Tel Aviv University
The Gershon H. Gordon Faculty of Social Science
The Eitan Berglas School of Economics

Research Proposal

Persistent Undershooting of the Inflation Target in Israel: Inflation-Avoidance Preferences or a Hidden Target?

by
Weitzman Nagar

Supervisor:
Prof. Alex Cukierman

September 2009
Abstract

The paper examines the following question: Is persistent undershooting of the inflation target - especially during a disinflation process – caused by, a) inflation avoidance preferences (IAP), namely asymmetric monetary policy with respect to inflation, or by, b) aiming at a hidden, lower than the announced, inflation target.

This examination involves introducing a Linex CB's behavior function into a New Keynesian (NK) framework, which facilitates explicitly distinguishing between the two hypotheses. Using data for Israel, 1994-2007, it is shown that a hidden target was indeed applied during the 1990s, and it amounted to, on the average, 4 percentage points (pp) in comparison to the announced inflation target of 8 pp. at the same time, asymmetric policy was also applied, suggesting a complementary relations, i.e. the hidden target was motivated by other reasons. During the 2000's, on the other hand, it is not conclusive whether hidden target or asymmetry behavior was conducted, suggesting different techniques to implement IAP policy.

1. Introduction

The Seminal work of Kydland & Prescott (1977) and Barro & Gordon (1983) – (KPBG) demonstrated that the employment or short-run Phillips Curve motive cause inflation bias due to time inconsistency. Following these foundations, Inflation targets regime (explicitly or implicitly) were widely adopted from the 1990s onwards. McCallum (1995, 1997) and Blinder (1998) denied the KPBG's inflation bias argument. At the same time, evidence of deflation bias was found to be, indicating asymmetric policy with respect to inflation. Cukierman (2000b, 2002) argued that under two conditions - If the CB is uncertain about the economic conditions and is more sensitive to employment below than above normal - inflation bias may occur even with the absence of KPBG's inflation bias argument, i.e. even when the CB targets the normal level of output. In the same coin, deflation bias may arise if the CB is more sensitive to inflation above than below the target. Cukierman and Muscatelli (2008) labeled these types of asymmetry as Recession Avoidance Preferences – RAP and Inflation Avoidance preferences – IAP, respectively. This hypothesis was estimated for US and UK for various sub-periods, concluding that the asymmetry properties change in line with the regime and the main macroeconomic problems of the day.

However, undershooting the inflation target may also be an outcome of intentionally hidden target of the CB which is lower than the announced target, rather than the asymmetric policy. Illustration 1 depicts the actual inflation versus the announced target. It shows that while until 1996 the actual inflation (the blue color) often overshooting the target, undershooting the inflation target (the thick dark color) was quite prevalent from 1997
onwards: especially so from mid 1997 to mid 1998, from the end of 1999 to 2001, and from 2003 onward. Also illustrated are the interest rates levels and changes (in bars) that show substantial shifts in the responses, besides the normal and smoothed changes.

Although this phenomenon may be viewed as IAP, it may also be due to a hidden target of the CB which is lower than the announced target. In this line, Sussman (2007) interpreted the Israeli experience, claiming that the hidden target level was zero since 1995. However, his outcome was attained in an implicit way only.

Yet, is there any difference in motivations between hidden targets versus asymmetric behavior? One can claim that building credibility and facing uncertainty may also be achieved by a hidden target. In such a case the question of complementary or substitute relations between hidden target and asymmetric is becoming relevant: if they have substitute relations, then hidden target may be also interpreted as motivated by uncertainty and building up credibility. However, in the case of complementary relations, a hidden target should have different motivations, like political economy reasons as follows: The inflation target is determined by politicians (and therefore it is exogenous to the CB). In a disinflation process, it may be more convenient for politicians to keep the same level of the actual current inflation as the next period target as well, in order to avoid higher unemployment.
This assumption is supported by the fact shown in the illustration above, demonstrating that through 1993-1998 the targets were more or less on the same level. Only in 1999 the target level declined sharply, following the decline of the inflation in 1998.

The implications of hidden target versus asymmetry may be a lack of policy transparency, which also affects the expectations: while IAS is natural, understandable, transparent to the market players, as well as efficient, hidden target mislead the public expectation and may be inefficient. Another outcome is the CB's non-authorized actions under hidden target.

This paper aims to differentiate explicitly between asymmetry and hidden target. This is done by introducing Linex behavior CB's function into a New Keynesian (NK) framework. The methodology designed here enables one to explicitly distinguish between the asymmetry and hidden ingredients. Using the data of Israel for the period 1994-2007, it is founded that hidden target was adopted during the 90s, and on average it was 5.5 pp in comparison with the announced target of 8 pp; at the same time, asymmetric policy was also applied. On the other hand, during the 2000 decade only asymmetric behavior was applied.

2. Literature Review

The dominant explanation for the 1970s inflation appeared to be the employment or short-run Phillips curve motive, as pioneered by the seminal articles by Kydland & Prescott (1977) and Barro & Gordon (1983) – (KPBG). Their models showed that under rational expectations, the CB's desire to achieve employment above the natural level leads to a persistent inflation bias, while output does not rise above the natural level (namely as it would be with zero

---

1 Cukierman (1992, Chapter 2) offers four possible reasons for an inflationary bias under perfect information, all of them leading to dynamic inconsistency: the employment or short-run Phillips curve motive; a fiscal (seigniorage) revenue motive; interest rate smoothing and financial stability motive; and a balance of payment (under fixed exchange rate) motive which may also be referred as an employment motive. However, the seigniorage revenues (which was noted already by Keynes, 1924, p46) did not appear to be the right explanation for the inflation bias in the 1970s for various reasons, because in developed economies the seigniorage was a relatively small share of the GNP (around 1%); because the developed economies have sophisticated tax system; and because of the institutional separation between the Government (or Treasury) and the CB.

Another related explanation for the inflationary trend in the 1960s and 1970s was nominal interest rate targeting without a nominal anchor - because the CB automatically accommodates any shock to money demand. However, as Fischer (1994, pp. 29) noted, this argument encountered some difficulties, since closer examination showed that nominal interest rate targeting has determinacy under rational expectations rather than adaptive expectations, and because combining any nominal anchor keep the price level determinate. This argument trace back to Wicksell ([1898] 1965) and developed by Friedman (1968).
inflation). This explanation formed also the notion of "dynamic inconsistency" which was labeled by KP and led to the perception that an inflation bias could be reduced if the CB is time consistent. Consequently, a vast body of literature examined ways of reducing the inflationary bias, e.g. through nominating conservative CB's Governor (Rogoff, 1985), enhancing the CB's independency, reliability, credibility and accountability (Cukierman, 1992, 2000a), and weighing discretion versus commitment or rules.²

Against this backdrop, during the 1990s many developed economies adopted inflation target regimes as a practical way of committing themselves to reducing inflation. Drawing on actual policy perspectives, another strand of literature highlights issues of how CBs operates in practice. Since the Governors are judged by their success in combating inflation, Fischer (1994, pp. 293) raised the possible outcome of deflationary bias. Evidence to support asymmetry with respect to inflation was offered by Laxton & Rose (1995) with regard to U.S. and Mishkin and Posen (1997) with regard to Canada and U.K., Clarida and Gertler (1997)³ and Dolado el al (2000).⁴ Ruge-Murcia (2001, 2003a)⁵ find also deflation bias in Canada, Sweden and U.K.

As CBs became more independent, some studies raised doubts regarding the realism of KPBG's theory of inflation bias. McCallum (1995, 1997) argued that CB's refrain from the attempt to systematically stimulating output even under discretion because they understand its futility outcome. On the basis of institutional evidence, Blinder (1998) denied the KPBG's inflation bias thesis, by claiming that Fed policymakers do not systematically try to maintain employment above the natural level. (He also argued that politically, tightening policy is more difficult than easing policy).

---

² On these issues see for example Walsh (1995), Persson & Tabellini (1993) that suggested optimal incentive contracts for central bankers, and Svensson (1997) who showed that such contracts can be implemented by means of a simple inflation target. For a description see also Clarida, Gali & Gertler (1999) (hereinafter – CGG).
³ Clarida & Gertler (1997) reviewed Germany's monetary policy (before the European monetary union) within the context of a Taylor-rule policy decision, and found evidence of asymmetry – deflationary bias.
⁴ Dolado et al (2000) reviewed this question by comparing four countries – the US, Germany, France and Spain. They found evidence of deflationary bias at various intensities (Germany showed the strongest). When reviewing the question with regard to asymmetry with respect to output, they found symmetry in the three European states, while the US showed a more aggressive response vis-à-vis depression. These papers used a quadratic objective function which is widespread in the literature (see discussion below).
⁵ Ruge-Murcia used the Linex CB's Objective Function suggested by Nobay and Peel (1998, 2000). These authors were the first to use the Linex for monetary policy purposes.
Regarding these views of the absence of a KPBG inflation bias mechanism under the new enhanced CBs autonomy, Cukierman (2000b, 2002) raised the argument that an inflation bias may arise even when the CB targets the normal level of output - due to some uncertainty about the future state of the economy\(^6\) and of being more sensitive to below than above normal employment. On the same coin, deflationary bias may appear when policymakers are more averse to positive then to a (same size) negative inflation gap, for examples during periods of inflation stabilization. Such asymmetric behavior was labeled later in Cukierman and Muscatelli (2008) as Recession Avoidance Preferences (RAP) and Inflation Avoidance Preferences (IAP), respectively. Ruge-Murcia (2003) tested this hypothesis empirically and found that it provided a better explanation for the behavior of US inflation than the KPBG model.\(^7\) Using a non-quadratic model, Cukierman & Muscatelli (2008) empirically tested the question of asymmetry regarding the US and UK, and found that the asymmetry properties change in line with the regime and the main macroeconomic problem of the day. Sussman (2007) interpreted the Israeli experience during the 1990's to imply that the Bank of Israel pursued a hidden target which was lower than the announced target – The issue to be investigated in this paper.

The uncertainty and the asymmetric behavior addressed also the related issue of the theoretical form of the CB objective function. While a quadratic form is widespread due to its convenient properties, it embodies uncertainty equivalence property, namely it ignores the uncertainty surrounding inflation.\(^8\) Therefore, such a function might seems to be unrealistic, and the situation in the real world is that CBs are not indifferent to uncertainty (see also Blinder, 1997).\(^9\) This and other criticism\(^10\) brought about the use of different objective functions. One of them is the Linex type, which we use in our model and will be described in section 3 below.

\(^6\) Uncertainty about the condition of the economy, the nature of the shocks and the long and variables lags of the policy impacts (see also Goodhart, 1999). See also CGG (1999) and blinder (1997) about the lack of monetary models that include uncertain behavior formulation.

\(^7\) Considering asymmetry with respect to unemployment, Ruge-Murcia (2004) found asymmetry indicating that in the US and France there is a greater aversion to higher-than-natural rates of unemployment than to inflation, whereas in Canada, Japan, Italy and England the assumption of symmetric behavior cannot be rejected.

\(^8\) CGG (1999) also address the weakness of the quadratic objective function (p. 1668): "One limitation of this approach, however, is that the models that are currently available do not seem to capture what many would argue is a major cost of inflation, the uncertainty that its variability generates for lifetime planning and for business planning ..."

\(^9\) On this issue see also Cukierman and Muscatelli (2008). For wider review see also Nagar (2007).

\(^10\) Among them are Brainard (1967), Theil (1966), Blinder (1997) and Goodhart (2001).
3. The Linex Function

Before we proceed, let’s briefly introduce the Linex function which will be used later on as our Central Bank (CB) objective function:\[ L = \left( e^{ax} - ax - 1 \right) / a^2 \] (1)

where \( e \) is the exponential function; \( x \) describes the deviations from the inflation target; and \( a \) describes the degree of asymmetric preferences. In the case of a deviation from the target, the percentage deviation is important as well as its direction – a result which is not part of the familiar quadratic function. Thus, where \( a \) is positive, there is a stronger aversion to a positive \((x > 0)\) than the aversion to a negative deviation from the target. For instance, if \( a = 1 \), then under a case of positive deviation, say of 2%, the loss for the policy-maker will be much higher than in the case of a negative deviation of the same magnitude (which tends to be linear). When \( a \to 0 \) we get the quadratic function, which is a private case within the Linex function, and the loss for a deviation of 2% is identical in both directions. The set of preferences is therefore described by the asymmetry parameter \( a \), and it is built into the function.\[^{12}\]

Another feature of the Linex function is its treatment of the uncertainty surrounding the target variable. If \( x \) is given by the process \( x = \bar{x} + \varepsilon \), where the expectation \( \bar{x} \) is conditional in the process and the distribution of the disturbance \( \varepsilon \) is conditionally normal with variance \( \sigma^2 \), Christofferson & Diebold (1994) show that the predicted loss expectation of (1) is:

\[ E(L) = \left( e^{a(x - \bar{x})/2\sigma^2} - a \bar{x} - 1 \right) / a^2. \] (2)

The meaning of equation (2) is that the optimum expectation of \( x \) is no longer given only by the average \( \bar{x} \) (as in the case of a quadratic function, which assumes certainty equivalence), but by \( \bar{x} + \frac{a}{2} \sigma^2 \).\[^{13}\] In other words: unlike the quadratic function, the Linex function takes into account uncertainty to be priced both by the degree of risk aversion, expressed by \( a \),


\[^{12}\] For illustration of the Linex function, see appendix A

\[^{13}\] See also Nobay & Peel, 1998.
and the amount of risk $\sigma^2$. Where $a \to 0$ a quadratic function is obtained, and the degree (and thus also the amount) of uncertainty becomes irrelevant.

4. The Model

4.1 The Economy

The basic economy is a New-Keynesian dynamic model (NK), with temporary nominal price rigidities. The model consists of two equations: the aggregate demand (AD or IS) equation, and the aggregate supply (AS) equation – an expectations-augmented Phillips curve.\(^{14}\)

4.1.1 The aggregate demand (AD) equation

The AD equation is as follows:

\[ x_t = E_t x_{t+1} - \varphi(i_t^l - E_t^l \pi_{t+1}) + g_t \]

where the output gap, $x_t$, is the difference between current output, $y_t$, and natural output, $y^\pi$ (the output obtained if prices and wages are fully flexible), $x_t = y_t - y^\pi$; $x_t$ is negatively affected by the real expected interest rate, which is the difference between the nominal monetary interest rate $i_t^l$ and the inflation rate expected today for the forthcoming period, $E_t^l \pi_{t+1}$. Therefore, the real expected interest rate embodies the natural (long run) interest rate, $r_t^\pi$.

The shock $g_t$ on the aggregate demand side can be induced by government or private sector (a shock that causes movement of the AD curve) and is serially correlated following AR(1) process, i.e.

\[ g_t = \mu g_{t-1} + \varepsilon_t^g \]  

where $1 > \mu > 0$. $\varepsilon_t^g$ is white noise and hence it is unforecastable on the basis of the information available to the CB at the beginning of period $t$.

4.1.2 The aggregate supply (AS) equation

The aggregate supply equation:

\[ \pi_t^l = \lambda x_t + \beta E_t^l \pi_{t+1} + \nu_t \]

\(^{14}\) See, for example, CGG (1999), Woodford (2003) and Gali (2008).
Assuming $\lambda > 0$, $\beta \in (0,1]$, where $\beta$ is a future discount factor. This (rather standard) equation describes the inflationary process by relating the inflation rate to the output gap and the expected inflation, and has the flavor of a traditional expectation augmented Phillips curve. However, the equation evolves from the Calvo price staggering process, when firms set nominal prices based on the expectation of future marginal cost. The shock $\nu_t$ is a "cost push" shock which captures all else, except pressure from the demand side, that may have an unexpected impact on the marginal cost, and is following AR(1) process, i.e. 
$$
\nu_t = \mu_\nu \nu_{t-1} + \varepsilon_t^{\nu} \quad \text{where} \quad 1 > \mu_\nu > 0. \quad \text{\cite{15}}
$$
Here again $\varepsilon_t^{\nu}$ is white noise and it is unforecastable on the basis of the information available to the CB at the beginning of period $t$.

### 4.1.3 Terminology and notations

Before proceeding with the model it is useful to introduce some terminology and notations.

The times notations here are much consistent with the standard NK model as in CGG (1999).\textsuperscript{16} The CB affect the economy through its choice of the nominal interest rate which, given the inflationary expectations, affects the real expected interest rate in equation (3); this affect in turn the output gap and then the inflation in equation (4), where $E_t x_{t+1}$ and $E_t \pi_{t+1}$ are the private sector expected values of these variables at the beginning of period $t$ for the beginning of period $t+1$. I assume that the policy horizon is one year ahead, which will be denoted by time $t$. Although during this year the CB can take many sub-year decisions - each month, for example - its decisions are always forward looking one year ahead. Therefore, the time of the decision is always at the beginning of one year ahead, and is conditioned on all the information set which is available at the time of decision-making, i.e. at the \textit{beginning} of time $t$. The set of information includes also the announced inflation

\textsuperscript{15} for elaboration on the AS equation See, for example, CGG (1999), Woodford (2003) and Gali (2008)

\textsuperscript{16}Recall that the NK model is micro founded. The AD equation is obtained by loglinearizing of the consumption Euler equation that arise from the household's optimal saving decision, and where $c_t$ and $E_t c_{t+1}$ refers to the current and next period consumption, respectively, and $E_t \pi_{t+1}$ is the expected inflation for the next period. The supply equation (4) is obtained by aggregation of the individual firm pricing decisions which involves Calvo's staggered nominal price setting process. In this feature, $\pi_t$ and $E_t \pi_{t+1}$ are defined as $\pi_t = p_t - p_{t-1}$ and $E_t \pi_{t+1} = E_t p_{t+1} - p_t$ (where $p$ denotes price). However, what evolve from this model is that the current inflation and consumption depends on the \textit{future course} of these variables, and hence the CB optimize (under discretion) the current inflation and output gap (which is variant of consumption gap since there is no government in the model), subject to the constraints on behavior implied by equations (3) and (4) which include those expected variables.
target for the year ahead, $\pi^*_t$, and the private expected inflation and output, $E_t\pi_{t+1}$ and $E_tx_{t+1}$, respectively, which also are conditioned on the information available at the beginning of period $t$. The model will be treated under discretion, and hence in each decision time the CB reoptimize the inflation and the output gap for the coming year; this is done before the realization of the white noise shocks, taking in account the available information set and use its interest rate tool to achieve these optimized variables.

However, the optimized inflation level $\pi^t_i$ for the coming year may be different than the inflation target $\pi^*_t$ due to shocks that have already realized and may take longer time than one year to converge to zero following the AR(1) process (and therefore $x_i$ also may differ from zero).

The model will be described hereafter in terms of deviations from the inflation target.\textsuperscript{17} This is done for convenience and for the empirical purposes latter, and does not affect any results. By subtracting $\pi^*_t$ from both sides of the Phillips equation (4) we get: (i) The inflation deviation will be $\pi_i = \pi^t_i - \pi^*_t$ where the symbol $l$ denotes levels. (ii) $E_t\pi_{t+1} = E_t\pi^t_{t+1} - \pi^*_t$ will be for expected inflation deviation and therefore also (iii) $i_i = i^t_i - \pi^*_t$ for nominal interest rate deviation in equation (3). Note also that the steady state (S.S.) of the nominal interest rate level is $i^* = \pi^*_t + r^*_n$, and hence in S.S. we get in term of deviation $i_t = r^*_n$ where $r^*_n$ is the real natural (long run) interest rate; However, $i_t \neq r^*_n$ in all the other cases that are not S.S.

\textbf{4.2 The Policy Objective}

The CB problem is to choose an interest rate path in order to guide the target variables – inflation rate and the output gap. In terms of the mathematical formulation – to reduce the loss function to a minimum, where the general formula is:

$$\min_{\{i_t\}} E_t \sum_{t=0}^{\infty} \beta^t L(\pi_t, x_t)$$

\textsuperscript{17}An analogue way to think about is as the case of zero inflation target: In such a case, equation (4) is actually determined in terms of deviations.
subject to the behavioral constraints in equations (3) and (4). This formulation contains two objectives, the output gap and the inflation gap relative to the target – a common formulation in the literature. The specific loss function is the Linex type for the inflation target variable, and quadratic for the output gap variable:

\[
L = \frac{1}{\alpha^2} \left[ (e^{a(\pi_t + k\pi_t^*)} - a(\pi_t + k\pi_t^*) - 1) + \frac{\alpha}{2} (x_t)^2 \right]
\]

reflected the weight that policy attributes to the output target. Where \( \alpha \in [0,1] \) Where

Note that in this expression the linex terms are different than the standard one shown in equation (1). The parameter \( k \) expresses the extent of the CB’s hidden inflation target – which is unknown to the public and is different from the announced target, \( \pi_t^* \). The total deviation of the inflation that the CB chooses from the hidden one is \( \pi_t + k\pi_t^* \) and it includes two parts: the first is the deviation of the inflation from the announced target, \( \pi_t - \pi_t^* \); and the second is the gap between the announced target and the hidden one, \( k\pi_t^* \), which we will refer to as “target-gap”. When \( k=0 \) there is no hidden target, and hence the target-gap is zero. However, \( k \neq 0 \) says that the CB adopts a hidden target. When \( k > 0 \), the level of the hidden target is lower than the announced one. For example, if the announced target is \( \pi_t^* = 2\% \) while the hidden target is 1.5\%, then \( k = 0.25 \) and the target gap is 0.5\%. Note that in general \( k \) can also be negative, and in such case the hidden target is above the announced target. However, since our interest focuses on why the CB undershoots the announced inflation target, the relevant range is \( k \in [0,1] \). The further \( k \) deviates from zero the lower is the hidden target (in comparison to the announced target) and the higher is the target-gap. When \( k=1 \), the hidden target is zero and the target-gap is in its maximum level (in our case).

18 The transition from here to a formulation of one objective only is simple and does not entail the loss of generality.
19 The output gap is quadratic and do not take the linex type behavior. This is done because my focus here is on the inflation policy, and because it complicates the model. Following the literature, however, this method is not exceptional – see Dool del al (2004), Ruge-Murcia (2001, 2002, 2003, and 2004) and CGG (1997) which tested asymmetric policy with respect to one policy variable.
20 Where \( \alpha = 1 \), the weight attributed by the policy to both targets – inflation and output – is the same; where \( \alpha = 0 \) the policy only operates to achieve inflation target. This means that \( \frac{1}{\alpha} \) is the weight attributed to inflation target. See also Svensson, 2003.
It should be emphasized that expression (5) contains two parameters that differentiate between asymmetric behavior policy - which is motivated by uncertainty and aversion to inflation (IAP) and expressed by the parameter $a$; and hidden target behavior - which may be motivated by other reasons and expressed by the parameter $k$.

Our goal is to minimize the CB’s expected loss function and obtain econometric equations which enable us to estimate the existence and size of these parameters.

### 4.3 Optimization

Optimizing (5) subject to the behavioral constraints in equations (4) yields the following Lagrangian:

\[
(L) = \left\{ \frac{1}{a^2} \left[ (e^{a(x_i + k\pi_i^*)} - a(x_i + k\pi_i^*) - 1) + \frac{\alpha}{2} \left( x_i \right)^2 + \gamma (\pi_i + k \pi_i^* - \hat{\lambda}_{x_i} - \beta E_{\pi_{i+1}} - k \pi_i^* - \nu_i) \right] \right\}
\]

where $\gamma$ is the Lagrange constraint and after adding the term $k\pi_i^*$ to both sides of (4). \(^{21}\)

We will consider the case of policy under discretion. In the discretion case (i.e. in the absence of commitment), the Central Bank cannot directly manipulate the private sector expectations and therefore they are taken as given in solving the optimization problem. Also, future inflation and output are not affected by today’s actions from the CB’s point of view (even though, conditional on the CB optimization, the private sector forms its expectations rationally).\(^{22}\) Hence, in each period, the central bank chooses the target variables $(x_i, \pi_i)$ and then the interest rate in an effort to optimize the objective function $L$ (minimum loss).

\(^{21}\) Note that since the derivation is by $(\pi_i + k\pi_i^*)$ rather than by $\pi_i$ as usual, we had to add $k\pi_i^*$ to both sides of the supply equation (4) when we use it as constrain in (6). However, this element is anyway dropping out through the derivations, and the results are the same as optimizing only with respect to $\pi_i$. This outcome is because even we optimize the total deviation of inflation from the hidden target, both ingredients of the hidden element, i.e. $k\pi_i^*$, are parameter or exogenous state variable, respectively, and hence they are not subjected to the CB’s optimization. However, The optimization is formulized anyway with respect to $(\pi_i + k\pi_i^*)$, and this is done for the sake of model consistency.

\(^{22}\) The case under discretion is different than the commitment case. Under commitment the CB commits over the course of its future monetary policy and hence the inflation expectations are directly manipulated by the policy. Consequently, the optimization will be multi-period rather than one period. See for example also CGG (1999) as well as Woodford (2003).
Deriving with respect to inflation yields:

\[
\frac{\partial (L)}{\partial (\pi_i + k\pi^*_i)} = \frac{1}{a^2} (e^{a(\pi_i + k\pi^*_i)} - a - a) + \gamma = 0
\]

(6.1) \quad \frac{1}{a} (e^{a(\pi_i + k\pi^*_i)} - 1) = -\gamma

Deriving with respect to the output-gap yields:

\[
\frac{\partial (L)}{\partial x_i} = \alpha x_i - \gamma \lambda = 0
\]

(6.2) \quad \gamma = \frac{\alpha}{\lambda} x_i

Combining (6.1) with (6.2) gives (6.3):

\[
\Rightarrow \frac{1}{a} (e^{a(\pi_i + k\pi^*_i)} - 1) = -\frac{\alpha}{\lambda} x_i
\]

(6.3) \quad x_i = \frac{-\lambda}{a\alpha} (e^{a(\pi_i + k\pi^*_i)} - 1).

Placing (6.3) in the supply equation (4) yields:

(6.4) \quad \pi_i = \frac{-\lambda^2}{a\alpha} (e^{a(\pi_i + k\pi^*_i)} - 1) + \beta E_i \pi_{t+1} + \upsilon_i.

Assuming that the inflation is normally distributed process, the exponent term is characterized by a log normal distribution as in equation (2). By taking expectations and rearranging the terms in (6.4) we get:

\[
\Rightarrow E(\pi_i) - \beta E_i \pi_{t+1} = \frac{-\lambda^2}{a\alpha} (e^{a(E(\pi_i) + k\pi^*_i + \frac{a}{2}\sigma^2_i)} - 1)
\]

\[
1 - \frac{a\alpha}{\lambda^2} E(\pi_i - \beta E_i \pi_{t+1}) = e^{a(E(\pi_i) + k\pi^*_i + \frac{a}{2}\sigma^2_i)} (6.4)'
\]

Taking logs from both sides of (6.4)' and rearranging gives:
For the purpose of solving this equation we will use the following approximation:

\[(\ref{eq:approx})\]

\[
\ln(1 - \frac{aa}{\lambda^2}) \cong -\frac{aa}{\lambda^2}.
\]

Applying it in (\ref{eq:equation}) provides:

\[(\ref{eq:approx})\]

\[-\frac{\alpha}{\lambda^2} \, E(\pi_i - \beta E_i, \pi_{t+1}) = E(\pi_i) + k\pi_i^* + \frac{a}{2} \sigma^2_{\pi}.
\]

Rearranging and moving terms gives:

\[
E(\pi_i) = -\frac{k}{1 + \frac{\alpha}{\lambda^2}} \pi_i^* - \frac{\alpha}{2} \frac{\sigma^2_{\pi}}{1 + \frac{\alpha}{\lambda^2}} + \frac{\alpha \beta}{\lambda^2 + \alpha} E_i, \pi_{t+1}, \text{ or}
\]

\[(\ref{eq:approx})\]

\[
E(\pi_i) = -\frac{k}{\theta} \pi_i^* - \frac{\alpha}{\theta} \frac{\sigma^2_{\pi}}{2} + \frac{(\theta - 1) \beta}{\theta} E_i, \pi_{t+1},
\]

where \[\theta = 1 + \frac{\alpha}{\lambda^2} > 1.\]

Equation (\ref{eq:approx}) describes the average inflation deviation as a function of three terms: (a) the inflation target-gap, which depends on \(k\), (b) the extent of the asymmetric behavior (the parameter \(a\)) and the "size" of the uncertainty (i.e. \(\sigma^2_{\pi}\)) and (c) the inflation expectation. The first two terms imply that inflation will be lower the higher are \(a\) and \(k\). If both of them equal zero, the expected inflation is determined only by the last term. Another important outcome which arises from equation (\ref{eq:approx}) is that asymmetry and hidden target may have complementary rather than substitutional relations. In other words, even when the CB adopts a hidden target, there is still place for asymmetric policy because of uncertainty (and vice-versa).

\[\text{23 This holds as long as we limit the inflation gap to a few percents, even up to 20%. This is not the case of a disinflation process of tens of percents to 2%, as in such instance the approximation is incorrect.}\]
Since all the variables in equation (6.8) are known it can also be econometrically estimated:

\[(6.8)\]  
\[\pi_t = c_0 - c_1 \pi^*_{t-1} - c_2 \frac{\sigma^2}{2} + c_3 E_t \pi_{t+1} + \epsilon_t\]

Where:  
\[c_1 = \frac{k}{\theta}; \quad c_2 = \frac{a}{\theta}; \quad c_3 = \frac{\left(\theta - 1\right)}{\theta} \beta.\]

After estimating the parameters of (6.8)', by calibrating \(\beta\) with the conventional values we can even extract the absolute values of the parameters \(a\) and \(k\), and not just their existence.\(^{24}\)

4.4 Policy response function

Expression (6.8) above describes the inflation process only, without the interest rate response rule. Introducing the interest rate here makes sense since: (i) It reflects the CB behavior since it is a policy variable and (ii) we can benefit from one more observed variable. We will now proceed to derive the interest rate response function.

Starting with the output-gap equation (3) \(x_t = E_t x_{t+1} - \varphi(i_t - E_t \pi_{t+1}) + g_t\) and isolating \(i_t\) we get:

\[(7)\]  
\[i_t = \frac{1}{\varphi}(E_t x_{t+1} - x_t + \varphi E_t \pi_{t+1} + g_t).\]

From the optimum condition (6.3) it emerges that:

\[(6.3')\]  
\[E_{t}x_{t+1} = -\frac{\lambda}{a} (e^{a(E_{t} \pi_{t+1} + k \pi^*_{t+1} + \frac{a}{2} \sigma^2_{t+1})} - 1),\]

where the inflation variance is expected for the period \(t+1\). By substituting in (7) the expected output gap in (6.3) together with (6.3') we get:

\[(7.1)\]  
\[i_t = -\frac{\lambda}{a \varphi} (e^{a(E_t \pi_{t+1} + k \pi^*_{t+1} + \frac{a}{2} \sigma^2_{t+1})} - 1) - \frac{\lambda}{a \varphi} (e^{a(E \pi_{t+1} + k \pi^*_{t+1} + \frac{a}{2} \sigma^2_{t+1})} - 1) + E_t \pi_{t+1} + \frac{g_t}{\varphi}\]

\(^{24}\) Note that in (6.8') the variance must be treated as a variable series rather than as a constant, because in the constant case (6.8') yields the following formula:

\[\Rightarrow (6.9) \pi_t = \delta_0 + \delta_1 \pi^*_{t} + \delta_2 E_t \pi_{t+1} + \zeta_t,\]

and the constant term \(\delta_0\) will include the term \(c_2 = \frac{a \sigma^2}{\theta} \frac{2}{2}\). Consequently, the parameter \(a\) is vanished in \(\delta_0\).
And after organizing we obtain:

\[
(7.1') \quad i_t = \frac{\lambda}{\alpha \alpha \phi} \left[ (e^{a(E(\pi_t^ *) + k\pi_t^ * + \frac{\alpha}{2} \pi_t^ *}) - (e^{a(E_1 \pi_t^ * + k\pi_t^ * + \frac{\alpha}{2} \pi_t^ *)}) + E_t \pi_t^ * + \frac{\alpha_t}{\phi} \right].
\]

In order to solve (7.1') we assume that the expected inflation variability for the next period is the same as the current period, even though the variance terms above the exponent is time varying.\(^{25}\) Using the approximation feature\(^{26}\) (7.2) \(e^{z_1} - e^{z_2} \cong e^{z_1 - z_2} - 1\) (see proof in appendix 2) and rearranging yields:

\[
(7.3) \quad i_t = \frac{\lambda}{\alpha \alpha \phi} (e^{a(E_1 \pi_t^ * + k(x_t - \pi_t^ *))}) + E_t \pi_t^ * + \frac{\alpha_t}{\phi}.
\]

The last expression reflects the interest rate response to a change in the expected inflation and shows that it is always greater than 1, even though the parameter \(a\) can also be negative. This condition is necessary for the NK model in the discretion case.\(^{27}\) In addition, the interest rate response also depends on the \(\pi_t^* - E_t \pi_t^*\) term, which is the spread between the current year and the expected official target for the next year (which will be or has already been announced). When the disinflation process has ended, in many economies including the Israeli one - the official target turned out to be a multi-period target; therefore, the current target is actually the next period target as well, i.e. \(\pi_t^* = E_t \pi_t^*\), and therefore the parameter \(k\) in equation (7.3) disappears. However, through the disinflation process, the official target gradually converges to the long-run target, and hence the term \(\pi_t^* - E_t \pi_t^*\) in expression (7.3) is positive (in general). In the case of Israel, for example, through the 1990’s until 2002 the official target, in most of the cases, was announced just for one (calendar) year; only from 2003-on the target was determined as a long run 1-3 percent. Hence, we have 11 years in which \(\pi_t^* \neq E_t \pi_t^*\) and thus the parameter \(k\) might be

---

\(^{25}\) This is a reasonable assumption since it is the expected variability around a random variable. The descriptive data also shows that the ex-post mean of this difference is zero.

\(^{26}\) It is correct where the \(z’s\) are small enough. In our case (and as in approximation (6.6) above), the inflation is measured in single percentage and decimal equation, so that for example, a 2% inflation target is 0.02 and the approximation has no significant deviation. Even where \(z1\) is 20% and \(z2\) is 10% the deviation is only one percent. Nevertheless, we should emphasize that this approximation is incorrect in the disinflation process where at its starting point the inflation is tens of percents.

\(^{27}\) See for example Woodford (2003) as well as CGG (1999).
estimated to be different than zero. Luckily the expression (7.3) can be used as an econometric equation.

By rearranging terms and taking logs from both sides we obtain:

\[
\ln(i_t - E_t \pi_{t+1} - \frac{g_t}{\phi}) = -\ln(\frac{\alpha \sigma \phi}{\lambda}) + a(\pi_t - E_t \pi_{t+1}) + ak(\pi_t^* - E_t \pi_{t+1}^*).
\]

For estimating purpose we will use the following econometric equation:

\[
\ln(i_t - E_t \pi_{t+1}) = d_0 + a(\pi_t - E_t \pi_{t+1}) + ak(\pi_t^* - E_t \pi_{t+1}^*) + \epsilon_t,
\]

where the disturbances term on the left hand side of expression (7.4) will be part of the residuals of equation (7.5). Comparing this equation to (6.8), the asymmetric parameter, \(a\), appears even though the variance has vanished, \(^{28}\) and here we can also explicitly estimate the parameters \(a\) and \(k\).

Notice that equation (7.5) is not a "classic" response function since the left term is the real expected interest rate rather than the usual nominal interest rate which the CB sets (even though the real interest rate is an outcome of its policy). In this equation we cannot separate the left hand side variables because of the (semi) logarithmic expression. Note also that (7.5) does not include the output as it is in the typical Taylor-Rule, \(^{29}\) because the first order condition of the output is expressed in terms of the inflation and the coefficients through equation (6.4) above.

4.5 Interest rate smoothing

The interest response function (7.5) is derived from optimization. However, it is widely known that the CBs are smoothing their response traditionally, i.e. only partially adjust the optimal interest rate in (7.5). CBs choose to act in this way for a number of reasons: (i) uncertainty about the reality - In reality the current condition of the economy and the source, size and persistence of shocks may be uncertain; thus policy-makers prefer to take moderate measures. (ii) Political-economic reasons – as Blinder (1998) noted, raising the interest rate turn to be more difficult than subtracting it. (iii) Financial stability reasons - big

\(^{28}\) Note that the variance has vanished because of our assumption that the expected variance for the current year is the same as for the next year, and therefore they vanish in equation (7.3).

and unexpected changes may result in undesirable shocks to the financial markets and intermediation.\footnote{See Cukierman (1992) among others.}

Smoothing the interest rate takes the following feature:

\begin{equation}
(7.6) \quad i_t^A = \rho(i_{t-1}^A) + (1 - \rho)i_t + \nu_t
\end{equation}

Where $i_{t-1}^A$ is the current actual interest rate (in which its decision took place in the past period) and it is weighted by fraction $\rho \in [0,1]$.

To obtain a similar expression like (7.5) let us subtract inflation expectations (for one period ahead) from both sides:

\begin{equation}
(7.6') \quad i_t^A - E_t \pi_{t+1} = \rho(i_{t-1}^A - E_{t-1} \pi_t) + (1 - \rho)(i_t - E_t \pi_{t+1}) + \nu_t,
\end{equation}

where $E_{t-1} \pi_t$ describes the expected inflation in the past period to the current period (which terminologically is parallel to $E_t \pi_{t+1}$ in time $t-1$). Placing this expression in equation (7.5) yields the following:

\begin{equation}
(7.7) \quad \ln(i_t^A - E_t \pi_{t+1}) = (1 - \rho)d_0 + (1 - \rho)a(\pi_t - E_t \pi_{t+1}) + (1 - \rho)ak(\pi_t^* - E_t \pi_{t+1}^*) + \rho(i_{t-1}^A - E_{t-1} \pi_t) + \nu_t + \epsilon_t^A
\end{equation}

where $z_t$ includes other variables that may be relevant to the policy and inflation behavior, like the exchange rate and lag variables.\footnote{See also Sussman (2007).} This is the final equation of the model and can be econometrically estimated where the parameters $a, k$ (and now also $\rho$) can be explicitly estimated.

To sum, we will proceed to econometric estimations of two equations – (6.8)' and (7.7). As a reminder, the first equation is:

\begin{equation}
(6.8') \quad \pi_t = c_0 - \frac{k}{\theta} \pi_t^* - \frac{a}{\theta} \sigma_\pi^2 (\theta - 1) \beta E_t \pi_{t+1} + \epsilon_t
\end{equation}

For estimation purposes, it is important to emphasize that (7.7) can be interpreted as forward looking view (ex-ante): each month the CB takes its decisions (the dependent
variable) according to the variables on the right hand side of the equation. Equation (6.8), on the other hand, can be interpreted as ex-post view: each month we compare the annual actual inflation to the explanatory variables on the right hand side as they were known one year ago, when the decision took place.

5. Estimation and data

We estimate the case of Israel for the period 1992:1-2007:06, dominated by BoI’s two Governors: Jacob Frankel, 1991-1999, and David Klein, 2000-2004. First we provide in short the background then the data.

5.1 Israel – background

An Inflation target was first announced by the Bank of Israel (in the end of 1991) for the calendar year of 1992 (14-15 percents). However, because of the prevailing exchange rate regime, it was actually aimed at backing the 9% slope of the new diagonal (crawling peg) exchange-rate band, which replaced the previous horizontal band. Hence, under this regime the meaning behind the interest rate decisions which took place were not clear: were they taken to serve the exchange-rate or inflation? From the official viewpoint, since the exchange rate regime is under the Government’s mandate, the monetary policy had to be restricted to serve the exchange-rate band as its first priority. The first inflation target to be announced by the government came to light (only) in Sep. 1994 and for the calendar year of 1995 (see table 1).

For this reason I will skip the data of 1992-1993. One can claim that also 1994’s data should be skipped. However, it should be noted that the usage of the interest rate instrument during 1994 was much more aimed at the inflation - since the actual exchange rate was not bounded and was about the middle of the band. Another reason for including the 1994’s data is the hike of the inflation rate in 1994 (see illustration 1), which seemed to increase the Governor's intention to fight inflation, as it was reflected also in the first government announcement.

Governors Stanley Fischer began his tenure in May 2005. His impact is only on the margin of this period, especially after taking into account also the lag impact of the policy.

In this respect, Israel was the third economy that announced inflation target (after New-Zealand and Canada). However, it was CB’s self-announcement, rather than Governmental.

See Nagar (2002) and Sussman (2007). Barro and Gordon (1983) claim that such unclear response behavior which is inconsistent with the self-declaration of monetary policy is quite known.
Since 1994, the need of the CB to achieve the inflation target, on the one hand, but the necessity to preserve the exchange rate (especially when it was bounded), on the other hand, was a source for much of the tension and debates between the CB and the government (Treasury)\textsuperscript{35}. In a few instances the tensions were resolved by reaching agreements (or "Deals") between these two authorities: in most of these Deals, the CB lowered the interest rate while widening at the same time the Exchange rate band, a fact that I will consider hereinafter.

Subtracting actual inflation depends also on the target level set by the government. As can be seen in illustration 1, the target did not change dramatically between the years 1994-1998. This also was a source of tension with the CB: while wishing to reduce the inflation, the government was not ready to ‘pay’ the price for it and subtract the target level. However, it is reasonable to assume that it was very convenient for politicians to retain the current inflation as the target level for the next period. For this reason, it may also be assumed that the CB had a hidden target which was lower than the announced one, aiming at achieving lower target announcement for the next period. Sussman (2007) claims that the CB had zero hidden target.

Two more points are relevant for our estimation and assumptions - the horizon of the target and the timing of the announcement. As it is shown in table 1, until 2000 the target level was announced for one calendar year. In addition, the timing of the announcement for the next (calendar) year generally was in the end of the current year. Consequently, taking into account the impact-lags of the monetary policy, it follows that many (monthly) interest rate decisions - especially during the second half of the calendar year - were relevant for the next year, but they were taken without knowing the official target.

Following this background, some assumptions and data adjustments were needed – they are introduced in the next section.

5.2 The data

5.2.1 The estimated period - The estimation will be for the period 1994:1-2007:06, with monthly data which total 162 observations. Two sub-periods were also used for comparison, 1994:1-1999:12\textsuperscript{36} and 2000:01-2007:06. These sub-periods were used not only because they

\textsuperscript{35}For more details in this issue see Nagar (2002).

\textsuperscript{36}It should be noted that the results were robust to slide into some months of 2000 also.
fit the tenures of governors Frankel and (roughly) Klein, respectively, but also because they are essentially different: while during the first sub-period the inflation converged from 10% to 2%, in the second sub-period it was already about its long run level (illustration 1), and the target levels were known also for the following periods, rather than just for the current (or next calendar) year (table 1). Hence, Generally speaking, this is to say that (with respect to inflation) in the second sub-period the CB job was to keep the inflation as it was, rather than lowering it as during the 90's.

5.2.2 Measurement terminology - The annual Inflation rate is measured in terms of moving years. Its variance is calculated for a moving 12 months of annual inflation rates. One can claim that the policy had to be judged according to the calendar year, rather than moving year. However, because of observations restrictions such a claim can't be applied. Expected inflation for one year ahead is as it was derived from the financial markets. 37

5.2.3 The target horizon - The inflation target is in the middle of the target range. The assumption here is that the horizon of the monetary policy is, at any decision time, 12 months ahead. Table 1 shows the timing of the target's announcements for the next year; it follows that the target for the policy horizon was unknown for many monthly policy decisions. Take for instance the 7-10 percents points (pp) target for 1998: The target for 1999 (of 4 pp) was announced only in August 1998. Standing in July 1998, the question is which target level the policy faced in month \( m \) for \( m+12 \), when the target for 1999 was not yet known: shall we take 8.5 or 4 pp or average? Therefore, we will consider different cases.

5.2.4 Other variables - In addition, I added some more variables that do not appear explicitly in the model (see Sussman 2007) - because they may affect the economy anyway - as well as some dummy variables as follows:

5.2.4.1 Real long run interest rate – represented by the yield of (8 year for maturity) CPI's indexed government bonds. This is mainly for two reasons: it is implied in the economy in equation (3.1) (which implicitly affects also the short interest rates; 38); equation (7.7) is in real terms anyway.

37 Since in Israel there is also government bonds which are inflation indexed, the derived inflation expectation is the yield gap between indexed and non-indexed bonds bearing the same maturity.

38 For equation (3.1) to hold – for example in S.S where the output gap is zero - it must include also the natural real interest rate on the right hand side. This is consistent also with Wicksell ([1898] 1965), and with Real Business Cycles (RBC) theory which is embodied in the New-Keynesian theory. See also Woodford (2003) and Gali (2008).
5.2.4.2 **The exchange rates** – the annual change in real and nominal NIS/$US terms, as well as nominal monthly changes.

### Table 1: Time of target announcement, the actual and expected inflation

<table>
<thead>
<tr>
<th>Time of Announcement</th>
<th>Target Year</th>
<th>Inflation target (%)</th>
<th>Current last inflation (%)(^1)</th>
<th>Inflation expectations (%)(^1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>11/1992 (beg.)(^2)</td>
<td>1993</td>
<td>10</td>
<td>8.1</td>
<td>8.6</td>
</tr>
<tr>
<td>07/1993 (end)(^2)</td>
<td>1994</td>
<td>8</td>
<td>10.6</td>
<td>8.0</td>
</tr>
<tr>
<td>09/1994(^3)</td>
<td>1995</td>
<td>8-11</td>
<td>13.8</td>
<td>12.4</td>
</tr>
<tr>
<td>10/1995 (beg.)(^4)</td>
<td>1996</td>
<td>8-10</td>
<td>8.7</td>
<td>10.0</td>
</tr>
<tr>
<td>27/12/1996(^4)</td>
<td>1997</td>
<td>7-10</td>
<td>10.6</td>
<td>8.9</td>
</tr>
<tr>
<td>8/8/1997(^3)</td>
<td>1998</td>
<td>7-10</td>
<td>9.2</td>
<td>9.0</td>
</tr>
<tr>
<td>12/8/1998</td>
<td>1999</td>
<td>4</td>
<td>3.0</td>
<td>3.1</td>
</tr>
<tr>
<td>16/8/2000</td>
<td>2001</td>
<td>2.5-3.5</td>
<td>1.0</td>
<td>1.6</td>
</tr>
<tr>
<td></td>
<td>2002</td>
<td>2-3</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>from 2003 on</td>
<td>1-3</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1. As it was known at the time of the announcement.  
2. (beg) and (end) denotes the beginning and the end of the month.  
3. The first time as Government (rather than CB) decision.  
4. The decision included also: to achieve the average OECD inflation in 2001.  
5. The decision included also: aiming to achieve long run price stability by converging the inflation gradually.  
6. The target was announced for two years (not just one as before).

5.2.4.3 **Dummy variables** – the first group is for "Deals" between the CB and Government – as it was described in the background above. Following a Deal - the timing as well as the size of the interest rate cuts were out of the ordinary monthly decisions and were the results of political-economy reasons, rather than just of "pure" economic conditions. The variable dum95 is for May 1995 Deal, which lead to 1 percent point (pp) interest rate cut against widening the exchange-rate to 7% on either side of the mid-point rate; dum97 is for June 1997 Deal, in which the interest rates were cut by 1.2 pp against widening the exchange rate band by another 14% (i.e., band of 28%); dum98 is for August 1998 Deal, which lead to 1.5 pp interest rate cut against lowering the slope of the lower band to 2% from 4%.\(^{39}\) Dum01 is

\(^{39}\) For more details on these points see Nagar (2002).
for the Deal of December 2001, in which the interest rate was cut by 2 pp against a promise of Prime-Minister Sharon to reduce the budget deficit.

Another question is how long such a Deal affected later the CB interest rate decision. The second dummies' group is for a possible delays of interest rate response following Deal. As an example let us take the Deal of Jun 1997 where the interest rate was cut by 1.2 pp: It is unreasonable to assume that the monetary decision in the same month - (or one month later - in July) was free and not affected by this interest rate cut. This is because such immediate increase after sharp interest rate cut could undermine the Governor credibility and independency. The dummies Dum(j), j={0,1,2,3,4,} control for the delay between the current month of the deal to four month after. All the dummy variables – for Deals and for delays - were applied both on the constant and the slopes of the regressions.

One more important point about the monthly monetary meetings and decisions will be described later with the estimations.

5.3 Estimation Method

The estimation was made using OLS, under Newey-West procedure and with autocorrelation of order 2 (i.e. AR(1)and AR(2)) to overcome the serial correlation and possible heteroskedasticity. I used the familiar General to Specific Approach – GTSA: starting with unrestricted regression using all the variables and their cross products; and gradually deleting the insignificant variables. The variable's names and their legends appear in Appendix 3.

6. Estimation Results – the base case

We estimate two equations: (6.8)' and (7.7).

6.1 The Estimated Results of Equation (6.8)'

After adjusting equation (6.8)' to monthly data series terms, it turns to be:

\[(6.8)'' \quad \pi_m = c_0 - k \pi_{{m+12}}^{*} - \frac{a}{\theta} \frac{\sigma^{2}_{\pi_m}}{2} + \frac{\theta - 1}{\theta} E_{m+12} \pi_m + \varepsilon, \]

where \(m\) denotes month \(m\) in annual terms. This equation can be interpreted as each month the policy success is judged in terms of inflation deviations. Note that the dependent variable in deviation terms is \(\pi_m = \pi_m^* - \pi^*_{{m+12}}\). For the base case, this is the gap between
the actual annual inflation level as measured in month \( m \) for the last 12 months, \( \pi_m^l \), and the official target level 12 month ago, \( \pi_{m-12}^* \), assuming that this was the target level that the policy faced in \( m-12 \) for month \( m \). (Another case \( \pi_m = \pi_m^l - \pi_m^* \) will also be analyzed.) The expected inflation term in \( (6.8)'' \) is treated in the same way, i.e. \( E_{-12} \pi_m - \pi_{-12}^* \). The inflation variance was calculated for an annually (12 months) moving inflation rates.

6.1.1 The Results

Table 2 shows the results of the estimated equation \((6.8)'\). The coefficients of the first two parameters are both negative, indicating that the parameters of the hidden target \( k \) and the asymmetry \( a \) are both positive. These results are in line with the model and as well as, we expect, the case in Israel. However, they are significant for the whole period and for the first sub-period, while for the second sub-period the asymmetry parameter is insignificant. Consider the first sub-period, the results provide evidence for both IAP and a hidden target and consequently they are complementary rather than substitutional. The result of a hidden target through the first sub-period is consistent with Sussman (2007); however, he concluded it implicitly while here it is specified and explicit. (He also claimed that the hidden target level was zero – a claim which we will consider later.) Regarding the second sub-period, where only the hidden target is found to be significant, one may ask if these results are reasonable, taking into account that during most of this period the target was already in its long-run level of 1-3 percents. A possible answer – beside the fact that during 2000-2002 the target was still above the long-run level (table 1) - is that the policy hidden target was in the lower range of 1-3 percents, rather than the mid-range of 2 percents.

Regarding the third coefficient - the inflation expectation – it is between zero and one, as we would expect: recall that \( \theta = 1 + \frac{\alpha}{\lambda^2} > 1 \), and therefore \( 0 < \frac{\theta - 1}{\theta} \beta < 1 \). However, it should be noted that this result was achieved by using a moving average of 9-11 month lags; applying exactly 12 month lags of the inflation expectations variable as the theory above says (eq. \((6.8)'\)) produces a negative coefficient for this variable. Interestingly, by calibrating the conventional parameters of the NK model\(^{40}\) we get approximately \( \theta = 1.6 \) and therefore the inflation expectation coefficient is 0.4, very similar to the results of table 2.

\(^{40}\) See for example Gali (2008), Ch. 3 page 52, and appendix 4 for details.
Table 2: The Regression Results of Equation (6.8) – Total and two sub-periods

<table>
<thead>
<tr>
<th>Explanation</th>
<th>The Variable</th>
<th>Coefficient</th>
<th>Total</th>
<th>Sub-period 1</th>
<th>Sub-period 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>constant</td>
<td>c</td>
<td>0.019 (0.9)</td>
<td>0.085 (5.6)</td>
<td>-0.007 (-0.5)</td>
<td></td>
</tr>
<tr>
<td>Dum2</td>
<td></td>
<td>-0.0031 (1.8)</td>
<td>-0.0033 (-1.9)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dum4</td>
<td></td>
<td>-0.0077 (-3.8)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2001's Deal dummy</td>
<td>Dum01</td>
<td>0.017 (4.9)</td>
<td>0.022 (5.4)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>The inflation target</td>
<td>$\pi^*_t$</td>
<td>$-\frac{k}{\theta}$</td>
<td>-0.95 (-13.24)</td>
<td>-0.97 (-14.3)</td>
<td>-0.78 (-7.4)</td>
</tr>
<tr>
<td>The inflation variance</td>
<td>$\sigma^2_{\pi} / 2$</td>
<td>$-\frac{a}{\theta}$</td>
<td>-0.58 (-2.3)</td>
<td>-1.36 (-3.2)</td>
<td>-0.26 (-0.9)</td>
</tr>
<tr>
<td>The Expected inflation</td>
<td>$E_{-12} \pi_t$</td>
<td>$\frac{(\theta - 1) \beta}{\theta}$</td>
<td>0.35 (-3.2)</td>
<td>0.34 (2.1)</td>
<td>0.47 (2.8)</td>
</tr>
<tr>
<td>Exchange-rate change</td>
<td>Log(ex(-3)/ex(-12))</td>
<td>0.063 (2.7)</td>
<td>0.077 (1.9)</td>
<td>0.059 (2.2)</td>
<td></td>
</tr>
<tr>
<td>$R^2_{\text{adj}}$</td>
<td>0.95</td>
<td>0.96</td>
<td>0.95</td>
<td></td>
<td></td>
</tr>
<tr>
<td>DW</td>
<td>1.9</td>
<td>1.87</td>
<td>2.03</td>
<td></td>
<td></td>
</tr>
<tr>
<td>LM (p Val.)</td>
<td>0.34</td>
<td>0.64</td>
<td>0.62</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Observations</td>
<td>151</td>
<td>61</td>
<td>90</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1 The results applying Newey-West procedure. t values are in parenthesis.
3 average of 9-11 month lags.

Given these results and calibrating $\beta = 0.96$ we can derive the exact size of $k$ and $a$. Table 3 present the parameters and their sensitivity for various month's lags of the exchange rate (IS/SUS) changes. On average, the hidden target parameter is 0.6 for the whole and the sub_1 periods. This is to say that in comparison to the announced target of 8.6 percent points (pp) for the sub-period 1, the hidden target was 60% less – 3.4 pp. Regarding Sussman's (2007) claim of a zero hidden target for the first sub period, the results here are essentially different. Part B of table 3 and illustration 2 depict the contribution of the hidden target policy to the inflation reduction for the second case of exchange rate changes (i.e. log(ex(-3)/ex(-12))): -3.1, -5.4 and -1.1 pp for the whole, sub 1 and sub 2 periods.

---

41 This size is common in the literature, implying a riskless annual return of about 4%. See for example Gali (2008). However, the results are not sensitive to different beta sizes.
respectively. The asymmetry policy’s contribution is -0.5 and -1.2 PP for the whole and sub 1 periods, respectively. (The total deviation of the inflation from the target is shown in illustration 2 above the Bars.) Surprisingly, the exchange rate changes contribute only slightly, in comparison to the Prominent 8.5 PP. contribution of the constant term for the sub 1 period; the last may indicate the intensity of price stickiness through the disinflation process, and therefore may justify the hidden target policy. Illustration 2 depicts also the contribution of 2001’ Deal, and the residual of the regressions.

Table 3: the derived parameters of eq. (6.8)’ for various Ex. Rate lags changes

<table>
<thead>
<tr>
<th></th>
<th>Total period</th>
<th>Sub-period 1</th>
<th>Sub-period 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>A.</td>
<td>k</td>
<td>a</td>
<td>θ</td>
</tr>
<tr>
<td>1. Log(Ex(-2)/ex(-12))’</td>
<td>0.71</td>
<td>0.44</td>
<td>1.35</td>
</tr>
<tr>
<td>2. Log(Ex(-3)/ex(-12))’</td>
<td>0.62</td>
<td>0.37</td>
<td>1.57</td>
</tr>
<tr>
<td>3. Log(Ex(-4)/ex(-12))’</td>
<td>0.54</td>
<td>0.34</td>
<td>1.71</td>
</tr>
<tr>
<td>average</td>
<td>0.62</td>
<td>0.38</td>
<td>1.53</td>
</tr>
<tr>
<td>B.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Contribution to inflation deduction (for ex(-3)/ex(-12), percents points)</td>
<td>-3.1</td>
<td>-0.5</td>
<td>-5.4</td>
</tr>
<tr>
<td>Memo: actual (12_month) Inflation</td>
<td>4.3</td>
<td>8.3</td>
<td>1.5</td>
</tr>
<tr>
<td>Memo: inflation Target</td>
<td>5.1</td>
<td>8.6</td>
<td>2.6</td>
</tr>
</tbody>
</table>

Memo: (IS/$US) Exchange rates changes. The brackets denotes lags of month: for example, for Dec. 1999 it is Ex(Oct)/Ex(Dec. 1998). The parameter fi is derived by calibrating beta=0.96.

Illustration No. 2: The variables’ contribution to inflation
6.1.2 Robustness

1. It is important to note that the significance of the results were also robust to changes of the estimated periods: by shortening gradually to 1996:12-2004:12 for the whole period, and by moving gradually to 1996:12-2002:12 for the sub 1 period. However, for the second sub period, shortening to 2001:06 onward it was found to be robust only with log (ex(0)/ex(-12)), instead of the three option in table 3 above.

2. Equation (6.8)' examines the success of the policy by measuring ex-post the actual annual inflation against the available target. The target specification of table 2 above is $\pi_{m-11}^*$, i.e. with 11 month lags. This specification implies that this was the target level that the policy faced 12 month ago to be valid 11 month later(one year ahead) – an assumption that is mainly relevant for the first sub period because the target one year ahead was generally unknown (see table 1). A different specification could be by applying $\pi_{m}^*$ instead, as if the policy knew any time the target 12 month ahead – an assumption which is relevant to the most of the second sub period. Table 4 apply the last specification and presents the derived $k$ and $a$ (as in table 3) from the regressions coefficients again when the exchange rate lags is specified as log(ex(-3)/ex(-12)). Compare to table 3 above, while the asymmetry parameter is similar, the parameter $k$ is essentially lowered from 0.63 above to 0.4; consequently, also its contribution diminished from -5.4 pp above to -3.2 pp here. Note that the average inflation target has also changed due to observations’ changes.

| Table 4: The hidden target and asymmetry parameters for different specification of Eq. (6.8)' |
|-----------------|-----------------|-----------------|-----------------|
|                | Total period    | Sub-period 1    | Sub-period 2    |
| parameters     | contributions   | Par. | Cont. | Par. | Cont. |
| $k$            | 0.47            | -2.2 | 0.4   | -3.2 | 0.27 | -0.65 |
| $a$            | 0.37            | -0.4 | 0.82  | -1.2 |      |       |
| $\theta$       | 1.58            | 1.68 | 1.85  |      |       |
| Memo: inflation Target | 4.6  | 7.9  | 2.4  |
| Observations   | 162             | 72   | 90   |
### 6.2 The Estimated Results of Equation (7.7)

The equation to be estimated is as noted in section 4.5 above:

\[
\ln(i_t^A - E_t \pi_{t+1}) = (1 - \rho)d_0 + (1 - \rho)a(\pi_t - E_t \pi_{t+1}) + (1 - \rho)ak(\pi_t^* - E_t \pi_{t+1}^*) + \rho(i_{t-1}^A - E_{t-1} \pi_{t+1}) + \varepsilon_t^A
\]

And after adjusting it to monthly terms we get:

\[
\ln(i_{m+1}^A - E_{m-1} \pi_{m+12}) = (1 - \rho)d_0 + (1 - \rho)a(\pi, / I_{m-1} - E_{m-1} \pi_{m+12}) + (1 - \rho)ak(\pi_{m+12}^* - E_m \pi_{m+13}^*) + \rho(i_{m-1}^A - E_{m-1} \pi_{m+12}) + \varepsilon_m^A
\]

As noted above, this equation is different than the conventional response function because it is defined in real terms (rather than nominal) in both sides of the equation, and because it is semi-logarithmic. The inflation target appears here in two forms: \(\pi_t^*\) is from now until 12 months ahead, i.e. \(\pi_{m+12}^*\) in annual terms; and \(E_t \pi_{t+1}^*\) is the next period of \(\pi_t^*\), i.e. \(E_m \pi_{m+13}^*\) which is interpreted as the annual expected inflation target for 13 month ahead onward from today. However, taking our assumption that the policy horizon is 12 month ahead (one year), it still raises the question of how to specify \(\pi_{m+12}^*\) due to the fact that for many years, especially through the 1990's, the official target was actually un-known (see table 1).

Note that (7.7) can be interpreted as forward looking. Therefore, the variables are conditional on the information set that is known at the decision time. The Israeli Central Bureau of Statistics (CBS) publishes the measured inflation at the 15th of each month, for the previous month, and the BoI’s interest rate setting meetings are held afterwards.\(^{42}\) This is to say that the available information when the decision takes place in month \(m\) is up to the previous month only. Consequently, for each decision time in month \(m\) the available information for the last annual inflation is \(\pi, / I_{m-1}\); and for the expected annual inflation it is \(E_{m-1} \pi_{m+12}\).

Regarding the interest rate, the CB’s decision for each month will hold from the next month \((m+1)\) onward. It follows that the actual interest rate in the time series for each month \(m\) is

\(^{42}\)To be more precise, partial information from the current month financial markets also flows; nevertheless, in general they don’t affect the decisions.
tied to the information that was available on a two-month's lags basis, i.e. \( m-2 \). Therefore, the time lag of the dependent variable is \( i_{m+1} - E_{m-1}\pi_{m+12} \). Regarding the inflation target, it is known at the time decision and hence there is no need for adaptation.

6.2.1 The Estimation Results

Table 5 below illustrates the main results regarding the parameters derived from the regression of equation (7.7)' – the parameters \( a, k \) and the interest rate smoothing parameter \( \rho \). These parameters are not provided straightforwardly from the regression, but necessitate some algebraic manipulation, because of two reasons: the first is that it is a semi logarithmic function and therefore it needs to be adapted. The second is that the asymmetry parameter is then derived from the regression coefficients. The regression and the calculations are shown in appendix 5. Note that the results of the response function (7.7) are in terms of real monetary expected interest rates, rather than nominal.

<table>
<thead>
<tr>
<th>Table 5: The Regression Results of Equation (7.7) – Total and sub-periods</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Specification A</strong></td>
</tr>
<tr>
<td><strong>1</strong></td>
</tr>
<tr>
<td><strong>Explanation</strong></td>
</tr>
<tr>
<td>Hidden target</td>
</tr>
<tr>
<td>asymmetry</td>
</tr>
<tr>
<td>Interest rate smoothing</td>
</tr>
<tr>
<td>Response coefficient</td>
</tr>
</tbody>
</table>

* For the second parameter (i.e. with the k) the specifications distinguish between the definitions of the target in the decision's month for \( m+12 \). Taking for example August 1997 while the target for 1997 was 8.5 pp and when the 4 pp target for 1998 was announced (see table 1): specification A is for the official target as it was for August 1997, i.e. 8.5 pp; while B is 4 pp already from August 1997- on.


The second term of eq. (7.7)' is the difference between: (1) the inflation target that the policy is facing in the decision month \( m \) for the month \( m+12 \), and (2) the inflation target for \( m+13 \) onward. While (2) is the same for both specifications A and B in table 5, the specifications distinguish (1): Taking the example of August 1997 where the target for 1997
was 8.5 pp and where the 4 pp target for 1998 was announced (see table 1), specification A
assumes that $\pi_{m+12}^* = 8.5$ while specification B assumes $\pi_{m+12}^* = 4$. These specifications are
similar to the specifications for equation (6.8)' in tables 3 and 4 above, respectively. However, recall that from January 2003 onward the long-run target of 1-3 range prevailed and therefore there is no difference between them. Consequently the lack of variance in the sub-period 2 (2000.1-2007.6) is the reason why the hidden target is insignificant. Hence it is not inclusive that there was no hidden target in sub-period 2.

The parameters have positive signs in line with preliminary expectations from the model and of Israeli experience – an impressive result by itself. Consider first specification A. The hidden target is found to be 0.5 compare to 0.62 for sub period 1. Probably surprising, these results are similar to the average results in table 3 above, 0.62 and 0.63, respectively. Regarding the asymmetry parameters, they are less similar to those of table 3 above: they found to be 0.57 for the total period (higher than 0.38 in table 3 above) and 0.43 for the sub-period 1 (lower than 0.85 in table 3 above), respectively. These results demonstrate again that the IAP and hidden target are mutually inclusive, i.e. complementary rather than substitutional. However, they may also have a substitutional property as sub-period 2 shows: the asymmetry parameter is found to be 0.49 while in table 3 above this parameter was insignificant and while $k$ was significant. The complementary property suggests that there were others reasons for asymmetric behavior beyond the reason of uncertainty, for example political instability during disinflation process which restrict the possibilities of reducing the official inflation target.

Regarding specification B, the main difference relates to the parameter $k$, which is found to be lower, 0.35 and 0.53 for the total and sub-period 1, respectively. The similar specification for equation (6.8)' in table 4 above shows a higher size for the total period (0.47) and a lower size for the sub-period 1.

Another important result is related to the interest rate smoothing parameter $\rho$. It is prominent that the smoothing parameter (of 0.55) in the second period was threefold in comparison to the first period (0.14). This is to say that in the first period the CB’s responses

---

43 The standard deviation of this variable is 2.98 and 2.45 for the whole period and sub period 1, in comparison to 0.37 in sub-period 2.

44 Recall that (7.7)' is formulated in real terms, and therefore $\rho$ cannot be compared to the smoothing parameter in nominal terms. Argov and Elkayam (2007) Estimated NK model for Israel for the period 1992q1-2005q4 and found the smoothing parameter in Israel to be 0.8 in nominal terms.
considered less its past responses and hence more aggressive in comparison to the second period. This foundation is consistent with the foundations of the existence of hidden target and asymmetry in the first sub-period, compare to hidden target or asymmetry in the second sub-period (tables 3 to 5 above). Other possible interpretations: From the financial stability perspective, the financial markets were much less liberalized compared to the second sub period, and hence the CB faced less constrains for such behavior. Another explanation is that the second sub period was characterized mostly with a declining interest rate path, suggesting that the higher smoothing coefficient is reflecting also a high degree of conservatism with respect to inflation.

Note also that the coefficient of the interest rate response to an increase in the inflation deviation from its expectations is 0.46 for the whole period, which means about 1.5 in nominal terms. This result (i.e. greater then 1) is necessary for determinacy in the case of New-Keynesian model under discretion (see Woodford 2003).

6.2.2. Robustness

Besides the two specifications in table 5 above which roughly exhibit similar results, others robustness tests were also applied. The results are generally robust inter alia to changes in the estimated periods. Note also that all the regressions were done under Newey-West procedure to overcome possible heteroskedasticity and serial correlation.

Table A-2 in appendix 5 presents the full regression results of table 5 above. It shows that dummy variables for Deals (i.e. dum95 and dum01) are added as constants, and dummy variables for CB’s response delay are added to the slope of the third variable (i.e. dum2. See also appendix 3). Regarding the dummy variables on the constant, their main contribution (when needed) was to overcome the LM probability test for serial correlation. The main regression parameters were robust also to changes in these dummies for the relevant periods, i.e. to dum97 or dum98 for the total and sup-period 1 and dum01 for the total period. Regarding the delay dummy variable, its main contribution was to the significance of the parameters. The main regression parameters found to be robust also to other delay dummies, i.e. dum, dum1, dum3 and dum4 where the serial correlation condition was not violated.
6.3 summing the equation's results

At this stage it is useful to sum the results of tables 3 to 5 above for the main parameters: the hidden target $k$ and the asymmetry parameter $a$. Table 6 bellow exhibit 2 specifications for each equation. Regarding the parameter $k$, the table shows that on average it is roughly the same for the total period and sub period 1, amounted to 0.5 and implying that the hidden target was 50% lower than the official target. The range of the results is also roughly the same: 0.35-0.62 for the whole period, in comparison to 0.4-0.62 for sub-period 1.

Considering the parameter $a$, it is lower and has less variation for the total period in comparison to sub-period 1. Regarding the second sub-period, there is not clear cut in favor of $k$ or $a$ at the first glance. However, recall that the specification of equation (7.7)' limits its ability to estimate the hidden target parameter for this period and therefore only $a$ is founded to be significant. This is in contrast to the specification of equation (6.8)" where both are capable, but $k$ was dominated. Therefore, it may be claimed that in the second sub period hidden target policy was adopted, rather than asymmetric. To conclude, in sub-period 2 they seems to have substitutional relations, i.e. different techniques for conducting IAP. The complementary relation in both other periods suggests that the hidden target was motivated by different reasons than IAP motives.

<table>
<thead>
<tr>
<th>Specifications</th>
<th>Total period</th>
<th>Sub-period 1</th>
<th>Sub-period 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>equation tables</td>
<td>$k$</td>
<td>$a$</td>
<td>$k$</td>
</tr>
<tr>
<td>(6.8)&quot; 3</td>
<td>0.62</td>
<td>0.38</td>
<td>0.59</td>
</tr>
<tr>
<td>(6.8)&quot; 4</td>
<td>0.47</td>
<td>0.37</td>
<td>0.4</td>
</tr>
<tr>
<td>(7.7)' 5-A</td>
<td>0.5</td>
<td>0.57</td>
<td>0.62</td>
</tr>
<tr>
<td>(7.7)' 5-B</td>
<td>0.35</td>
<td>0.54</td>
<td>0.53</td>
</tr>
<tr>
<td>average</td>
<td>0.49</td>
<td>0.47</td>
<td>0.54</td>
</tr>
</tbody>
</table>

Given these results, a question that emerged through this work is how hidden target policy, especially where it is lasting for few years, is settled with the rational expectation theory. Obviously, they cannot contradict each other, suggesting that the hidden target policy was motivated by the desire to reduce the inflation via the inflation expectations mechanism. Hence, it is "hidden" in the sense that it was different than the announced target, but the
public perceived it through CB’s reactions and behavior. Interestingly, going back to the stylized fact of illustration 1, it shows that through the first sub period the inflation expectations line (red) is co-moving with the actual inflation and always lower than it. This is far to be the same through the second sub-period. These evidences are consistent with complementary relations between hidden target and asymmetry – where the CB desires to affect the inflation expectations through a different lower target - and substitutional relations where the CB policy behavior is characterized by IAP behavior. Theoretically, it is true that as the NK model is applied here, the CB cannot manipulate expectation under discretion (but under commitment only). However, since policy under commitment is not practical, a lower target under discretion, although hidden one, may be a practical way to manipulate expectation.

7. Summary and Conclusions

This paper investigates the question whether the persistent undershooting of the inflation target during the disinflation process in Israel was due to Inflation Avoidance Preferences-IAP or due to a hidden target inflation rates. To the best of my knowledge, this distinction has not yet had any consideration in the literature. The Central Bank (CB)’s behavior under IAP is motivated by the uncertainty regarding the inflation and the economic conditions, combined with aversion with respect to inflation, but aiming to achieve the target - hence reflected by asymmetric policy with respect to inflation (Cukierman 2000, 2002, and Cukierman and Muscatelli, 2008). A Hidden target behavior, on the other hand, may be aiming to undershoot the target, and can be motivated by other reasons, like credibility and political-economy kind of decisions. The paper is also motivated by the claim of Sussman (2007) that during the disinflation process in Israel in the 90’s the CB had a zero hidden inflation target. Another question is whether both – asymmetry and hidden target - are substitutional or complementary behavior.

The model developed here is based on New-Keynesian feature economy, under the case of discretion and with non-linear Linex CB objective function. After developing the model it was empirically tested for Israel, 1994-2007, using monthly data. This era was also divided into two sub-periods. The first was for the 90’s - under the tenure of Governor Jacob Frankel - an era that was characterized by the disinflation process. The second was from 2000, mainly under the tenure of Governor David Klein (until 2004).
The main results here are: (a). During the 90's (the first sub-period) the CB had hidden target which roughly was 4 percents points (pp), on average, in comparison to average announced target of 8 pp. (b). During the whole period and the first sub period the CB had together hidden target and asymmetry policy, suggesting complementary relations between hidden target and IAP. (c) On the second sub period, on the other hand, it is not conclusive whether hidden target or IAP policy was conducted, suggesting substitutional techniques for implementing IAP. (d) The foundation of complementary relations suggests that the hidden target was motivated by different reasons than IAP motives. Obviously, the hidden target foundation cannot contradict the rational expectation, i.e. it cannot last to be hidden from the public notion even it is not announce. Consequently, the public perceived the lower hidden target than the announced target via the CB's reactions and behavior, suggesting a motive to manipulate expectations even under discretion.

References


Cukierman, Alex (2000b). "The Inflation bias result revisited". Mimeo, Tel Aviv University.


Nagar, Weitzman (2002)."Monetary Policy in Israel in an Era of Inflation Targets – Principles and Implementation", Bank of Israel, Monetary Department (April).


Appendices

Appendix 1: illustration of the Linex function
Illustration A-1 describes aversion to inflation (for \(a=1\)): the loss (the \(y\) axis) from a deviation above the target (the \(x\) axis) is considerably higher than to below the target of the same magnitude. For comparison it shows also the quadratic case (when \(a\) approach zero) which has symmetric losses for positive or negative deviations. Illustration A-1a show the mirror picture of aversion to negative (for \(a=-1\)) compare to positive deviation. Illustration A-2 shows the impact of uncertainty: Even when the expected deviation is zero, there is positive loss.

Appendix 2: proof of the approximation: (7.2) \(e^{z_1} - e^{z_2} + 1 \equiv e^{z_1 - z_2}\) where the \(Z's\) are small enough.

1. Expanding the Taylor series of the function \(e^{z_1}\) of order 3 around \(z_2\) yields the following expression:

\[
(A2) \quad e^{z_1} = e^{z_2} + e^{z_2}(z_1 - z_2) + \frac{1}{2!}e^{z_2}(z_1 - z_2)^2 + \frac{1}{3!}e^{z_2}(z_1 - z_2)^3.
\]

The reason for order 3 is that higher orders add only negligible values that can be ignored. Rearranging (A2) yields the following expression:

\[
(A2.1) \quad e^{z_1} - e^{z_2} = e^{z_2}(z_1 - z_2) + \frac{1}{2!}e^{z_2}(z_1 - z_2)^2 + \frac{1}{3!}e^{z_2}(z_1 - z_2)^3.
\]
2. We turn now to develop the Taylor expansion series of the function \( e^{z_1 - z_2} \) around zero (i.e. \( (z_1 - z_2)(0) = 0 \)).

\[
(A2.2) \quad e^{z_1 - z_2} = 1 + (z_1 - z_2) + \frac{1}{2!}(z_1 - z_2)^2 + \frac{1}{3!}(z_1 - z_2)^3, \text{ or}
\]

\[
(A2.3) \quad e^{z_1 - z_2} - 1 = (z_1 - z_2) + \frac{1}{2!}(z_1 - z_2)^2 + \frac{1}{3!}(z_1 - z_2)^3.
\]

3. Subtracting (A2.3) from (A2.1) yields the following expression:

\[
(A2.4) \quad e^{z_1} - e^{z_2} = (e^{z_1} - 1)(z_1 - z_2) + \frac{1}{2!}(e^{z_1} - 1)(z_1 - z_2)^2 + \frac{1}{3!}(e^{z_1} - 1)(z_1 - z_2)^3.
\]

The terms on the right hand side of equation (A2.4) are very small. Note also that higher order is smaller than its predecessor and therefore this series converges to zero. By assuming that \( z_1 \geq z_2 \) and taking \( \lim_{z_1 \to 0} \) we get that both sides of the equation are zero. Hence, \( e^{z_1} - e^{z_2} + 1 \approx e^{z_1 - z_2} \).

Q.E.D.

Table A-1 gives some examples for the right hand side of equation (A2.4). Taking the first example where \( z_1 \) and \( z_2 \) are 20% and 1%, respectively, the error is only 0.2%.

| Table A-2: An example for (A2.4) for Z’s small enough |
|-----------|-----------|-----------|-----------|-----------|
| (1) | (2) | (3) | (4) | (5)=(3)-(4) |
| \( z_1 \) | \( z_2 \) | \( e^{z_1} - e^{z_2} + 1 \) | \( e^{z_1 - z_2} \) | The difference |
| 0.2 | 0.01 | 1.211352591 | 1.20925 | 0.002102993 |
| 0.2 | 0.05 | 1.170131662 | 1.161834 | 0.008297419 |
| 0.2 | 0.1 | 1.11623184 | 1.105171 | 0.011060922 |
| 0.1 | 0.01 | 1.095120751 | 1.094174 | 0.000946467 |
| 0.1 | 0.05 | 1.053899822 | 1.051271 | 0.002628725 |
Appendix 3: Table A-2: The variables

<table>
<thead>
<tr>
<th>The variable</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>agach8</td>
<td>The real yield of inflation indexed Treasury bonds, 8 years to maturity (proxy for real potential interest rate).</td>
</tr>
<tr>
<td>dexx</td>
<td>NIS/$US exchange rate annual change.</td>
</tr>
<tr>
<td>dexxm</td>
<td>NIS/$US exchange rate monthly change.</td>
</tr>
<tr>
<td>dexr</td>
<td>Real effective exchange rate change. [CHECK]</td>
</tr>
<tr>
<td>dum95</td>
<td>Dummy for the Deal (an agreement between the CB and the Government) of May 1995.</td>
</tr>
<tr>
<td>dum97</td>
<td>Dummy for the Deal of July 1997.</td>
</tr>
<tr>
<td>dum0</td>
<td>Dummy for current month CB’s response delay (as result of a Deal).</td>
</tr>
<tr>
<td>dum1</td>
<td>Dummy for 2 months CB’s response delay after a Deal</td>
</tr>
<tr>
<td>dum2</td>
<td>Dummy for 3 months CB’s response delay after a Deal.</td>
</tr>
<tr>
<td>dum3</td>
<td>Dummy for 4 months CB’s response delay after a Deal.</td>
</tr>
<tr>
<td>dum4</td>
<td>Dummy for 5 months CB’s response delay after a Deal.</td>
</tr>
<tr>
<td>Infx</td>
<td>The annual inflation (last 12 months).</td>
</tr>
<tr>
<td>Sinfexpx</td>
<td>The deviations of the expected inflation from the target.</td>
</tr>
<tr>
<td>Sinf</td>
<td>The deviation of the annual inflation (last 12 months) from the target which was known 12 month ago.</td>
</tr>
<tr>
<td>Sribx</td>
<td>The gap between the CB interest rate and the inflation target.</td>
</tr>
<tr>
<td>Targmx</td>
<td>The Inflation target which is known for the current year.</td>
</tr>
<tr>
<td>Targm13x</td>
<td>The Inflation target for the next calendar year (13 months onward).</td>
</tr>
<tr>
<td>Varinfav</td>
<td>The inflation variance – measured annually for the last 12 months.</td>
</tr>
</tbody>
</table>

¹All the variables are in decimal terms, i.e. 2% is 0.02.
Appendix 4: calibrating the third coefficient of Eq. (6.8)' by the conventional NK parameters

The third coefficient of equation (6.8)' is \( \frac{(\theta - 1)\beta}{\theta} \), where \( \theta = 1 + \frac{\alpha}{\lambda^2} > 1 \). We are interested to find the size of this coefficient - or in other words, to find \( \frac{\alpha}{\lambda^2} \) - by using the conventional parameters for NK model, for example as in Gali (2008), Ch. 3 pp 52.

\( \frac{\alpha}{\lambda^2} \) is the weight that the CB gives to output-gap in his behavior, were \( \lambda = 6 \) denotes the elasticity of substitution between goods (a-la Dixit-Stieglitz utility function)

\[ \lambda = \lambda_G \left( \sigma + \frac{\varphi + \alpha_G}{1 - \alpha} \right), \quad \lambda_G = \frac{(1 - \theta_G)(1 - \beta \theta_G)}{\theta_G} \quad \text{and} \quad \Theta = \frac{1 - \alpha_G}{1 - \alpha_G - \alpha_G \varepsilon} \] where the symbol G denotes the parameters as in Gali (2008) which are different then the parameters here. I use here \( \beta = 0.96 \) while the other following parameters are as in Gali (2008): \( \sigma = 1 \) is the substitution coefficient of the consumer utility function, \( \alpha_G = 0.33 \) is the production function parameter, \( \theta_G = 0.667 \) is the Calvo parameter and \( \varphi_G = 3 \) is the labor substitution parameter in the consumer utility function. Computing these parameters gives \( \Theta = 0.25 \), \( \lambda_G = 0.045 \), \( \lambda = 0.27 \), \( \frac{\alpha}{\lambda^2} = 0.617 \) and the coefficient \( \frac{(\theta - 1)\beta}{\theta} = 0.366 \). Probably surprisingly, we got similar size in our estimation as table 2 shows.
### Appendix 5: the econometric results of equation (7.7)

Table A-3: The Regression Results of Equation (7.7) – Total and sub-periods (Dependent variable: \( \ln(i_t^A - E_t\pi_{t+1}) \))

<table>
<thead>
<tr>
<th>The variable</th>
<th>parameters</th>
<th>Total period</th>
<th>Sub-period 1</th>
<th>Sub-period 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>original</td>
<td>Translated</td>
<td>original</td>
<td>Translated</td>
</tr>
<tr>
<td>constant</td>
<td>((1 - \rho)d_0)</td>
<td>-3.5</td>
<td>-3.66</td>
<td></td>
</tr>
<tr>
<td>Dum95</td>
<td>0.31*</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dum01</td>
<td></td>
<td></td>
<td>-0.29</td>
<td></td>
</tr>
<tr>
<td>((\sin(-1) - \sin\exp(-1)))</td>
<td>((1 - \rho)a)</td>
<td>9.68</td>
<td>0.46</td>
<td>7.88*</td>
</tr>
<tr>
<td>Targmx(-1)-targm13x(-1)</td>
<td>((1 - \rho)ak)</td>
<td>4.83</td>
<td>0.23</td>
<td>4.87</td>
</tr>
<tr>
<td>(((\sin\exp(-1))-1)%(1+\text{dum2}))</td>
<td>(\rho)</td>
<td>4.16</td>
<td>0.2</td>
<td>3.05*</td>
</tr>
<tr>
<td>AR[1]</td>
<td>0.9</td>
<td>0.55</td>
<td>0.83</td>
<td></td>
</tr>
<tr>
<td>adjR²</td>
<td>0.88</td>
<td>0.66</td>
<td>0.94</td>
<td></td>
</tr>
<tr>
<td>DW</td>
<td>2.01</td>
<td>2.07</td>
<td>1.81</td>
<td></td>
</tr>
<tr>
<td>LMprob. test for serial correlation.</td>
<td></td>
<td>0.85</td>
<td>0.37</td>
<td>0.56</td>
</tr>
<tr>
<td>Observations after adjustment</td>
<td>150</td>
<td>72</td>
<td>78</td>
<td></td>
</tr>
</tbody>
</table>

#### 2. The extracted parameters

| asymmetry   | \(a\) | 0.59 | 0.43 | 0.49 |
| Hidden target | \(k\) | 0.57 | 0.62 | insignificant |
| Average Dependent variable | | 0.047 | 0.0561 | 0.042 |

* The significance level is 1% unless denoted * for 5%. Translated from the logarithmic function; the extracted parameters in pert 2 of the table above are as follows. As a reminder, equation (7.7) is:

\[
\begin{align*}
\ln(i_t^A - E_t\pi_{t+1}) &= (1 - \rho)d_0 + (1 - \rho)a(\pi_t - E_t\pi_{t+1}) + \\
& (1 - \rho)ak(\pi_t^* - E_t\pi_{t+1}^*) + \rho(i_{t-1}^A - E_{t-1}\pi_{t+1}) + \gamma_t + \varepsilon^A_t 
\end{align*}
\]

1. Extracting the hidden target parameter for the first sub-period, 1994.1-1999.12 is as follows: 

\[
k = \frac{(1 - \rho)ak}{(1 - \rho)a} = \frac{4.87}{7.88} = 0.62.
\]
2. For the semi logarithmic equation like (7.7), the marginal effect \( \frac{d(y)}{d(x)} \) is:

\[
\beta = \frac{d(\ln y)}{d(x)} = \frac{1}{y} \cdot \frac{d(y)}{d(x)} \Rightarrow \beta y = \frac{d(y)}{d(x)}.
\]

Therefore each "original" regression's coefficients \( \beta \) have to be multiplied by the average value of the dependent variable (which appears also in the table A-3). The outcome of this calculation appears under the column "translated" in table A-3 above.

The regression coefficients appear under the column "original" in the table. Since the equation is semi logarithmic, the results are translated.

3. Regarding the response coefficient \((1 - \rho)a\) with respect to inflation, it should be emphasized that it is in real expected terms, rather than the conventional nominal terms. It means that in nominal terms it is greater than one – the necessary conditions for determinacy in NK model (see Woodford 2003).