Brain Drain and Development Traps

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Abstract

This paper links the two fields of “development traps” and “brain drain”. We construct a model which integrates endogenous international migration into a simple growth model. As a result the dynamics of the economy can feature some underdevelopment traps: an economy starting with a low level of human capital can be caught in a vicious circle where low level of human capital leads to low wages, and low wages leads to emigration of valuable human capital. We also show that our model displays a rich array of different dynamic regimes, including the above traps, but other regimes as well, and we link explicitly the nature of the regimes to technology and policy parameters.

Keywords: brain drain; development traps; human capital; migration.

JEL classification: F22, J61, O11, O15

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1 Introduction

The analysis of migration has become an important branch of the development economics literature. One of its main domains is concerned with the phenomenon of brain drain, which emphasizes that an essential cause of impoverishment for developing countries is the flight of skilled elites towards countries with higher standards of living. However, recent research has emphasized that brain drain can also generate positive dynamic forces for development (see Docquier and Rapoport, 2008, 2012, and Gibson and McKenzie, 2011, for overviews of this literature)\(^1\).

Another important domain in development economics is concerned with development traps, also coined poverty traps. These models show how an economy can be characterized by multiple equilibria, and find itself historically trapped in an inferior equilibrium. This is a very rich area, and recent synthetic accounts of the literature can be found, for example, in Azariadis and Stachurski (2005), Bowles, Durlauf and Hoff (2006) and Matsuyama (2008).

The purpose of this paper is to link the brain drain phenomenon to development traps. For that we construct a model which integrates endogenous international migration into a simple growth model. We show that the existence of brain drain can lead to multiple dynamic regimes, and that the type of regimes displayed is notably affected by both technology and policy parameters.

The will to link these two literatures is not new. The first elaborated model implementing such integration for brain drain appears in De la Croix and Docquier (2010), which actually inspired this paper. They combine a migration function based on wage differentials with a production function inspired by Lucas (1988), and exhibiting positive externalities. They obtain

\(^1\)Further references to the literature are found in section 2 below.
two different types of trajectories: a “vicious circle” one, with high poverty and high brain drain, and a “virtuous circle” one, with low poverty and low brain drain. The actual dynamics is led by a “sunspot” mechanism through which the economy somehow “alternates” stochastically between the two types of trajectories. That article belongs to the category of “expectations driven poverty traps”, where unexpected shocks and coordination failures play a central role.

The model in this paper belongs, instead, to the alternative category of “history dependent poverty traps”\(^2\). In our model we have no sunspots, no coordination failures, and the dynamics are fully deterministic. Yet, we shall nevertheless exhibit a very rich set of dynamics, and notably vicious circle type dynamics. The dynamics will result from the combination of two main elements: (1) The first is that, as in Romer (1990), there are positive externalities between different lines of production using skilled workers. These positive externalities will result in productivity and real wages being possibly increasing with the skilled population. (2) The second, inspired from De la Croix and Docquier (2010, 2012), is the migration mechanism: If the real wage in the home country is low compared to the real wage abroad, then a part of the highly skilled population will emigrate.

The intuition as to why the economy can be led into a vicious circle is the following: If the skilled population is low to start with, their resulting wage is low\(^3\) and therefore many workers emigrate abroad. This in turns

\(^2\)An interesting comparison between “history dependent dynamics” and “expectations dependent dynamics”, their differences and similarities, appears in Krugman’s (1991) stimulating article.

\(^3\)Although this will be treated formally in section 4.3, we can already briefly explain intuitively why skilled wages may be high when there are many skilled workers, and low when there are few skilled. Consider the “high number” case. Two conflicting effects are at work. First, as in Solow-type models, diminishing returns to labor (equation 1 below) will lead to lower wages, the traditional result. But conversely, as in Romer-type models, there are positive externalities between skilled (equations 2 and 3), so that a high number of skilled workers increases the productivity of each of them, and thus their wage. As we shall see analytically below, this last effect dominates when, somehow, the Romer effect is stronger than the Solow effect.
reduces productivity and the real wage further, which will lead to further migration and so on. We clearly have a vicious circle, which can create an underdevelopment trap, because of the loss of skilled workers who had accumulated valuable human capital.

In the contrary, if the economy starts with a high level of skilled population and therefore high productivity, most skilled workers will choose to stay in the home country, which leads itself to high productivity, and we now have some sort of a virtuous circle.

As it turns out, we shall find that our model displays many potential different dynamic regimes, as it includes the above development traps, but other regimes as well. The nature of the regimes and the type of dynamic equilibrium depend on two sets of parameters, technology parameters and policy parameters. Let us now examine these in turn.

There are two important technology parameters: the returns to scale (à la Solow, 1956) for skilled labor and the degree of productive externalities (à la Romer, 1990), these two parameters being linked to the functional form of the production function. This will be developed in section 3.1 below.

When positive externalities à la Romer (1990) dominate, we get the possibility of unstable equilibria, vicious circles, or virtuous circles, as we outlined above. If, however, the diminishing returns to scale dominate, the economy is much more stable and converges to some sort of “Solowian” equilibrium.

So at this stage of the reasoning we would thus have, depending on the value of the above technology parameters, two types of economies: (a) Some “traditional” economies with a single long run equilibrium, towards which dynamic trajectories converge. (b) Economies where this central equilibrium may be unstable, and development traps may occur.

However, the type of economies and dynamics are not linked only to technology parameters, but also to “policy parameters”. In our model, we introduce one such parameter, denoted $z$, which is meant to summarize all
possible influences through which government can influence human capital formation.

A typical “academic” example of $z$ is the size of higher education in the country, and investment of government in education. An increase in that variable should normally increase the number of skilled workers. This paper shows that this policy variable has a substantial effect on the dynamics. The multiple equilibria with vicious and virtuous circles actually occur for median values of $z$. For low values of $z$, only the bad equilibrium, the “trap”, survives, whereas with a high value of $z$ only the high equilibrium remains.

So our model displays a rich variety of dynamics, going from fully stable economy to multiple equilibria, development traps, vicious and virtuous circles. In the next sections we will develop a more formal presentation of the model and results.

The paper is divided into nine sections. We outline in section 2 some related literature. Section 3 describes the model. Section 4 studies the short run equilibrium. Section 5 describes the dynamics and long run equilibria. Section 6 begins, somehow as a benchmark, with the traditional “stable” model. Section 7 studies the converse case, and shows when and how it can lead to development traps. Section 8 emphasizes the role of policy. Section 9 concludes.

2 Related literature

The literature on brain drain has, from the beginning, proposed a balanced view between the negative effects (essentially the loss of human capital) and positive ones (such as remittances or contribution to “international knowledge”). Two important early articles are Grubel and Scott (1966) and Bhagwati and Hamada (1974).

We shall ourselves be emphasizing the negative effects of brain drain, but we must mention that lately a number of authors have shown that the
possibility of migration could create some positive effects on the emigration
country. This has been called a “brain gain” effect. This line of research
has been studied by Mountford (1997), Stark, Helmenstein and Prskawetz
(1997, 1998), Beine, Docquier and Rapoport (2001) and Stark (2004). Beine,
Docquier and Rapoport (2008) and Easterly and Nyarko (2009) derive the
theoretical effects of migration on human capital creation, and test these
effects empirically.

This debate on the brain drain vs. brain gain has stimulated the devel-
opment of this field. As emphasized by Gibson and McKenzie (2011), the
number of studies on this subject has increased in the last decade. Many
contributions are empirical, although Docquier and Rapoport (2012) present
in their survey a model permitting to discuss the conditions under which we
get brain drain or brain gain.

The empirical literature has touched various angles of the questions, and
is focusing mainly on the assessment of the size of the phenomenon, as well as
on the elements affecting the flows, which are notably the size of the country,
political instability, and low levels of human capital. The importance of
migration costs should also be emphasized (see McKenzie and Rapoport,
2010). Still, data limitations continue to be a huge challenge to work in this
area, and there is a need for better data which tracks the flows of high-skilled
workers back and forth.

One outcome of all this research, that has been emphasized recently, is
that brain drain is much more complex than the one way migration as ana-
yzed until now. In particular the “return migration”, by which the migrants
end up returning to their home country, might change not only the per-
spective on the data but also on the models of migration (see Docquier and
Rapoport, 2012).

A particular case of this “return migration” is students, who go abroad for
more education and come back to their home country. Lately these student
flows have substantially increased, in line with large increases in tertiary enrolment rates. This is already happening in Europe, notably due to the Bologna Process (see Brezis and Soueri, 2011). These phenomena are likely to modify the interpretation and research on brain drain.

The literature on development traps is extremely vast, and builds on many different mechanisms. We indicated a few useful surveys in the introduction. A well-known contribution based on human capital accumulation is Azariadis and Drazen (1990).

There are very few papers linking the two issues of development traps and brain drain. We already described De la Croix and Docquier (2010). Some other papers link migration and multiple equilibria, but in a different context than ours. Kwok and Leland (1982) have a model with multiple equilibria in migration, based on asymmetric information. Brezis and Krugman (1996) also present a multiple equilibria migration model, based on a Romer type model, but where the focus is on the host country and not on the country of origin. Mountford and Rapoport (2011) introduce endogenous fertility in a brain-drain environment and find potential multiple equilibria. There are similarities in the building blocks of these models, but they are focusing on different economic contexts. We now turn to our model.

3 The model

As we indicated above, the two main building blocks of our model are the technology and population dynamics due to migration. We shall now describe both more formally.

3.1 Technology

Final output $Y_t$ consists of a homogeneous good, which is produced by competitive firms through the following Cobb-Douglas production function:
\[ Y_t = AX_t^\gamma L_t^{1-\gamma} \quad \gamma \leq 1 \] (1)

where \( A \) is an exogenous productivity parameter, \( L_t \) unskilled labor and \( X_t \) an index of skilled labor, which we describe in detail in the next paragraph. The parameter \( \gamma \) represents the extent of decreasing returns to scale in \( X_t \). A low \( \gamma \) means strong decreasing returns. This parameter will play an important role below.

The variable \( X_t \) is an index of the productivity of the skilled workers, which reflects some positive externalities between them in the tradition of Dixit and Stiglitz, 1977, Ethier, 1982, and Romer, 1990. More precisely assume that skilled workers work in intermediate industries indexed by \( j \in [1, N_t] \), where \( N_t \) is the number of intermediate goods. Then \( X_t \) is given by the standard C.E.S. formula:

\[ X_t = \left( \sum_{j=1}^{N_t} x_{jt}^\theta \right)^{1/\theta} \quad \theta < 1 \] (2)

The parameter \( \theta \) depicts the extent of positive externalities between the skilled workers working in the various intermediate industries. In the limit case \( \theta = 1 \), there are no externalities. For \( \theta < 1 \), there are positive externalities. To show this, let us consider the simple particular case where all \( x_{jt} \)'s are equal to the same \( x_t \). Then (2) becomes:

\[ X_t = N_t^{1/\theta} x_t \] (3)

Since \( \theta < 1 \), a higher number of intermediates \( N_t \) increases average productivity. The lower \( \theta \), the higher the positive externalities. The parameter \( \theta \) will be also important in the analysis below.

Let us now relate inputs \( x_{jt} \) to the rest of the economy. The intermediate input \( x_{jt} \) is produced by intermediate firm \( j \) according to the production function:
\[ h_{jt} = x_{jt} + f \]  \hspace{1cm} \text{(4)}

where \( h_{jt} \) is the amount of skilled labor employed in firm \( j \) and \( f \) is a fixed labor cost. These firms operate in a framework of monopolistic competition with free entry, through which the number \( N_t \) will be determined endogenously, as we shall see below.

Finally we must express that the sum of skilled workers employed in all firms \( j \) is equal to the aggregate amount of skilled labor \( H_t \):

\[ H_t = \sum_{j=1}^{N_t} h_{jt} \]  \hspace{1cm} \text{(5)}

### 3.2 Population dynamics

We now turn to the dynamics of skilled and unskilled workers. Since the number of unskilled workers will play little role in what follows, we shall assume to simplify that it is constant in time:

\[ L_t = L \quad \forall t \]  \hspace{1cm} \text{(6)}

It is assumed that the number of high skilled workers, \( H_t \), evolves according to:

\[ H_{t+1} = a (1 - e_t) H_t + z \quad a < 1 \quad z > 0 \]  \hspace{1cm} \text{(7)}

The first term in the right hand side represents the evolution of already existing skilled. First, there is a natural attrition rate \( a \) for skilled workers. Secondly, a fraction \( e_t \leq 1 \) of skilled workers emigrates between periods \( t \) and \( t + 1 \). This is the brain drain, which we will study in detail in the next subsection.

The second term, \( z \), represents the influx of skilled workers not related to \( H_t \). This parameter should be thought of as influenced by government
policy, in many possible different manners. For example immigration quotas for skilled labor, a tool used by many governments, will directly influence this influx. Also, a greater level of higher education in the country will typically increase the number of skilled workers. All these heterogeneous influences are subsumed in the simple policy parameter $z$.

3.3 The migration function

For notational convenience, our working variable will not be the fraction of migrants $e_t$, but rather $s_t = 1 - e_t$, where $s$ stands for “stayers”, i.e. the fraction of those skilled workers who stay in the country. We shall assume that $s_t$ (and therefore $e_t$) is determined endogenously by:

$$1 - e_t = s_t = S(\omega_t) \quad 0 \leq s_t \leq 1$$

where $\omega_t$ is the real wage of skilled workers in the home country. We assume:

$$S'(\omega_t) \geq 0$$

A typical function $S(\omega_t)$ is pictured in figure 1.

**Figure 1**

We will actually use in what follows migration functions of the form:

$$S(\omega_t) = \min(\lambda\omega_t^\alpha, 1) \quad \alpha \leq 1$$

We now show that this possibility of a brain drain can modify dramatically the dynamics. We start by studying the short run equilibrium.

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4We may note that the parameter $z$ influences both the ratio of skilled to unskilled, and population growth. We could have introduced an extra parameter to disentangle the two, but this would complicate things without adding conceptual gain from the point of view of this model, so we keep a single parameter for the sake of simplicity.
4 The short run equilibrium

4.1 The demand for intermediates

Let us denote as $P_t$ the price of output $Y_t$, $q_{jt}$ the price of intermediate good $j$ and $V_t$ the wage of unskilled labor. The objective of final output producing firms is to maximize profits:

$$P_tY_t - \sum_j q_{jt}x_{jt} - V_tL_t$$

(11)

subject to the production functions (1) and (2). We can decompose the problem in two parts. First, for any given value of $X_t$ firms will choose the amounts of intermediate goods $x_{jt}$ so as to minimize costs, i.e. so as to solve:

Minimize $\sum_j q_{jt}x_{jt}$ s.t. $X_t = \left(\sum_{j=1}^{N_t} x_{jt}^\theta\right)^{1/\theta}$

(12)

The first order conditions with respect to the $x_{jt}$’s yield:

$$\frac{x_{jt}}{X_t} = \left(\frac{q_{jt}}{Q_t}\right)^{-1/(1-\theta)}$$

(13)

where $Q_t$ is the traditional C.E.S. aggregate index:

$$Q_t = \left(\sum_{j=1}^{N_t} q_{jt}^{\theta/(\theta-1)}\right)^{(\theta-1)/\theta}$$

(14)

The cost of producing $X_t$ is $Q_tX_t$. The level of $X_t$ will be chosen itself by maximizing profits:

$$P_tY_t - Q_tX_t - V_tL_t$$

(15)

subject to production function (1), which yields the first order condition:

$$\frac{Q_t}{P_t} = \frac{\partial Y_t}{\partial X_t} = \gamma AX_t^{\gamma-1}L^{1-\gamma}$$

(16)
4.2 Monopolistic competition equilibrium

Let us now move to the intermediate firms. We denote as $W_t$ the wage of the skilled workers. A monopolistically competitive firm producing intermediate good $j$ maximizes profits subject to the demand curve (13), i.e. it solves the program:

Maximize $\quad q_{jt}x_{jt} - W_t h_{jt}$ \quad s.t.

$h_{jt} = x_{jt} + f$

$$x_{jt} = \left( \frac{q_{jt}}{Q_t} \right)^{-1/(1-\theta)}$$

We find the following first order condition, traditional in monopolistic competition:

$$q_{jt} = \frac{W_t}{\theta} = q_t$$ \quad (17)

The equilibrium is symmetrical, so that $x_{jt} = x_t$, $q_{jt} = q_t$, and:

$$X_t = N_t^{1/\theta} x_t \quad q_t = N_t^{(1-\theta)/\theta} Q_t$$ \quad (18)

There is free entry into the intermediate firms’ industry, so the number of firms $N_t$ is determined by the zero profit condition for intermediate firms:

$$q_t x_t - W_t (x_t + f) = 0$$ \quad (19)

Combining (3), (17) and (19) we obtain:

$$x_{jt} = x_t = \frac{\theta f}{1 - \theta} = x \quad h_{jt} = h_t = \frac{f}{1 - \theta} = h$$ \quad (20)

and since $N_t (x_t + f) = N_t h_t = H_t$, we find the endogenous number of firms:
4.3 The real wage of skilled workers

Since this is a central element in the migration phenomenon, we now compute the real wage $\omega_t$, i.e. the wage of skilled workers $W_t$ deflated by the price of output $P_t$:

$$\omega_t = \frac{W_t}{P_t}$$  \hspace{1cm} (22)

We can decompose it as:

$$\omega_t = \frac{W_t}{P_t} = \frac{W_t}{q_t} \cdot \frac{q_t}{Q_t} \cdot \frac{Q_t}{P_t}$$  \hspace{1cm} (23)

We know already from previous computations (equations 16, 17 and 18):

$$\frac{W_t}{q_t} = \theta \quad \frac{q_t}{Q_t} = N_t^{(1-\theta)/\theta}$$  \hspace{1cm} (24)

$$\frac{Q_t}{P_t} = \gamma A X_t^{\gamma-1} L^{1-\gamma}$$  \hspace{1cm} (25)

Combining (21), (23), (24) and (25), we find:

$$\omega_t = \Lambda H_t^{(\gamma-\theta)/\theta}$$  \hspace{1cm} (26)

with:

$$\Lambda = A \gamma \theta^\gamma L^{1-\gamma} h^{-\gamma(1-\theta)/\theta}$$  \hspace{1cm} (27)

We see that the real wage of skilled workers is increasing in $H_t$ if $\gamma > \theta$, i.e. if the effect of externalities dominates the diminishing returns to scale in skilled labor. If in the contrary $\gamma < \theta$, i.e. if diminishing returns dominate, then the real wage of skilled workers is decreasing in $H_t$, the traditional result.
5 Dynamics and long run equilibria

Combining (7), (8), (10) and (26) we find that the dynamics of \( H_t \) is given by:

\[
H_{t+1} = aS(\omega_t)H_t + z = a \min(1, \xi H_t^\nu)H_t + z = F(H_t)
\]  

(28)

with:

\[
\nu = \frac{\alpha(\gamma - \theta)}{\theta} \quad \xi = \lambda \Lambda^\alpha
\]  

(29)

Long run equilibria will be solution of the equation:

\[
F(H_t) = a \min(1, \xi H_t^\nu)H_t + z = H_t
\]  

(30)

We see that the long run equilibria and the dynamics of equation (28) will be different depending on whether \( \nu \) is greater than or smaller than zero or, using equation (29), on whether \( \gamma \) is greater or smaller than \( \theta \). We start with the traditional case.

6 The traditional case

The “traditional” case is that where the diminishing returns to scale to skilled labor dominate the positive external effects, i.e. where:

\[
\gamma < \theta
\]  

(31)

We can first compute the derivative of the function \( F \):

\[
F'(H_t) = \frac{dH_{t+1}}{dH_t} = aS(\omega_t) \left[ 1 + \varepsilon(\omega_t) \frac{\partial \log \omega_t}{\partial \log H_t} \right]
\]  

(32)

where \( \varepsilon_t = \varepsilon(\omega_t) \) is the elasticity of \( S(\omega_t) \):

\[
\varepsilon_t = \varepsilon(\omega_t) = \frac{\partial \log S(\omega_t)}{\partial \log \omega_t} \geq 0
\]  

(33)
Further using formula (26), (32) becomes:

\[ F'(H_t) = aS(\omega_t) \left[ 1 + \varepsilon(\omega_t) \frac{\gamma - \theta}{\theta} \right] \tag{34} \]

Note that, from (10), \( \varepsilon(\omega_t) \leq 1 \). Since moreover \( \gamma < \theta \) and \( S(\omega_t) \leq 1 \), we have:

\[ 0 \leq F'(H_t) \leq a < 1. \tag{35} \]

The equation \( F(H_t) = H_t \) has a single long run equilibrium, which is dynamically stable. We denote it as \( H_S \) (\( S \) for stable). This case is represented in figure 2.

**Figure 2**

### 7 Development traps

Let us now assume that:

\[ \gamma > \theta \tag{36} \]

i.e. the positive externalities between skilled workers are dominant. In that case we shall now see that several patterns of development traps can occur. There are actually three distinct possibilities, which we now explore in turn.

#### 7.1 Multiple equilibria (case A)

The first possibility, which corresponds to figure 3, is that where the initial “central” equilibrium, is unstable, so we denote it as \( H_U \). Two new stable equilibria, high and low, and denoted \( H_h \) and \( H_l \), now appear.

**Figure 3**

The dynamics of \( H_t \), which is represented in figure 3, displays multiple equilibria and strong history dependence. The actual path will fundamentally
depend on whether the initial level of skilled workers $H_0$ is above or below $H_U$.

1. If $H_0 > H_U$, the high skilled workers mostly stay in the home country and $H_t$ converges toward the high value $H_h$.

2. If, however, $H_0 < H_U$, the picture changes dramatically and we have a clear development trap, coming from the following vicious circle: a low initial $H_t$ means a low $\omega_t$. This triggers emigration of the skilled workers and therefore lowers $H_t$, leading to lower $\omega_t$ and so on. Finally the system will end up in the low value $H_t$, the trap.

### 7.2 An unavoidable trap (case B)

But the matter can be even worse, as exemplified by the dynamics in figure 4. In such a case the high stable equilibrium $H_h$ and the unstable intermediate equilibrium $H_U$ have disappeared, and all that remains is the low equilibrium $H_l$. Whatever the starting point $H_0$, $H_t$ will converge toward this low trap value.

**Figure 4**

### 7.3 Escaping the trap (case C)

The last possibility is actually the most favorable of the three. Although the combination of parameters $\gamma$ and $\theta$ is consistent with development traps, we see that in this case it is the low stable and middle unstable equilibria, $H_l$ and $H_U$, that have disappeared, so that the economy will always converge toward a high equilibrium $H_h$. This is represented in figure 5.

**Figure 5**
8 Policy and dynamic regimes

We saw in section 7 that for $\gamma > \theta$ the economy becomes prone to development traps. We could distinguish, however, in such a case three different dynamic regimes (cases A, B and C above). We shall now show that the value of the policy parameter $z$ will be instrumental in determining which of the three regimes the economy is in.

8.1 Parameters and regimes

It will be convenient for a first characterization of the regimes to use the auxiliary function:

$$G(H_t) = a\xi H_t^{1+\nu} - H_t + z$$

(37)

The function $G(H_t)$ starts at $z$, and is convex since $\nu > 0$. One possibility is represented in figure 6, which actually corresponds to regime A.

Figure 6

It is easy to see that the long run equilibria of the dynamics, i.e. the values of $H_t$ such that $F(H_t) = H_t$ (equation 30) are equivalently the solutions of the equation:

$$\min \left[ G(H_t), z - (1 - a) H_t \right] = 0$$

(38)

Now consider the equation $G(H_t) = z - (1 - a) H_t$, and define as $H^*$ the (nonzero) root of that equation (cf figure 6).

Figure 7

Figure 8
Using figure 6, associated to case A, and figures 7 (associated to case B) and 8 (associated to case C), we obtain immediately the following proposition:

**Proposition 1:** We will obtain cases $A$, $B$ and $C$ under the following respective conditions:

(a) Case $A$ will occur if $G(H^*) > 0$ and the equation $G(H_t) = 0$ has two real roots.

(b) Case $B$ will occur if $G(H^*) < 0$ and the equation $G(H_t) = 0$ has two real roots.

(c) Case $C$ will occur if the equation $G(H_t) = 0$ has two complex roots.

### 8.2 Policy and traps

Now, using the above definition of the function $G$, (equation 37), we will show that the conditions of proposition 1 can be transformed into simple conditions based on the value of the policy parameter $z$. So let us define the following two threshold values for $z$:

\[
    z_1 = (1 - a) \left( \frac{1}{\xi} \right)^{1/\nu} 
\]

(39)

\[
    z_2 = \nu \left( \frac{1}{a\xi} \right)^{1/\nu} \left( \frac{1}{1 + \nu} \right)^{(1+\nu)/\nu} 
\]

(40)

One can compute that for all values of the parameters:

\[
    z_1 \leq z_2 
\]

(41)

Accordingly the separation conditions of proposition 1 translate into:

**Proposition 2:** We obtain cases $A$, $B$ and $C$ under the following values of the parameter $z$:

(a) Case $A$ occurs if $z_1 < z < z_2$. 

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(b) Case $B$ occurs if $z < z_1$.

(c) Case $C$ occurs if $z > z_2$.

**Proof:** In proposition 1 we replace the function $G(H_t)$ by its expression (equation 37). Simple calculations using (39), (40) and (41) show that conditions (a), (b) and (c) of proposition 1 become conditions (a), (b) and (c) of proposition 2.

So we see that, when $\gamma > \theta$, the policy parameter $z$ is fundamental in determining, among the three cases $A$, $B$ or $C$, the exact nature of “trap dynamics”.

9 Conclusion

This paper has developed a simple model where brain drain can lead to development traps, even though the same economy without workers’ mobility would be stable. Insipite of its simplicity this model displays multiple different dynamic regimes, including a “Solowian” one, vicious and virtuous circles, multiple equilibria and also cases where the economy can be trapped in a single low or high state. We saw that the type of regime that obtains can be related to both technological factors and policy.

Indeed, there are two important technological parameters which will have an influence on the type of equilibrium we get. The first one which we have denoted $\gamma$, is depicting the importance of decreasing returns to scale in skilled labor. A small $\gamma$ means strong decreasing returns.

The second parameter $\theta$ depicts the extent of positive externalities between various sorts of skilled labors. A low value of $\theta$ means that these positive externalities are strong, which, ceteris paribus, increases productivity.

When $\gamma$ is smaller than $\theta$, which means that, relatively, the externalities are not strong, we are in a model akin to the classical model, and, not
surprisingly, we get then a “Solowian” dynamic regime. Policy, as well as history, has no substantial long run influence.

However, when we have a strong externalities effect, with $\gamma > \theta$, then this will lead to several possible different dynamics, and the type of dynamics depends on government’s policy variable $z$.

When the government has a low level of intervention $z$, then we have one single low equilibrium of human capital, wages and output, the trap. When the government has a high level of intervention, then we have a high equilibrium, with high human capital, wages and output. When the size of the policy parameter is neither high or low, then we are in the case of multiple dynamic equilibria, and history matters.

The model is compact enough to serve as a simple basic framework for further research in the field of dynamics in brain drain models. Scholars who want to have an environment of multiple dynamic equilibria would concentrate on the $A$ case. Those who want to always be in a trap should adopt our $B$ case.

This paper has kept the framework simple in order to make clear the elements which trigger the different possibilities. However, of course, future work could make the policy parameter $z$ endogenous, with the purpose of influencing dynamics in such a way that we can move from the (unsatisfactory) cases $A$ or $B$ to a better equilibrium like $C$. We leave that to future work.
References


Figure 1
Figure 2

$H_{t+1} = aH_t + z$

$H_{t+1} = H_t$

$H_{t+1} = F(H_t)$
$H_{t+1} = H_t$
Figure 4

\[ H_{t+1} = F(H_t) \]
\[ H_{t+1} = H_t \]

\[ H_{t+1} = F(H_t) \]
Figure 6
Figure 7

\[ G(H_t) \]

\[ z - (1 - a) H_t \]
Figure 8