Introduction to VARs and Structural VARs:

Estimation & Tests Using Stata

Bar-Ilan University 26/5/2009
Avichai Snir

Background: VAR

- **Background:**
- **Structural simultaneous equations**
  - Lack of Fit with the data
  - Lucas Critique (1976)

- **VAR: Vector Auto Regressions**
  - Simple
  - Non Structural
  - All Variables are treated identically
  - Better Fit with the Data

Simple VAR: Sims (1980)

- **Symmetric**
  - **Lags of the dependent variables**
  - **Same Number of Lags**

$$y_{1,t} = \alpha_0 + \alpha_1 y_{1,t-1} + \alpha_2 y_{2,t_{-1}} + \alpha_3 y_{3,t-1} + \alpha_4 y_{1,t-2} + \alpha_2 y_{2,t_{-2}} + \alpha_3 y_{3,t-2} + \ldots + \varepsilon_{1,t}$$

$$y_{2,t} = \beta_0 + \beta_1 y_{1,t-1} + \beta_2 y_{2,t_{-1}} + \beta_3 y_{3,t-1} + \beta_4 y_{1,t-2} + \beta_2 y_{2,t_{-2}} + \beta_3 y_{3,t-2} + \ldots + \varepsilon_{2,t}$$

$$y_{3\,1,t} = \gamma_0 + \gamma_1 y_{1,t-1} + \gamma_2 y_{2,t_{-1}} + \gamma_3 y_{3,t-1} + \gamma_4 y_{1,t-2} + \gamma_2 y_{2,t_{-2}} + \gamma_3 y_{3,t-2} + \ldots + \varepsilon_{3,t}$$

$$E(\varepsilon_{i,t}, \varepsilon_{j,\tau}) = \begin{cases} \sigma_{i,j} & if \quad t = \tau \\ 0 & if \quad t \neq \tau \end{cases}$$

$$i, j \in (1,2,3)$$

Simple VAR: Matrix Form

- In Matrix Form:

$$\mathbf{y_t} = \alpha + \Gamma_{t-1}\mathbf{y_{t-1}} + \Gamma_{t-2}\mathbf{y_{t-2}} + \ldots + \varepsilon_\mathbf{t}$$

$$or \;\; Simply:$$

$$\left[\mathbf{I} - \Gamma(L)\right]\mathbf{y_t} = \mathrm{A} + \varepsilon_\mathbf{t}$$

- $\mathbf{y_t}$ is a vector of the Dependent Variables
- $\Gamma_{\mathbf{t}-i}$ is a Matrix of Coefficients
- $\Gamma(L)$ is a Matrix in Lagged Variables
- $\varepsilon_\mathbf{t}$ is a Vector of White Noise Errors
- A is a Matrix of exogenous variables (constant,…)

Covariance Matrix

$$
\boldsymbol{\varepsilon}_{\mathbf{t}} \times \boldsymbol{\varepsilon}_{\mathbf{t}}' =
\begin{pmatrix}
\varepsilon_{1,1} \\ \varepsilon_{1,2} \\ \varepsilon_{1,3} \\ \vdots \\ \varepsilon_{2,1} \\ \varepsilon_{2,2} \\ \varepsilon_{2,1} \\ \vdots \\ \varepsilon_{3,1} \\ \varepsilon_{3,2} \\ \varepsilon_{1,1} \\ \vdots
\end{pmatrix}
\times
\begin{pmatrix}
\varepsilon_{1,1} & \varepsilon_{1,2} & \varepsilon_{1,3} & \ldots & \varepsilon_{2,1} & \varepsilon_{2,2} & \varepsilon_{2,3} & \ldots & \varepsilon_{3,1} & \varepsilon_{3,2} & \varepsilon_{3,3} & \ldots
\end{pmatrix}
=
$$

$$
\begin{pmatrix}
\sigma_{1,1} & 0 & 0 & \ldots & \sigma_{1,2} & 0 & \ldots & \sigma_{1,3} & 0 & \ldots \\
0 & \sigma_{1,1} & 0 & \ldots & 0 & \sigma_{1,2} & \ldots & 0 & \sigma_{1,3} & \ldots \\
0 & 0 & \sigma_{1,1} & \ldots & 0 & 0 & \ldots & 0 & 0 & \ldots \\
\vdots & & & \vdots & & & \vdots & & & \vdots \\
\sigma_{2,1} & 0 & 0 & \ldots & \sigma_{2,2} & & & & & \\
0 & \sigma_{2,1} & 0 & \ldots & & \sigma_{2,2} & \ldots & \sigma_{2,3} & 0 & \ldots \\
 & 0 & \sigma_{2,1} & \ldots & 0 & 0 & \ldots & 0 & \sigma_{2,3} & 0 \\
\vdots & & & \vdots & & & \vdots & & & \vdots \\
\sigma_{3,1} & 0 & 0 & \ldots & \sigma_{3,2} & 0 & \ldots & \sigma_{3,3} & 0 & \ldots \\
0 & \sigma_{3,1} & 0 & \ldots & 0 & \sigma_{3,2} & \ldots & 0 & \sigma_{3,3} & 0 \\
\vdots & & & & & & & & &
\end{pmatrix}
$$

Contemporary Variance Matrix

$$\Omega = \begin{pmatrix} \sigma_{1,1} & \sigma_{1,2} & \sigma_{1,3} \\ \sigma_{2,1} & \sigma_{2,2} & \sigma_{2,3} \\ \sigma_{3,1} & \sigma_{3,2} & \sigma_{3,3} \end{pmatrix}$$

Issues Before Estimation

- **Stationarity:**
  - Constant expected value
  - Constant Variance
  - Constant Covariances
- **Granger Exogeneity:**
  - Order of variables
- **Lag Length**
  - Optimal lag length

Testing Stationarity

- We have data on Canada 1966Q1-2002Q1
  - GDP
  - Consumer Price Index (CPI)
  - Household Consumption (consumption)

Consumption	CPI	GDP	Descriptor
36.91	18.04	62.53	1966Q1
37.47	18.24	64.58	1966Q2
38.46	18.41	65.47	1966Q3

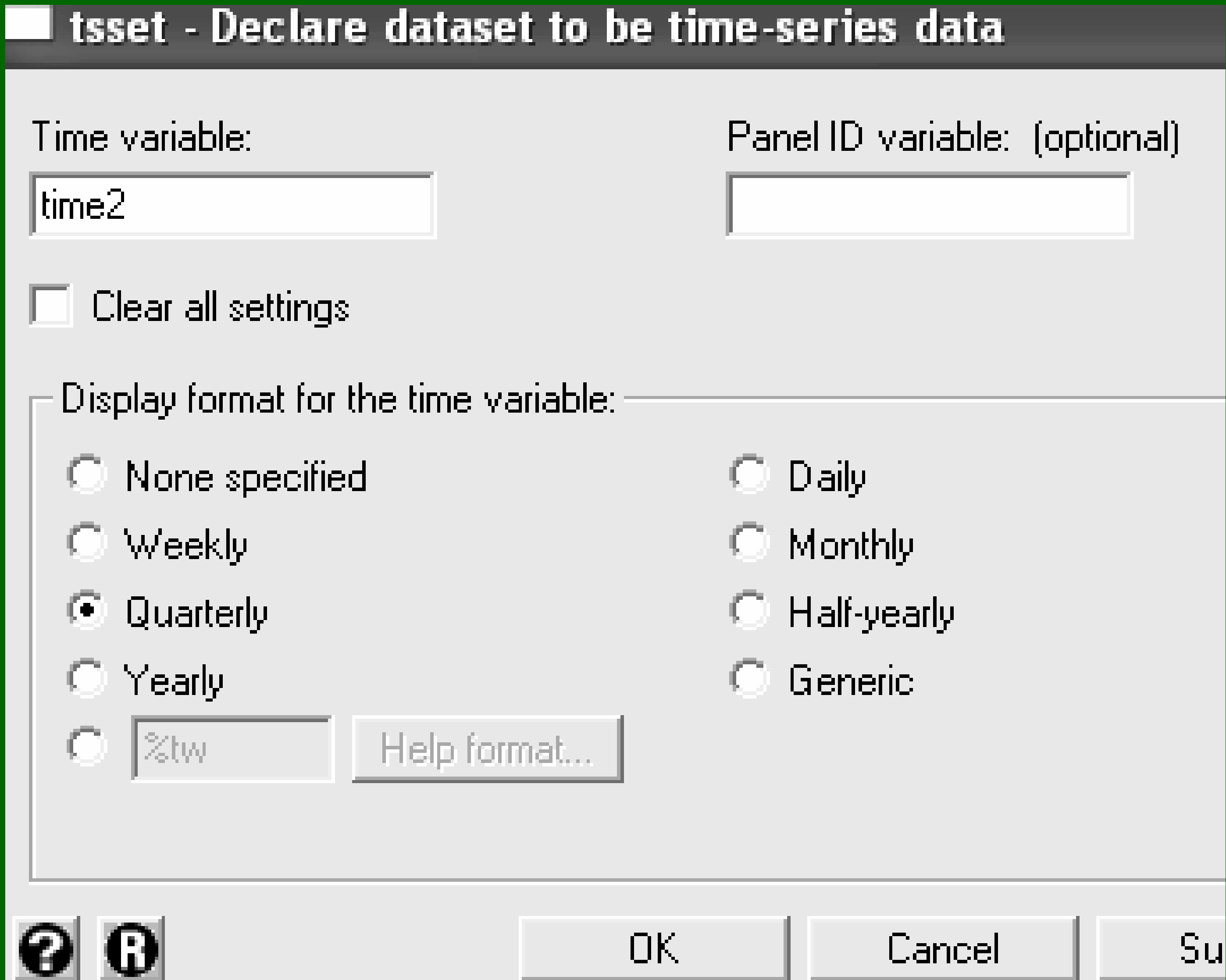
Declare: Time Series

- Define and format: time variable
  - date(var_name,"dmy") or
  - Quarterly(var_name, "yq")
  - format: format var_name %d
- Declare database as time series
  - Menu: statistics → time series → setup & utilities → declare dataset to be time series data

```
. generate time2 = quarterly(descriptor, "yq")

. format time2 %tq

. list time2 if _n<=3

     +--------+
     |  time2 |
     |--------|
  1. | 1966q1 |
```

Declaring Time series

Declare Time Series



Transformations

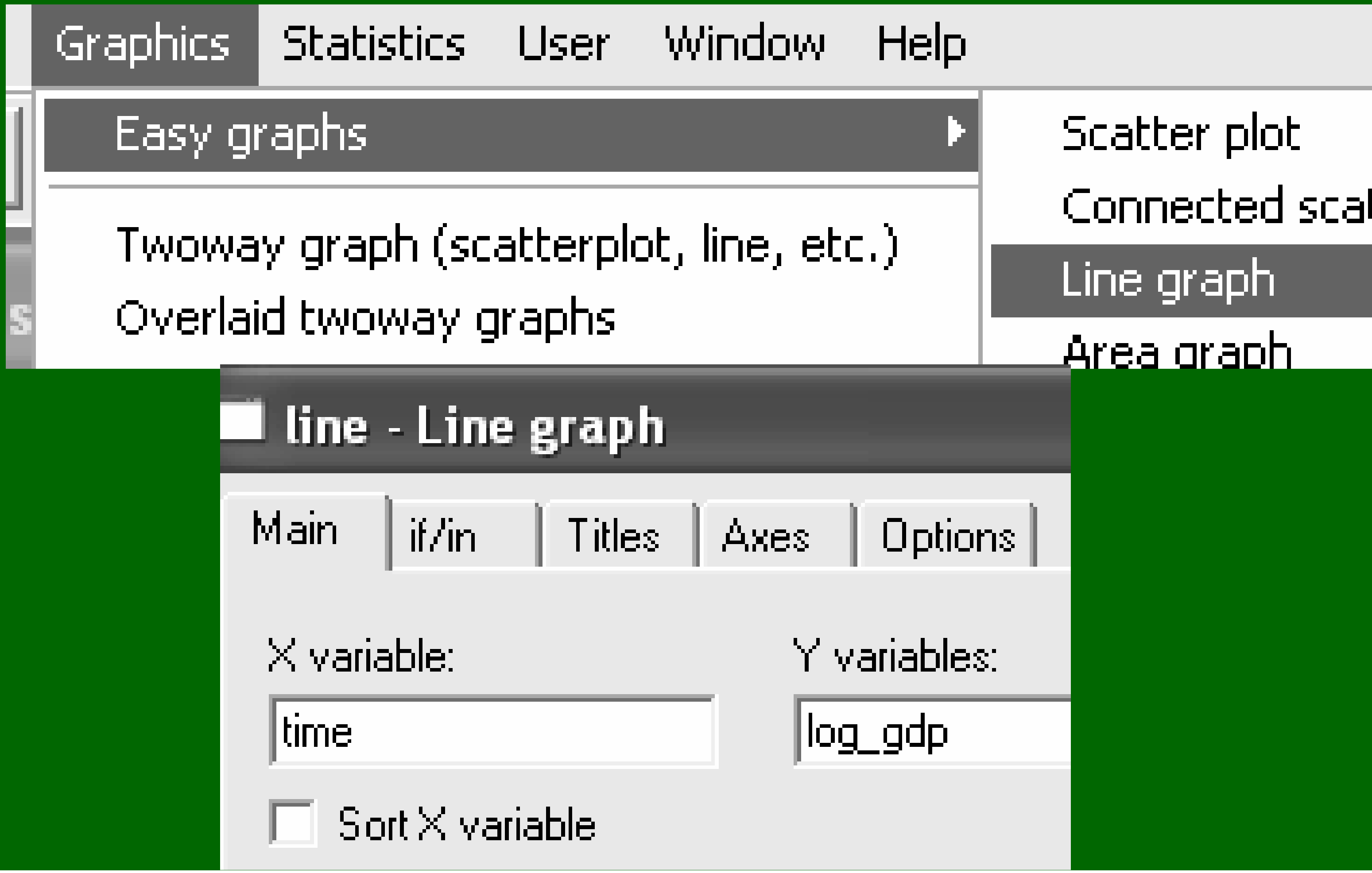
- Convenience
  - taking logs: gen var_name=log(var_name)

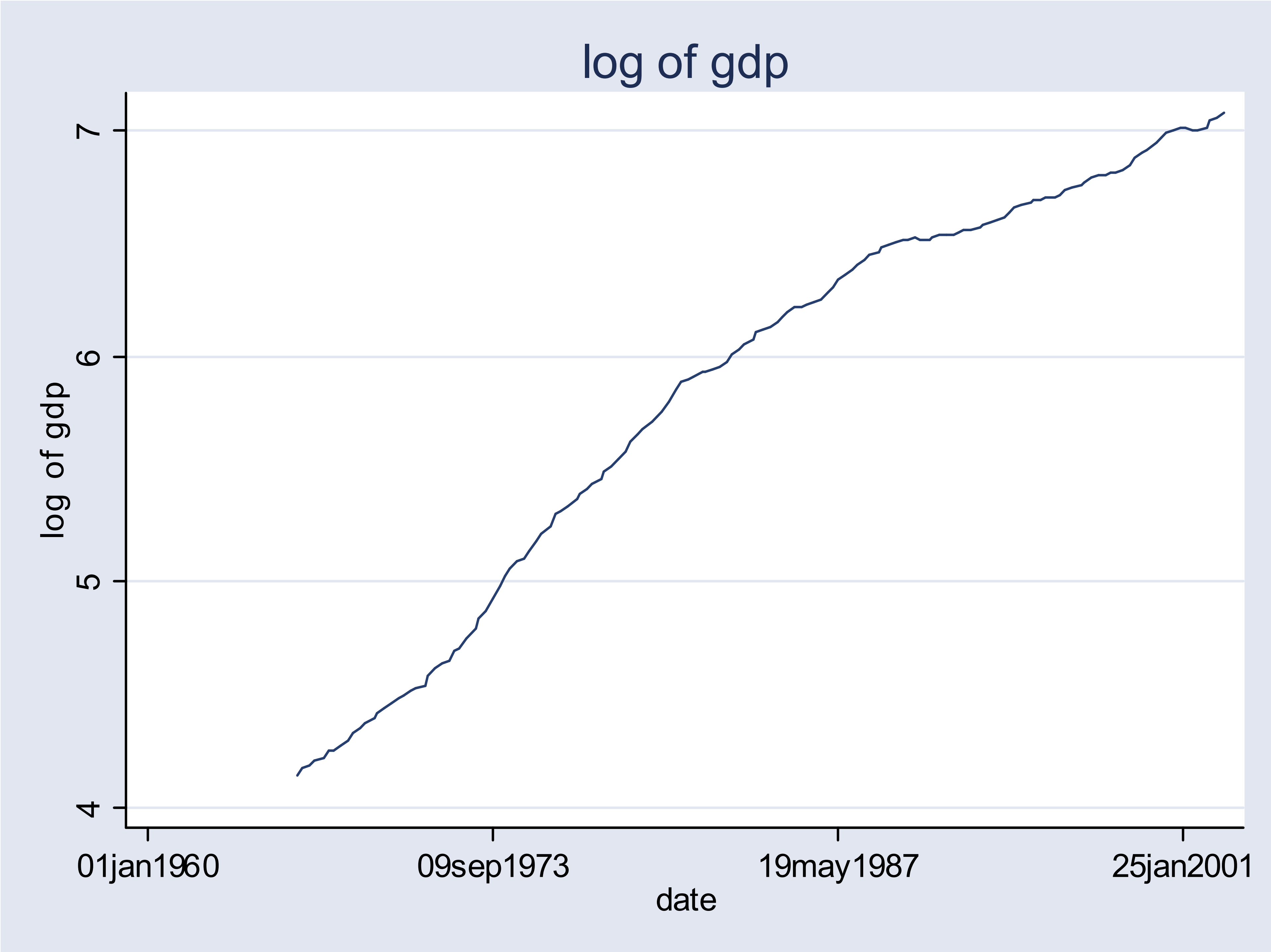
```
gen log_gdp=ln(gdp_sa)

gen log_cons=log(household_cons)

gen log_cpi=log(cpi)
```

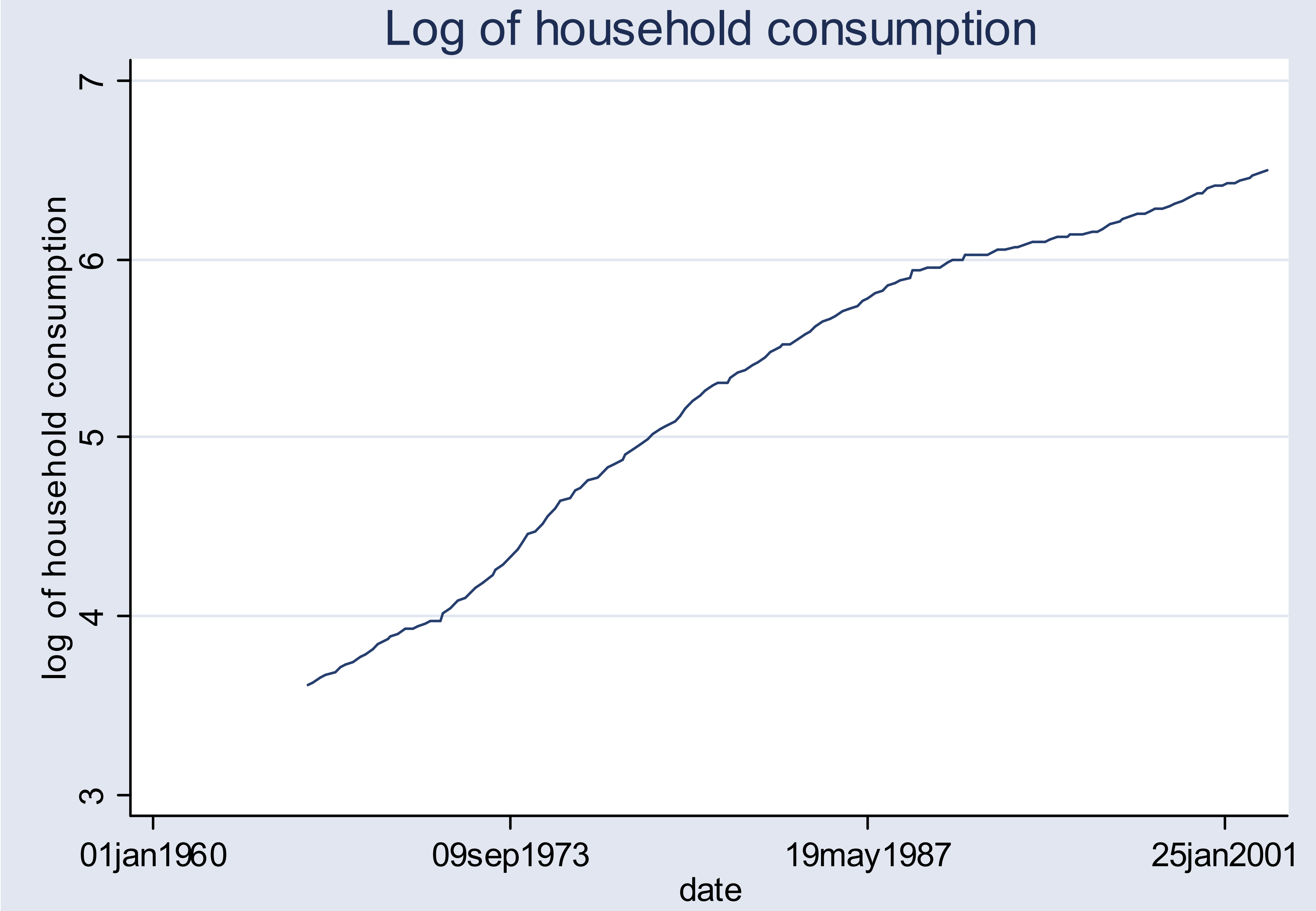
- Differences are in percentage

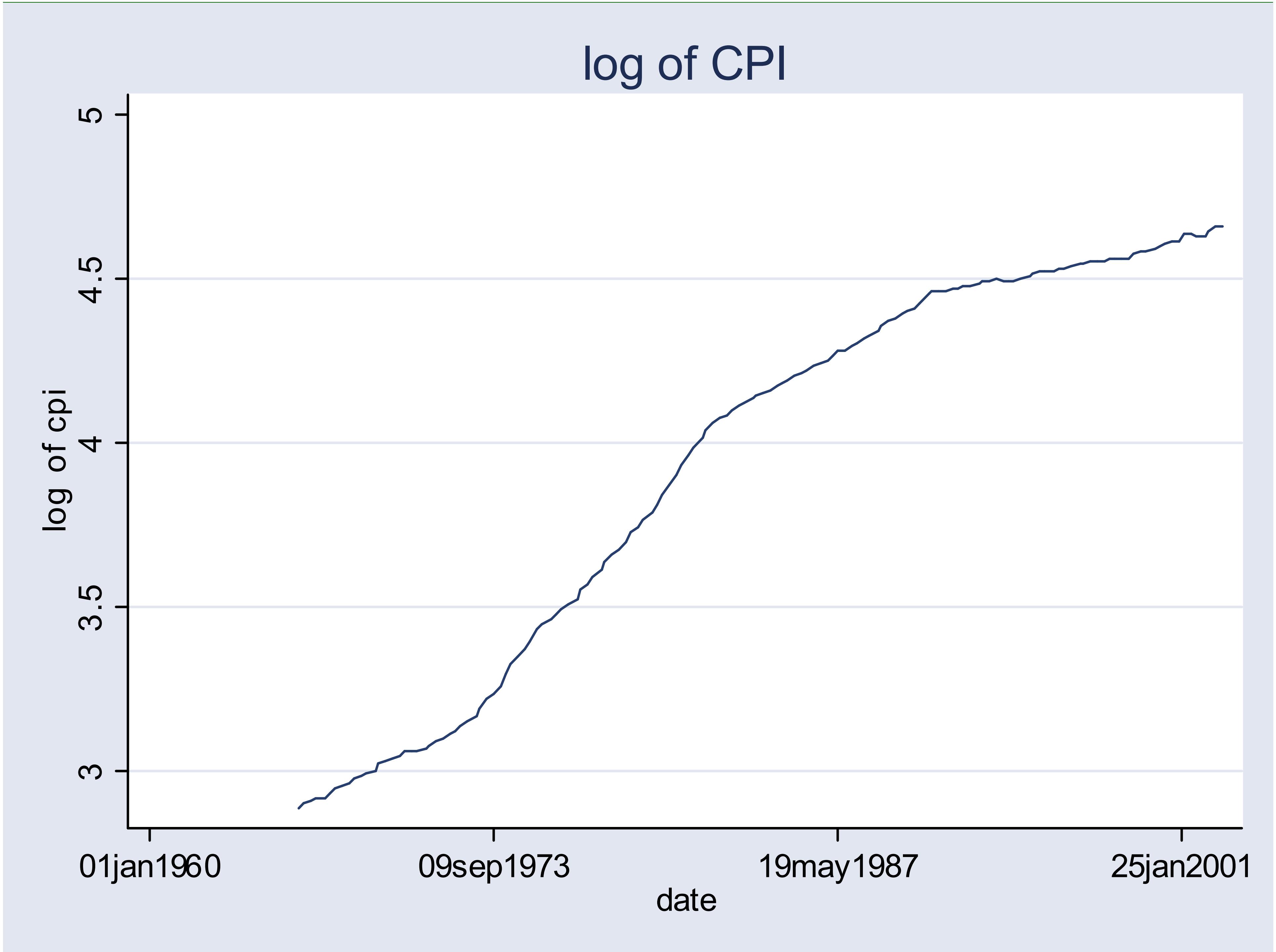
Plots

- Menu: Graphics → easy graphs → line graph
- Follow the wizard…




log of gdp
log of gdp
4
5
6
7
01jan1960
09sep1973
19may1987
25jan2001
date


Log of household consumption
log of household consumption
3
4
5
6
7
01jan1960
09sep1973
19may1987
25jan2001
date



log of CPI
5
4.5
4
3.5
3
log of cpi
01jan1960
09sep1973
19may1987
25jan2001
date

Stationarity

- Data don't look stationary
- Formal test required

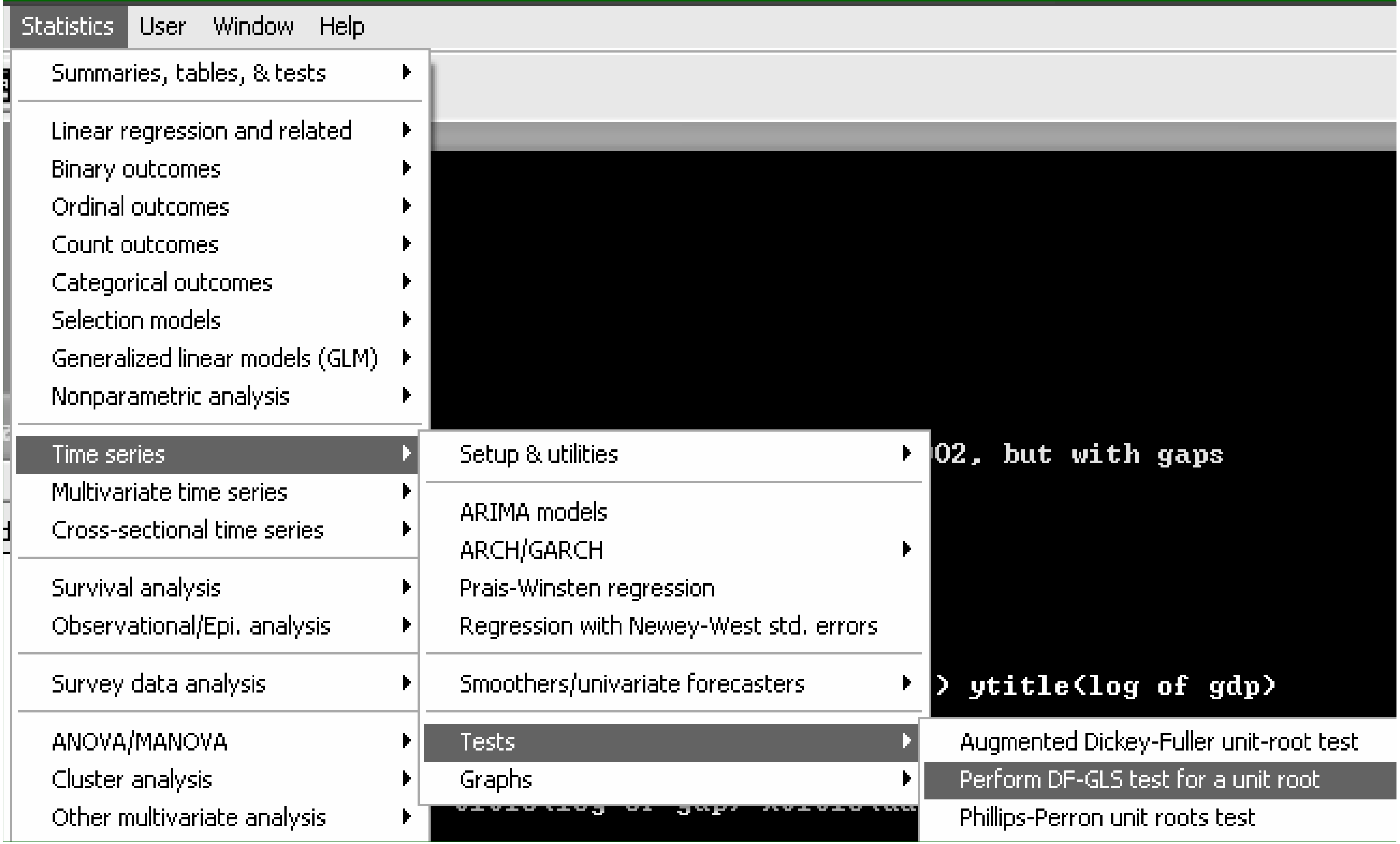
- Common tests (Greene, 636-646):
  - Dickey Fuller:
    - H0: Variable has a unit root
  - Philips Peron
    - H0: Variable has a unit root
  - Dickey Fuller – GLS
    - H0: Variable has a unit root

Testing:

- Menu: Statistics→Time Series→Tests

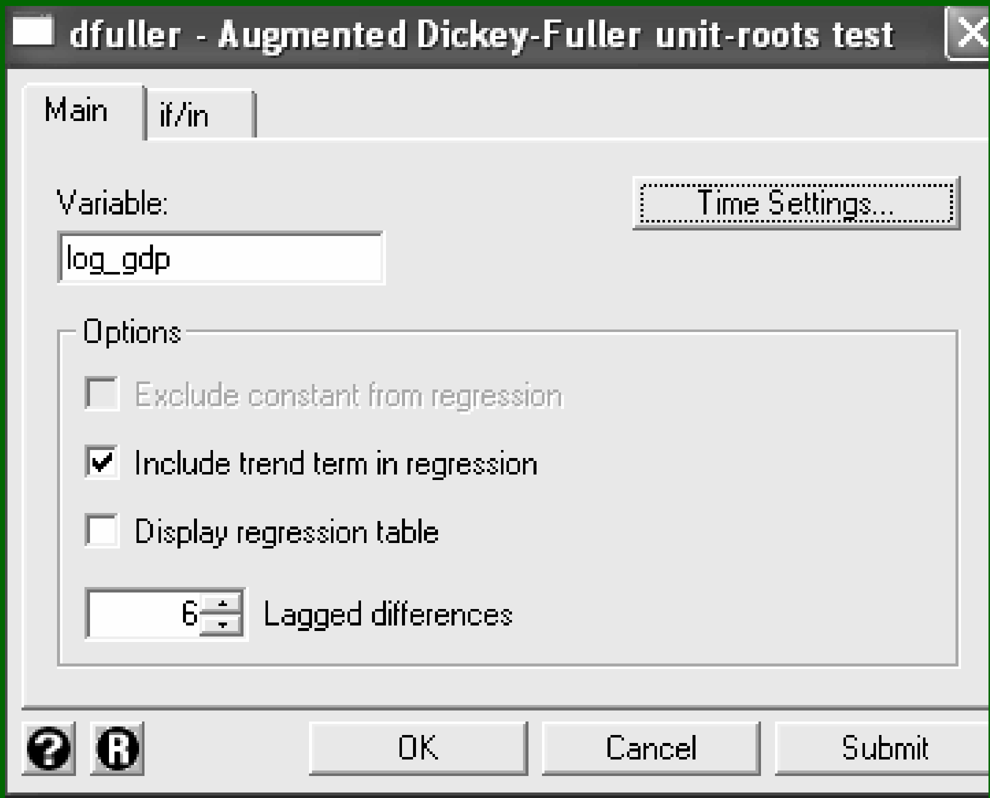
- Choose a test and follow the menu
  - Augmented Dickey Fuller
  - DF-GLS for a Unit root
  - Phillips-Peron unit root

Choosing a test



Running a Test

- Augmented Dickey-Fuller Test
  - 6 lags
  - Including Trend



Result

- **Cannot reject the null at 5%**

Result **Critical Values**

Augmented Dickey-Fuller test for unit root Number of obs = 141

	Test Statistic	Interpolated Dickey-Fuller 1% Critical Value	5% Critical Value	10% Critical Value
Z(t)	-1.074	-4.026	-3.445	-3.145

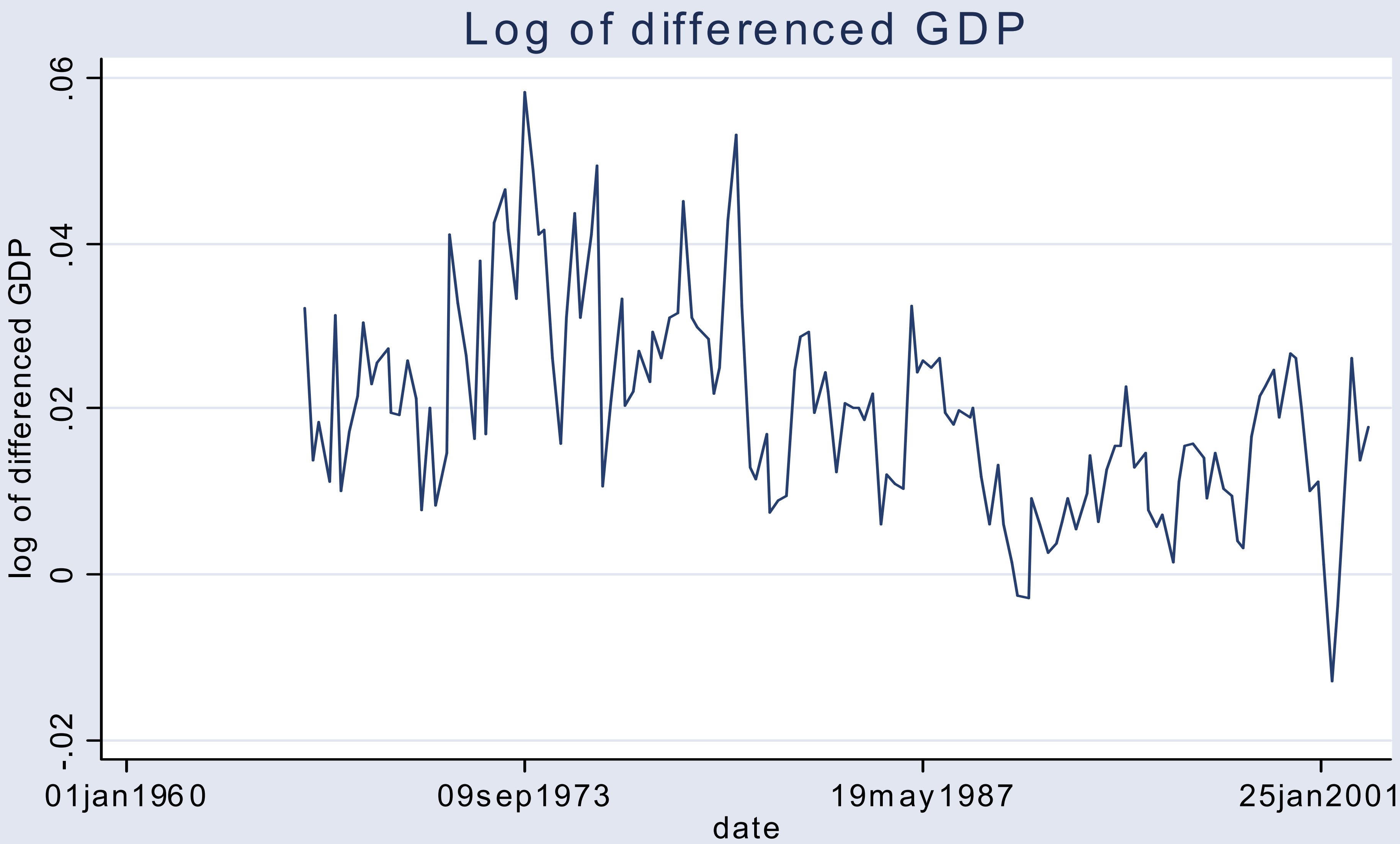
* MacKinnon approximate p-value for Z(t) = 0.9330

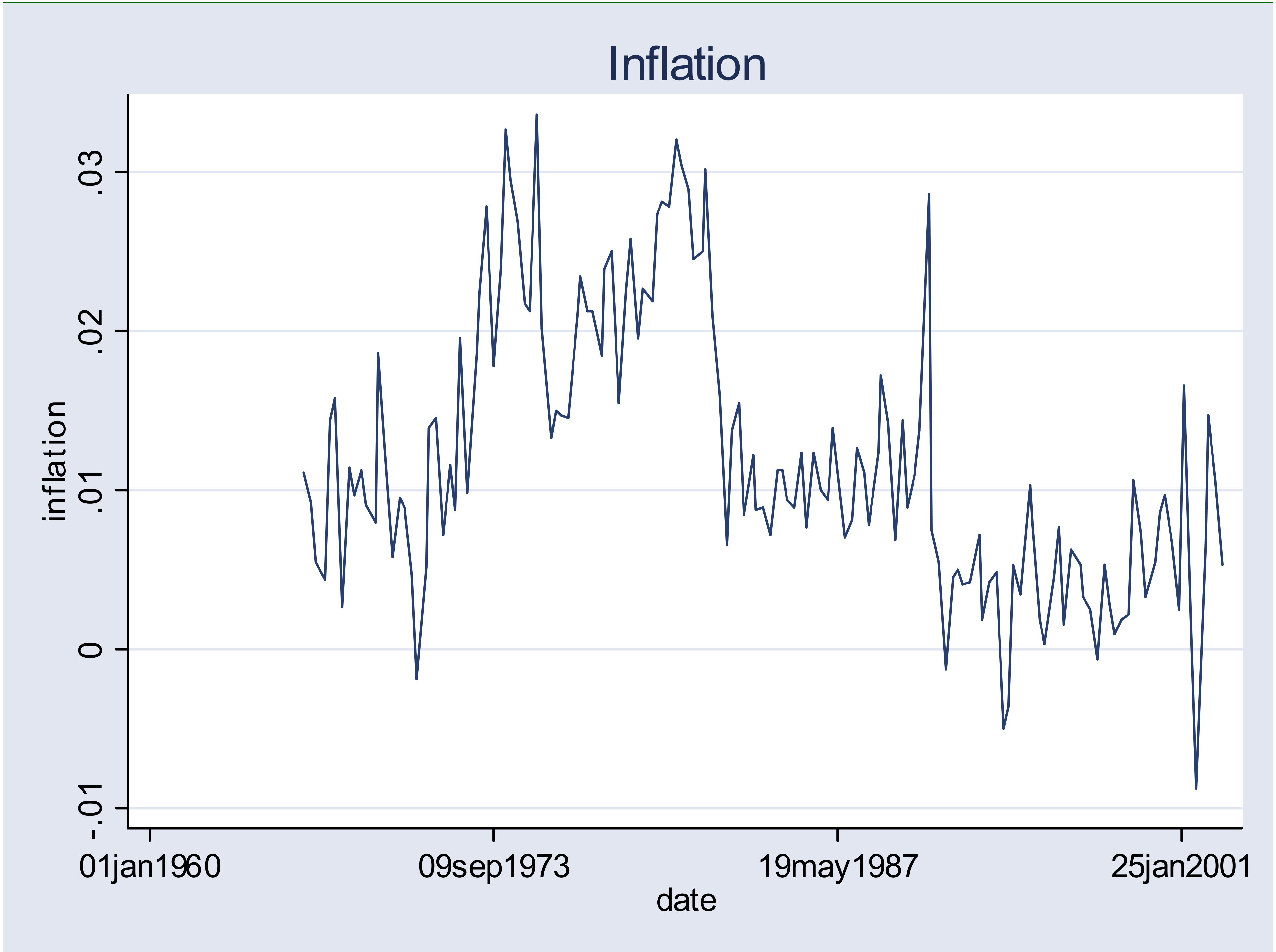
Create First Differences

- **Cannot Reject Unit root: Data is I(1)**
- **Create First Differences of the data**:

```
. gen y=log_gdp-log_gdp[_n-1]
(1 missing value generated)

. gen inflation=log_cpi-log_cpi[_n-1]
(1 missing value generated)

. gen dcons=log_cons-L.log_cons
(1 missing value generated)
```

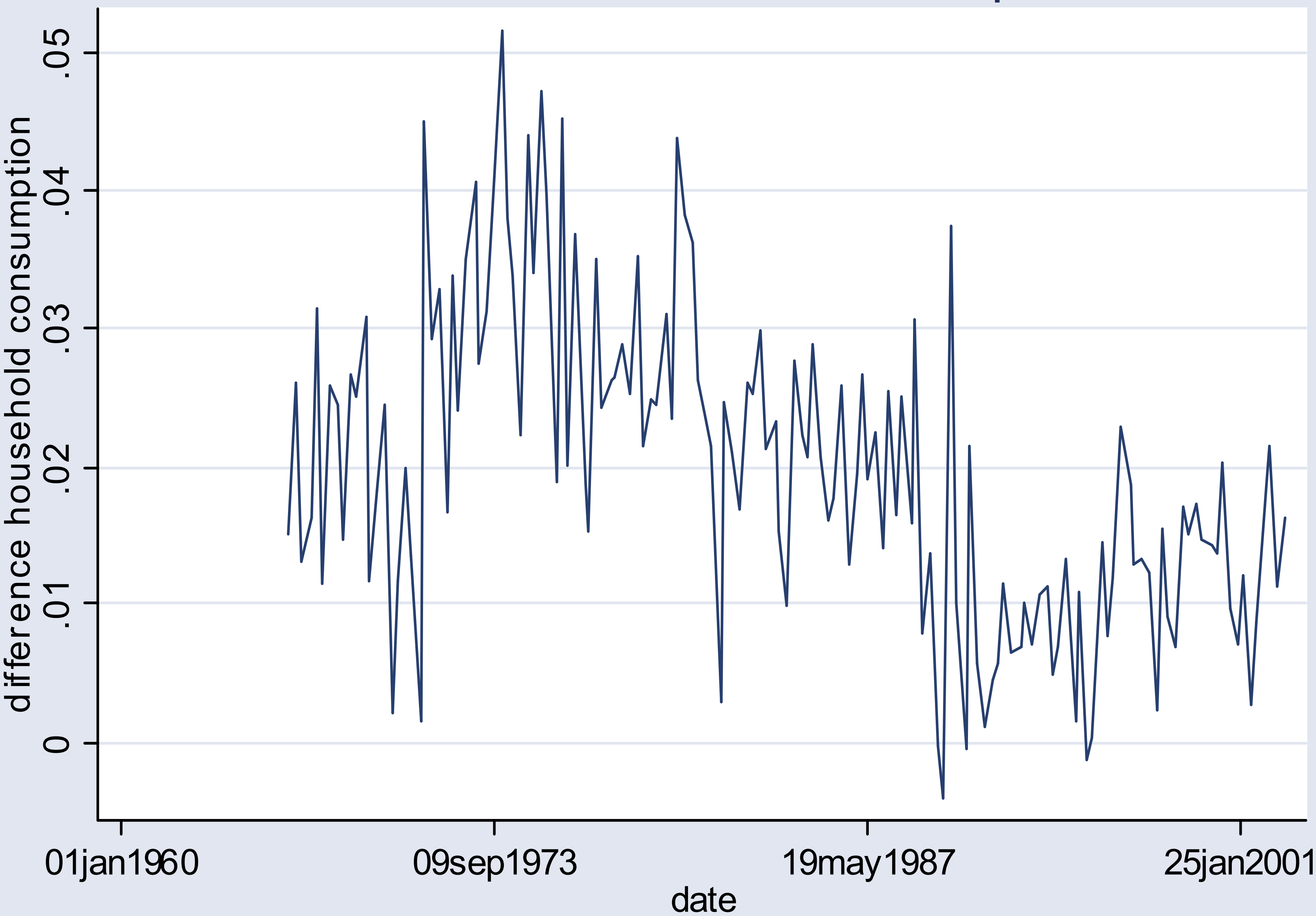
Check the new graphs





Inflation
inflation
-.01
0
.01
.02
.03
01jan1960
09sep1973
19may1987
25jan2001
date

Log differenced household consumption



Is it stationary now? (PP test)

```
. pperron y, lags(6)

Phillips-Perron test for unit root                 Number of obs   =       146
                                                   Newey-West lags =         6

                              ---------- Interpolated Dickey-Fuller ---------
                  Test        1% Critical       5% Critical      10% Critical
               Statistic         Value             Value             Value
------------------------------------------------------------------------------
 Z(rho)         -49.247           -19.953           -13.792           -11.061
 Z(t)            -5.403            -3.495            -2.887            -2.577
------------------------------------------------------------------------------
```

The differenced data seems to be stationary

Granger Causality

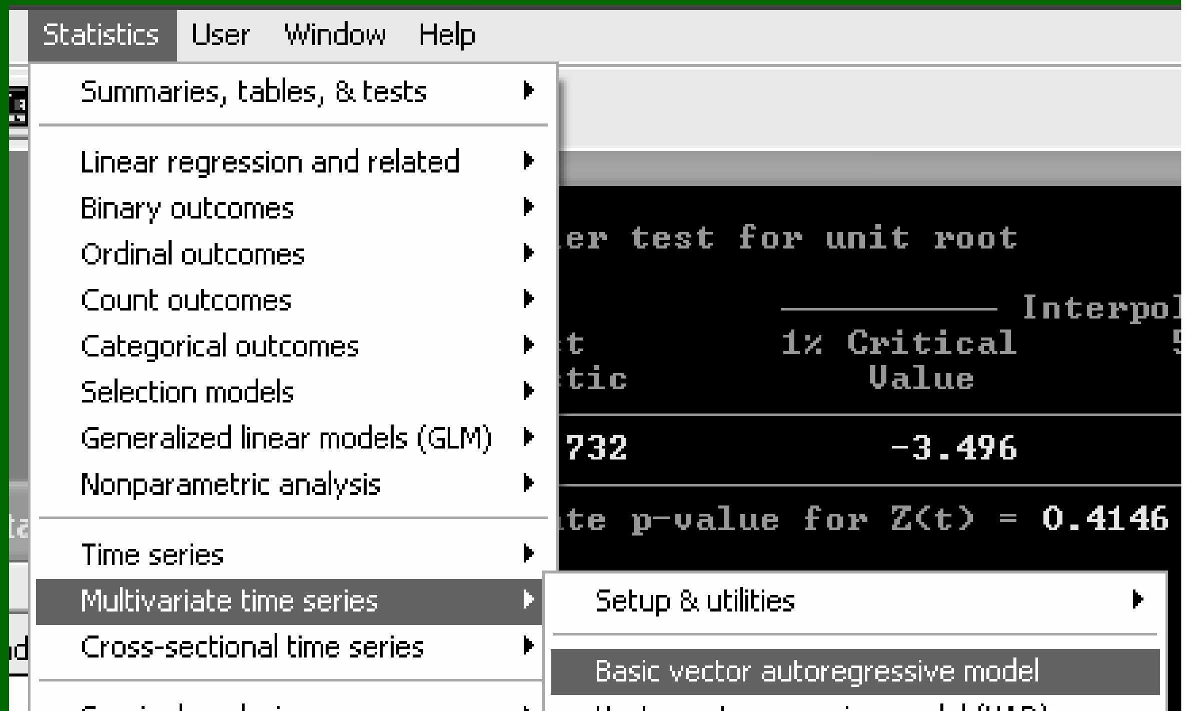
- X Does not Granger Cause Y if:

$$E(y_t \mid y_{t-1}, y_{t-2}, \ldots, x_{t-1}, x_{t-2} \ldots) = E(y_t \mid y_{t-1}, y_{t-2} \ldots)$$

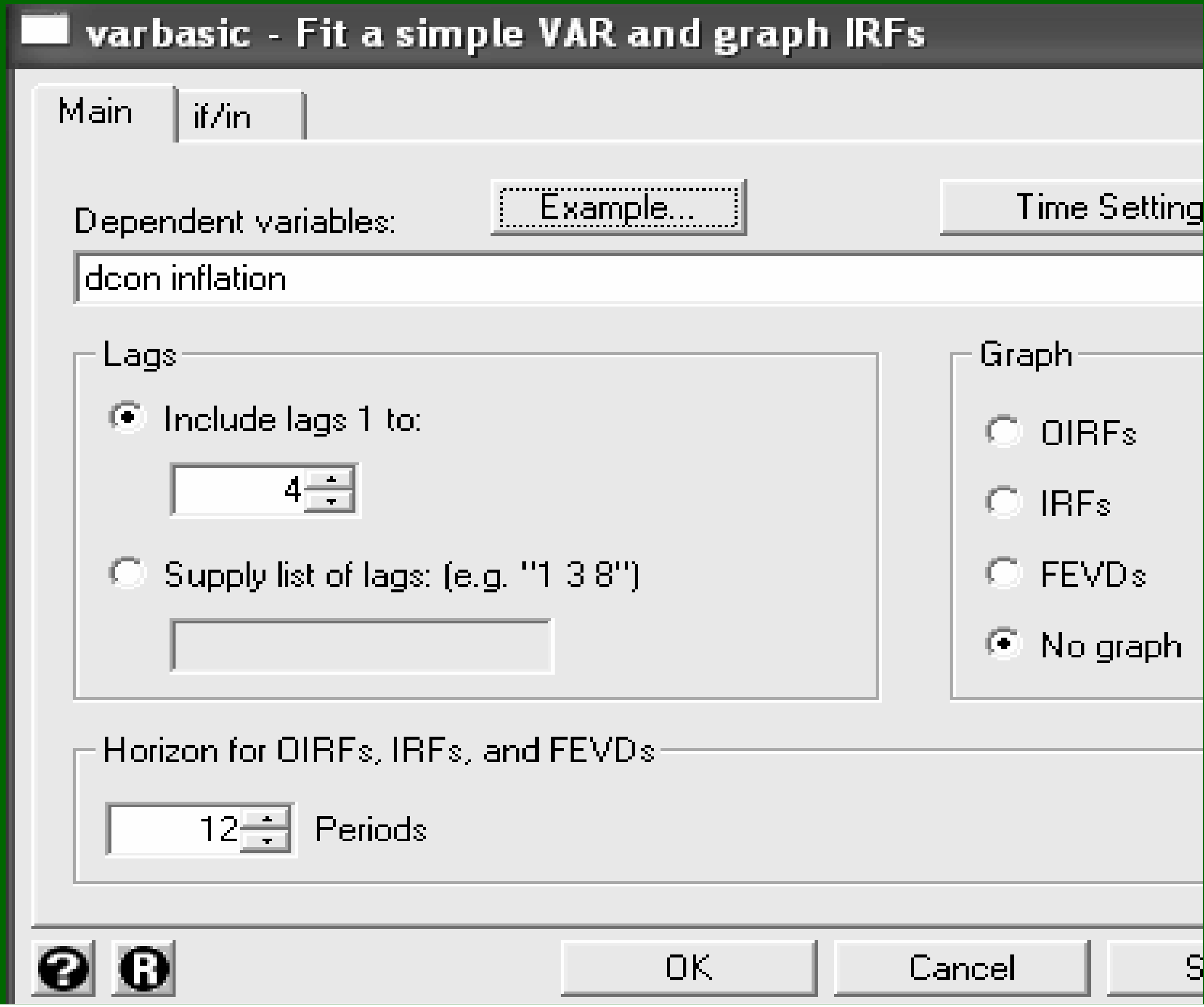
- Test (Greene, p.592):
  - Regress Y on lags of X and Y
  - Regress Y on lags of Y
  - Test if the restricted model is significantly outperformed by the non restricted model
  - Either χ^2 or F test

Granger Test

- **Run simple VAR between the variables of interest**
- **Menu: Statistics→multivariate time series→Basic Vector Autoregression Model**
- **Choose**
  - **Variables**
  - **Lag Length**

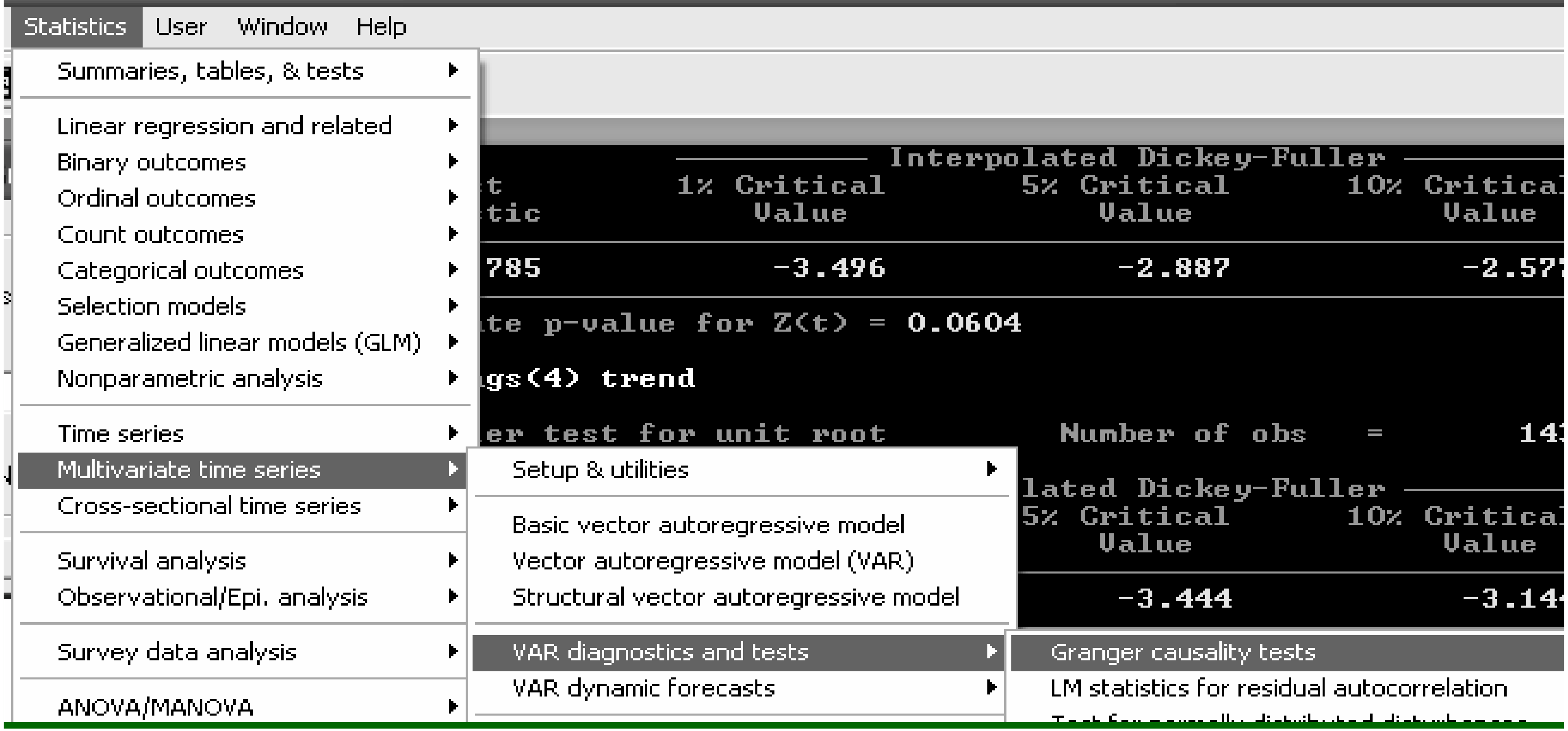


Granger Test: Running VAR



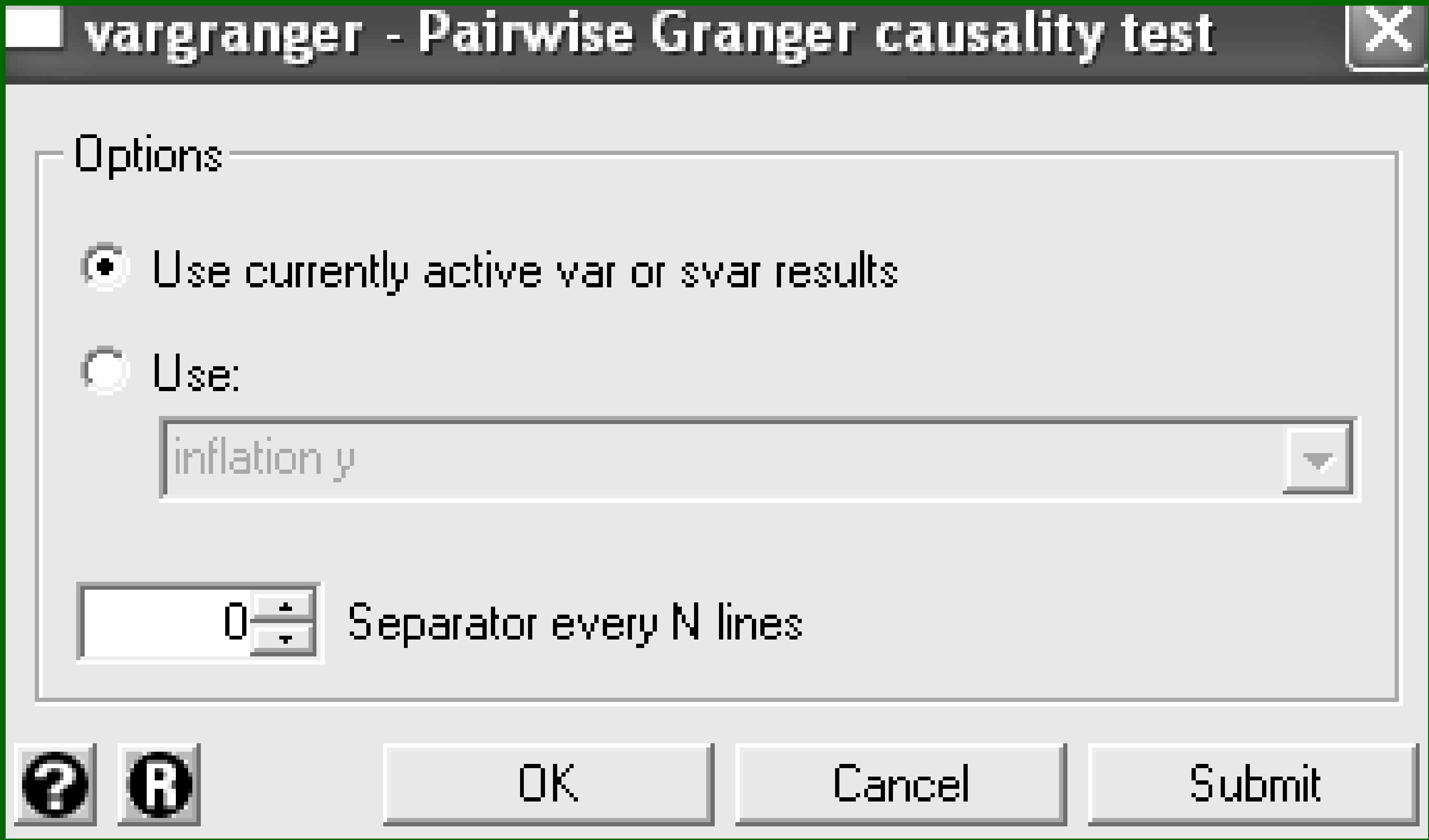
Testing in Stata

- Statistics→multivariate time series→ var diagnostics and tests→ Granger causality test



Granger Test

- Choose variables



Granger Test: Results

- **We can reject that Inflation Granger Cause Household Consumption**
- **We cannot reject that Household Consumption Granger Cause Inflation**

Granger causality Wald tests

Equation	Excluded	chi2	df	Prob > chi2
dcons	inflation	2.4424	4	0.6550
dcons	ALL	2.4424	4	0.6550
inflation	dcons	22.7235	4	0.0001
inflation	ALL	22.7235	4	0.0001

Optimal Lag Length

- Sometimes, we have theory to guide us
- Often, we do not
- Three common tests (Greene, 589):
  - Likelihood Ratio Test (LR)
  - Akaike Information Criterion
  - Bayesian (Schwartz) Information Criterion
-

Likelihood Ratio (LR) test

General to simple approach: Run VAR with p lags. Use the LR test. If the test rejects the null, then stop. Otherwise run p-1 lags and compare with p-2…

$$\lambda = T\left(\ln\left|W_{res}\right| - \ln\left|W_{unres}\right|\right) \rightarrow \chi^{2}\left(M^{2}\right)$$

$$W_{res} - restricted\ \operatorname{cov}ariance\ matrix$$

$$W_{unres} - unrestricted\ \operatorname{cov}ariance\ matrix$$

$$M - Number\ of\ equations$$

Information Criteria

- **Two information Criteria: Akaike (AIC) and Bayesian (BIC). Find the information criteria for lag length 1 to p. Choose the lag length that minimizes the information criteria that you chose.**

$$\lambda = \ln(|W|) + \frac{(pM^2 + M)IC(T)}{T}$$

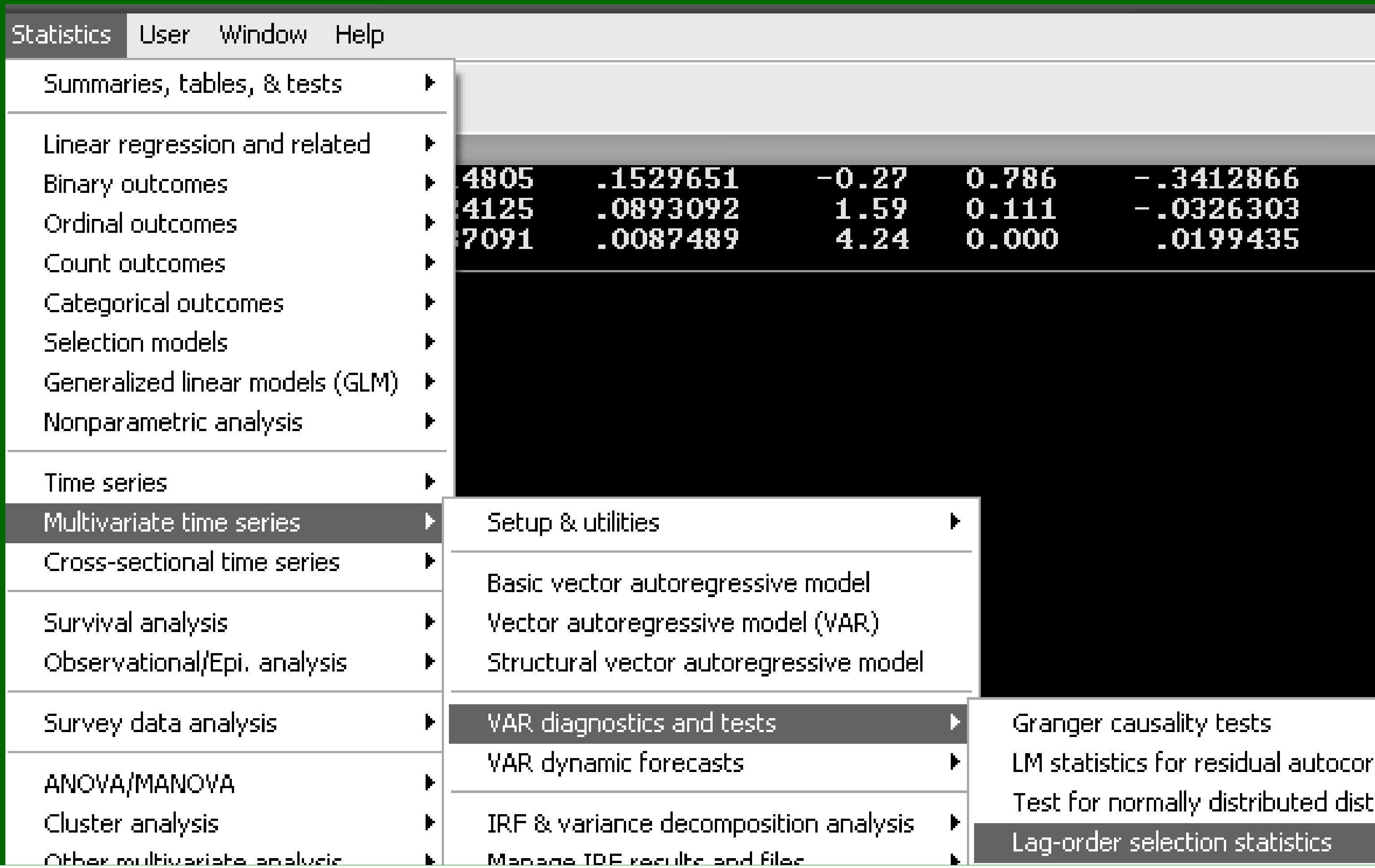
$W - The\ covariance\ Matrix,\ p - number\ of\ lags,$

$T - number\ of\ observations\ M - number\ of\ equations,$

$IC(T) - 2\ for\ AIC,\ T\ for\ BIC,$

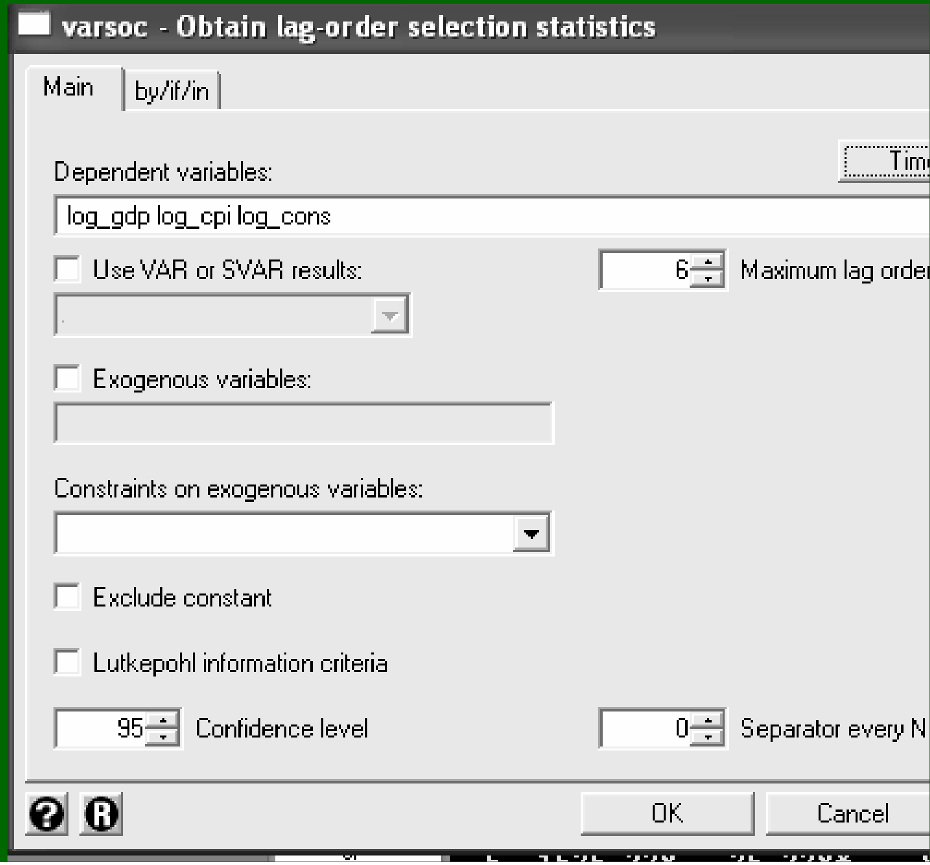
Tests in Stata

- **Menu: Statistics→multivariate time series→ var diagnostics and tests→ Lag-Order Selection statistics**



Running test

- Choose Variables
- Choose maximum lags



Lag Length: Results

LR AIC BIC

Lag	LL	LR	df	p	FPE	AIC	HQIC	SBIC
0	389.636	.	.	.	8.66e-07	-5.44557	-5.42019	-5.38312
1	1482.751	2186.231	9	0.000	2.02e-13	-20.7148	-20.6133	-20.465
2	1533.067	100.631	9	0.000	1.13e-13	-21.2967	-21.1191*	-20.8596*
3	1542.128	18.123	9	0.034	1.13e-13	-21.2976	-21.0438	-20.6731
4	1551.512	18.768	9	0.027	1.13e-13	-21.303	-20.9731	-20.4912
5	1562.889	22.755	9	0.007	1.09e-13	-21.3365	-20.9305	-20.3373
6	1575.778	25.778*	9	0.002	1.03e-13*	-21.3912*	-20.9091	-20.2047

We go with the LR and AIC and say 6 (why not?)

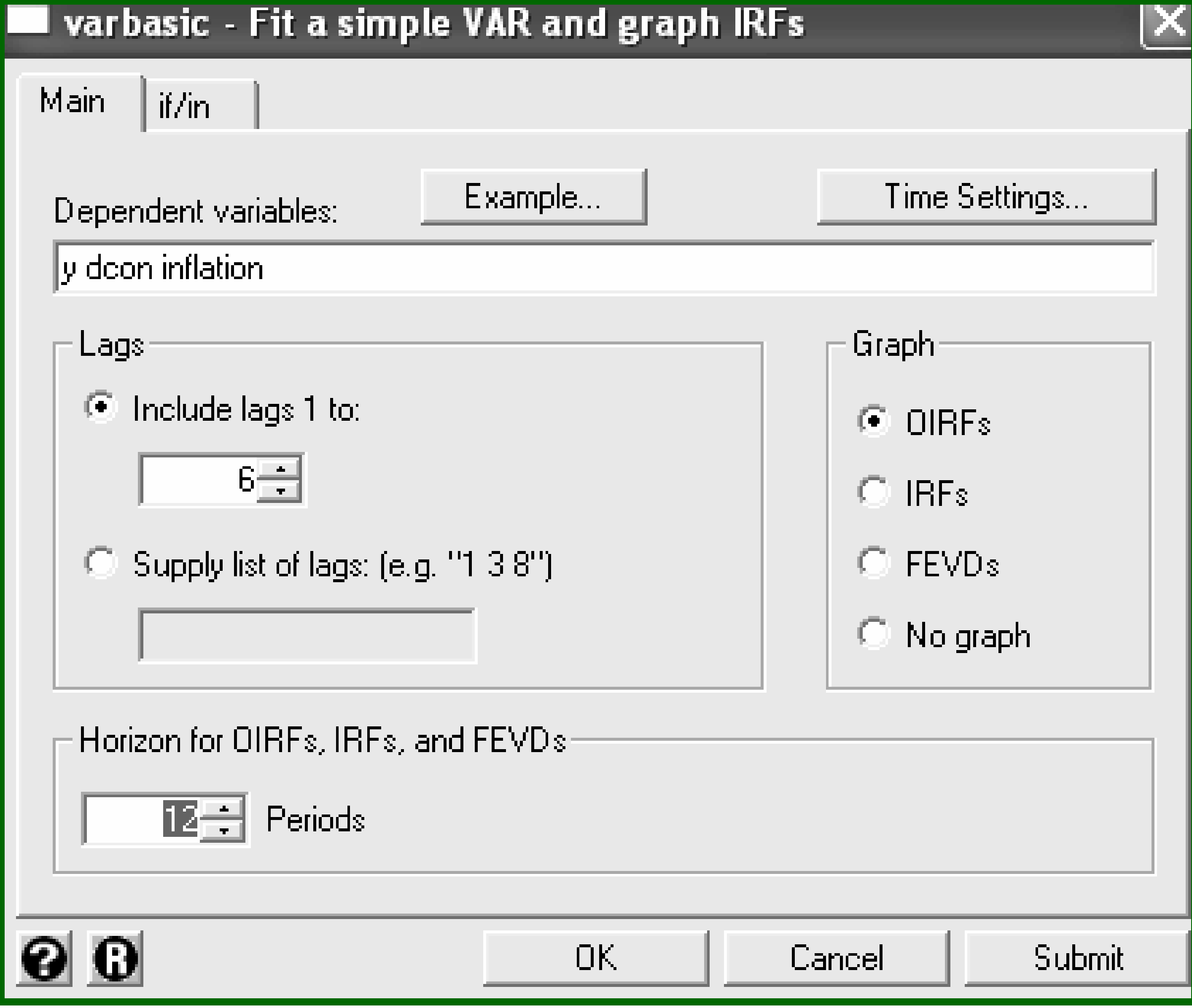
Run Simple VAR

- We run a simple VAR (not structural, no assumptions on order of variables) between Household Consumption, Inflation and GDP
- To do so:
- Menu: Statistics→multivariate time series→Basic Vector Autoregression Model

Simple VAR

- **Choose**
  - **Variables**
  - **Lag Length**
- Choose how to plot the response functions:
  - **Irf (simply uses the covariance matrix, minimum order)**
  - **Orf (orthogonalized the Covariance matrix to set order)**
  - **FEVD: Variance Decomposition Tables (In a graph form)**

Simple VAR


varbasic - Fit a simple VAR and graph IRFs
Main
if/in
Dependent variables:
Example...
Time Settings...
y dcon inflation
Lags
Include lags 1 to:
6
Supply list of lags: (e.g. "1 3 8")
Graph
OIRFs
IRFs
FEVDs
No graph
Horizon for OIRFs, IRFs, and FEVDs
12
Periods
OK
Cancel
Submit

Results: Table of Coefficients

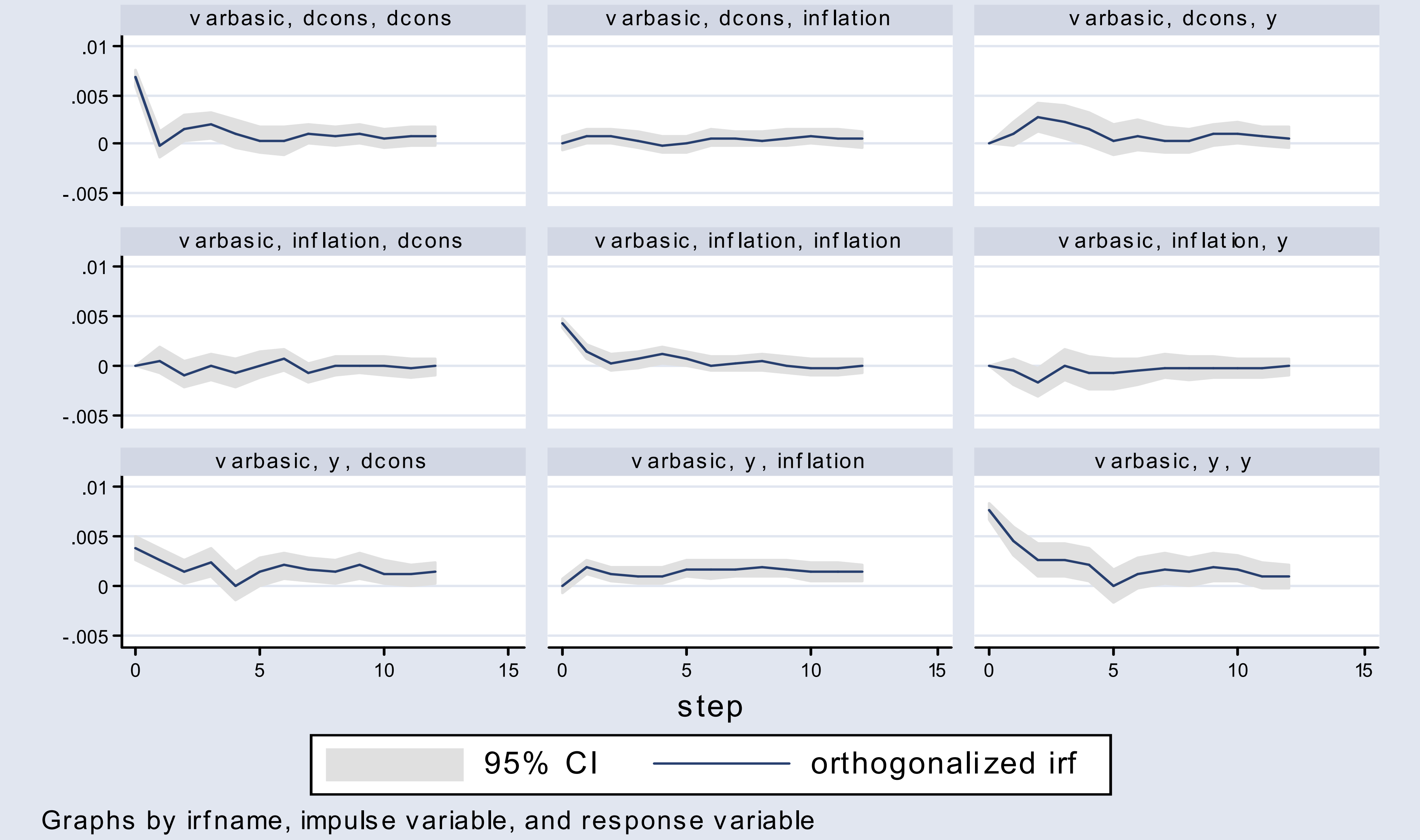
Model lag order selection statistics

FPE	AIC	HQIC	SBIC	LL	Det(Sigma_ml)
1.144e-13	-21.290209	-20.8058	-20.098157	1557.9597	5.073e-14

	Coef.	Std. Err.	z	P>|z|	[95% Conf. Interval]	
y						
y						
L1	.5240169	.094267	5.56	0.000	.3392569	.7087769
L2	-.1716709	.1008584	-1.70	0.089	-.3693498	.026008
L3	.1312994	.0988067	1.33	0.184	-.0623581	.3249569
L4	-.0964825	.098346	-0.98	0.327	-.2892371	.0962721
L5	-.0531829	.0987135	-0.54	0.590	-.2466577	.1402919
L6	.2467331	.0968582	2.55	0.011	.0568944	.4365717
dcons						

—more—

Impulse Response Function



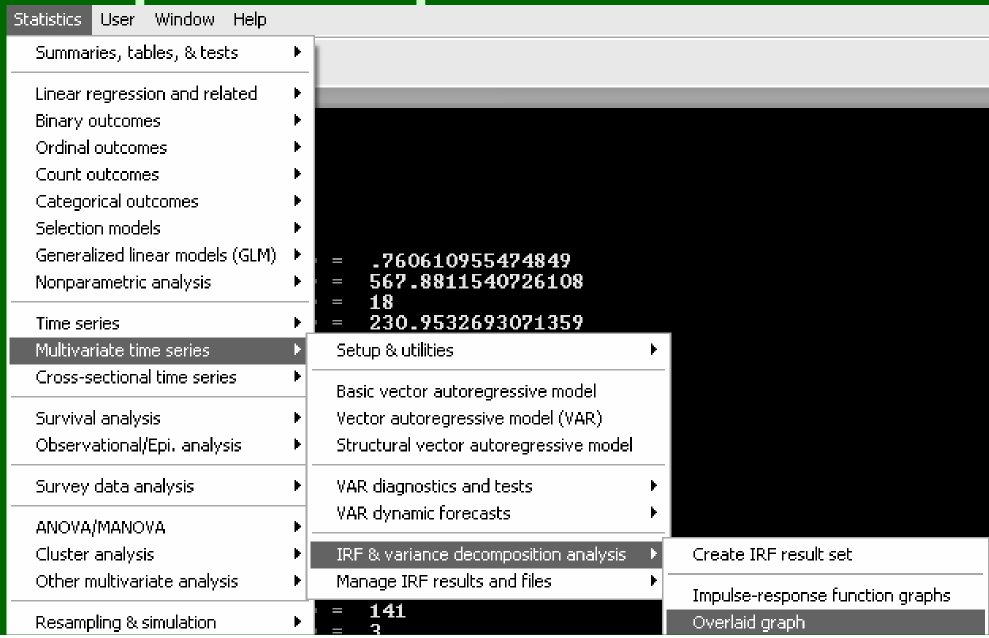
Simple VAR: Variance Decomposition Table

step	(9) fevd	(9) Lower	(9) Upper
0	0	0	0
1	.997849	.982572	1.01313
2	.821666	.707423	.935909
3	.758885	.620901	.896868
4	.729669	.579145	.880193
5	.714909	.555406	.874413
6	.653683	.482896	.82447
7	.594682	.41942	.769945
8	.543658	.364066	.723249
9	.501872	.316881	.686863
10	.463959	.277265	.650654
11	.44034	.252745	.627934

```
5% lower and upper bounds reported
(1) irfname = varbasic, impulse = y, and response = y
(2) irfname = varbasic, impulse = y, and response = dcons
(3) irfname = varbasic, impulse = y, and response = inflation
(4) irfname = varbasic, impulse = dcons, and response = y
(5) irfname = varbasic, impulse = dcons, and response = dcons
(6) irfname = varbasic, impulse = dcons, and response = inflation
(7) irfname = varbasic, impulse = inflation, and response = y
(8) irfname = varbasic, impulse = inflation, and response = dcons
(9) irfname = varbasic, impulse = inflation, and response = inflation
```

Generating After Estimation

- **generate after estimation:**
  - **Choose:**
    - **Menu: Statistics→Multivariate time series → IRF & Variance Decomposition Analysis**
    - **Choose the table or impulse response function that you need**



To get the results

- If you want to use some of the results:
    - Coefficients
    - Number of observations
    - Etc…
  - Stata keeps them under the ereturn command

  - To get them type e(variable_name)
  - To see all the variables that you can choose from:
    - ereturn list

Examples

```
. ereturn list

scalars:
                 e(r2_3) =   .760610955474849
                 e(ll_3) =   567.8811540726108
                e(df_m3) =   18
               e(chi2_1) =   230.9532693071359
               e(chi2_2) =   161.69054276 4445
               e(chi2_3) =   447.9993849956065
                   e(ll) =   1557.959742978141
           e(detsig_ml) =   5.07253323216e-14
```

```
.
. matrix list e(Sigma)

symmetric e(Sigma)[3,3]
                   y       dcons   inflation
        y  .00005872
    dcons  .00002995   .00006185
inflation  9.504e-07   1.555e-06   .00001859
```

More than simple VAR

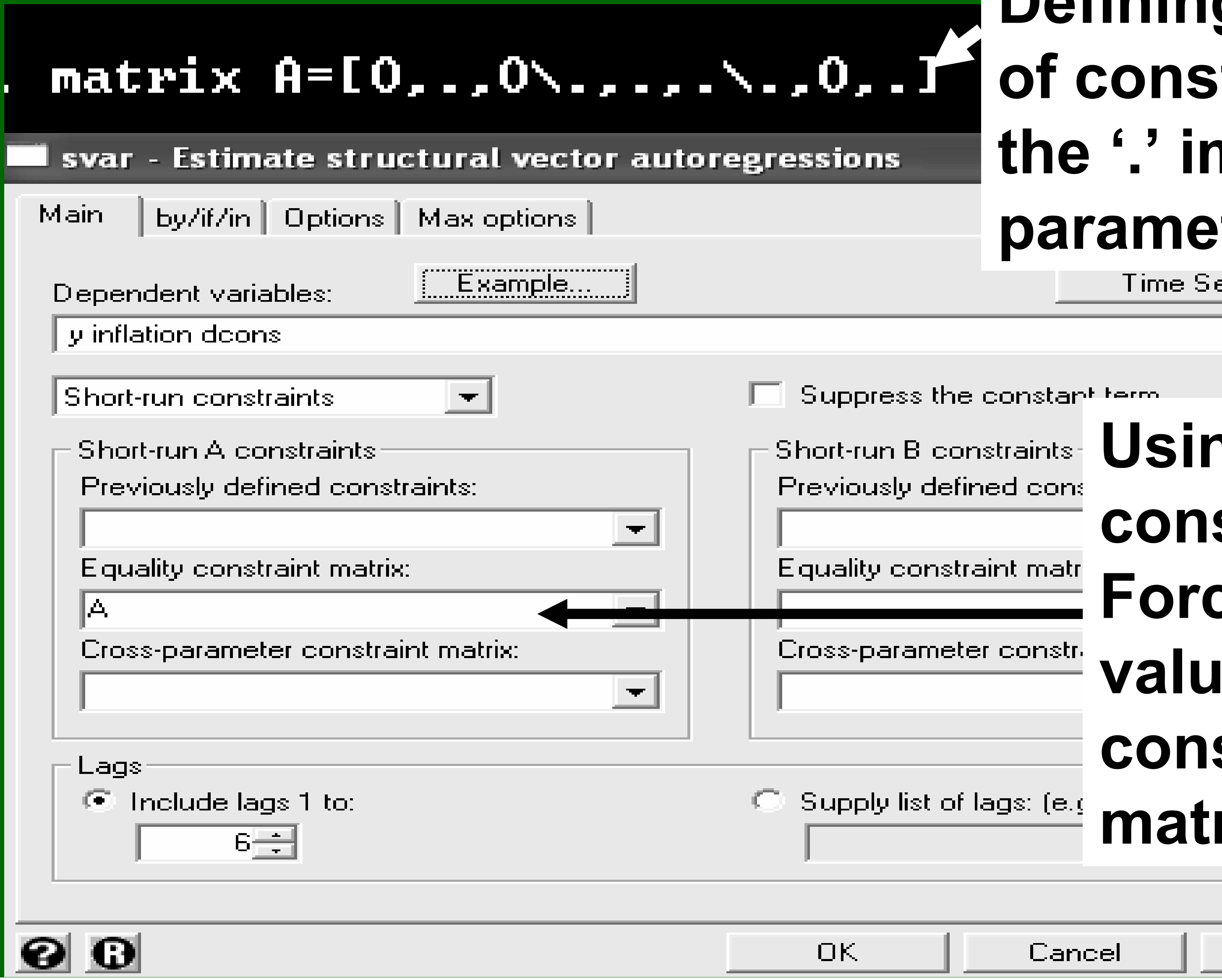
- **More than a simple VAR**:
  - Adding Exogenous Variables
  - Constraining blocks of variables to equal zero
- **Use Menu: Statistics→multivariate time series→ Vector Autoregression Model**

- **Generating Impulse Responses:**
  - **Menu: Statistics→Multivariate time series → IRF & Variance Decomposition Analysis**

More than simple VAR

- **Adding constraints on the A or B matrix**
  - A: y Matrix, B: errors matrix
  - Short and long run constraints
- **skip lags**
- **Menu: Statistics→multivariate time series→ Structural Autoregression Model**
- **Stata runs the VAR with the restrictions**
- **Caveat 1: Too many constraints can lead to failures in the convergence process**
- **Caveat 2: You need enough constraints to allow identification.**

Structural VAR



Defining a matrix of constraints: the '.' imply a free parameter

Using the constraints: Forcing the values in the constrained matrix

Structural VAR: Results

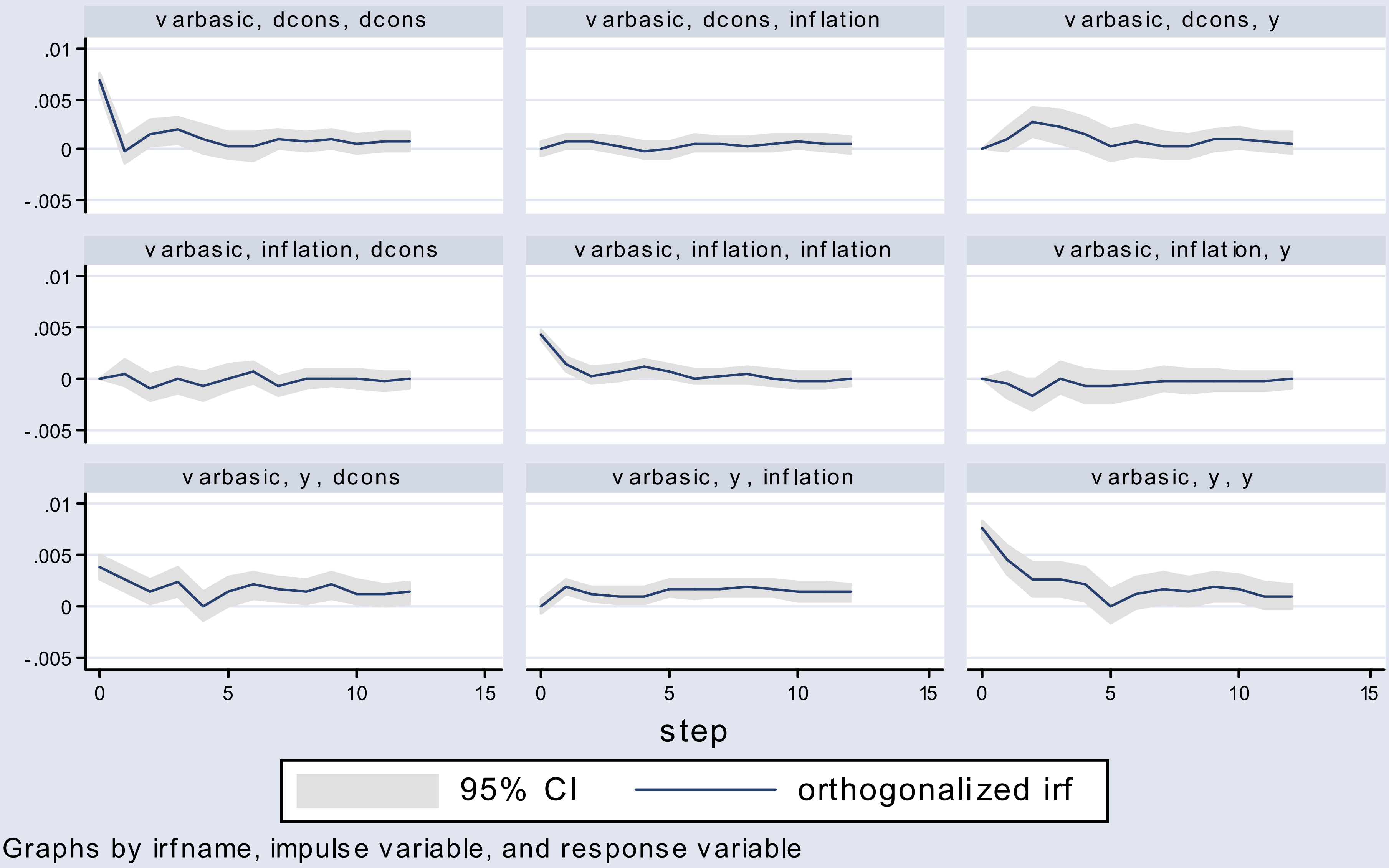
Equation	Obs	Parms	RMSE	R-sq	chi2	P
y	141	19	.008238	0.6209	230.9533	0.0000
inflation	141	19	.004635	0.7606	447.9994	0.0000
dcons	141	19	.008455	0.5342	161.6905	0.0000

VAR Model lag order selection statistics

FPE	AIC	HQIC	SBIC	LL	Det(Sigma_ml)
1.144e-13	-21.290209	-20.8058	-20.098157	1557.9597	5.073e-14

	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
a_2_1						
_cons	22.33818	270.0692	0.08	0.934	-506.9877	551.664
a_3_1						
_cons	148.7274	41.47564	3.59	0.000	67.43667	230.0182
a_1_2						
_cons	231.9367	13.81163	16.79	0.000	204.8664	259.007
a_2_2						
_cons	-10.76834	19.54312	-0.55	0.582	-49.07215	27.53546
a_2_3						

Structural VAR: Results



Structural VARs

- **Structural VAR: VAR that is the result of a structural model**

- **Goal: Obtaining the Structural parameters out of the Estimated Reduced Form**

- **Required: Number of Constraints**

Model: Inflation and GDP

- **Assume we have a simple model of the form:**

$$y_t = \alpha_0 + \alpha_1 y_{t-1} + \alpha_2 \pi_{t-1} + \varepsilon_t$$

$$\pi_t = \beta_0 + \beta_1 y_t + \beta_2 y_t + \beta_3 \pi_{t-1} + \upsilon_t$$

$$y_t - GDP$$

$$\pi_t - \inf lation$$

$$\upsilon_t,\ \varepsilon_t - White\ noise,\ independent\ random\ \text{var}iables$$

We can write it:

$$y_t = \alpha_0 + \alpha_1 y_{t-1} + \alpha_2 \pi_{t-1} + \varepsilon_t$$

$$\pi_t - \beta_1 y_t = \beta_0 + \beta_2 y_t + \beta_3 \pi_{t-1} + \upsilon_t$$

In Matrix Form:

$$\begin{pmatrix} 1 & 0 \\ -\beta_1 & 1 \end{pmatrix}\begin{pmatrix} y_t \\ \pi_t \end{pmatrix} = \begin{pmatrix} \alpha_{0_t} \\ \beta_0 \end{pmatrix} + \begin{pmatrix} y_{t-1} \\ \pi_{t-1} \end{pmatrix} + \begin{pmatrix} \varepsilon_t \\ \upsilon_t \end{pmatrix}$$

OR:

$$\begin{pmatrix} y_t \\ \pi_t \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -\beta_1 & 1 \end{pmatrix}^{-1}\begin{pmatrix} \alpha_{0_t} \\ \beta_0 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ -\beta_1 & 1 \end{pmatrix}^{-1}\begin{pmatrix} y_{t-1} \\ \pi_{t-1} \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ -\beta_1 & 1 \end{pmatrix}^{-1}\begin{pmatrix} \varepsilon_t \\ \upsilon_t \end{pmatrix}$$

Inverting the Matrix gives

$$\begin{pmatrix} 1 & 0 \\ -\beta_1 & 1 \end{pmatrix}^{-1} = \begin{pmatrix} 1 & 0 \\ \beta_1 & 1 \end{pmatrix}$$

So we can substitute this in the equations:

We find:

$$y_t = \alpha_0 + \alpha_1 y_{t-1} + \alpha_2 \pi_{t-1} + \varepsilon_t$$

$$\pi_t = (\beta_1 \alpha_0 + \beta_0) + (\beta_1 \alpha_1 + \beta_2) y_{t-1} + (\beta_1 \alpha_2 + \beta_3) \pi_{t-1} + (\beta_1 \varepsilon_t + \upsilon_t)$$

So we can write in VAR form:

$$y_t = \alpha_0 + \alpha_1 y_{t-1} + \alpha_2 \pi_{t-1} + \varepsilon_t$$

$$\pi_t = \theta_0 + \theta_1 y_{t-1} + \theta_2 \pi_{t-1} + \eta_t$$

Almost there

- **After estimating the VAR we can find:**

$$\left(\beta_1\alpha_0 + \beta_0\right) = \theta_0$$
$$\left(\beta_1\alpha_1 + \beta_2\right) = \theta_1$$
$$\left(\beta_1\alpha_2 + \beta_3\right) = \theta_2$$

So we have three equations and four unknowns…

Hakuna Matata

- **We also have the covariance matrix:**

$$\begin{pmatrix} \sigma_{\varepsilon,\varepsilon} & \sigma_{\varepsilon,\eta} \\ \sigma_{\varepsilon,\eta} & \sigma_{\eta,\eta} \end{pmatrix} = \begin{pmatrix} \sigma_{\varepsilon,\varepsilon} & \beta_1 \sigma_{\varepsilon,\varepsilon} \\ \beta_1 \sigma_{\varepsilon,\varepsilon} & \left(\beta_1 \sigma_{\varepsilon,\varepsilon} + \upsilon_t\right)^2 \end{pmatrix}$$

- **So we have a fourth equation:**

$$\beta_1 \sigma_{\varepsilon,\varepsilon} = \sigma_{\varepsilon,\eta}$$

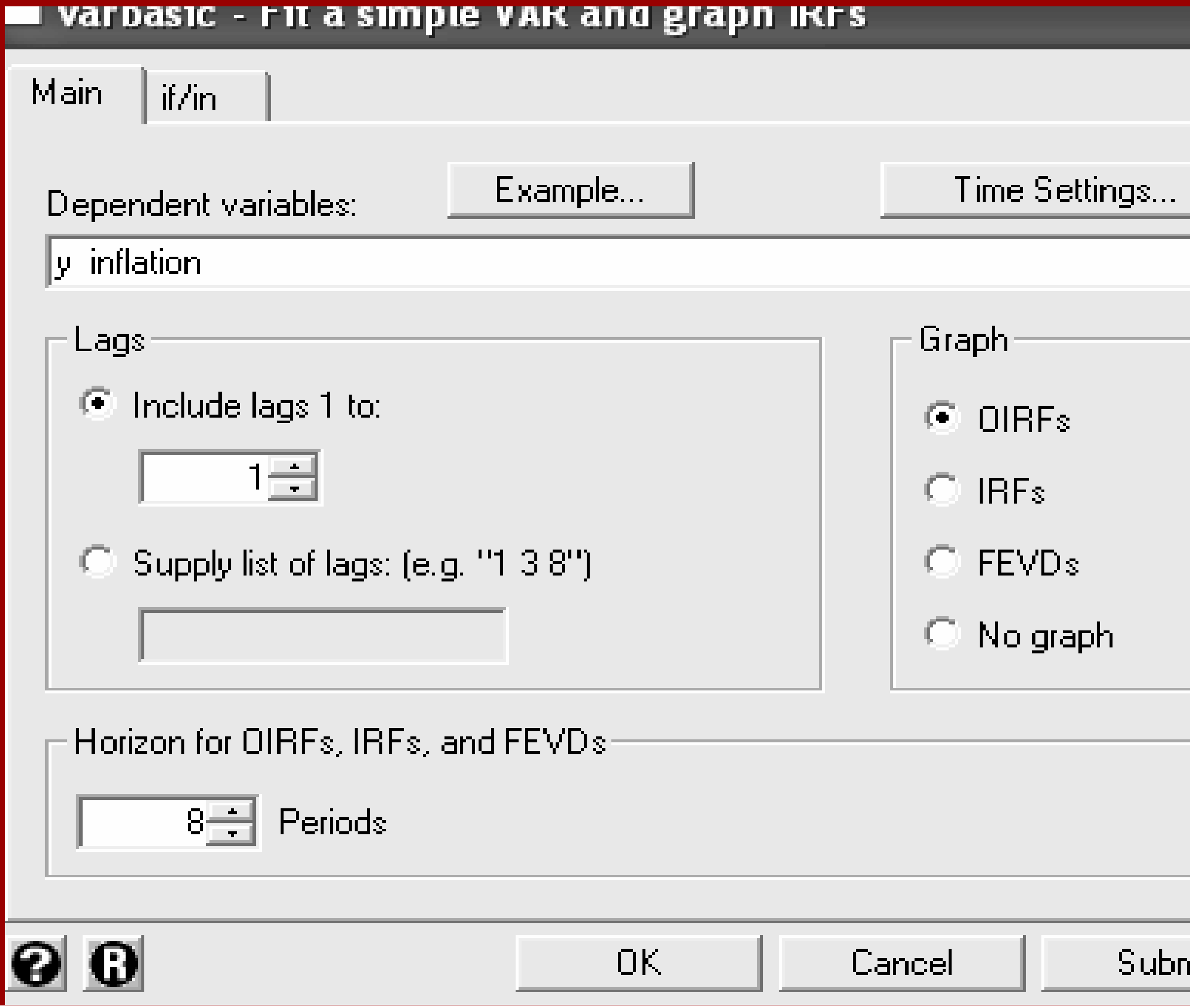
Run the VAR

- **Note that because we assume that the "real" covariance matrix has the triangular form:**

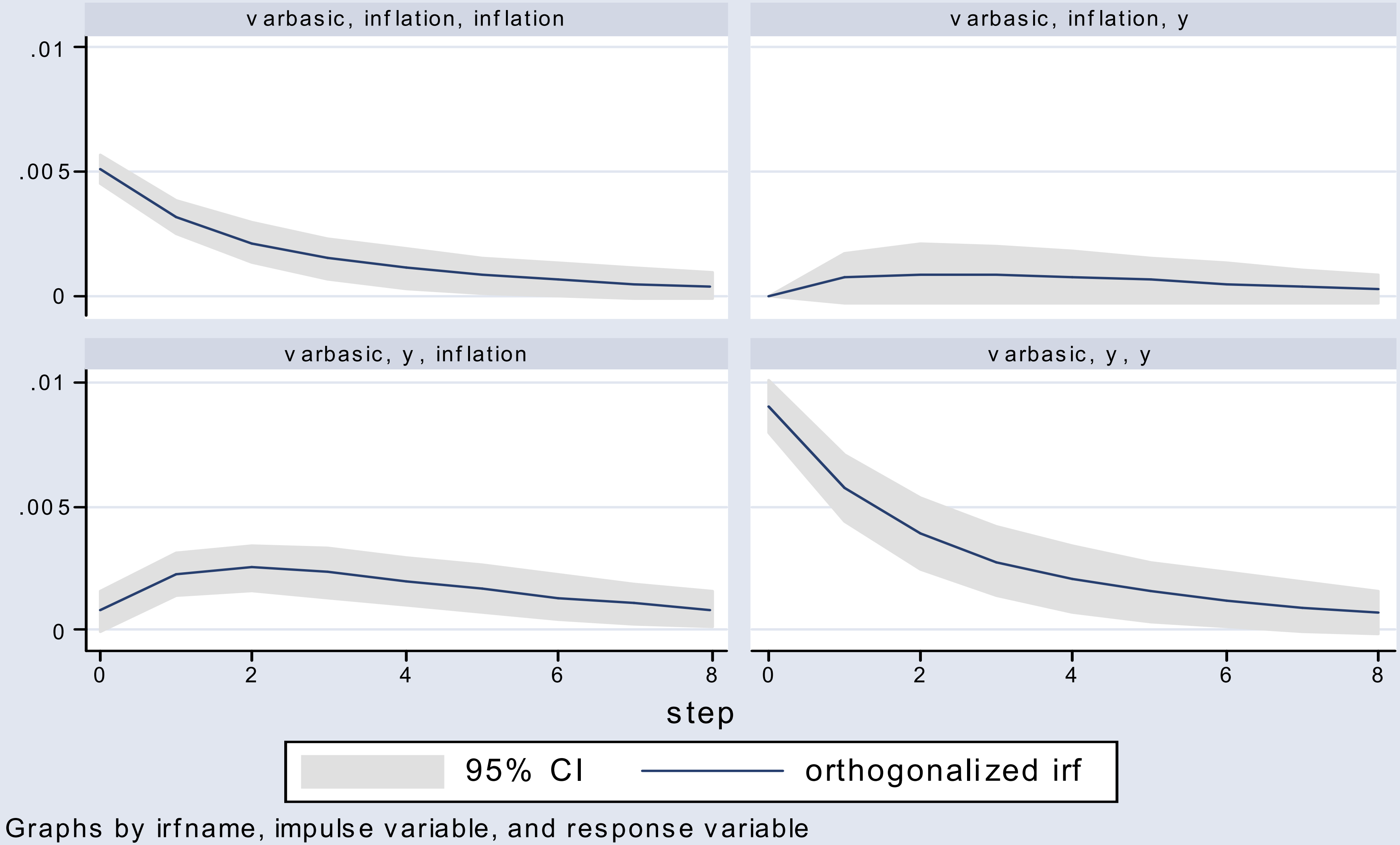
$$\begin{pmatrix} \sigma_{\varepsilon,\varepsilon} & 0 \\ \beta_1 \sigma_{\varepsilon,\varepsilon} & \sigma_{\varepsilon,\varepsilon} \end{pmatrix}$$

- **We can use the OIRF that Stata gives us (Cholesky factorization) to watch the Structural impulse functions.**

Run the VAR (1 lag)



Study the Impulse Responses



Get the coefficients

		Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]	
y								
y								
	L1	.622347	.070297	8.85	0.000	.4845675	.7[illegible]66	α_1
inflation								
	L1	.1421922	.0999996	1.42	0.155	-.0538034	.3[illegible]78	α_2
_cons		.0057446	.0015036	3.82	0.000	.0027976	.0[illegible]16	α_0
inflation								
y								
	L1	.1950256	.040598	4.80	0.000	.1154549	.2[illegible]52	θ_1
inflation								
	L1	.6247529	.0577519	10.82	0.000	.5115613	.7[illegible]45	θ_2
_cons		.0005972	.0008684	0.69	0.492	-.0011047	.0[illegible]2	θ_0

Get the Errors matrix

```
. matrix list e(Sigma)

symmetric e(Sigma)[2,2]
                    y   inflation
        y   .00008145
inflation   7.018e-06   .00002716
```

$\sigma_{\varepsilon,\varepsilon}$ → .00008145

$\beta_1\sigma_{\varepsilon,\varepsilon}$ → 7.018e-06

We find:

$$\beta_1 = \frac{\text{cov}(\varepsilon,\eta)}{\sigma_{\varepsilon,\varepsilon}} = \frac{0.000007018}{0.00008145} = 0.086$$

$$\beta_0 = \theta_0 - \beta_1\alpha_0 = 0.0005972 - 0.086 \times 0.0574 = -0.0043$$

$$\beta_2 = \theta_1 - \beta_1\alpha_1 = 0.195 - 0.086 \times 0.622 = 0.142$$

$$\beta_3 = \theta_2 - \beta_1\alpha_2 = 0.625 - 0.086 \times 0.142 = 0.614$$

Conclusion

- **Enough restrictions**
- **Exact Identification**
- **Possible to deduce the Structural Parameters**

To test a restricted Model

- Run a non restricted model
- Test the null by using the LR test on the difference between the restricted and unrestricted model

$$\lambda = T\left(\ln\left|W_{res}\right| - \ln\left|W_{unres}\right|\right) \rightarrow \chi^{2}(M)$$

$$W_{res} - restricted\ \text{cov}ariance\ matrix$$

$$W_{unres} - unrestricted\ \text{cov}ariance\ matrix$$

$$M - Number\ of\ restrictions$$

Caveat

- With the data we used, it is likely that the variables are cointegrated (consumption and GDP)
- One should (theoretically) check for that option