, "Coarse grades: Informing the public by withholding information," *American Economic Journal: Microeconomics*, 2018, *10* (1), 210–35.

Hart, Sergiu, Ilan Kremer, and Motty Perry, "Evidence games: Truth and commitment," *American Economic Review*, 2017, *107* (3), 690–713.

Jung, Woon-Oh and Young K Kwon, "Disclosure when the market is unsure of information endowment of managers," *Journal of Accounting research*, 1988, pp. 146–153.

Kamenica, Emir and Matthew Gentzkow, "Bayesian Persuasion," *American Economic Review*, 2011, *101* (6), 2590–2615.

Lichtig, Avi and Ran Weksler, "Information Transmission in Voluntary Disclosure Games," *Working Paper*, 2022.

Lizzeri, Alessandro, "Information revelation and certification intermediaries," *The RAND Journal of Economics*, 1999, pp. 214–231.

Marschak, Jacob and Koichi Miyasawa, "Economic Comparability of Information Systems," *International Economic Review*, 1968, *9* (2), 137–174.

Mas-Colell, Andreu, Michael D. Whinston, and Jerry R. Green, *Microeconomic theory*, Oxford university press New York, 1995.

Maskin, Eric and Jean Tirole, "The Principal-Agent Relationship with an Informed Principal: The Case of Private Values," *Econometrica: Journal of the Econometric Society*, 1990, pp. 379–409.

_ **and** _ , "The Principal-Agent Relationship with an Informed Principal, II: Common values," *Econometrica: Journal of the Econometric Society*, 1992, pp. 1–42.

Milgrom, Paul R, "Good news and bad news: Representation theorems and applications," *The Bell Journal of Economics*, 1981, pp. 380–391.

Shavell, Steven, "Acquisition and disclosure of information prior to sale," *The RAND Journal of Economics*, 1994, pp. 20–36.

Sher, Itai and Rakesh Vohra, "Price discrimination through communication," *Theoretical Economics*, 2015, *10* (2), 597–648.

Verrecchia, Robert E, "Discretionary disclosure," *Journal of accounting and economics*, 1983, *5*, 179–194.

Viscusi, W Kip, "A note on" lemons" markets with quality certification," *The Bell Journal of Economics*, 1978, pp. 277–279.

Weksler, Ran and Boaz Zik, "Disclosure in Markets for Ratings," *Working Paper*, 2022.