

The Effect of Exogenous Information on Voluntary Disclosure and Market Quality

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Abstract

We analyze a disclosure model in which information may be disclosed by a firm as well as a third party. We refer to this third party as analyst coverage. Under plausible assumptions, analyst coverage crowds out disclosure by the firm. Despite the crowding out effect, we argue that an increase in analyst coverage increases aggregate information. While ranking based on Blackwell informativeness cannot be obtained we base this claim on two measures of information in prices. The first is statistical in nature while the second is based on liquidity in a trading stage that follows the information disclosure.

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1 Introduction

There is a growing literature on voluntary disclosure. It reflects the understanding that agents or firms strategically decide whether to disclose private information or withhold it. While public companies, for example, are mandated to disclose certain information, there are many pieces of information that are disclosed at the discretion of the manager. For example a firm does not have to publish that a major customer is negotiating a deal with one of its competitors. Information that is disclosed voluntarily is considered to be a major information source in financial markets.¹

Another example is an entrepreneur who seeks funding from investors (VC funds, “angels”, etc.); such entrepreneur can choose whether to disclose or conceal the results of previous attempts to raise funding and acquire new customers. An incumbent politician may obtain private information about the success or failure of his previously enacted policies, and can choose whether to disclose or conceal these results. In all the examples above, agents are less likely to lie because of severe or even criminal punishment or because once disclosed the information can be easily verified. Instead because there is uncertainty about the existence of this information they can hide it.

An open question is the effect of additional sources of information on voluntary disclosure. For example, financial analysts and rating agencies provide additional information about public firms; Investors can gather information through their social network about an entrepreneur; and the media, as well as independent think-tanks, may also be able to assess the success of public policies. In this paper we focus on sources of information that may discover and reveal the agent’s private information, even in the case where the agent chooses to withhold it. The effect is not immediately clear, because the mere presence of additional source can affect the agent’s disclosure policy and shift the entire environment to a new equi-

¹Beyer et al. (2010) find that approximately 66% of the accounting-based corporate information is provided by voluntary disclosures, 22% are provided by analyst forecasts, 8% are provided by earnings announcements, and 4% are provided by SEC filings.

librium. The main question that this paper addresses is whether the introduction of such external source of information adds to the aggregate amount of information that becomes public. In other words, what would be the effect once we account for the reaction by agent?

Our model's starting point is a standard voluntary disclosure setting with uncertainty about information endowment (a la [Dye, 1985](#) and [Jung and Kwon, 1988](#)). A manager of public firm may be endowed with private value-related information. She is facing a financial market that prices the firm based on public information, and wishes to maximize that price. An informed manager can credibly and costlessly disclose her information to the market. Our model's novelty is the presence of an additional external source of information, an analyst, who may find and publish the information held by the manager. We assume the analyst may publish information when the manager is informed and when it is uninformed, and allow for correlation in the endowment of information.

We first show that, as standard in this literature, the game has a unique equilibrium, in which the firm's manager discloses her information if and only if it's "type" is higher than an equilibrium threshold. We then turn to the effect of the analyst, and specify how the firm's disclosure strategy changes in response to an increase in analyst coverage, that is, an increase the probabilities that information is exogenously disclosed. We show that in plausible information environments analyst coverage typically crowds-out corporate voluntary disclosure, i.e., firms respond to an increase in analyst coverage by decreasing the amount of information they disclose. This result, which is new to the theoretical literature, is consistent with the empirical evidence in [Anantharaman and Zhang \(2011\)](#) and [Balakrishnan et al. \(2014\)](#) (see more details on the empirical literature below).

Given the crowding-out result, a second, and more challenging question, is regarding the overall effect of an increase in analyst coverage on the amount of public information. We indeed find that the information cannot be ranked using Blackwell informativeness criterion, because more exogenous information does not lead to a finer information partition. Instead,

we use two separate measures to capture the overall information available to the market. First, we consider a quadratic loss, which equals expected difference between the firm's actual and perceived value squared. This measure has a natural interpretation in terms of price efficiency or ex-post return volatility. It also fits a utility of a receiver who minimizes quadratic loss function by setting prices to be equal to the expected value conditional on all available information.

Our second measure of information quality is more specific to the financial example, and can be linked to empirical findings. We use the expected bid-ask as a measure that reflects the extent of information asymmetry in the market. We do so by augmenting the model and introducing a trading stage a la [Glosten and Milgrom \(1985\)](#) that follows the disclosure stage by the manager and the analyst. The trade and pricing in this stage are affected by the information revealed in the disclosure stage.

We show that both price efficiency and liquidity increase as a result of an increase in analyst coverage, and thus the overall effect on market quality is positive. This result arises from the qualitative difference between voluntary disclosure and information provided by third parties. Firms always choose to disclose news that are strictly good and hide news that are strictly bad. Thus, the threshold for disclosure is when the news are neutral. A slight change in the firm's disclosure threshold will have only a minor effect on the way the firm is valued, because the pricing of the affected types will not change by much. In contrast, third parties (such as financial analysts) may find and reveal even news that are very good or (more plausibly) very bad, and thus their reports have a more significant effect on prices. Third party information will, on the margin, affect prices more than the change in voluntary disclosure.

Our results has the potential to contribute to the assessment of different policies that aim to increase market transparency. Such policies often focus on increasing the information

provided by one market participant.² While financial analysts are a major mechanism in the capital market to discover firm's private information, exogenous revelation can arrive from other sources, such as media, social media, competitors, suppliers or governmental bodies. Our results show that an improvement in one information source may crowd-out information of another source, and that different parties affect public information differently. Our model suggests that to the extent that increasing transparency of such sources is not too costly, it is beneficial in terms of price efficiency and liquidity.

Unlike the theoretical literature that we review in the next subsection, the empirical literature has studied the effect of analyst coverage on firm's voluntary disclosure and on the liquidity of the firm's stock. The predictions of our model are supported by this literature. For example, [Kelly and Ljungqvist \(2012\)](#) show that following an exogenous decrease in the supply of public information, due to closing of brokerage houses that resulted in decreased analyst coverage, the affected firms' information asymmetry increased and their stocks' liquidity decreased.

[Anantharaman and Zhang \(2011\)](#) and [Balakrishnan et al. \(2014\)](#) use the same exogenous shock to analyst coverage that is used in [Kelly and Ljungqvist \(2012\)](#) to establish the effect of decrease in analyst coverage on firms' voluntary disclosure. [Balakrishnan et al. \(2014\)](#) show that one quarter following the decrease in analyst coverage, the affected firms increase their voluntary disclosure (earning guidance) to mitigate the increased information asymmetry and the decreased liquidity. This increased disclosure partially reverses the decrease in liquidity, although the overall effect remains negative, consistent with our model's prediction.

There is a more extensive empirical literature that studies how disclosure and transparency

²Here are just some examples of regulations that focus on information provision. The Sarbanse-Oxley Act attempted to increase the mandated reporting of firms. The Williams Act of 1968 limits the ability of investors to anonymously trade on their private (optimistic) information by requiring investors to disclose information regarding cash tender offers and large positions (more than five percent of the outstanding firm's shares). The regulation on analyst certification (Reg AC) requires analysts to disclose whether they were paid by a company in an attempt to reveal possible conflicts of interests and prevent biased reports. The Dodd-Frank Act includes several measures aimed at improving the transparency and viability of credit ratings. See also [Goldstein and Yang \(2017\)](#).

affect informational environment in general and in particular the bid-ask spread. While the results are mixed, many papers find that increased disclosure increases informativeness and decreases bid-ask spread (e.g. [Welker 1995](#); [Healy et al. 1999](#); [Leuz and Verrecchia 2000](#); [Heflin et al. 2005](#)).

1.1 Related Theoretical Literature

By studying voluntary disclosure in the presence of potentially informed traders, our paper contributes to two separate streams of the theoretical literature. The first stream is the voluntary disclosure literature. To the best of our knowledge, there are only few theoretical papers that study voluntary disclosure in the presence of potentially informed trader/receiver. [Langberg and Sivaramakrishnan \(2008, 2010\)](#) offer two models with a firm that can voluntarily disclose information and strategic analysts that can disclose additional information. In these papers, the analyst and the firm obtain two different pieces of information, in contrast to our proposed model, in which analysts and the firm potentially reveal the same information to the public. This makes the analysis significantly different. Moreover, in these papers, by construction, greater firm disclosure encourages the analysts to obtain more information. [Einhorn \(ming\)](#) also explores the effect of additional information sources on voluntary disclosure. She focuses on an equilibrium where the firm's disclosure strategy is independent of the fundamental value and thus cannot be affected by other sources of information. The closest work with an informed receiver is [Ispano \(2016\)](#), whose model can be seen as a simplified example of our model with three states and a specific analyst technology. He proves, in his discrete example, that the utility of the receiver, equivalent to price efficiency in our setting, is increasing with the probability that the receiver is informed. He does not discuss liquidity. [Dutta and Trueman \(2002\)](#) study a setting in which a firm's manager can credibly disclose verifiable private information, but not how to interpret this information. Since the manager is uncertain whether the market will interpret the disclosed information as good

or bad news, the manager faces uncertainty regarding the market reaction. In this setting, [Dutta and Trueman \(2002\)](#) show that the equilibrium disclosure strategy is not necessarily a threshold strategy. [Banerjee and Kim \(2017\)](#) explore a model where a manager may disclose information to the public and may also use private cheap-talk communication to contact her employees. With some probability, private communication is “leaked” and becomes public. It turns out that the possibility of private communication becoming public has very different implications than the possibility of the manager’s information becoming public, as in our model. Finally, several papers deal with disclosure of two strategic firms/experts ([Bhattacharya and Mukherjee \(2013\)](#); [Bhattacharya et al. \(2018\)](#); [Kartik et al. \(2017\)](#)). In those papers, as in ours, agents consider the possibility of additional information due to disclosure by their peers; those papers, however, do not focus on the change in overall information due to an increase in the quality of information by one of the agents.

The second related stream of literature studies how changes in one source of information affect information acquisition incentives by other parties, and the resulting overall effect on information available. [Goldstein and Yang \(2017\)](#) present a noisy Rational Expectation Equilibrium (REE) model with a public signal that can be interpreted either as corporate mandatory disclosure or as disclosure by a third party, such as an analyst. They show that when keeping traders’ information constant, a more precise public signal improves market liquidity and price efficiency. However, better public information undermines the incentives of traders to acquire information, and thus the overall effect is ambiguous and depends on the measure used to identify market quality. By contrast, we endogenize the corporate disclosure decision and allow for voluntary rather than mandatory disclosure. Another related paper is [Gao and Liang \(2013\)](#), which studies how firm’s commitment to disclosure policy affects investors incentives to acquire information. Their focus is on the feedback effect, by which the firm can benefit from the information provided from prices.

The remainder of the paper is organized as follows. In the next section, we describe the

setting of our model. Our objective in this paper is to address three questions pertaining to the above voluntary disclosure setting with a potentially informed receiver. First, how the introduction of a potentially informed receiver affects the equilibrium of the disclosure game, in particular the likelihood of voluntary disclosure and the manager's disclosure threshold and the price given no disclosure. This analysis is conducted in Section 3. Second, since the presence of a potentially informed receiver affects the manager's disclosure strategy, what is the effect of changes in the probability that the market becomes informed, i.e., analyst coverage, on the overall price efficiency. We address this question in Section 4. Finally, we study how changes in analyst coverage affect the liquidity of the firm's stock, as captured by the expected bid-ask spread. To do that, we introduce in Section 5 a stylized trading stage à la [Glosten and Milgrom \(1985\)](#). In this market microstructure model the trader in the firm's stock may be either an uninformed liquidity trader or an informed trader that perfectly knows the firm value, x . The details of this setting are deferred to Section 5.

2 Setting

Our model builds on the voluntary disclosure literature starting with [Grossman \(1981\)](#), [Milgrom \(1981\)](#), and [Dye \(1985\)](#). We consider a firm that is involved in a project, e.g., an R&D project such as drug development, which may eventually succeed or fail. We denote by \tilde{x} the outcome of the project. To simplify the model we assume \tilde{x} to follow a binary distribution so that $x \in \{0, 1\}$ where $x = 1$ denotes success and $x = 0$ denotes failure. The ex-ante probability of success is $p_0 \equiv \Pr(x = 1)$ and the probability of failure is $1 - p_0 \equiv \Pr(x = 0)$. Note that most of our results easily extend to an arbitrary distribution of \tilde{x} , including all the results in Sections 3 and 4. The binary structure is used only to simplify the trading stage in Section 5.

Information With probability $q \in (0, 1)$, the manager of the firm observes additional information about the possible outcome of the project in form of a signal s . With probability $1 - q$ the manager does not observe s . Information endowment is independent of the realization of s , and therefore the ex-ante expected value of s (or x) conditional on an information event equals the expected value conditional on no information event and equals p_0 . The signal may represent, for example, the results of a clinical trial or the results of an oil exploration, information about competing projects or information about relevant macroeconomic conditions. We assume that all players in the game are risk neutral, and thus it is without loss of generality to assume that the signal s is simply the updated probability of success. That is, the posterior beliefs given the realization of s is $\Pr(\tilde{x} = 1 | s) = s$. Hence, we assume that $\tilde{s} \in [0, 1]$, with a PDF $f(s)$, a CDF $F(s)$, and $E[\tilde{s}] = p_0$.

Disclosure and Pricing If the manager learns the realization of the private signal s , she can voluntarily disclose it to the market. Disclosure is assumed to be costless and credible (verifiable at no cost). As standard in the voluntary disclosure literature, if the manager does not obtain the private signal, she cannot credibly convey that she is not informed. The manager seeks to maximize the market value, or price, of the firm. For now, assume that risk neutral investors set the market price as the expected value conditional on all the available public information, which we denote by I . That is, $P = E[s | I] = E[x | I]$. Later, in Section 5, we introduce a trading stage that follows [Glosten and Milgrom \(1985\)](#) where prices are set by a centralized market maker.

The setting introduced so far is similar to a standard voluntary disclosure setting endowment that has been studied extensively. The only minor difference is that we assume that when the manager is informed he still faces uncertainty regarding the asset's value. This in the classic model can be assumed away without loss of generality but has a bite when we consider the presence of an additional source of information. A more significant innovation

of our model is the possibility that the signal s will become public by an external source.

Analyst (Exogenous Signal) We use financial analysts as our main motivating example, however, any alternative mechanism that induces stochastic public supply of the firm’s information, such as media, competitors, suppliers, social media, regulators etc., will have a similar effect in our model’s findings.

To study the interaction between firm’s voluntary disclosure and the potentially informed market, we add to the above setting the presence of financial analyst, who may also learn the realization of the updated probability of success, s . We abstract from strategic considerations of the analyst, and assume that analysts simply publish the information they find.³

The likelihood that the analyst will discover this information may depend on whether the manager is informed or not. For example, if the information s is the result of a drug development clinical trial, it is unlikely that the analyst will discover this information when the manager herself has not yet received this information. However, if the signal s is information about market conditions, it is natural to assume that the analyst may learn this information even if the manager is not aware of it. In order to allow for both types of information, we assume a relatively non-restrictive analyst’s information production technology. In particular, assume that the analyst’s information production technology is reflected by a pair of conditional probabilities $(g_I(r), g_U(r))$, where $g_I(r) \in [0, 1)$ and $g_U(r) \in [0, 1)$ are the probabilities that the analyst discovers s conditional on the manager being informed and uninformed, respectively. We introduce the parameter r to capture the overall quality and/or quantity of analysts that cover the firm. We refer to r as “analyst coverage.” An increase in analyst coverage weakly increases the probability that the analyst becomes informed when

³We abstract from potential incentives by the analyst to bias his forecast, and assume truthful disclosure by the analyst. It is immediate to see that all of our results are robust to an analyst’s reporting strategy that is potentially biased, as long as the analyst always issues a report when obtaining information and the analyst’s forecast follows a separating strategy. For an example and additional references see [Beyer and Guttman \(2011\)](#).

the manager is informed and when the manager is uninformed. For simplicity, we assume that g_I and g_U are differentiable, and thus assume $g'_U(r) \geq 0$ and $g'_I(r) \geq 0$. Note that the ex-ante probability that the analyst issues a report is $q \cdot g_I(r) + (1 - q) \cdot g_U(r)$.

Timeline To summarize our disclosure game, the timeline is as follows.

1. With probability q the manager privately learns the signal s .
2. If the manager is informed, she decides whether to publicly disclose s or not.
3. Analysts learn the signal s with probabilities $g_I(r)$ or $g_U(r)$, depending on the outcome of stage 1. An informed analyst immediately discloses s to the market.
4. Following the disclosure or lack of disclosure by both the manager and the analyst, market participants update their beliefs about the expected value of the project.
5. The price of the firm is determined, and the manager is compensated accordingly. We first assume risk neutral pricing, and in Section 5 we specify a market mechanism generates the price.

The setting and all the parameters of the model are common knowledge.

Remark 1. The manager and the analyst disclose the same exact information. Thus, the manager's disclosure is relevant only in the case the analyst has not published a report. This implies that even if the manager observes the report by the analyst before making her disclosure decision (that is, even if stage 3 is before stage 2), the equilibrium is essentially the same: following a disclosure by the analyst the manager is indifferent whether to disclose or not, and following no report the manager's strategy is identical to her strategy in the current model.

3 Equilibrium of the Disclosure Game

3.1 Disclosure Strategy

The manager seeks to maximize the expected price. If s is publicly disclosed (an event denoted by “D”), the price of the firm simply equals its expected value

$$P^D(s) \equiv E[\tilde{x}|s] = s.$$

Denote by ND the event that neither the manager nor the analyst disclosed s , and the price following no disclosure by $P^{\text{ND}} \equiv E[\tilde{x}|\text{ND}]$. P^{ND} represents the beliefs of the market regarding the firm’s value following no-disclosure. In case the analyst observes the signal s and publishes a report, the manager’s action has no effect on the price. Thus, an informed manager’s decision depends only on the case where the analyst does not publish a report, and she will choose to disclose if and only if $P^D(s) > P^{\text{ND}}$. While $P^D(s)$ is increasing in s , P^{ND} is independent of the manager’s type. Therefore, the manager’s equilibrium strategy is characterized by a threshold signal σ , such that an informed manager discloses if and only if $s \geq \sigma$.

The price following no-disclosure by the manager or the analyst P^{ND} depends on the beliefs on the market regarding the manager’s disclosure strategy. If the market believes the manager uses a threshold σ , then we obtain

$$P^{\text{ND}}(\sigma) \equiv E[\tilde{x}|\text{ND}, \sigma] = \frac{(1-q) \cdot (1-g_U(r)) E[\tilde{s}] + qF(\sigma) \cdot (1-g_I(r)) \cdot E[\tilde{s}|s < \sigma]}{(1-q)(1-g_U(r)) + qF(\sigma)(1-g_I(r))}. \quad (1)$$

Note that for any threshold σ the price given no disclosure is lower than the prior mean, that is, $P^{\text{ND}}(\sigma) < E[\tilde{s}] = p_0$. This is because managers with low signals withhold information, and thus no-disclosure is bad news.

Our disclosure model generalizes [Dye \(1985\)](#) and [Jung and Kwon \(1988\)](#) to a setting that contains an additional stochastic public revelation mechanism. Formally, those models are a particular case of our setting in which $g_I(r) = g_U(r) = 0$. It is easy to extend the analysis in [Jung and Kwon \(1988\)](#) and show that a threshold equilibrium exists, and that it is unique. This is formalized by the lemma below.

Fact 1. *There exists a unique equilibrium to the disclosure game, in which an informed manager discloses if and only if the signal s is greater than a disclosure threshold σ^* . σ^* is the unique solution of the indifference condition*

$$\sigma^* = P^{\text{ND}}(\sigma^*). \tag{2}$$

An additional useful property of disclosure games that also holds in our model is the minimum principle property, first described by [Acharya et al. \(2011\)](#). This property shows that $P^{\text{ND}}(\sigma)$ is minimized under the equilibrium threshold.

Fact 2 (“The Minimum Principle,” ([Acharya et al., 2011](#), Proposition 1)). *The equilibrium threshold σ^* is the unique threshold that minimizes the price given no disclosure, that is, $\sigma^* = \min_{\sigma} P^{\text{ND}}(\sigma)$.*

An immediate corollary of the minimum principle, is that a change in any parameter, for example r , that increases or decreases the function $P^{\text{ND}}(\sigma)$ for any threshold σ , will increase or decrease the equilibrium threshold σ^* , respectively. If, for example, a change in r increases the price following no-disclosure for any threshold, then, by the the minimum principle, it must increase the equilibrium threshold (that is, decrease disclosure). This formalized in the following Corollary.

Corollary 1. *The equilibrium disclosure threshold σ^* is increasing (decreasing) in r , if and only if $P^{\text{ND}}(\sigma)$ is increasing (decreasing) in r .*

3.2 The Effect of Analyst Coverage

In this section we analyze the main comparative static of the disclosure game, that is, how the equilibrium threshold is affected by the level of analyst coverage, r .

The equilibrium disclosure threshold is given by the solution to the fixed point problem defined by the indifference condition (2). Based on Corollary 1, in order to study the effect of analyst coverage on corporate disclosure, we need to study how analyst coverage affects the price given no disclosure for an exogenous disclosure threshold σ , i.e., $\frac{\partial P^{\text{ND}}(\sigma)}{\partial r}$. Note from (1) that, for any exogenous disclosure threshold σ , $\frac{\partial P^{\text{ND}}(\sigma)}{\partial g_I(r)} > 0$ and $\frac{\partial P^{\text{ND}}(\sigma)}{\partial g_U(r)} < 0$. Higher $g_I(r)$ means that the analyst is more likely to discover and disclose s when the manager is informed. Thus, no-disclosure when $g_I(r)$ is high implies that it is less likely that the manager is informed and withholds negative information. Therefore, P^{ND} is higher. In contrast, higher $g_U(r)$ means that the analyst is more likely to discover and disclose s when the manager is uninformed. Thus, no-disclosure when $g_U(r)$ is high implies that it is more likely that the manager is informed and withholds negative information. Therefore, P^{ND} is lower. The overall effect of an increase in r on the price given no-disclosure is

$$\frac{\partial P^{\text{ND}}(\sigma)}{\partial r} = \frac{\partial P^{\text{ND}}(\sigma)}{\partial g_I(r)} g'_I(r) + \frac{\partial P^{\text{ND}}(\sigma)}{\partial g_U(r)} g'_U(r);$$

because both $g_I(r)$ and $g_U(r)$ increase in r , the overall effect of changes in r is ambiguous. Without further assumptions on the functions $g_I(r)$ and $g_U(r)$, one cannot conclude whether an increase in analyst coverage increases or decreases managerial voluntary disclosure. Next, we provide the condition that determines whether disclosure is increasing or decreasing in r , and show this condition is satisfied in plausible information environments.

3.2.1 The Effect of Analyst Coverage on Voluntary Disclosure

It is useful to consider the following function

$$m(r) \equiv \frac{\Pr(\text{analyst knows nothing} \mid \text{manager uninformed})}{\Pr(\text{analyst knows nothing} \mid \text{manager informed})} = \frac{1 - g_U(r)}{1 - g_I(r)}. \quad (3)$$

$m(r) \in [0, \infty)$ is the ratio between the likelihood that the analyst does not discover news when the manager is uninformed and the likelihood the analyst does not discover news when the manager is informed. For convenience, we refer to $m(r)$ below as the “*informed analyst ratio*.”

Denote by σ_D^* the disclosure threshold in a model where there is no analyst available, that is, the classic [Dye \(1985\)](#) model. σ_D^* satisfies [Fact 1](#), when the price following no disclosure P^{ND} is calculated by substituting $g_I = g_U = 0$ in [\(1\)](#). We first establish the fact that the size of $m(r)$ determines whether the existence of analyst coverage increase or decreases disclosure compared to the standard [Dye \(1985\)](#) model.

Lemma 1. *In a model where an analyst can discover information, the firm discloses more information compared to the case where an analyst is not available if and only if the informed analyst ratio is greater than unity; that is*

$$\sigma^*(r) < \sigma_D^* \iff m(r) > 1.$$

Proof. Rewrite [1](#) as

$$P^{\text{ND}}(\sigma, r) = \frac{(1 - q)E[\tilde{x}] + q \cdot m(r)^{-1} \cdot F(\sigma) E[\tilde{x} \mid s < \sigma]}{1 - q + q \cdot m(r)^{-1} \cdot F(\sigma)}. \quad (4)$$

Clearly from [\(3\)](#), when $g_I = g_U = 0$ then $m = 1$. Thus $P^{\text{ND}}(\sigma, r) > P^{\text{ND}}(\sigma, r) |_{g_I=g_U=0}$ if and only if $m(r) > 1$. The lemma then follows from [Corollary 1](#). \square

We now turn to the effect of analyst coverage on disclosure. The following Proposition shows that this effect depend on the directional change in $m(r)$ when r changes.

Proposition 1. *In equilibrium, analyst coverage crowds out corporate voluntary disclosure if and only if $m'(r) > 0$, that is,*

$$\frac{\partial \sigma^*}{\partial r} > 0 \iff m'(r) > 0.$$

Proof. First we show that for any exogenous disclosure threshold σ , $\frac{\partial P^{\text{ND}}(\sigma)}{\partial r} > 0$ iff $m'(r) > 0$. By (4) and the fact that $E[\tilde{s}] > E[\tilde{s} \mid s < \sigma]$, it is clear that $P^{\text{ND}}(\sigma, r)$ is increasing in $m(r)$. Thus $\frac{\partial P^{\text{ND}}(\sigma)}{\partial r} > 0$ iff $m'(r) > 0$. This, together with Corollary 1, imply the desired result. \square

Higher $m(r)$ means that the analyst is *relatively* more likely to miss news regarding the firm when the manager is uninformed than when she is informed. Thus, no-report by the analyst signals that the manager is more likely to be uninformed. Formally, as shown by 4,

$$\Pr(\text{manager uninformed} \mid \text{ND}) = \frac{1 - q}{1 - q + q \cdot m(r)^{-1} \cdot F(\sigma)}.$$

Therefore, higher m gives the manager a higher payoff in case the analyst does not publish a report, and thus a higher incentive to withhold. Note that, as discussed above, the actual probability that the analyst discovers news does not enter the manager's payoff function in any way except through P^{ND} , because the manager, in her decision, only cares about the state of the world where the analyst does not discover any news.

3.2.2 Information Structure Examples

To judge whether it is reasonable or not to assume that disclosure is increasing or decreasing in analyst coverage, we offer two relatively simple examples of information structures, which we find appealing and realistic.

Example 1 (Private Inquiry and Leaks). Suppose that the manager learns \tilde{s} with probability q . The analyst has two potential sources of information, one within the firm and the other external. Examples for external sources could be information about the industry or macro economic conditions, or in the case of a clinical trial it could be that the analyst has direct access to one of the researchers conducting the clinical trial. Further assume that the probability that the analyst learns s from an external source is r and this probability is independent of whether the manager is informed or not.

The inside source of information captures information that is “leaked” to the analyst from within the firm. Such information can be observed by the analyst only when the manager is informed. Suppose that the probability that the analyst learns s from insiders, conditional on the manager being informed, is $\delta(r) \in (0, 1)$. Naturally we assume that an increase in analyst coverage increases the probability of leakages. For simplicity, we assume that $\delta(r)$ is differentiable, and $\delta'(r) > 0$.

In the setting of this example, we obtain $g_U(r) = r$ and $g_I(r) = r + (1 - r)\delta(r)$.

Example 2 (Conditionally Independent Information Endowment). Suppose that with probability $\omega \in (0, 1)$ some information event occurs and with probability $1 - \omega$ no information event occurs. If no information event occurs, the firm’s expected value remains the prior mean (p_0). However, if an information event occurs, it generates an outcome s which is the posterior expected value of the firm.

Conditional on the occurrence of an information event, the probability that the analyst discover s is r , and the probability of the manager to discover s is $\frac{q}{\omega}$ (so the overall probability that the manager discovers s is q). Assume that the information endowment events of the manager and the analyst are independent conditional on an information event. This structure

implies ⁴

$$g_U(r) = \frac{\omega - q}{1 - q} \cdot r \text{ and } g_I(r) = r.$$

One can easily verify that $m'(r) > 0$ in both examples.⁵ Given Proposition 1, this implies that an increase in analyst coverage increases the manager's disclosure threshold, i.e., $\frac{\partial \sigma^*}{\partial r} > 0$. In other words, in both of these examples an increase in analyst coverage crowds-out voluntary disclosure.

3.3 Assumption on Analyst's Information Production

Following the two examples above, in the remaining of the paper we will focus our attention on the case where analyst coverage crowds-out disclosure. That is, we shall assume the following regarding the analyst's information production technology $(g_I(r), g_U(r))$:

Assumption 1. *The informed analyst ratio $m(r)$, as calculated in (3), is increasing in r .*

Note that this assumption is supported by the empirical literature presented above (Anant-haraman and Zhang, 2011; Balakrishnan et al., 2014). Moreover, in the opposite case, where disclosure is increasing with analyst coverage, the main results of the paper regarding price efficiency and liquidity will trivially hold.

⁴The conditional probabilities that the analyst learns s are given by

$$g_U(r) = \Pr(\text{analyst learns} \mid \text{manager uninformed}) = \frac{\omega(1 - \frac{q}{\omega})r}{1 - q} = \frac{\omega - q}{1 - q}r$$

$$g_I(r) = \Pr(\text{analyst learn} \mid \text{manager informed}) = \frac{\omega \frac{q}{\omega}r}{q} = r.$$

⁵Note $m'(r) > 0$ if and only if $\frac{g'_U(r)}{1 - g_U(r)} < \frac{g'_I(r)}{1 - g_I(r)}$.

4 Price Efficiency

An increase in analyst coverage, r , increases, by definition, the probability that the signal will be disclosed, and thus directly provides more public information. However, in the previous section we have established that an increase in analyst coverage affects voluntary disclosure. In particular, given Assumption 1, an increase in r decreases corporate voluntary disclosure (Proposition 1). As such, the overall effect of changes in analyst coverage on investors' information, or price informativeness, is not clear. In this section we assess the overall effect of an increase in analyst coverage. This effect can be decomposed into two parts:

- A change in the probability that the signal s becomes public, either by the manager and/or by the analyst. The probability of this event is given by:

$$q \cdot g_I(r) + q(1 - g_I(r))(1 - F(\sigma^*)) + (1 - q)g_U(r).$$

As mentioned before, since the manager's equilibrium disclosure threshold, σ^* , is increasing in analyst coverage r , it is not clear whether the probability that the signal s becomes publicly available increases or decreases following an increase in r .

- Market uncertainty regarding s in case it does not become public. An increase in r affects the manager's disclosure strategy and as a result also affects the uncertainty given no disclosure.

As one might guess, due to the effect on disclosure strategy, one cannot use Blackwell informativeness criteria as a way to measure the effect of an increase in analyst coverage on the amount of public information. This is because more coverage increases the probability that the value of some types will be disclosed, but decreases this probability for other types. Instead, in the next section we define a measure of price information efficiency that is the inverse of the expected square distance of prices to the fundamental value. Since prices equal

the posterior mean, this measure also equals the expected posterior variance. Thus, it can be viewed as either the extent which prices reflect the fundamental value of the firm and or as a measure of the remaining uncertainty. We show that an increase in analyst coverage always increases price efficiency according to this measure. In section 5 we obtain a similar result when develop a liquidity measure in an extended model.

4.1 A Measure of Price Efficiency

In our model, in case information becomes public, either by the manager or by the analyst, the price perfectly reflects the underlying value, i.e., the price is $P = E[\tilde{x}|s] = s$. In case information does not become public the price is $P = P^{\text{ND}} = E[s | \text{ND}]$. In this case, although the price is on average correct, it is a noisy measure of the signal s (the manager might be withholding).

In order to measure how efficiently prices reflect information about future cash flows, we adopt the commonly used expected squared deviation between the market price and the signal s . Our price efficiency measure, which we refer to as PEF, is given by

$$\text{PEF} \equiv -E[(s - P)^2]. \quad (5)$$

One can think of PEF as representing the “social” benefit from having a price that is close to the fundamental, or the externalities and gains that are obtained from the informativeness of prices. Note that this measure is in line with our assumption of risk neutral pricing: a social planner who wishes to maximize efficiency, will indeed choose $P = E[s | I]$, where I is all the available information.

Another interpretation for PEF is that it is the variance of the noise in the price relative to the true underlying value s . Thus, higher price efficiency means a decrease in the future volatility of prices (the future movement of prices when the real cash flows x will be realized

or revealed).

4.2 Analyst Coverage and Price Efficiency

As indicated earlier, it is hard to determine even the directional effect of changes in analyst coverage, r , on price efficiency. One of our main results, is that an increase in analyst coverage always increase price efficiency.

Proposition 2. *Price efficiency increases in analyst coverage, i.e.,*

$$\frac{d\text{PEF}(r)}{dr} > 0.$$

The formal proof of the Proposition is quite involved and hence, is delegated to the appendix. The intuition for the result in the Proposition is as follows. In equilibrium, whenever the manager obtains a signal below the disclosure threshold, $s < \sigma^*$, she does not disclose, and if the analyst does not reveals s , the resulting price is $P^{\text{ND}} = \sigma^*$. Consider the various effects of a small increase in analyst coverage, r .

Consider a change from r to $r + \Delta$ for some small $\Delta > 0$. This will lead to a change in the disclosure threshold from σ^* to $\sigma^* + \Delta'$, where Δ' reflects the strategic effect on the disclosure strategy following this increase in r . Given Assumption 1, $\Delta' > 0$. One can examine the total effect on price efficiency by looking at each of the following effects separately: (i) the effect of changing r to $r + \Delta$ without changing σ^* (ii) changing σ^* to $\sigma^* + \Delta'$ without changing r .

Our claim follows from the fact that the first effect is positive and is of the order of $O(\Delta)$ while the second is negative but in the order of at most $O(\Delta^2)$. The fact that $\Delta' = O(\Delta)$ implies a positive effect so the derivative is positive. The first effect is clear: an increase in the probability that s is revealed by the analysts, increases the probability that the price is equal to the true type, s . This has a first order effect $O(\Delta)$.

The second effect is an increase in the manager's disclosure threshold, that is, a decrease

in corporate disclosure. This increase in the disclosure threshold means that signals $s \in (\sigma^*, \sigma^* + \Delta')$, which originally were disclosed and priced correctly when the manager was informed, are now not disclosed, and, thus receive, with some positive probability, a price P^{ND} . For the moment, suppose that P^{ND} does not change and remains σ^* . There is a decrease in price efficiency because types $s \in (\sigma^*, \sigma^* + \Delta')$ are not always priced correctly. However, this is a $O(\Delta'^2)$ effect, because even when types $s \in (\sigma^*, \sigma^* + \Delta')$ are not priced correctly, they obtain a price of σ^* that is still very close to their fundamental value. Moreover, the price following no-disclosure, P^{ND} , changes, and thus the pricing of all types whose value is not disclosed changes. By definition, the price following no disclosure $P^{\text{ND}} = E[\tilde{s} \mid \text{ND}]$ maximizes price efficiency following no disclosure. Thus, the new P^{ND} , that reflects the additional types who do not disclose, as described above, increases the overall price efficiency as compared to keeping the old price. This means that the negative effect is even smaller.

Proposition 2 implies that although analyst coverage has adverse effect on corporate voluntary disclosure, the overall informational effect of analyst coverage is positive.

5 Analyst Coverage, Informed Trading and Liquidity

The results in the previous section examine the effect of analyst coverage on a theoretical measure of price efficiency. Empirically measuring or estimating price efficiency, however, is not an easy task. Liquidity is a common measure of information asymmetry in the empirical literature. One measure of it, the bid-ask spread, is relatively easy to estimate and directly reflects the uncertainty of investors. Our intention is to analyze how the information asymmetry that remains after the disclosure game manifests itself in the liquidity of the firm's stock, as measured by the bid-ask spread, and to further study the informational implication of analyst coverage on the expected bid-ask spread.

We therefore extend our disclosure model by adding a stylized trading stage. Trading

takes place after the manager’s potential voluntary disclosure decision and after the potential release of the analyst’s report. The trading stage is a static version of the [Glosten and Milgrom \(1985\)](#) model (henceforth GM). There are two players: a competitive market maker and a single trader. The trader can either buy or sell one unit (share) of the firm’s stock. With probability $\gamma \in (0, 1)$ the trader is strategic and *informed*, and knows the firm’s terminal value, x . An informed trader maximizes his value from trading. With probability $1 - \gamma$ the trader is a “*liquidity trader*”, who sells or buys independently of the firm’s value (for example, due to a liquidity shock). The liquidity trader chooses to sell or to buy one unit with equal probabilities.

The risk neutral market maker does not have private information about the firm value or whether the trader is informed or not. He operates in a competitive market (which is not modeled), and, as a result, sets prices that lead to zero expected profit. The bid price, b , equals the expected value of the asset conditional on the trader selling a share. The ask price, a , equals the expected value of the asset conditional on the trader buying a share. The term $a - b$ is the bid-ask spread.

5.1 Disclosure Decision in the Extended Model

The analysis of the manager’s disclosure strategy in Section 3 assumes risk neutral pricing. That is, the price is $P = E[\tilde{x} | I]$: the expected terminal value given all publicly available information I . In the extended model, the price is determined by the trading that takes place, and is either a or b .⁶ The expected price following a trade, however, equals the price that is determined in the basic model. To see why note that $a = E[\tilde{x} | I, \text{buy}]$ and $b = E[\tilde{x} | I, \text{sale}]$, where “buy” and “sale” denote that events where the trader buys and sells one unit, respectively. Note the market maker uses all public information when she prices.

⁶The assumption that the strategic trader is always informed assures that there trade always takes place. Adding an uninformed strategic trader results in a possibility of no trade and a third price ($E[\tilde{x} | I, \text{no-trade}]$), but does not change our results.

Thus, we can use the law of iterated expectation to determine that the expected price at the beginning of the trading stage is

$$\Pr(\text{buy}) \cdot a + \Pr(\text{sale}) \cdot b = E[\tilde{x} | I],$$

exactly as in the basic model. Because the manager is risk neutral, this result means that we do not have to solve the trading stage in order to solve the equilibrium of the disclosure stage. To the contrary, we can use all the results of Section 3, and specifically the threshold found in this section when we calculate the prices and the spreads in the trading stage.

5.2 Prices and the Bid-Ask Spread

We first provide a short derivation of the bid and ask prices and the bid-ask spread in a standard static GM setting. Readers who are familiar with this derivation can skip directly to Lemma 2. The appendix provides a more detailed derivation.

Let the public belief about the firm's terminal value at the beginning of the trading stage, after the disclosure stage, be $p = \Pr(x = 1 | I)$. From Section 3 we know $p \in (0, 1)$. Given that $\gamma < 1$, it is clear that the bid and ask prices are also strictly between zero and one. Thus, an informed trader always buys if $\tilde{x} = 1$ and sells if $\tilde{x} = 0$. The uninformed market maker believes a "buy" event occurs with probability $\gamma p + (1 - \gamma) \frac{1}{2}$, and that, conditional on such event, the probability that the trader is informed is $\Pr(\text{Informed} | \text{buy}) = \frac{\gamma p}{\gamma p + (1 - \gamma) \frac{1}{2}}$. Thus, he sets an ask price that equals

$$\begin{aligned} a = E[\tilde{x} | I, \text{buy}] &= \frac{\gamma p}{\gamma p + (1 - \gamma) \frac{1}{2}} \cdot 1 + \frac{(1 - \gamma) \frac{1}{2}}{\gamma p + (1 - \gamma) \frac{1}{2}} \cdot p \\ &= \frac{1 + \gamma}{1 + \gamma(2p - 1)} \cdot p. \end{aligned}$$

A similar calculation result in a bid price of

$$b = E[\tilde{x} \mid I, \text{sale}] = \frac{1 - \gamma}{1 - \gamma(2p - 1)} \cdot p.$$

It is easy to see that $b < p < a$ for any $p \in (0, 1)$.

The bid-ask spread, which we denote by $\Psi(p)$, is the difference between the ask and the bid prices above, and is given by

$$\Psi(p) \equiv a - b = \frac{4\gamma(1 - p)p}{1 - \gamma^2(2p - 1)^2}. \quad (6)$$

The following Lemma provides some properties of the bid-ask spread.

Lemma 2. *The bid-ask spread, $\Psi(p)$, has the following properties:*

1. *It is a strictly concave inverse U-shape function of p .*
2. *for any $\gamma \in (0, 1)$, the spread is maximized at $p = 0.5$.*
3. $\Psi(p = 0) = \Psi(p = 1) = 0$.

The proof is trivial and merely involves differentiation of (6) and thus is omitted. The main characteristic of the bid-ask spread that we will be using is the concavity in the beliefs, p .⁷

5.3 Disclosure and Liquidity

The public information available to the market maker is the result of the disclosure model we studied before. We now study how the parameters of the disclosure game affect the expected bid-ask spread, which is the measure of information asymmetry and illiquidity.

⁷For simplicity we assume that the probability that a liquidity trader buys and sells with equal probabilities. Relaxing this assumption and allowing for this probability to be anywhere between zero and one, does not affect our main results. In particular, the bid-ask spread remains a concave inverse U-shape function of p .

Our measure of illiquidity, $\text{IL}(q, r)$, which depends on the parameters of the disclosure game, q and r , is given by

$$\text{IL}(q, r) \equiv E[\Psi(p) \mid q, r].$$

When we refer to liquidity we refer to $\text{L}(q, r) = -\text{IL}(q, r)$.

If neither the manager nor the analyst disclosed s , the market's expectation of \tilde{x} is given by $p = \sigma^* = E[\tilde{x} \mid \text{ND}]$. If the manager and/or the analyst disclose the signal s , then the market's expectation is $p = s$. We can identify three mutually exclusive events that lead to public information following the disclosure stage:

1. With probability $q \cdot g_I(r) + (1 - q) g_U(r)$ the analyst observes and published s . In this case all signals become public.
2. With probability $q(1 - g_I(r))(1 - F(\sigma^*))$ the analyst is uninformed, but the manager is informed and discloses signals above σ^* .
3. With probability $1 - (q \cdot g_I(r) + q(1 - g_I(r))(1 - F(\sigma^*)) + (1 - q) g_U(r))$ there is no disclosure; the analyst is uninformed, and the manager is either informed, or withholds signals that are below σ^* . The public belief in this case is σ^* (Fact 1).

Given these events, and the resulting distribution of beliefs, we can write the expected bid-ask spread as

$$\begin{aligned} \text{IL}(q, r) = & [1 - (q \cdot g_I(r) + q(1 - g_I(r))(1 - F(\sigma^*)) + (1 - q) g_U(r))] \cdot \Psi(\sigma^*) \quad (7) \\ & + [q \cdot g_I(r) + (1 - q) g_U(r)] \cdot E[\Psi(s)] \\ & + q(1 - g_I(r))(1 - F(\sigma^*)) \cdot E[\Psi(s) \mid s \geq \sigma^*]. \end{aligned}$$

We are interested in the effect of analyst coverage, r , on liquidity. The difficulty in proving this is similar to the one we described in the previous section, and stems from the fact that

an increase in r has an ambiguous effect on the probability that the signal becomes public, as well as the effect of the underlying uncertainty following no disclosure. IL , however, captures a different economic construct than PEF. In particular, expected liquidity is not a linear function of PEF, and hence Proposition 2 does not imply that the expected liquidity increases in r . For example, if a certain signal s is disclosed with higher probability following an increase in r , then this clearly effects positively on price efficiency, because disclosure results in “correct” pricing. However, since the spread is non-monotone (Lemma 2), such an effect may actually decrease liquidity if $\Psi(\sigma^*) < \Psi(s)$. Thus, the effect on IL is far less immediate than on PEF. Nevertheless, it is possible to show that analyst coverage has a total positive effect on liquidity:

Proposition 3. *The expected bid-ask spread, $IL(q, r)$, is decreasing in r for any $q \in (0, 1)$, that is*

$$\frac{dIL(q, r)}{dr} < 0.$$

The proof of the proposition appears in the Appendix. The key part of the proof is to show that an increase in r has a first order effect of decreasing illiquidity, despite the fact that for some values s , as described above, $\Psi(\sigma^*) < \Psi(s)$. This proof relies on the concavity of the spread function (Lemma 2). It is similar to assuming that the disclosure policy does not change and then use Blackwell informativeness criterion. The proof then uses the same intuition as in the proof of Proposition 2 to show that the change in voluntary disclosure plays a second order effect.

The result of Proposition 3, which provides additional motivation for the informational benefit of analyst coverage, is consistent with the empirical findings of Kelly and Ljungqvist (2012). Kelly and Ljungqvist (2012) find that following an exogenous negative shock to analyst coverage, there is a decrease in the liquidity of the firms that were affected by the decreased in analyst coverage.

A Appendix

Proof of Proposition 2

Proof. Denote by $P^{\text{ND}}(\sigma, r)$ the price given no disclosure by the firm or the analyst, as a function of a given disclosure threshold, σ , and a given analyst coverage r . $P^{\text{ND}}(\sigma, r)$ is given by (1). In addition, define $G(r, \sigma)$ as the PEF function (Equation (5)) for a given disclosure threshold σ and analyst coverage r :

$$\begin{aligned} G(r, \sigma) &= -E[(s - P(\sigma, r))^2] \\ &= -\text{Pr ND}(r) \cdot E[(\tilde{s} - P^{\text{ND}}(\sigma, r))^2 \mid \text{ND}], \end{aligned}$$

where

$$\text{Pr ND}(r, \sigma) = (1 - q)(1 - g_U(r)) + q(1 - g_I(r))F(\sigma)$$

is the probability of no disclosure. Note that in equilibrium the manager's disclosure threshold is $\sigma = \sigma^*(r)$ and hence, $\text{PEF}(r) = G(r, \sigma^*(r))$.

We need to show that in equilibrium, PEF is increasing in r , that is $\frac{d\text{PEF}}{dr} > 0$. This equals to

$$\frac{d\text{PEF}}{dr} = \frac{dG(r, \sigma^*(r))}{dr} = \frac{\partial G(r, \sigma)}{\partial r} \Big|_{\sigma=\sigma^*(r)} + \frac{\partial G(r, \sigma)}{\partial \sigma} \Big|_{\sigma=\sigma^*(r)} \frac{d\sigma^*(r)}{dr}.$$

A sufficient condition for $\frac{\partial \text{PEF}}{\partial r} > 0$ is that (1) $\frac{\partial G}{\partial r} \Big|_{\sigma=\sigma^*(r)} > 0$ and (2) $\frac{\partial G}{\partial \sigma} \Big|_{\sigma=\sigma^*(r)} = 0$. We prove those two properties below.

1. Proof that $\frac{\partial G}{\partial r} \Big|_{\sigma=\sigma^*(r)} > 0$:

$\frac{\partial G(r, \sigma)}{\partial r}$ is given by

$$\begin{aligned} \frac{\partial G(r, \sigma)}{\partial r} &= ((1 - q)g'_U(r) + q \cdot g'_I \cdot F(\sigma)) E \left[(\tilde{s} - P^{\text{ND}}(\sigma, r))^2 \mid \text{ND} \right] \\ &\quad + 2 \Pr \text{ND}(r, \sigma) \cdot E \left[\tilde{s} - P^{\text{ND}}(\sigma, r) \mid \text{ND} \right] \frac{\partial P^{\text{ND}}(\sigma, r)}{\partial r}. \end{aligned}$$

At $\sigma = \sigma^*(r)$, it becomes

$$\begin{aligned} \frac{\partial G(r, \sigma)}{\partial r} \Big|_{\sigma=\sigma^*(r)} &= [(1 - q)g'_U(r) + q \cdot g'_I \cdot F(\sigma^*(r))] E \left[(\tilde{s} - P^{\text{ND}}(\sigma^*(r), r))^2 \mid \text{ND} \right] \\ &\quad + 2 \Pr \text{ND}(r, \sigma) (E[\tilde{s} \mid \text{ND}] - P^{\text{ND}}(\sigma, \sigma^*(r))) \frac{\partial P^{\text{ND}}(\sigma, r)}{\partial r} \Big|_{\sigma=\sigma^*(r)}. \end{aligned}$$

Since in equilibrium the price conditional no disclosure equals the expected value given no disclosure, i.e., $P^{\text{ND}}(\sigma, \sigma^*) = E[\tilde{s} \mid \text{ND}, \sigma]$, the second term equals zero. Since, by definition, $g'_U(r) > 0$ and $g'_I(r) > 0$, we obtain

$$\frac{\partial G(r, \sigma)}{\partial r} \Big|_{\sigma=\sigma^*(r)} = [(1 - q)g'_U(r) + q \cdot g'_I \cdot F(\sigma^*(r))] E \left[(\tilde{s} - P^{\text{ND}}(\sigma^*(r), r))^2 \mid \text{ND} \right] > 0.$$

2. Proof that $\frac{\partial G}{\partial \sigma} \Big|_{\sigma=\sigma^*(r)} = 0$:

We can rewrite $G(r, \sigma)$ as

$$\begin{aligned} G(r, \sigma) &= - (1 - q) (1 - g_U(r)) E \left[(s - P^{\text{ND}}(\sigma, r))^2 \right] \\ &\quad - q (1 - g_I(r)) F(\sigma) E \left[(s - P^{\text{ND}}(\sigma, r))^2 \mid s \leq \sigma \right] \\ &= - (1 - q) (1 - g_U(r)) \int_0^1 (s - P^{\text{ND}}(\sigma, r))^2 f(s) ds \\ &\quad - q (1 - g_I(r)) \int_0^\sigma (s - P^{\text{ND}}(\sigma, r))^2 f(s) ds. \end{aligned}$$

Differentiating with respect to σ we obtain

$$\begin{aligned} \frac{\partial G(r, \sigma)}{\partial \sigma} = & -2(1-q)(1-g_U(r)) \int_0^1 (s - P^{\text{ND}}(\sigma, r)) f(s) ds \cdot \left(-\frac{\partial P^{\text{ND}}(\sigma, r)}{\partial \sigma} \right) \quad (8) \\ & -q(1-g_I(r)) 2 \int_0^\sigma (s - P^{\text{ND}}(\sigma, r)) f(s) ds \cdot \left(-\frac{\partial P^{\text{ND}}(\sigma, r)}{\partial \sigma} \right) \\ & -q(1-g_I(r)) (\sigma - P^{\text{ND}}(\sigma, r))^2. \end{aligned}$$

To obtain $\frac{\partial G}{\partial \sigma} |_{\sigma=\sigma^*(r)}$ observe that: (i) by Fact 1, $\sigma^*(r) = P^{\text{ND}}(\sigma^*(r), r)$. Thus, the third term in (8) equals zero; and (ii) by the “minimum principle”, $\frac{\partial P^{\text{ND}}(\sigma, r)}{\partial \sigma} |_{\sigma=\sigma^*(r)} = 0$.

Therefore, the first two terms in (8) also equal zero. Thus $\frac{\partial G}{\partial \sigma} |_{\sigma=\sigma^*(r)} = 0$.

□

Proof of Proposition 3

Proof. For a given and constant value of q , define a function $H(r, \sigma)$ that equals the expected spread conditional on analyst coverage r and a given disclosure threshold σ (which may not be the equilibrium threshold), as follows:

$$\begin{aligned} H(\sigma, r) \equiv & \Pr \text{ND}(r, \sigma) \Psi(P^{\text{ND}}(\sigma, r)) + ((1-q)g_U(r) + q \cdot g_I(r)) \cdot E[\Psi(s)] \quad (9) \\ & + q \cdot (1-g_I(r)) \int_\sigma^1 \Psi(s) \cdot f(s) ds, \end{aligned}$$

where

$$\Pr \text{ND}(r, \sigma) \equiv (1-q)(1-g_U(r)) + q(1-g_I(r)) F(\sigma) \quad (10)$$

is the probability of no disclosure, and $P^{\text{ND}}(\sigma, r)$, given in (1), is the price following no-disclosure by the manager or the analyst. When evaluated at the equilibrium disclosure threshold, $H(\sigma, r)$ is our measure of Illiquidity, that is, $\text{IL}(q, r) = H(\sigma^*(r), r)$. Thus, the

total derivative of $\text{IL}(q, r)$ with respect to r is:

$$\frac{d\text{IL}(q, r)}{dr} = \frac{\partial H(\sigma, r)}{\partial r} \Big|_{\sigma=\sigma^*(r)} + \frac{\partial H(\sigma, r)}{\partial \sigma} \Big|_{\sigma=\sigma^*(r)} \frac{d\sigma^*(r)}{dr}. \quad (11)$$

To obtain $\frac{d\text{IL}(q, r)}{dr} < 0$ it is sufficient to show that $\frac{\partial H(\sigma, r)}{\partial r} \Big|_{\sigma=\sigma^*(r)} < 0$ and $\frac{\partial H(\sigma, r)}{\partial \sigma} \Big|_{\sigma=\sigma^*(r)} = 0$.

We establish these sufficient conditions in the two Lemmas below.

Lemma 3. $\frac{\partial H(\sigma, r)}{\partial r} \Big|_{\sigma=\sigma^*(r)} < 0$.

Proof. We show that $\frac{\partial H(\sigma, r)}{\partial r} < 0$ for any given σ , and hence it also holds for $\sigma = \sigma^*(r)$. Given the continuity of $H(r, \sigma)$ in r , it is sufficient to show that $H(r_h, \sigma) < H(r_l, \sigma)$ for any $r_h > r_l$ and any σ .

1. Using (14), we compute $H(r_l, \sigma) - H(r_h, \sigma)$:

$$\begin{aligned} H(r_l, \sigma) - H(r_h, \sigma) &= \text{Pr ND}(r_l, \sigma) \Psi(P^{\text{ND}}(\sigma, r_l)) - \text{Pr ND}(r_h, \sigma) \Psi(P^{\text{ND}}(\sigma, r_h)) \\ &\quad + [(1 - q)(g_U(r_l) - g_U(r_h)) + q \cdot (g_I(r_l) - g_I(r_h))] \cdot E[\Psi(s)] \\ &\quad + q \cdot (g_I(r_h) - g_I(r_l)) \int_{\sigma}^1 \Psi(s) \cdot f(s) ds \\ &= \text{Pr ND}(r_l, \sigma) \Psi(P^{\text{ND}}(\sigma, r_l)) - \text{Pr ND}(r_h, \sigma) \Psi(P^{\text{ND}}(\sigma, r_h)) \\ &\quad - (1 - q)(g_U(r_h) - g_U(r_l)) \cdot E[\Psi(s)] \\ &\quad - q \cdot (g_I(r_h) - g_I(r_l)) F(\sigma) \cdot E[\Psi(s) \mid s < \sigma] \end{aligned}$$

We can therefore establish that $H(r_l, \sigma) - H(r_h, \sigma) > 0$ if and only if

$$\begin{aligned} \text{Pr ND}(r_l, \sigma) \Psi(P^{\text{ND}}(\sigma, r_l)) &> \text{Pr ND}(r_h, \sigma) \Psi(P^{\text{ND}}(\sigma, r_h)) \\ &\quad + (1 - q)(g_U(r_h) - g_U(r_l)) \cdot E[\Psi(s)] \\ &\quad + q \cdot (g_I(r_h) - g_I(r_l)) F(\sigma) \cdot E[\Psi(s) \mid s < \sigma]. \quad (12) \end{aligned}$$

2. Now observe from (1) that

$$\Pr \text{ND}(r, \sigma) \cdot P^{\text{ND}}(\sigma, r) = (1 - q)(1 - g_U(r)) E[s] + qF(\sigma)(1 - g_I(r)) \cdot E[s | s < \sigma].$$

This equation, applied to r_l and r_h , together with some algebra, leads to

$$\begin{aligned} \Pr \text{ND}(r_l, \sigma) \cdot P^{\text{ND}}(\sigma, r_l) &= \Pr \text{ND}(r_h, \sigma) \cdot P^{\text{ND}}(\sigma, r_h) \\ &\quad + (1 - q)(g_U(r_h) - g_U(r_l)) E[s] \\ &\quad + q(g_I(r_h) - g_I(r_l)) F(\sigma) \cdot E[s | s < \sigma]. \end{aligned} \quad (13)$$

Observe the similarity between the LHS and RHS of (12) and (13); in the next step we use (13) to prove that (12).

3. We can use (10) to rewrite (13) explicitly as

$$P^{\text{ND}}(\sigma, r_l) = A \cdot P^{\text{ND}}(\sigma, r_h) + B \cdot E[s] + (1 - A - B) \cdot E[s | s < \sigma]$$

where $A = \frac{\Pr \text{ND}(r_h, \sigma)}{\Pr \text{ND}(r_l, \sigma)}$ and $B = \frac{(1-q)(g_U(r_h) - g_U(r_l))}{\Pr \text{ND}(r_l, \sigma)}$. This representation presents $P^{\text{ND}}(\sigma, r_l)$ as an average of $P^{\text{ND}}(\sigma, r_h)$ and various signals. In order to obtain (12) remember that $\Psi(\cdot)$, is a strictly concave function (Lemma 2). Thus, by definition,

$$\Psi(P^{\text{ND}}(\sigma, r_l)) < A \cdot \Psi(P^{\text{ND}}(\sigma, r_h)) + B \cdot E[\Psi(s)] + (1 - A - B) \cdot E[\Psi(s) | s < \sigma].$$

This inequality is simply (12), and thus implies that $H(r_l, \sigma) > H(r_h, \sigma)$.

□

Lemma 4. $\frac{\partial H}{\partial \sigma} \big|_{\sigma=\sigma^*(r)} = 0$.

Proof. Differentiating (9) with respect to σ we obtain

$$\frac{\partial H}{\partial \sigma} = q(1 - g_I(r)) f(\sigma) [\Psi(P^{\text{ND}}(\sigma, r)) - \Psi(\sigma)] + \Pr ND(r, \sigma) \Psi'(\cdot) \frac{\partial P^{\text{ND}}}{\partial \sigma}. \quad (14)$$

To obtain $\frac{\partial H}{\partial \sigma} |_{\sigma=\sigma^*(r)}$ observe that: (i) by Fact 1, $\sigma^*(r) = P^{\text{ND}}(\sigma^*(r), r)$. Thus, the first term in (14) equals zero; and (ii) by the “minimum principle”, $\frac{\partial P^{\text{ND}}(\sigma, r)}{\partial \sigma} |_{\sigma=\sigma^*(r)} = 0$. Therefore, the second term in (14) also equals zero. Thus $\frac{\partial H}{\partial \sigma} |_{\sigma=\sigma^*(r)} = 0$. \square

This completes the proof of Proposition 3 \square

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