Bankruptcy Risk and the Great Recession*

Oren Levintal
Bar-Ilan University
oren.levintal@biu.ac.il
March 2013

Abstract

The steep rise in CDS premiums of financial institutions since 2007 suggests that bankruptcy risk played an active role in the Great Recession. Nevertheless, macroeconomic models with financial frictions usually abstract from bankruptcy risks. This paper studies a new balance-sheet channel that operates through the impact of bankruptcy risk on precautionary savings. Negative shocks to banks’ balance sheets raise the risk of bank default, thereby increasing the likelihood of financial disasters. Consequently, demand for safe assets rises, credit spreads widen and the interest rate falls. A flight-to-quality effect reduces the supply of funds to the productive sector, exacerbating the recession.

Keywords1: Great recession, financial crisis, rare disasters, financial disasters, equity capital, leverage, bankruptcy risk.

JEL classification: E32, E43, E44, E52, G12, G32.

*I would like to thank Joseph Zeira for his invaluable advice throughout this project. I benefited from discussions with Ramon Marimon, Franklin Allen, Elena Carletti, Piero Gottardi, Russell Cooper, David Levine, Árpád Ábrahám, Saverio Simonelli and Raphael Franck. Finally, I thank the European University Institute for the Jean Monnet fellowship and the Hebrew University for the Blazuska fellowship and the Stoessel fellowship. All errors are solely mine.
1 Introduction

In recent years, the global economy has witnessed a steep rise in bankruptcy risks, particularly within the financial sector. The CDS premiums of the largest financial institutions increased from 10-20 bp before 2007 to a range of 100-300 bp in the following years (Figure 1). The jump in bankruptcy risks reflected severe deterioration in the stability of the financial sector, as evident by the sharp fall in banks’ equity-asset ratios (Figure 1). At the same time, interest rates in all major economies have fallen to zero and the global economy entered the "Great Recession".

The rise in banks’ leverage and bankruptcy risk was a prominent feature also in previous recessions at the magnitude of the Great Recession. For instance, the US Great Depression has seen massive failures of banks and commercial businesses (Bernanke 1983). The market equity-asset ratios of New York banks declined between 1929 and 1933 from 27% to 12% and the probability of default increased (Calomiris and Wilson 2004). The Japanese recession in the 1990s was another example of a significant contraction in equity capital, both in the financial sector (Hoshi and Kashyap 2010) and the nonfinancial sector, as implied by the surge in bad loans and write-offs (Hoshi 2001, Fukao 2003). Eventually, the chronic shortage of capital has led to a wave of bank failures (Imai 2009, Hoshi and Ito 2004).

The role of the financial sector in generating and amplifying business fluctuations has been studied in the literature on the balance sheet channel (see below). Surprisingly, most of this literature abstracts from bankruptcy risk, which seems to play an important role in the Great Recession. The present paper studies a new balance sheet channel that works through the impact of bankruptcy risks on precautionary
savings. The paper establishes a link between banks’ balance sheets, the probability of default and the real economy. Banks’ equity capital serves as a shock buffer ensuring that deposits could be paid in bad times. When a negative shock hits, equity capital falls, leverage rises and the risk of bank default increases. At the aggregate, the economy becomes more vulnerable to external shocks, which makes financial disasters more likely to happen. As a result, demand for safe assets increases, credit spreads rise and the risk-free interest rate declines. A flight-to-quality effect reduces the supply of funds to the productive sector, thereby exacerbating the recession.

The balance sheet channel proposed in this paper builds on the rare disaster literature developed by Rietz (1988) and Barro (2006). These studies show that low probabilities of economic disasters can have significant impact on asset prices. The same principle is applied here. A fall in banks’ equity capital raises the probability of a massive wave of bank defaults, which is defined as a financial disaster. Such a disastrous event is usually very rare, but when equity capital falls it becomes more probable. Note that the variable of interest is not the disaster itself, but rather the probability that it will occur. The novel feature of the model is that the disaster probability is endogenous and determined by banks’ equity-debt ratio. By contrast, in Rietz (1988) and Barro (2006) the disaster probability is exogenous and fixed. Later studies have explored the effect of time-varying disaster risk, but the disaster probability was still determined exogenously (e.g. see Gabaix 2008, Gourio 2012, Wachter 2012).

The paper is mostly related to the literature on the balance-sheet channel advanced by Bernanke and Gertler (1989), Kiyotaki and Moore (1997), Holmstrom
and Tirole (1997) and Bernanke, Gertler and Gilchrist (1999). Recent contributions to this literature include He and Krishnamurthy (2009), Gertler and Karadi (2011), Gertler and Kiyotaki (2010, 2012) and Jermann and Quadrini (2012). These studies incorporate asymmetric information to establish a role for corporate finance. They show that variations in equity capital (net worth) affect the cost of capital, thereby producing business fluctuations and asset price effects.

The present paper proposes a new version of the balance sheet channel, which has not been studied before. The channel works through the impact of banks’ balance sheets on aggregate bankruptcy risk and hence on the probability of financial disasters. Previous studies usually abstract from bankruptcy risk by constructing models where the equilibrium probability of default is zero, e.g. see Kiyotaki and Moore (1997), He and Krishnamurthy (2012), Gertler and Karadi (2011) and Jermann and Quadrini (2012). Hence, these models are not suitable for studying macroeconomic effects of aggregate bankruptcy risks. Gertler and Kiyotaki (2012) introduce bank runs in a macroeconomic model with a financial sector. They assume that bank runs are completely unanticipated, so the ex ante probability of a run is always zero. In Bernanke, Gertler and Gilchrist (1999) firms may default, but savers are able to eliminate this risk and earn a perfectly safe return on their savings. Thus, their model does not contain a precautionary saving effect, which is the core of the present paper.

The proposed balance sheet channel generates asset price effects that differ in their dynamics from previous models. In standard business cycle models asset prices are determined primarily by consumption dynamics. For instance, in these models
the interest rate tends to rise above its steady state level when the economy is recovering, *e.g.* see Gertler and Karadi (2011). By contrast, in the present model asset prices are affected by the precautionary saving motive. Consequently, the interest rate may stay at low levels even if the economy is growing, provided that leverage and bankruptcy risks are high. The model is able to produce long periods of high leverage, low interest rates and high credit spreads in response to an initial negative shock. This type of economic dynamics is consistent with the recent global recession, where the recovery path is a slow process coupled with low interest rates and high bankruptcy risks.

The paper proceeds as follows: Section 2 describes the model and section 3 derives the general equilibrium conditions. Section 4 elaborates on the choice of parameter values used to simulate the model. The simulation results are presented in Sections 5 and 6. Section 7 concludes.

2 The Model

The basic setup is an overlapping generation model with heterogeneous agents. Agents consume in the first period of life and leave a bequest to their offspring in the second period. Their utility function is defined over consumption and bequest. Acemoglu (2009) provides a detailed discussion on these models and their application in the literature. The main advantage of this setup is its tractability, as agents optimize over two periods only. This is particularly useful in models with heterogeneous agents and a corporate finance problem, *e.g.* Bernanke and Gertler (1989), Holmstrom and
Tirole (1997). The two-period setup simplifies the dynamic dimension of the model and allows to focus on the corporate finance issues, which provide the main insights of the model.

Subsection 2.1 presents the optimization problem of the agents, which is fairly standard. Subsection 2.2 introduces demand for liquidity as in Gertler and Kiyotaki (2012). The banking sector and the possibility of bank runs are modelled in subsections 2.3-2.6.

2.1 Agents

There are two types of agents, Bankers and Depositors, denoted $B$ and $D$, respectively. The quantity of each type is normalized to 1. Both types live for two periods, consume in the first period and bequeath their wealth to newly born agents of the same type in the second period. Agents differ in their risk preferences. Specifically, bankers are risk-neutral with respect to their future bequest and depositors are risk averse. Bankers save only in bank equity and depositors can save in deposits and in safe storage. These participation constraints will not bind in equilibrium. It is shown that in equilibrium bankers prefer to save in equity over deposits and depositors are indifferent between the two. This result stems from the heterogeneity in risk preferences which creates demand for risk sharing. Hence, in equilibrium more risk-averse agents tend to save in safer securities (Allen and Gale 1988). However, the participation constraint simplifies the analysis considerably, so it is used here for exposition clarity.

Young bankers ($B$) born in period $t$ maximize the following utility function:
\[ U_t^B = c_t^B + \beta E_t W_{t+1}^B, \quad 0 < \beta < 1 \quad (1) \]

subject to the budget constraints:

\[ c_t^B = W_t^B - e_{t+1}, \quad (2) \]
\[ W_{t+1}^B = e_{t+1} R_{t+1}^e. \quad (3) \]

Superscript \( B \) stands for bankers, \( c_t^B \) denotes consumption in period \( t \) of a single consumption good, \( W_t^B \) is wealth inherited from an old banker, and \( W_{t+1}^B \) denotes bequest in \( t + 1 \). The variable \( e_{t+1} \) denotes savings in bank equity in period \( t \) which pay the (state dependent) gross return \( R_{t+1}^e \) in \( t + 1 \). We assume that short sales are prohibited, so \( e_{t+1} \) must be non-negative.

The first order condition is:

\[ 1 + \phi = \beta E_t R_{t+1}^e \geq 0, \quad \text{if } e_{t+1} = 0, \quad (4) \]

where \( \phi \) denotes the Lagrange multiplier associated with the constraint \( c_t^B \geq 0 \).

The FOC implies that bankers do not consume if \( E_t R_{t+1}^e > \beta^{-1} \), and do not save if \( E_t R_{t+1}^e < \beta^{-1} \).

Turning now to depositors (\( D \) type), these agents maximize:
subject to:

\[ c_t^D = W_t^D - d_{t+1} - m_{t+1}, \]
\[ W_{t+1}^D = d_{t+1} R_{t+1}^d + m_{t+1}, \]

where \( d_{t+1} \) denotes deposits issued in period \( t \) that pay \( R_{t+1}^d \) in \( t + 1 \). The deposit return can be state dependent as banks may default on their debts. Saving in deposits must be non-negative.

Depositors can save also in storage, which is denoted \( m_{t+1} \). Storage is a perfectly safe asset with a gross return normalized to 1. The storage technology serves as an outside saving option. Normally, depositors will not save in storage due to its low return. However, when risk is too high depositors may prefer to shift their portfolio towards the safe outside option, creating a flight-to-quality effect as in Bernanke and Gertler (1989). The counterpart of storage in real life would be any investment opportunity that is perceived to be safer than bank deposits. For example, under certain circumstances foreign currency might be viewed as safer than domestic deposits, especially in small countries with fixed exchange rate regimes. Commodities such as gold may occasionally play the role of a ”safe haven”. Moreover, monetary assets such as money or short term government debt can also be perceived as default free, provided that inflation is under control. In this case, accumulating storage could
be interpreted as hoarding cash.

The first order conditions of the depositor’s problem are:

\[ 1 + \psi = \delta E_t \frac{R_{t+1}^d}{(W_{t+1}^D)^\theta} \quad \geq \quad \text{if } d_{t+1} = 0, \quad (8) \]
\[ 1 + \psi = \delta E_t \frac{1}{(W_{t+1}^D)^\theta} \quad \geq \quad \text{if } m_{t+1} = 0, \quad (9) \]

where \( \psi \) is the Lagrange multiplier of the constraint \( c_t^D \geq 0 \). These FOC yield the following saving rule for depositors:

\[ W_t^D - c_t^D = \min \left( \left\{ \delta E_t (\Pi_{t+1}^D)^{1-\theta} \right\}^{\frac{1}{\theta}}, \ W_t^D \right) . \quad (10) \]

\( \Pi_{t+1}^D \equiv \frac{W_{t+1}^D}{W_t^D - c_t^D} \) denotes the total return on depositors’ savings\(^1\). When the inherited wealth of depositors is less than \( \left\{ \delta E_t (\Pi_{t+1}^D)^{1-\theta} \right\}^{\frac{1}{\theta}} \), they save all their wealth. When their wealth exceeds that level, they consume the surplus.

### 2.2 Liquidity Risk

Following Gertler and Kiyotaki (2012), I introduce a liquidity shock à la Diamond and Dybvig (1983) to create demand for liquidity. Specifically, suppose that each depositor may have a need for emergency expenditure at the beginning of period

\(^1\)To get (10), multiply (8) and (9) by \( d_{t+1} \) and \( m_{t+1} \), respectively, and sum them together to have \( (W_t^D - c_t^D) (1 + \psi) = \delta E_t (W_{t+1}^D)^{1-\theta} \). When \( c_t^D \) is positive \( \psi = 0 \) so \( W_t^D - c_t^D = \left\{ \delta E_t (\Pi_{t+1}^D)^{1-\theta} \right\}^{\frac{1}{\theta}} \), which yields (10).
The probability of getting a liquidity shock is denoted $\pi$. Depositors that are hit by the liquidity shock derive utility only from the emergency expenditure. They consume all their wealth and leave no bequest. Other depositors that do not have an emergency need for liquidity bequeath their wealth at the end of period $t + 1$. The consumption and saving decisions of their offspring are also taken at the end of $t + 1$. Hence, the beginning and the end of period $t + 1$ should be viewed as distinct periods, corresponding to Diamond and Dybvig (1983) timing framework.

The utility function of depositors remains as in (5). The only difference is that a fraction $\pi$ of depositors do not leave a bequest, but consume all their future wealth as an emergency expenditure. For simplicity, assume that these depositors do not have offspring, and each of the other depositors have $1/(1 - \pi)$ descendants. Thus, the total size of the continuum of depositors is fixed at 1 and the aggregate bequest that each generation of depositors inherits from the previous generation is $(1 - \pi)W_t^D$, where $W_t^D$ is the aggregate wealth of depositors born in $t - 1$.

The demand for liquidity implies that bank deposits must be payable on demand. This opens up the possibility of a bank run, as explained by Gertler and Kiyotaki (2012). Bank runs occur when the liquidation value of the bank is lower than the liabilities to the depositors. This situation puts the bank in a fragile position. Namely, a bank that is fundamentally solvent may become insolvent if depositors run on the bank. The next section develops the optimization problem of the bank and studies the possibility of bank runs.
2.3 Banks, Firms and Bank Runs

Each bank is established by a young banker that contributes her own wealth to the bank by holding the bank equity. The bank borrows additional funds from depositors and lends its total capital to firms, which have access to investment technology. The firms live for two periods and start off the first period without capital. The lending problem follows Holmstrom and Tirole (1997). Namely, firms have shirking opportunities that require monitoring. This gives a role for banking, as banks are better monitors than depositors.

To model the moral hazard problem, suppose that investment of the firm in period $t$ yields in $t + 1$ a return $R_{t+1} > 1$ if the firm succeeds and zero otherwise. The probability of success depends on the management of the firm, which cannot be observed without monitoring. Bad management yields low success probability but generates private benefit to the firm. Good management yields higher success probability, denoted $\rho$, and no private benefit to the firm. Hence, to ensure that the firm chooses good management it must be monitored$^2$. Assume that banks are able to monitor at zero cost, whereas agents monitor at a unit cost $\kappa$.

Banks lend to many firms, monitor them at zero cost and earn the average return on loans, which is denoted $u_{t+1} \equiv \rho R_{t+1}$. In what follows, $u_{t+1}$ is assumed to be an aggregate shock identical across all banks. Thus, bank $i$ that gives $a_i$ loans in period $t$ will have an asset value of $a_i u_{t+1}$ in period $t + 1$. Throughout, variables that are controlled by the bank are denoted by subscript $i$, and aggregate variables are

$^2$Since the firm owns no capital initially, the moral hazard problem cannot be solved with a mixture of inside and outside capital.
denoted by a time subscript.

Since a fraction $\pi$ of depositors consume at the beginning of period $t + 1$, we assume that some of the bank loans mature at the beginning of period $t + 1$, while the other loans mature at the end of the period. We refer to the early maturing loans as "liquid assets" and the late maturing loans as "illiquid assets". Specifically, suppose that the liquid assets equal $\alpha a_i$ and the illiquid assets are $a_i (u_{t+1} - \alpha)$ \(^3\). Assume that the amount of liquid assets exceeds the demand for emergency expenditure ($\alpha > \pi x_i$). Hence, the bank can always supply the demand for emergency expenditures with its own liquid assets. However, if all depositors run on the bank at the beginning of $t + 1$, the bank will have to liquidate its illiquid assets by selling them in the market. The liquidation price will determine if the bank is solvent or not.

The important assumption in this model, which follows Gertler and Kiyotaki (2012), is that banks can monitor at zero costs, whereas agents monitor at positive costs\(^4\). This assumption generates fluctuations in the liquidation price of the bank assets. Specifically, when all banks fail, agents must monitor the bank assets at a unit cost $\kappa$. Thus, the liquidation price of the bank assets is $1 - \kappa$. By contrast, if only one bank fails and all the other banks are solvent, the assets of the failed bank can be sold to the other banks at face value without any loss. The liquidation price in this case is 1. Hence, the liquidation value of the bank assets depends on the state of the economy. It is low in bad times (when all banks fail) and high in good times\(^5\).

\(^3\)The lowest value of $u_{t+1}$ is assumed to be larger than $\alpha$.

\(^4\)We assume that monitoring always provides higher return on loans than not monitoring, even at the high monitoring costs. Hence, firms are always monitored, either by the banks or by the agents.

\(^5\)Gertler and Kiyotaki (2012) obtain a similar result by using a slightly different monitoring cost function.
(when all banks are solvent).

To study the possibility of bank runs, consider bank \( i \) that gives \( a_i \) loans in period \( t \). The bank finances the loans with its own equity capital \( e_i \) and by issuing deposits in period \( t \) and selling them to depositors. Let \( x_i \) denote the debt-asset ratio of the bank. Namely, the bank has to pay to its depositors \( a_i x_i \) in period \( t + 1 \), so \( a_i x_i \) is the bank’s debt. If the bank is not able to pay out its debt it is liquidated.

The liquidation of the bank depends on the realization of \( u_{t+1} \). The simple case is when \( u_{t+1} < x_i \). In this case the bank is insolvent and must be liquidated, no matter what the liquidation price is, because the total debt of the bank exceeds its total asset value even at the high liquidation price \( (a_i x_i > a_i u_{t+1}) \). This case is summarized by the following proposition:

**Proposition 1** When \( u_{t+1} < x_i \) the bank is liquidated.

The more interesting case is when \( u_{t+1} \geq x_i \). In principle, the bank is solvent and should not be liquidated. However, Gertler and Kiyotaki (2012) show that the bank will be forced to liquidate under certain circumstances. We distinguish between two types of states. The first type refers to states in which all other banks are solvent. These states will be called normal. The second type refers to states in which all other banks default. These states will be called financial disasters.

Starting with normal states, suppose that in \( t + 1 \) all other banks are solvent. In this case the liquidation price is 1, so the market value of the bank assets is \( a_i u_{t+1} \). Hence, if a bank run occurs, the bank can liquidate its assets at no cost and pay out all its debt (because \( u_{t+1} \geq x_i \)). Thus, there is no reason to run on the bank. Only
the fraction $\pi$ of depositors that need an emergency expenditure will withdraw their accounts at the beginning of period $t + 1$.

The result is different in a financial disaster, where all the other banks fail. The liquidation price of the bank’s assets falls to $1 - \kappa$ because the assets can be sold only to non-banks. The depositors of bank $i$ have to decide if they run on the bank. As in Diamond and Dybvig (1983), when depositors run they form a line and are served sequentially. Thus, the first comers are paid out fully and the last ones in the line lose all their money. The sequential service constraint implies that running on the bank is a weakly dominant strategy when the liquidation value of the bank assets does not cover the debt. Formally, a run occurs when:

$$\alpha a_i + a_i (u_{t+1} - \alpha) (1 - \kappa) < a_i x_i.$$ 

The left hand side is the liquidation value of the bank assets. The liquid portion of the bank assets is priced at face value, and the illiquid portion is priced at the liquidation price $1 - \kappa$. The right hand side is total debt to depositors. A depositor who do not run will lose all her money if other depositors run at the beginning of the period, because the bank cannot fully pay out all depositors. By contrast, if the depositor runs she might be paid with some probability (depending on her place in the line). If the other depositors do not run, an individual depositor is indifferent between running and not running. Hence, a strategy of not running is weakly dominated by a running strategy. Therefore, all depositors will run on the bank. The first depositors in the line will be fully paid and the last ones will lose all
their money. The probability of being paid in a run is \( \frac{u_{t+1} (1 - \kappa) + \alpha \kappa}{x_i} \).

To save notation, we will henceforth assume that the fraction of emergency expenditure \( \pi \) and the fraction of liquid assets \( \alpha \) are infinitesimally small so they can be ignored. None of the results depend on this assumption. The following proposition summarizes the bank run conditions when \( u_{t+1} \geq x_i \):

**Proposition 2** When \( u_{t+1} \geq x_i \) and the liquidation price is \( 1 - \kappa \), depositors run on the bank at the beginning of \( t + 1 \) if \( u_{t+1} (1 - \kappa) < x_i \). Each depositor is paid with probability \( \frac{u_{t+1} (1 - \kappa)}{x_i} \).

Finally, note that the sequential service constraint generates ex post heterogeneity among depositors, because some depositors are paid in a run and others are not. To fix this technical issue, assume that a system of lump sum taxes and transfers redistributes wealth among depositors so that all depositors end up with the same amount of wealth. This assumption is not necessary, but it simplifies the analysis as it enables to work with a representative depositor.

The next section studies the capital structure decision of the bank. This part goes beyond Gertler and Kiyotaki (2012) by studying how the probability of a bank run affects the bank’s capital structure. By contrast, Gertler and Kiyotaki (2012) assume that bank runs are completely unanticipated. Namely, in their analysis the ex ante probability of a bank run is assumed to be zero.

### 2.4 The Capital Structure of the Bank

The goal of the bank is to maximize the value of its equity shares. Since the shares are held by bankers, their value is defined by:
\[ V^e(a_i, x_i) = \frac{\beta}{1 + \phi} \int_{\bar{u}_i}^{\infty} a_i(u_{t+1} - x_i) \, dF(u_{t+1}), \quad (11) \]

where \( F(\cdot) \) denotes the CDF of \( u_{t+1} \), which is the only aggregate shock in the model. This expression follows directly from the first order condition of bankers given by (4). It integrates the present value of the shareholders’ payout across all states in which the bank is solvent (no bank run equilibria). The solvency threshold is denoted \( \bar{u}_i \) and is derived explicitly below. When the bank is solvent, depositors are paid \( a_i x_i \) and shareholders receive \( a_i (u_i - x_i) \).

The bank takes its equity capital \( e_i \) as given and borrows additional funds from depositors. The total assets of the bank \( (a_i) \) are determined by the balance sheet constraint:

\[ a_i = e_i + V^d(a_i, x_i). \quad (12) \]

The function \( V^d(a_i, x_i) \) provides the value of the issued deposits, given \( a_i \) and \( x_i \). This is the amount of funds that the bank borrows by issuing deposits and selling them to depositors.

The bank maximizes its stock market value (11) with respect to \( a_i \) and \( x_i \), subject to the balance sheet constraint (12), taking its equity capital \( e_i \) as given. To see how (12) works, suppose that the bank increases its debt-asset ratio \( x_i \) by selling more deposits. The amount of funds that the bank can borrow depends on the value of the
bank deposits \( V^d \). On the one hand, selling more deposits enables to borrow more funds. On the other hand, the rise in the debt-asset ratio increases the probability that the bank will default. Therefore, depositors will charge a higher interest rate on the bank deposits. This second effect will tend to reduce the value at which the deposits are sold to depositors, and hence the funds borrowed by the bank.

### 2.5 The First Order Condition of the Bank

To derive the first order condition of the bank, I focus on a symmetric equilibrium in which all banks pick the same debt-asset ratio \( x_{t+1} \). Hence, when \( u_{t+1} \geq x_{t+1} \) all banks are solvent. These states are defined as normal states. Conversely, when \( u_{t+1} < x_{t+1} \) all banks default. I call these states financial disasters.

The first order condition is:

\[
E_t \left( \frac{R^e_{t+1}}{(W^D_{t+1})^\theta} \right) = E_t \left( \frac{R^d_{t+1}}{(W^D_{t+1})^\theta} \right).
\]  

(13)

When this condition holds, an individual bank cannot raise its equity share value by choosing \( x_i \neq x_{t+1} \), given that all other banks choose \( x_{t+1} \). The proof is provided in the rest of this section. As before, variables that are controlled by the bank are denoted by subscript \( i \), and aggregate variables are denoted by a time subscript \( t \) or \( t + 1 \).

For exposition clarity, I introduce the following notation for depositors’ future wealth. In normal states \( (u_{t+1} \geq x_{t+1}) \) all banks are solvent and depositors are fully
paid out. Hence, depositors’ wealth is fixed and denoted $\bar{W}_{t+1}^D$. Note that $\bar{W}_{t+1}^D$ is the highest value of depositors’ wealth across all realizations of $u_{t+1}$:

$$\bar{W}_{t+1}^D \geq W_{t+1}^D. \quad (14)$$

In disaster states $u_{t+1} < x_{t+1}$ so all banks default. Depositors receive a lower pay out, which depends on the realization of $u_{t+1}$. In these states their wealth is denoted by $\bar{W}_{t+1}^D$, which is a stochastic variable.

The discontinuity between normal and disaster states requires to solve the optimization problem of the bank separately for two regions, $x_i \geq x_{t+1}$ and $x_i \leq x_{t+1}$. Consider first the region $x_i \geq x_{t+1}$, in which bank $i$ picks a higher debt-asset ratio than the other banks. Using (11), the equity share value of the bank is:

$$V^e(a_i, x_i) \mid_{x_i \geq x_{t+1}} = \frac{\beta}{1 + \phi} \int_{x_i}^{\infty} a_i (u_{t+1} - x_i) dF(u_{t+1}). \quad (15)$$

The solvency threshold is $x_i$. When $u_{t+1} \geq x_i$, all the other banks are solvent because $u_{t+1} \geq x_i \geq x_{t+1}$. The liquidation price is 1 since the bank assets can be sold to other banks. Thus, the liquidation value of the bank exceeds its debt, so depositors do not run on the bank. Conversely, when $u_{t+1} < x_i$ the bank defaults due to Proposition 1. Hence, $x_i$ is the solvency threshold of the bank.

The deposit value of the bank is:
\[ V^d (a_i, x_i) |_{x_i \geq x_{t+1}} = \frac{\delta a_i}{1 + \psi} \left\{ \int_0^{x_{t+1}} \frac{u_{t+1} (1 - \kappa)}{\bar{W}_{t+1}^D} dF (u_{t+1}) + \int_{x_{t+1}}^{\infty} \frac{x_i}{\bar{W}_{t+1}^D} dF (u_{t+1}) \right\} . \] (16)

This expression is derived from the first order condition (8). It provides the value at which depositors are indifferent between buying and not buying zero deposits of bank \( i \). At this value, the return on deposits of bank \( i \) satisfies the first order condition (8).

The RHS of (16) integrates the present value of the payoffs to bank depositors across all the realizations of \( u_{t+1} \). The first integral runs over disaster states (\( u_{t+1} < x_{t+1} \)). In these states all banks default so depositors’ wealth is \( \bar{W}_{t+1}^D \). Bank \( i \) also defaults because \( u_{t+1} < x_{t+1} \leq x_i \). Depositors of bank \( i \) run on the bank and the bank liquidates its assets. A share \( u_{t+1} (1 - \kappa) / x_i \) of depositors are fully paid and the rest are not paid at all. The payment to each depositor depends on her place in the line which is purely random. The probability of being paid times the amount paid is \( a_i u_{t+1} (1 - \kappa) \), which is the expected depositors’ payoff in the run. The second integral runs over normal states in which \( x_{t+1} \leq u_{t+1} < x_i \). In these states all banks are solvent and the wealth of depositors is \( \bar{W}_{t+1}^D \), which is fixed\(^6\). Bank \( i \) is still insolvent because \( u_{t+1} < x_i \) (Proposition 1). Since all the other banks are solvent, the liquidation price is 1 and the expected payoff to depositors is \( u_{t+1} \). The last

\(^6\)Bank \( i \) is assumed to be small and competitive. It does not take into account its impact on the aggregate wealth of depositors.
integral runs over normal states in which \( u_{t+1} \geq x_i \). In these states all banks are solvent, including bank \( i \). Hence, depositors are fully paid out.

To solve the optimal capital structure in the range \( x_i \geq x_{t+1} \), maximize (15) with respect to \( a_i \) and \( x_i \) subject to (12), (16) and the constraint \( x_i \geq x_{t+1} \). The first order condition is (see appendix for details):

\[
\frac{a_i}{e_i} \int_{x_i}^{\infty} (u_{t+1} - x_i) \, dF(u_{t+1}) \leq (W_{t+1}^D) \frac{\theta}{\delta} \left( 1 + \psi \right),
\]

(17)

for a corner solution \( x_i = x_{t+1} \). For an interior solution \( x_i > x_{t+1} \) the FOC holds with equality.

We turn now to the optimization problem of the bank in the range \( x_i \leq x_{t+1} \), where bank \( i \) chooses a lower debt-asset ratio than the other banks. It is sufficient here to study the interval:

\[
x_{t+1} (1 - \kappa) \leq x_i \leq x_{t+1}.
\]

(18)

The range \( x_i \leq x_{t+1} (1 - \kappa) \) is discussed in the appendix.

The stock market value of the bank in the range defined by (18) is:

\[
V^e(a_i, x_i) \bigg|_{x_{t+1}(1-\kappa) \leq x_i \leq x_{t+1}} = \frac{\beta}{1 + \phi} \int_{x_{t+1}}^{\infty} a_i (u_{t+1} - x_i) \, dF(u_{t+1}).
\]

(19)
Note that the solvency threshold is now $x_{t+1}$ and not $x_i$. Namely, reducing the debt-asset ratio below $x_{t+1}$ does not insulate the bank against default in disaster states\(^7\). This is due to the bank run effect. To see this, note that when $u_{t+1} < x_{t+1}$, all banks default and the liquidation price falls to $1 - \kappa$. The liquidation value of the assets of bank $i$ is $a_i u_{t+1} \left(1 - \kappa\right)$ and the bank’s debt is $a_i x_i$. Note that:

$$a_i u_{t+1} \left(1 - \kappa\right) < a_i x_{t+1} \left(1 - \kappa\right) \leq a_i x_i,$$

where the second inequality follows from (18). Hence, the liquidation value of the bank’s assets is lower than the bank’s debt. As explained, depositors will run on the bank and force its liquidation. Thus, the bank can be solvent only when $u_{t+1} \geq x_{t+1}$ (normal states).

The value of the bank deposits is:

$$V_d^d (a_i, x_i) \big| x_{t+1}(1-\kappa) \leq x_i \leq x_{t+1} = \frac{\delta a_i}{1 + \psi} \left\{ \int_0^{x_{t+1}} \frac{u_{t+1} \left(1 - \kappa\right)}{W_{t+1}^D} \theta \, dF \left(u_{t+1}\right) \right\}$$

$$+ \int_{x_{t+1}}^{\infty} \frac{x_i}{(W_{t+1}^D)^\theta} \theta \, dF \left(u_{t+1}\right) \right\}.$$  

Compared with (16), there are now two relevant cases: financial disasters and normal periods. In financial disasters ($u_{t+1} < x_{t+1}$) the bank defaults due to (20). Depos-

\(^7\)The bank can stay solvent in some disaster states only if it chooses debt-asset ratio in the range $x_i < x_{t+1} \left(1 - \kappa\right)$, see the appendix for details.
itors run on the bank, and their expected payoff is $a_i u_{t+1} (1 - \kappa)$. These states are integrated by the first integral in (21). The second integral refers to normal states ($u_{t+1} \geq x_{t+1}$). In these states all banks are solvent, including bank $i$, and depositors are fully paid out.

Maximizing (19) subject to (12), (21) and (18) yields the first order condition (details in the appendix):

$$\frac{a_i}{e_i} \int_{x_{t+1}}^{\infty} (u_{t+1} - x_i) dF(u_{t+1}) \geq (\bar{W}_{t+1}^D) \theta \frac{1 + \psi}{\delta},$$  \hspace{1cm} (22)

for a corner solution $x_i = x_{t+1}$. For an interior solution the FOC holds with equality, and for a corner solution $x_i = x_{t+1} (1 - \kappa)$ it holds with inequality $\leq$.

Conditions (17) and (22) imply that $x_i = x_{t+1}$ is optimal if and only if:

$$\frac{a_{t+1}}{e_{t+1}} \int_{x_{t+1}}^{\infty} (u_{t+1} - x_{t+1}) dF(u_{t+1}) = (\bar{W}_{t+1}^D) \theta \frac{1 + \psi}{\delta}.$$  \hspace{1cm} (23)

Since this is an equilibrium condition, the asset-equity ratio $a_i/e_i$ was replaced with the equilibrium ratio $a_{t+1}/e_{t+1}$. Note that the LHS is the expected return on the bank equity $E_t R_{t+1}^e$. Substitute (8) in the RHS to get:

$$E_t R_{t+1}^e = E_t \left[ R_{t+1}^d \left( \frac{\bar{W}_{t+1}^D}{W_{t+1}^D} \right)^\theta \right] \geq E_t R_{t+1}^d.$$

The inequality follows from (14). Hence, in equilibrium the expected return on the
bank equity is (weakly) higher than the expected return on deposits. This implies that bankers would always prefer to save in equity shares rather than deposits. Thus, the initial assumption that bankers do not save in bank deposits is not binding in equilibrium.

To get (13) note that:

\[
E_t \frac{R_{t+1}^e}{(W_{t+1}^D)^\theta} = \frac{a_{t+1}}{e_{t+1}} \int_{x_{t+1}}^{\infty} \frac{u_{t+1} - x_{t+1}}{(W_{t+1}^D)^\theta} dF(u_{t+1})
\]

\[
= \frac{a_{t+1}}{e_{t+1}} \frac{1}{(W_{t+1}^D)^\theta} \int_{x_{t+1}}^{\infty} (u_{t+1} - x_{t+1}) dF(u_{t+1}).
\]

This expression follows from the equilibrium result that depositors’ wealth is fixed at \( \bar{W}_{t+1}^D \) whenever banks are solvent, namely, when \( u_{t+1} \geq x_{t+1} \). Substituting in (23) together with (8) yields (13). Note that the initial assumption that precluded depositors from saving in equity is also not binding in equilibrium, because (13) implies that depositors are indifferent between saving in deposits and equity.

### 2.6 Equilibrium Returns on Equity and Deposits

The returns on equity and deposits are defined by the ratio of period \( t+1 \) payoffs to period \( t \) savings. Total savings in equity and deposits are \( e_{t+1} \) and \( d_{t+1} \), where \( a_{t+1} = e_{t+1} + d_{t+1} \). Hence, the respective return distributions in equilibrium are:
\[
R^c_{t+1}, R^d_{t+1} = \begin{cases} 
\frac{u_{t+1} - x_{t+1}}{1 - \lambda_{t+1}}, \frac{x_{t+1}}{\lambda_{t+1}} & u_{t+1} \geq x_{t+1} \quad \text{(normal states)} \\
0, \frac{u_{t+1}(1-\kappa)}{\lambda_{t+1}} & u_{t+1} < x_{t+1} \quad \text{(financial disasters)}
\end{cases}
\]

where \( \lambda_{t+1} \equiv \frac{d_{t+1}}{a_{t+1}} \).

The variables \( x_{t+1} \) and \( \lambda_{t+1} \) are two closely related measures of leverage. \( x_{t+1} \) denotes the payout that the bank promises to make to its depositors as a share of total assets. \( \lambda_{t+1} \) is slightly different. It is the ratio between the amount borrowed by the bank \( (d_{t+1}) \) and total assets. Note that the promised interest rate on the bank deposits is \( x_{t+1}/\lambda_{t+1} \), which will be denoted \( \bar{R}_{t+1} \). In the rest of the paper I will refer to \( \lambda_{t+1} \) as the bank leverage.

## 3 General Equilibrium

We can now summarize the system of equilibrium conditions and discuss how to solve the variables of the system. The budget constraints of bankers and depositors provide the following conditions:

\[
W^B_t - c^B_t = (1 - \lambda_{t+1})a_{t+1},
\]

\[
W^D_t - c^D_t = \frac{\lambda_{t+1}a_{t+1}}{1 - \mu_{t+1}},
\]

\[
W^D_t - c^D_t = \min \left( \left\{ \delta E_t(\Pi^D_{t+1})^{1-\theta} \right\}^{\frac{1}{\theta}}, W^D_t \right).
\]
The leverage $\lambda_{t+1}$ is the ratio of deposits to total assets. The new variable $\mu_{t+1} \equiv m_{t+1}/(W_t^D - c_t^D)$ denotes the share of storage in the savings of depositors. The total return on their savings is denoted $\Pi_{t+1}^D = (1 - \mu_{t+1}) R_{t+1}^d + \mu_{t+1}$.

Condition (25) states that bankers hold their savings in bank equity shares. Note that the value of equity shares issued in period $t$ is $e_{t+1} = (1 - \lambda_{t+1}) a_{t+1}$. Condition (26) states that depositors hold a share $1 - \mu_{t+1}$ of their savings in bank deposits and the rest in storage (the value of deposits is $d_{t+1} = \lambda_{t+1} a_{t+1}$). The total savings of depositors are given by (27) which replicates (10).

In addition, we have three more equilibrium conditions:

\begin{align*}
E_t \frac{R_{t+1}^e}{(\Pi_{t+1}^d)^\theta} &= E_t \frac{R_{t+1}^d}{(\Pi_{t+1}^d)^\theta}, \quad (28) \\
E_t \frac{R_{t+1}^d}{(\Pi_{t+1}^d)^\theta} &\geq E_t \frac{1}{(\Pi_{t+1}^d)^\theta} \quad \text{when } \mu_{t+1} > 0, \quad (29) \\
E_t R_{t+1}^e &\geq \frac{1}{\beta} \quad \text{when } c_{t}^B > 0. \quad (30)
\end{align*}

Condition (28) follows from (13), after multiplying both sides of (13) by $(W_t^D - c_t^D)^\theta$. Condition (29) follows from (8) and (9). Finally, condition (30) replicates (4).

It is convenient to describe the system with the new variable $\mu$, which denotes the share of storage in depositors’ savings, instead of the level of storage $m$. Thus, the endogenous state variables of the system are $a_t$, $\lambda_t$, $\mu_t$ and $x_t$, which are determined one period in advance. Together with the realization of $u_t$ we can solve the six variables of the system: $c_t^B$, $c_t^D$, $a_{t+1}$, $\lambda_{t+1}$, $\mu_{t+1}$ and $x_{t+1}$ through the six conditions (25) to (30). To do so, we first have to calculate $R_t^e$ and $R_t^d$ through (24). Then we
can calculate the wealth inherited by the two types of agents by adding the returns on their parents’ portfolios:

\[ W^B_t = (1 - \lambda_t) a_t R^c_t \]
\[ W^D_t = \frac{\lambda_t a_t}{1 - \mu_t} \left\{ (1 - \mu_t) R^d_t + \mu_t \right\}. \]

In the next step, we express the expectation terms that appear in the system in terms of \( \lambda_{t+1} \), \( x_{t+1} \) and \( \mu_{t+1} \). Then we get a system of six conditions with exactly six variables to solve.

The appendix shows that a solution exists for the set of parameter values that are used to simulate the model. When storage is ruled out (the no-storage case) the solution is unique. When storage is allowed (the full model), there can be at most three equilibria, but only one of them is economically plausible. This equilibrium is studied in the analysis below. The other two equilibria imply that bankers consume all (or almost all) their wealth in one period leaving no bequest. This is a technical result arising from the assumption of linear utility from consumption. It is not robust to other functional forms, hence I will not elaborate on these solutions. For further discussion see the technical appendix.

\(^8\text{Recall that we assume that the share of depositors that leave no bequest (\( \pi \)) is infinitesimally small, so we can ignore it.}\)
3.1 The risk-free interest rate

The risk-free interest rate is the return on a perfectly safe asset. It determines whether depositors save in storage or not. When the risk-free rate falls below the return on storage (which is normalized to 1), depositors shift some of their wealth to storage. It follows from the depositors’ first order condition (8) that the risk-free rate, denoted $R^f_{t+1}$, is:

$$R^f_{t+1} = \frac{E_t \left( \frac{R^d_{t+1}}{(W^D_{t+1})^\theta} \right)}{E_t (W^D_{t+1})^\theta}.$$ (31)

4 The Choice of Parameter Values

The results of the model are demonstrated through a series of simulations, which are presented in the next two sections. The goal is to illustrate the model results and point to the potential effects of the proposed balance sheet channel. To this end, the choice of parameter values is based on real data which makes the simulations more meaningful. Some of the model parameters estimated in this section are of particular interest for the study of financial stability issues, as they determine the probability of bank default. To my knowledge, these parameters have not been estimated previously, so I use raw data to derive their values.

I start by evaluating the distribution of bank returns ($u_{t+1}$), which is assumed to be log normal. The distribution parameters are the mean and standard deviation of log $u_{t+1}$, denoted $\mu$ and $\sigma$ respectively. The time period is interpreted as one year, so the distribution of log $u_{t+1}$ refers to annual asset returns.
Table 1 presents annual bank returns in 30 OECD countries during the period 1980-2003. The data is aggregated at the country level, so these statistics reflect aggregate variations. The table presents the aggregate returns on bank assets and bank equity, corresponding to log $u$ and log $R^e$. A note of caution is in order. The variable $u$ that appears in the model is different from the accounting term Return on Assets (ROA), which measures the ratio of bank profits to total assets. Its model counterpart is $u - x$. The difference between the accounting ROA and $u$ is interest payments, represented in the model by $x$. Therefore, $u$ was calculated by adding interest payments to the accounting ROA (before taxes)\(^9\). The result was adjusted to year end CPI inflation and transformed to log of the gross return.

The mean and standard deviation of log $u$ in the OECD sample are 0.015 and 0.053, respectively. A large portion of the variation in log $u$ is due to inflation shocks, because bank assets are mostly nominal. Banks hedge against inflation shocks by issuing liabilities that are also nominal. Thus, the standard deviation of log $u$ in the full sample overstates the actual risks that banks are exposed to, because banks hedge against some of these risks (the inflation shocks). Hence, I exclude from the sample periods of large inflation shocks\(^10\). The mean and standard deviation of log $u$ become 0.025 and 0.03, which are used in the simulation.

The next parameter to be evaluated is $\kappa$, which denotes the liquidation costs in a financial disaster. A financial disaster is defined as a complete failure of the

\(^9\)More precisely, the accounting ROA equals $(u - 1) - \lambda (\bar{R} - 1)$, where $\bar{R}$ is the gross interest rate promised by the bank, namely $\bar{R} \equiv x/\lambda$. The term $\lambda (\bar{R} - 1)$ equals interest payments as a ratio of total bank assets. Hence, $u$ can be reconstructed by adding interest payments to the accounting ROA.

\(^10\)Large inflation shocks are defined as changes in annual CPI inflation that are larger than 3 percentage points
banking sector. The closest example of such a rare event is the massive wave of US bank failures during the Great Depression. Out of the 25 thousand banks operating in 1929, about 9 thousand banks suspended operation during the years 1930-1933. Depositors of those suspended banks lost approximately 20% of their money (Board of Governors of the Federal Reserve System 1943). Moreover, the liquidation of insolvent banks took around 4 to 5 years (Treasury Department, Comptroller of the Currency 1940), so depositors’ money was practically frozen for a long period of time, which implies an additional pecuniary loss.

Depositors’ loss in a bank failure corresponds to the expression $1 - (1 - \kappa) \frac{u}{x}$. This could be a proxy of $\kappa$ if $\frac{u}{x}$ is sufficiently close to 1 when banks default, which is reasonable given the low standard deviation of log $u$. However, a more direct evidence on banks’ liquidation costs is provided by James (1991), who studied 412 bank failures during the US savings and loan crisis in the 1980s. The assets of the failed banks were liquidated by the FDIC in several ways. James (1991) calculated the difference between the book value of a bank’s assets at the time of its closure and the value of the assets in an FDIC receivership or the value of the assets to an acquirer in cases that the assets were sold by the FDIC. The average liquidation costs amounted to around 30% of bank assets. James (1991) provides some evidence that the book value of the bank’s assets at the moment of closure might be overestimated due to unrealized past losses. This implies that the estimated liquidation costs are somewhat overstated. For this reason, I evaluate the $\kappa$ parameter more conservatively at 0.2. I also experiment with different values of $\kappa$ to gauge the sensitivity of the results to this parameter (see table 3 and the discussion in Section 5).
Finally, the parameter \( \theta \) denotes the relative risk-aversion coefficient of depositors’ utility from bequest. The asset pricing literature usually calibrates this coefficient in the range of 2 to 5 (Barro 2006). In this literature the relative risk-aversion coefficient refers to consumption in an infinitely-lived agent model. The Bellman representation of these models is similar to the present model, where the risk aversion coefficient relates to future wealth (e.g. Levhari and Srinivasan 1969). Hence, I approximate \( \theta \) by drawing from the asset pricing literature. The parameter is calibrated at \( \theta = 3 \), as in Barro (2006).

5 Results Under No Storage

The results of the model are presented under two alternative assumptions. This section assumes that depositors are not allowed to invest in storage so \( \mu \) (or equivalently \( m \)) is always zero. This assumption enables to concentrate on the asset price effects of bankruptcy risk. The next section removes the no-storage assumption and studies the full model.

5.1 Leverage and asset prices

Leverage is defined by the ratio of deposits to total assets and denoted \( \lambda \equiv d/a \). The leverage is an endogenous variable that depends on the wealth ratio of the two agents. This subsection demonstrates the asset pricing effects of different leverage rates. For this purpose, \( \lambda \) is treated here as an "exogenous" variable, so that asset prices are calculated for given values of \( \lambda \). In subsection 5.2 we study the model
dynamics, so leverage will be determined endogenously.

Figure 2 depicts the effect of $\lambda$ on three variables: the expected return on equity, the deposit interest rate and the risk-free interest rate. These variables are derived through (28), which provides the asset pricing condition in equilibrium. Under the no-storage assumption ($\mu = 0$), this condition provides a unique solution to $x$ given $\lambda$ (see technical details in the appendix). Having $x$ and $\lambda$, we know the entire distribution of $R^e$ and $R^d$ through (24). Thus, we can calculate the expected return on equity ($ER^e$) and the risk-free interest rate ($R^f$). We can also calculate the interest rate on deposits which is denoted $\bar{R} \equiv x/\lambda$.

The main result shown in Figure 2 is the negative impact of leverage on the risk-free interest rate ($R^f$) and the positive impact on the credit spread ($\bar{R} - R^f$). To understand the source of the leverage effect, Table 2 calculates the other variables of the model for different leverage rates. The variable of interest which drives the results is the probability of a financial disaster. Financial disasters occur in this model when all banks default. This happens when $u < x$, so the disaster probability is simply $Pr (u < x)$.

The model produces disaster probabilities that are fairly low. Take for instance a leverage of 0.91 that corresponds to an equity-asset ratio of 9 percent. This was the equity-asset ratio of large financial institutions prior to 2007 (Figure 1). The disaster probability implied by the model for this leverage rate is 0.1%. This low probability seems plausible given the definition of a financial disaster in this paper, which is a complete failure of the banking sector.

Barro (2006) provides data on economic disasters that took place during the
twentieth century. Of these disasters, the Great Depression is the closest example of a financial disaster. Out of the 20 OECD countries surveyed by Barro (2006), 8 countries were severely hit by the Great Depression and experienced a decline of more than 15 percent in GDP per capita. Caprio and Klingebiel (2003) describe two additional events that are close to the definition of financial disaster: Japan’s banking crisis in the 1990s with an estimated loss of 24 percent of GDP, and Spain banking crisis in 1977-1985 with a loss estimated at 17 percent of GDP. This gives us 10 financial disasters in 20 OECD countries over a century, with an annual disaster probability of 0.5%. These data suggest that financial disasters are indeed rare events, which is consistent with the model result.

While the disaster probability is usually very low, it can have significant asset price effects when it rises. The disaster probability affects asset prices in two ways. First, there is a general effect on the mean return of bank assets net of liquidation costs, which we denote by $R^a$. In normal times $R^a = u$ because liquidation costs are zero. In financial disasters $R^a = u (1 - \kappa)$. Hence, when disasters become more likely $ER^a$ falls\(^\text{11}\). Table 2 shows that $ER^a$ drops 0.8 percentage points when leverage jumps from 0.91 to 0.95. This is caused by the rise in the disaster probability from 0.1 to 4.3 percent. The effect on $ER^a$ spreads out to other assets through equilibrium conditions.

The second effect comes from the convexity of the marginal utility of bequest of depositors, which creates the precautionary saving motive. Namely, the utility loss of depositors in states of low bequest is not offset by states of high bequest. Hence,\(^\text{11}\)To see this, note that the distribution of $R^a$ implies that $E \log R^a = E \log u + P \cdot \log (1 - \kappa)$, where $P$ is the disaster probability and $\log (1 - \kappa) < 0$.\(^{63}\)
these agents have a natural demand for safe assets, which can hedge against states of financial disasters. A rise in the disaster probability raises the demand for safe assets as a hedge against disaster events, reducing the risk-free interest rate. In the above example, the risk-free rate falls 1.8 percentage points when leverage rises from 0.91 to 0.95. Part of this fall is due to the 0.8 percentage point decline in $ER^a$ and the rest is due to the rise in the demand for safe assets.

To further explore the contribution of the various parameters of the model, Table 3 provides a sensitivity analysis. The upper panel presents the effect of different values of $\sigma$, which is the standard deviation of log $u$. Higher $\sigma$ implies higher volatility of asset returns. Overall, it produces a larger disaster probability (denoted $P$ in the table) and a lower risk-free rate ($R^f$). On the other hand, changes in the liquidation cost parameter $\kappa$ have only moderate impact on the disaster probability but a large impact on the risk-free rate. In contrast, the impact of the risk-aversion parameter $\theta$ on the disaster probability and on the risk-free rate is fairly weak.

### 5.2 Dynamics

I turn now to analyse the dynamics of the model under the no-storage assumption ($\mu = 0$). The dynamics is governed by the saving rules of the two types of agents. Bankers save in equity capital and depositors save in deposits. The accumulation of wealth of the two types of agents determines the bank’s assets and leverage. Hence, the model dynamics can be presented through the evolution of two endogenous variables: assets ($a$) and leverage ($\lambda$), depicted by the vertical and horizontal axes in Figure 3.
Equilibrium condition (30) requires that $ER_e \geq \beta^{-1}$. Since $ER_e$ is rising with $\lambda$ (Figure 2), there is a specific leverage rate for which $ER_e = \beta^{-1}$. Denote this rate by $\lambda^S$. It is depicted in Figure 3 by the vertical solid line at $\lambda^S$. The model can be in equilibrium only in the range $\lambda \geq \lambda^S$ in which $ER_e \geq \beta^{-1}$.

The second line in Figure 3 (denoted DD) is derived from (26) and (27) for the case $W_t^D \geq \{\delta E_t(\Pi_t^{D+1})^{1-\theta}\}^{1/\theta}$:

$$\{\delta E (R^d)^{1-\theta}\}^{1/\theta} = \lambda a.$$  

(32)

This condition provides the maximum amount of deposits that can be issued in equilibrium ($\lambda a$). This amount is attained when depositors inherit more wealth than they wish to save, so they save $\{\delta E_t(\Pi_t^{D+1})^{1-\theta}\}^{1/\theta}$ and consume the rest. It can be shown that the LHS of (32) is a function of $\lambda$. Thus, condition (32) represents a line on the ($\lambda$, $a$) plane. It is depicted in Figure 3 by the DD line. Any equilibrium must be in the area below or on the DD line. A point above the DD line violates the saving rule of depositors defined in (27).

The general tendency of the model is to converge to point $S$. When the model is in the area below the DD line and to the right of $\lambda^S$, both agents save all their wealth so $a$ is likely to rise. Normally, the return on equity would be higher than the interest rate on deposits ($R_t^e > \bar{R}_t$) so shareholders get higher return on their wealth than depositors. Hence, equity capital would rise faster than deposits, and the model would move north-west towards $S$. Once the model reaches point $S$ it stays there as long as $R_t^e > 1$ and $R_t^d > 1$. This implies that both types of agents have sufficient
wealth to save the same amounts they have inherited, which means that the model stays at point S. When the realized shock is too low, banks incur a loss ($R_t^e < 1$) so equity capital falls and leverage rises. The model moves to the right of point S and then converges back to S in the subsequent periods.

6 Results with Storage

I now remove the no-storage restriction to allow depositors to save in storage ($\mu \geq 0$). The availability of storage enables depositors to respond to the rise in bankruptcy risk by shifting funds from bank deposits to safe assets. This generates a flight-to-quality effect as in Bernanke and Gertler (1989).

The results are presented through the following simulation. The model starts at point S of Figure 3. At this point $ER^e = \beta^{-1}$. The leverage rate at S depends on the value of $\beta$, which is calibrated at $1.0263^{-1}$ to target an initial leverage rate of 0.91. This leverage rate corresponds to the equity-asset ratio of 9% observed in large financial institutions prior to 2007 (Figure 1).

Starting at S, the model is hit by a negative shock of 1.9 standard deviations to log $u_t$ in period 0. The initial shock is chosen to target a negative return on equity of -60% ($R_0^e = 0.4$). This equity loss is similar in magnitude to the loss documented during the Great Recession, as evident by indices of total stock market return (Table 4). The effects of this shock are summarized by the impulse response functions depicted in Figure 4. For comparison, the figure presents the results of the full model as well as the version without storage.

35
The first graph on the upper-left of Figure 4 presents the impact on assets ($a_t$). The units are ratios of the initial level. When storage is allowed (the solid line), assets respond to the initial shock by falling 18 percent. Leverage rises to 0.956 and the disaster probability jumps from 0.1% to 6.7%. The risk-free return falls to one and the credit spread ($\bar{R}_t - R^f_t$) jumps from zero to 2.5%. The new equilibrium is depicted by point R in Figure 3, which presents $\lambda_1$ and $a_1$. Conversely, when storage is not allowed bank assets fall only 3.5 percent, as shown by the dashed line in Figure 4. However, the rise in leverage and in the disaster probability is much larger creating stronger asset price effects. The new equilibrium in the no-storage version is depicted by point Q in Figure 3.

The difference between the full model and the no-storage version is explained by the steep rise in storage in the full model. The share of storage in depositors’ savings (variable $\mu_t$) rises from zero to 13 percent. On the one hand, the rise in storage helps to mitigate the jump in the disaster probability and the asset price effects. On the other hand, the supply of bank loans to the production sector shrinks. Hence, the rise in the disaster probability induces agents to invest in safe storage rather than making productive (albeit risky) investments. This "flight to quality" effect exacerbates the initial negative shock.

After the initial shock and the rise in storage, storage starts to decline gradually for the next 7 years until it returns back to zero. The pace of recovery depends on the rate of wealth accumulation of bankers (the shareholders). These agents lose 60 percent of their wealth in the first year, and then gain an average of 3.0 percent each year. It takes 7 years for equity capital to reach a sufficiently high level that would
induce depositors to shift their wealth back to bank deposits. Thus, the recovery in this model is a slow deleveraging process coupled with low interest rates and high credit spreads.

The asset price dynamics generated by the model differs from previous models with financial frictions, such as Gertler and Karadi (2011) and Cúrdia and Woodford (2010). In these models asset prices are governed primarily by consumption dynamics through an Euler condition. This implies that the asset price effects tend to reverse once the economy starts to recover. For instance, the interest rate tends to rise above the steady state level when the economy is recovering, even if the recovery is very slow. By contrast, in the present model asset prices are affected by the precautionary saving motive that takes effect when bankruptcy risks are high. Hence, as long as leverage and bankruptcy risks are high, the precautionary saving effect is strong. Since deleveraging can take a long time, the model is able to generate long periods of low interest rates and high credit spreads, during which the economy is slowly recovering from the initial negative shock.

The type of recovery suggested by this model characterized the Great Depression in the 1930s and the Japanese recession in the 1990s, which lasted for a long period and were associated with severe financial distress and low interest rates. The recent Great Recession shares similar characteristics. One of the problems during these episodes was the sharp interest rate decline, which made it difficult for central banks to stimulate the economy through conventional monetary policy (Krugman 1998). The present model suggests that the rise in aggregate bankruptcy risk may have contributed to this problem. Hence, policy measures that alleviate those risks may
help raising the (natural) interest rate, which would make conventional monetary policy more effective.

7 Conclusions

The steep rise in bankruptcy risk of financial institutions was a prominent feature of the Great Recession. To understand the macroeconomic effects of this phenomena we need a model in which aggregate bankruptcy risk evolves endogenously. The present paper offers such a model by building on the rare disaster literature (Barro 2006). The result is a new balance sheet channel that operates through the impact of aggregate bankruptcy risk on the precautionary demand for saving. By comparison, previous models that incorporated endogenous bankruptcy risk studied the effects on the external finance premium, leaving aside issues of volatility and risk (Bernanke, Gertler and Gilchrist 1999).

These issues have been at the center of a recent literature on uncertainty shocks, e.g. Bloom (2009), Bloom, Floetotto, Jaimovich, Saporta-Eksten and Terry (2012) and the references therein. This literature postulates a stochastic process for total factor productivity with time varying volatility, and studies its impact on the macro economy. The present paper suggests a rationale for these "uncertainty shocks". Specifically, uncertainty may increase as a result of balance sheet shocks that raise the probability of financial disasters. In addition to the rise in uncertainty, higher disaster probability also implies lower expected growth. Hence, the effect studied in the present paper involves changes in both the first and second moments of the
growth distribution.

The model suggests that elevated bankruptcy risk contributed to the slow recovery observed during severe recessions such as the US Great Depression, Japan’s recession in the 1990s and the recent Great Recession. One way to reduce bankruptcy risks is through direct government injection of equity capital into financially distressed corporations. Nevertheless, if the shortage in equity capital is too large, the required amount of public support might put the government itself at risk. An alternative way is to use tax incentives to encourage banks to issue more equity capital and households to buy this capital. Assessing the effects of these policy measures is left for future research.

References


conomics, John B. Taylor and Michael Woodford, eds. (Amsterdam: North-Holland, 1999).


*Journal of Monetary Economics*, 58 (2011), 17-34.


Table 1: Bank Financial Statistics in OECD Countries (1980-2003)*

<table>
<thead>
<tr>
<th></th>
<th>Return on Bank Assets(^1) (log (u))</th>
<th>Return on Bank Equity(^2) (log (R^e))</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>N</td>
<td>mean</td>
</tr>
<tr>
<td>All years</td>
<td>549</td>
<td>0.015</td>
</tr>
<tr>
<td>Excl. inflation shocks(^3)</td>
<td>434</td>
<td>0.025</td>
</tr>
</tbody>
</table>

* Source: OECD, Bank Profitability: Financial Statements of Banks (2004 edition). Data is annual at the country level. Countries included: US, UK, Austria, Belgium, Denmark, France, Germany, Italy, Luxembourg, Netherlands, Norway, Sweden, Switzerland, Canada, Japan, Finland, Greece, Iceland, Ireland, Portugal, Spain, Turkey, Australia, New Zealand, Mexico, Korea, Czech Republic, Slovak Republic, Hungary, Poland. For further details about this sample see Levintal (2013).

1 The return on bank assets in this table is different from the accounting measure of ROA (see explanation in the main text). It is calculated by annual profits before interest expenses and taxes over (previous year end) total assets. The result is adjusted for (year end) CPI inflation and transformed to log of gross return.

2 The return on bank equity is calculated by annual profits before taxes over (previous year end) equity capital. The result is adjusted for (year end) CPI inflation and transformed to log of gross return.

3 This sub-sample excludes years in which the inflation rate increased or decreased by more than 3 percentage points compared to the previous year.
<table>
<thead>
<tr>
<th>Leverage λ</th>
<th>Asset Return ERₐ</th>
<th>Equity Return ERₑ</th>
<th>Interest Rate R</th>
<th>Risk Free Rate Rᶠ</th>
<th>Disaster Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.90</td>
<td>1.026</td>
<td>1.026</td>
<td>1.026</td>
<td>1.026</td>
<td>0.000</td>
</tr>
<tr>
<td>0.91</td>
<td>1.026</td>
<td>1.026</td>
<td>1.026</td>
<td>1.025</td>
<td>0.001</td>
</tr>
<tr>
<td>0.92</td>
<td>1.025</td>
<td>1.027</td>
<td>1.026</td>
<td>1.025</td>
<td>0.003</td>
</tr>
<tr>
<td>0.93</td>
<td>1.024</td>
<td>1.030</td>
<td>1.026</td>
<td>1.022</td>
<td>0.008</td>
</tr>
<tr>
<td>0.94</td>
<td>1.022</td>
<td>1.037</td>
<td>1.025</td>
<td>1.017</td>
<td>0.020</td>
</tr>
<tr>
<td>0.95</td>
<td>1.018</td>
<td>1.051</td>
<td>1.025</td>
<td>1.007</td>
<td>0.043</td>
</tr>
<tr>
<td>0.96</td>
<td>1.009</td>
<td>1.077</td>
<td>1.025</td>
<td>0.991</td>
<td>0.084</td>
</tr>
<tr>
<td>0.97</td>
<td>0.996</td>
<td>1.122</td>
<td>1.025</td>
<td>0.966</td>
<td>0.154</td>
</tr>
</tbody>
</table>

The table presents equilibrium values of the model variables for different leverage rates, under the no-storage assumption (μ = 0). The parameter values are g = .025, σ = .030, d = .2 and θ = 3, see section 4 for details.
Table 3: Sensitivity analysis

<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>$P$</th>
<th>$ER^e$</th>
<th>$\bar{R}$</th>
<th>$R^f$</th>
<th>$\sigma = 0.02$</th>
<th>$\sigma = 0.04$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.93</td>
<td>0.000</td>
<td>1.026</td>
<td>1.026</td>
<td>1.026</td>
<td>0.035</td>
<td>1.047</td>
</tr>
<tr>
<td>0.94</td>
<td>0.001</td>
<td>1.026</td>
<td>1.026</td>
<td>1.025</td>
<td>0.060</td>
<td>1.063</td>
</tr>
<tr>
<td>0.95</td>
<td>0.005</td>
<td>1.029</td>
<td>1.025</td>
<td>1.023</td>
<td>0.098</td>
<td>1.087</td>
</tr>
<tr>
<td>0.96</td>
<td>0.020</td>
<td>1.037</td>
<td>1.025</td>
<td>1.017</td>
<td>0.153</td>
<td>1.124</td>
</tr>
<tr>
<td>0.97</td>
<td>0.062</td>
<td>1.062</td>
<td>1.025</td>
<td>1.000</td>
<td>0.235</td>
<td>1.181</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\kappa = 0.1$</th>
<th>$\kappa = 0.3$</th>
<th>$\theta = 2$</th>
<th>$\theta = 4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.93</td>
<td>0.008</td>
<td>1.028</td>
<td>1.026</td>
</tr>
<tr>
<td>0.94</td>
<td>0.020</td>
<td>1.031</td>
<td>1.026</td>
</tr>
<tr>
<td>0.95</td>
<td>0.045</td>
<td>1.038</td>
<td>1.026</td>
</tr>
<tr>
<td>0.96</td>
<td>0.090</td>
<td>1.051</td>
<td>1.026</td>
</tr>
<tr>
<td>0.97</td>
<td>0.168</td>
<td>1.075</td>
<td>1.027</td>
</tr>
</tbody>
</table>

The table presents the effect of parameter values on the disaster probability ($P$), the expected return on equity ($ER^e$), the deposit interest rate ($\bar{R}$) and the risk-free rate ($R^f$), under the no-storage assumption ($\mu = 0$). Parameter values in each panel are equal to the benchmark values in Table 2 except for a single parameter stated at the top of the panel.

Table 4: Indices of Total Stock Market Return of the Banking Sector

<table>
<thead>
<tr>
<th></th>
<th>US</th>
<th>UK</th>
<th>Japan</th>
<th>France</th>
<th>Germany</th>
<th>Italy</th>
</tr>
</thead>
<tbody>
<tr>
<td>2006</td>
<td>100.0</td>
<td>100.0</td>
<td>100.0</td>
<td>100.0</td>
<td>100.0</td>
<td>100.0</td>
</tr>
<tr>
<td>2007</td>
<td>76.5</td>
<td>82.5</td>
<td>73.6</td>
<td>82.7</td>
<td>92.6</td>
<td>90.7</td>
</tr>
<tr>
<td>2008</td>
<td>42.4</td>
<td>39.0</td>
<td>43.8</td>
<td>31.5</td>
<td>31.1</td>
<td>42.3</td>
</tr>
<tr>
<td>2009</td>
<td>42.1</td>
<td>48.7</td>
<td>35.2</td>
<td>55.8</td>
<td>47.4</td>
<td>56.6</td>
</tr>
<tr>
<td>2010</td>
<td>47.3</td>
<td>51.3</td>
<td>34.3</td>
<td>48.5</td>
<td>43.9</td>
<td>40.5</td>
</tr>
<tr>
<td>2011</td>
<td>35.0</td>
<td>35.3</td>
<td>28.4</td>
<td>28.2</td>
<td>30.6</td>
<td>23.6</td>
</tr>
</tbody>
</table>

The table presents yearend values of the total stock market return index (dividends included) of the banking sector (100=2006 yearend). Source: Datastream, global equity indices.
Figure 1: Equity Capital and Bankruptcy Risk of Large Financial Institutions

The figure presents asset weighted means of the equity/asset ratio (at market value) and the CDS premium (5 year term) of the following 25 Large Financial Institutions: Deutsche Bank, HSBC Holdings, BNP Paribas, Barclays Plc, JPMorgan Chase, Bank of America, Citigroup, Banco Santander, Societe Generale, UBS, Lloyds Banking Group, Wells Fargo, Credit Suisse, Goldman Sachs, Allianz, Metlife, Morgan Stanley, National Australia Bank, Westpac Banking Corp, Danske Bank, Australia and New Zealand Bank, American International Group, Aegon NV, Nomura Holdings and Prudential Plc. The market value of bank assets is estimated by the book value of assets less the book value of equity plus the market value of equity. Data on equity and assets are from Compustat North America and presented at year end values. Data on CDS premiums are from Datastream and presented at month end values.
Figure 2: The leverage effect on returns under no-storage ($\mu = 0$)

The figure shows the effect of leverage on returns, under the no-storage assumption ($\mu = 0$). The expected return on equity ($ER^e$), the interest rate on deposits ($\bar{R}$) and the risk-free rate ($R^f$) are presented as functions of leverage ($\lambda$). The parameter values are $g = .025$, $\sigma = .03$, $\kappa = .2$ and $\theta = 3$, see section 4 for details.
Figure 3: The dynamics of assets and leverage

The figure presents the model dynamics on the $(\lambda, a)$ plane, starting at point $S$. Point $Q$ presents the model response under no-storage. Point $R$ describes the model response when storage is allowed. The parameter values are $g = .025$, $\sigma = .03$, $\kappa = .2$ and $\theta = 3$, see section 4 for details. The model starts at $\lambda^S = .91$, which corresponds to the average leverage rate of large financial institutions prior to the Great Recession, see section 6.
The figure depicts the dynamic response of the model variables to a shock of 1.9 standard deviations to $\log u$. Parameter values are as in Figure 3. The variables $a_t$ and $e_t$ are presented as ratios of their initial levels. The other variables are presented at their natural units. Solid lines present the model response when storage is allowed. Dashed lines present the model response when storage is not allowed. The periods are presented on the horizontal axis.